Ad hoc Coulomb corrections to the radiation tail

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We give an empirically derived correction factor to an *ad hoc* procedure of including Coulomb corrections in the plane-wave calculation of the radiation tail accompanying elastic electron scattering from the atomic nucleus. Apart from regions of the radiation tail with very large Coulomb distortion effects, our corrected *ad hoc* results agree with a distorted-wave Born approximation calculation of the radiation tail to better than 5%.

Uncertainties in the evaluation of the radiation tail resulting from bremsstrahlung of electrons which elastically scatter from the atomic nucleus have long caused problems in single-arm inelastic electron scattering experiments. The major uncertainty is the Coulomb corrections to the electron plane wave or Bethe-Heitler results. A recent paper by Talwar *et al.*[1] reported success in carrying out a distorted-wave Born approximation (DWBA) calculation of the radiation tail. In DWBA, the static spherically symmetric Coulomb field of the target nucleus is included in the electron basis functions and the bremsstrahlung radiation is treated to first order in the electromagnetic coupling constant.

While the formulation of the DWBA result is reasonably straightforward, the evaluation of the result requires a supercomputer and thus is not suitable for routine use in "subtracting" the radiation tail from experimental data. However, Talwar *et al.*[1] showed that an *ad hoc* modification of the radiation tail formula calculated in the electron plane-wave approximation included most of the Coulomb distortion effects.

The plane-wave result can be written as an integral over the square of the ground state form factor $F^2(q)$ where q is the momentum transfer to the nucleus. In the *ad hoc* procedure this Born approximation form factor squared is replaced by a "distorted" form factor

TABLE I. The first line of each row is the Coulomb distortion factor D, the second line is the ratio $R = D^{\mathrm{DW}}/D^{ad\ hoc}$, while the third line is the same ratio but with a corrected *ad hoc* result. We show these results for electron scattering angles from 30° to 150° and for various energy loss ratios for the case $A=238$, $Z=92$, and $T_i=50$ MeV.

ω/T_i	30	40	50	60	70	80	90	100	110	120	130	140	150
	1.57	1.69	1.76	1.76	1.72	1.66	1.61	1.58	1.58	1.65	1.75	1.89	2.04
0.3	0.99	0.98	0.99	0.99	0.98	0.98	0.99	0.99	0.98	0.98	0.98	0.97	0.96
	1.00	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99
	1.55	1.70	1.81	1.86	1.87	1.86	1.85	1.85	1.86	1.91	1.98	2.04	2.10
0.4	0.99	0.97	0.98	0.98	0.98	0.98	0.98	0.98	0.97	0.97	0.96	0.95	0.93
	1.00	0.98	0.99	0.99	0.98	0.98	0.99	0.99	0.99	0.99	1.00	0.99	1.00
	1.53	1.69	1.86	1.96	2.04	2.09	2.14	2.19	2.22	2.29	2.35	2.36	2.33
0.5	0.98	0.96	0.97	0.97	0.97	0.97	0.97	0.97	0.96	0.96	0.95	0.92	0.88
	1.00	0.98	0.99	0.99	0.99	0.99	0.99	0.99	0.99	1.00	1.00	1.00	1.00
	1.53	1.67	1.89	2.07	2.22	2.35	2.48	2.59	2.67	2.76	2.81	2.78	2.63
0.6	0.99	0.94	0.95	0.95	0.95	0.95	0.95	0.95	0.93	0.93	0.91	0.88	0.82
	1.04	0.99	1.00	1.00	0.99	0.99	0.99	1.00	1.00	1.02	1.02	1.01	0.99
	1.34	1.64	1.93	2.17	2.40	2.62	2.83	3.02	3.15	3.27	3.29	3.19	2.91
0.7	0.88	0.92	0.94	0.93	0.92	0.92	0.92	0.91	0.90	0.88	0.85	0.81	0.73
	0.96	1.00	1.02	1.01	1.00	1.00	1.00	1.01	1.01	1.03	1.03	1.01	0.97
				2.11	2.47	2.81	3.12	3.39	3.57	3.69	3.67	3.48	3.06
0.8				0.84	0.86	0.86	0.86	0.85	0.83	0.80	0.76	0.70	0.61
				0.98	0.99	1.00	0.99	1.01	1.02	1.02	1.02	0.99	0.93

$$F_D^2(q)=\frac{\sigma_{\text{elastic}}(\theta_e,p)}{\sigma_{\text{Mott}}(\theta_e,p)}\ ,$$

where σ_{elastic} is the elastic scattering differential cross section from the target nucleus calculated by phase shift analysis for electrons with momentum p as a function of scattering angle θ_e and σ_{Mott} is the plane-wave elastic scattering cross section from a point nucleus. Our *ad hoc* procedure is to choose θ_e in this expression to be the electron scattering angle in the bremsstrahlung process and the momentum p to be given by $p=q/(2\sin\theta_e/2)$ for each value of the momentum transfer variable q. Physically this amounts to assuming that the momentum change of the electron is effected by elastic scattering from the nucleus. Talwar *et al.*[1] found that the *ad hoc* procedure is not so accurate at large electron scattering angles and at larger percentage energy losses. Additional details may be found in Ref. 1.

We have evaluated the radiation tail in DWBA for a number of nuclei ($Z=28$, 50, and 92), for a number of incident electron energies ranging from 50 to 160 MeV, for energy loss percentages up to 80%, and for a range of electron scattering angles from 30° to 150°. We define the Coulomb distortion D^{DW} to be the ratio of $d^2\sigma/d\Omega\,d\omega$ calculated in DWBA to the same result in the plane-wave approximation, and $D^{ad\ hoc}$ to be the same ratio except that the *ad hoc* procedure is used to calculate the numerator. As a measure of the success of the *ad hoc* procedure we evaluate the ratio $R=D^{\text{DW}}/D^{ad\ hoc}$. The DWBA distortion D^{DW} and the ratio R for the cases we

TABLE II. Same as Table I for $A=238$, $Z=92$, and $T_i=100$ MeV.

ω/T_i	30	40	50	60	70	80	90	100	110	120	130	140	150
	1.53	1.52	1.44	1.40	1.45	1.79	2.71	4.89	9.61	17.0	19.6	16.4	12.8
0.3	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.97	0.96	0.94	0.93	0.94	0.98
	0.99	0.99	0.99	1.00	0.99	0.99	0.99	0.98	0.96	0.95	0.94	0.97	1.02
	1.53	1.57	1.52	1.43	1.36	1.34	1.45	1.81	2.63	4.31	7.51	12.9	20.4
0.4	0.98	0.99	0.99	0.99	0.99	0.99	0.98	0.97	0.96	0.95	0.95	0.94	0.92
	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.98	0.97	0.97	0.99	0.99	1.00
	1.53	1.62	1.61	1.55	1.46	1.36	1.30	1.30	1.44	1.76	2.29	3.05	3.96
0.5	0.97	0.98	0.98	0.98	0.98	0.99	0.98	0.97	0.96	0.95	0.93	0.91	0.88
	0.98	0.99	0.99	0.99	1.00	1.00	1.00	0.99	0.99	0.99	0.99	0.99	1.00
	1.52	1.65	1.70	1.70	1.65	1.58	1.50	1.44	1.43	1.48	1.59	1.74	1.90
0.6	0.97	0.97	0.97	0.97	0.97	0.98	0.98	0.98	0.97	0.95	0.93	0.90	0.84
	0.99	0.99	0.99	0.99	1.00	1.00	1.00	1.00	1.01	1.01	1.01	1.01	1.00
	1.47	1.72	1.81	1.88	1.92	1.92	1.91	1.89	1.88	1.88	1.88	1.87	1.79
0.7	0.94	0.98	0.96	0.95	0.96	0.96	0.96	0.96	0.95	0.94	0.91	0.87	0.80
	0.98	1.02	1.00	0.99	1.00	1.00	1.00	1.01	1.02	1.03	1.03	1.03	1.01
				2.03	2.22	2.36	2.47	2.56	2.62	2.65	2.61	2.49	2.04
0.8				0.91	0.92	0.92	0.92	0.91	0.90	0.88	0.85	0.79	0.69
				0.98	0.99	0.99	0.99	1.00	1.01	1.01	1.01	1.00	0.89
					$A=238$, $Z=92$, and $T=160$ MeV								
ω/T_i	30	40	50	60	70	80	90	100	110	120	130	140	150
	1.51	1.59	2.16	5.69	16.2	6.36	2.35	1.25	1.17	2.27	5.87	15.2	25.9
0.3	0.99	0.99	0.98	0.98	0.90	0.98	0.98	1.00	0.96	0.96	1.00	0.99	1.03
	0.99	0.99	0.98	0.98	0.90	0.98	0.98	1.00	0.96	0.97	1.00	1.01	1.07
	1.51	1.51	1.43	1.39	1.60	2.72	7.46	28.3	26.4	9.86	4.85	3.04	2.35
0.5	0.98	0.99	0.98	0.98	0.97	0.96	0.94	0.89	0.90	0.97	0.98	0.95	0.91
	0.98	0.99	0.99	0.99	0.98	0.96	0.95	0.89	0.91	0.99	1.01	1.00	0.99
	1.62	1.57	1.51	1.40	1.31	1.33	1.56	2.28	4.03	8.22	17.8	33.6	35.7
0.6	1.03	0.98	0.98	0.99	0.99	0.99	0.97	0.96	0.94	0.93	0.92	0.89	0.83
	1.04	0.99	0.99	1.00	1.00	1.00	0.98	0.97	0.96	0.96	0.96	0.96	0.92
				1.83	1.87	1.86	1.83	1.79	1.77	1.75	1.75	1.72	1.63
0.8				0.94	0.96	0.97	0.97	0.97	0.96	0.94	0.90	0.85	0.75
				0.98	0.99	1.00	1.00	1.00	1.00	1.00	0.98	0.95	0.88

have investigated are shown in the top two lines of each entry in Tables I–III.

After examining these results in addition to a few more special cases, we constructed a correction factor C to be multiplied times the cross section for the radiation tail calculated with the *ad hoc* procedure outlined above. The correction factor contains an electron scattering angle dependent term and an energy loss term, both of which depend on the incident electron energy. The correction C is given by

$$C=\left[1-\frac{\alpha Z}{m_e R_0}C_\theta C_\omega\right], \quad (1)$$

where $R_0=1.12A^{1/3}/197.32$ is in units of MeV^{-1} and depends on the mass number A, while m_e is the electron rest mass in MeV. The angular correction factor C_θ is given in terms of θ_e measured in degrees and incident electron energy $E_i=T_i+m_e$ where T_i is the incident electron kinetic energy by

$$C_\theta=\begin{cases}1, & \theta_e\le 90^\circ, \\ \exp[0.162(\theta_e-90^\circ)\exp(-1.96\omega/E_i)\exp(-40/E_i)], & \theta_e>90^\circ,\end{cases} \quad (2)$$

while the energy loss correction factor C_ω is given by

$$C_\omega=0.019(\omega/E_i)^4\exp(-E_i/70), \quad (3)$$

where E_i is in MeV. The third line in each entry of Tables I–III gives the corrected *ad hoc* ratio R using this prescription. Note that apart from kinematic conditions where the Coulomb distortion exceeds a factor of 5–10 or very large electron scattering angles at large energy losses, the corrected *ad hoc* ratio agrees with the DWBA ratio to better than 5%. Actually for most kinematic

TABLE III. Same as Table I for $A=56$, $Z=28$, and $T=100$ MeV.

ω/T_i	30	40	50	60	70	80	90	100	110	120	130	140	150
	1.11	1.13	1.13	1.12	1.11	1.08	1.06	1.03	1.00	0.97	0.94	0.90	0.85
0.5	0.99	1.00	1.00	0.99	1.00	1.00	1.00	1.00	1.00	1.00	0.99	0.98	0.96
	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.01	1.01	1.02	1.02	1.02
	1.10	1.15	1.19	1.22	1.24	1.25	1.25	1.25	1.24	1.22	1.18	1.12	1.03
0.8	0.95	0.96	0.97	0.97	0.97	0.97	0.96	0.95	0.94	0.92	0.90	0.86	0.79
	0.99	0.99	1.00	1.00	1.00	1.00	0.99	0.99	0.99	0.99	0.98	0.96	0.91

$A=120$, $Z=50$, and $T=100$ MeV

ω/T_i	30	40	50	60	70	80	90	100	110	120	130	140	150
	1.19	1.15	1.07	0.96	0.85	0.75	0.73	0.91	1.51	3.10	5.76	9.01	10.4
0.2	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99	1.01	0.98	1.00	0.99
	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99	1.01	0.99	1.00	1.00
	1.20	1.18	1.12	1.03	0.94	0.84	0.76	0.71	0.73	0.86	1.14	1.62	2.27
0.3	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99	0.98	0.98	0.97
	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99	1.00	1.00
	1.21	1.21	1.18	1.12	1.04	0.96	0.87	0.79	0.73	0.68	0.67	0.70	0.75
0.4	0.99	1.00	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99	0.98	0.95
	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.01	1.01	1.01	1.01	1.00	1.00
	1.23	1.25	1.24	1.21	1.16	1.10	1.03	0.96	0.90	0.83	0.77	0.73	0.69
0.5	0.99	0.99	0.99	0.99	0.99	1.00	1.00	1.00	1.00	1.00	0.99	0.98	0.94
	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.01	1.01	1.02	1.03	1.03	1.03
	1.24	1.28	1.30	1.30	1.28	1.25	1.21	1.17	1.12	1.06	1.00	0.95	0.87
0.6	0.98	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.97	0.95	0.90
	1.00	1.00	1.00	1.00	1.00	1.00	1.01	1.01	1.02	1.03	1.03	1.03	1.01
	1.22	1.31	1.37	1.39	1.41	1.41	1.40	1.38	1.35	1.31	1.26	1.18	1.06
0.7	0.95	0.97	0.98	0.98	0.98	0.98	0.98	0.98	0.97	0.96	0.94	0.90	0.83
	0.98	1.00	1.01	1.00	1.00	1.00	1.00	1.01	1.01	1.02	1.02	1.00	0.97
			1.38	1.46	1.51	1.54	1.56	1.57	1.56	1.53	1.46	1.36	1.19
0.8			0.94	0.95	0.95	0.95	0.95	0.94	0.93	0.90	0.87	0.81	0.72
			1.00	1.00	1.00	1.00	0.99	1.00	1.00	1.00	0.98	0.95	0.88

conditions the agreement is within 2%. Note also that we have not shown the comparison at forward scattering angles for 80% energy loss since for such large energy losses, our DWBA calculation has not converged well at forward electron angles.

We have determined this empirical correction factor to the *ad hoc* procedure up to incident energies of 160 MeV, but clearly the *ad hoc* procedure is improving with increasing energy. The exponential in C_ω will effectively reduce the correction factor to 1 for incident energies greater than 300 MeV. We have not shown a comparison at electron angles smaller than 30°, but suspect that the *ad hoc* procedure works well there. At large electron angles the *ad hoc* procedure does not work so well, but the empirical correction helps. However, this procedure should be used with caution for scattering angles greater than 150°. We have also not investigated energy losses greater than 80% of the incident energy since our DWBA calculation does not converge as rapidly there. Within the kinematic domain described, the empirically corrected *ad hoc* procedure describes the Coulomb correction to the plane-wave calculation to within 5% apart from the kinematic regions where the Coulomb effect is very large (greater than factors of 5–10). In these regions, the discrepancy between this approximate procedure and the DWBA calculation climbs as high as 12% in some cases we have examined.

We recommend that our corrected *ad hoc* procedure for subtracting the radiation tail accompanying elastic electron scattering from the nucleus be used in all single-arm inelastic electron scattering experiments. If the kinematic conditions are such that very large Coulomb distortion effects are present, the full DWBA calculation is now available,[2] although it does require a supercomputer to run.

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[1]Indu Talwar, L. E. Wright, D. S. Onley, and C. W. Soto Vargas, Phys. Rev. C **35**, 510 (1987).

[2]Indu Talwar and L. E. Wright, Comput. Phys. Commun. **55**, 367 (1989).