PHYSICAL REVIEW C VOLUME 41, NUMBER 1 JANUARY 1990

Spin observables and reconstruction of transverse amplitudes in *p-d* elastic scattering

S. N. Sandhya
Department of Physics, University of Mysore, Mysore 570 006, India
(Received 28 June 1989)

A suitable set of observables, a few of which have already been measured, is suggested. Using these, the *p-d* elastic scattering amplitudes may be completely reconstructed.

The study of spin observables in elastic scattering of protons on deuterons, being the simplest of few-body collisions, is expected to provide insight into several dynamical aspects. To list[1] a few: (i) The deuteron vector analyzing power seems to show sensitivity to the *p*-wave *N-N* interaction,[2] while (ii) the tensor analyzing power has a dominant contribution from the tensor force.[2] (iii) Information on the Coulomb interaction could be obtained by studying the proton analyzing power.[2,3] (iv) The spin rotation parameters provide an additional test for noneikonal multiple scattering[1-4] theories apart from the deuteron asymmetries, which detect any deviations[5] from the eikonal approximation. Lastly (v) spin correlation parameters like $C_{y,y;00}$ help in the understanding of off-shell effects.[6]

Spin observables have been studied theoretically[7,8] and experimentally at low[9] and at intermediate energies,[4,5,10-14] and data are now available on the deuteron vector and tensor analyzing powers,[5,11] the nucleon analyzing power,[12] and the proton spin rotation parameters.[13] Recently[14] a set of triple spin correlation parameters (SCP's) has also been measured which, indeed, is essential in view of the fact that one would obtain as many as 2^{11} sets of solutions for the scattering amplitudes (due to discrete ambiguities) if the measurements were to be restricted to double SCP's only. A complete knowledge of the twelve independent *p-d* scattering amplitudes would provide grounds for testing various theoretical models. To that end a systematic study of the spin observables in *p-d* elastic scattering is presented in this paper, and an appropriate set of SCP's (at a given energy and angle) is suggested using which the scattering amplitudes could be reconstructed without any ambiguities. In suggesting the set, care has been taken to include as many measured SCP's as possible.

The analysis is carried out in the special transverse frame[15] (STF) which is defined by two Cartesian coordinate systems $(\mathbf{X},\mathbf{Y},\mathbf{Z})$ corresponding to the initial and final *p-d* systems; $\mathbf{X}$ is chosen along the initial (final) proton momentum directions $\mathbf{p}_i$ ($\mathbf{p}_f$) and $\mathbf{Z}$ chosen normal to the reaction plane, along $\mathbf{p}_i\times\mathbf{p}_f$. The twelve invariant *p-d* scattering amplitudes may alternatively be expressed in terms of twelve independent[8] [see Eq. (1)] transverse amplitudes (TA's),

$$F_{m_a',m_b';m_a,m_b}\equiv\langle m_a';m_b'|T|m_a;m_b\rangle, m_a(m_a')$$

and

$$m_b(m_b')$$

being the spin projections of the initial (final) proton and deuteron, respectively. They satisfy the constraints[16]

$$F_{m_a',m_b';m_a,m_b}=(-1)^{m_a'+m_b'-m_a-m_b}F_{m_a',m_b';m_a,m_b} \quad (1)$$

(intrinsic parity $\eta=\eta_a'\eta_b'/\eta_a\eta_b=1$) due to parity conservation and

$$F_{m_a,m_b;m_a',m_b'}=(-1)^{m_a'-m_b'-m_a+m_b}F_{m_a',m_b';m_a,m_b} \quad (2)$$

due to time reversal invariance. Since some of the TA's vanish due to parity conservation, the analysis becomes much simpler[17] in this frame. The TA's may then be represented by a block diagonal symmetric matrix,

$$\mathcal{F}=\begin{bmatrix}\mathcal{A} & 0\\ 0 & \mathcal{B}\end{bmatrix},$$

where

$$\mathcal{A}=\begin{bmatrix}f_1 & f_2 & f_3\\ f_2 & f_7 & f_8\\ f_3 & f_8 & f_{11}\end{bmatrix};\quad \mathcal{B}=\begin{bmatrix}f_4 & f_5 & f_6\\ f_5 & f_9 & f_{10}\\ f_6 & f_{10} & f_{12}\end{bmatrix}, \quad (3)$$

where $\mathcal{F}$ is expressed in the basis

$$(|+,+\rangle,|+,-\rangle,|-,0\rangle,\ |+,0\rangle,|-,+\rangle,|-,-\rangle).$$

The Cartesian coordinate systems $(\mathbf{X},\mathbf{Y},\mathbf{Z})$ defining the STF may conveniently be identified with the right-handed coordinate systems, $(\mathbf{L},\mathbf{S},\mathbf{N})$, used by experimentalists.[14] The SCP's $C_{i,\mu;j,\nu}$, are defined[14] in this frame as

$$C_{i,\mu;j,\nu}=\mathrm{Tr}(\mathcal{F}\sigma_i S_\mu\mathcal{F}^\dagger\sigma_j S_\nu)/Y_0, \quad (4)$$

where $Y_0=\mathrm{Tr}(\mathcal{F}\mathcal{F}^\dagger)$; σ_i are the spin-$\frac{1}{2}$ operators and S_μ are the spin-1 operators with $\mu=$i denoting the vector components and $\mu=ij$ the tensor components; $i,j=L,S,N$. Recall (Madison convention) that the tensor operators are given by

$$S_{ij}=3(S_iS_j+S_jS_i)/2-2\delta_{ij}. \quad (5)$$

 ©1990 The American Physical Society

For purposes of solving for the TA's it is preferable to rescale the spin parameters as

$$C'_{i,\mu;j,\nu}=g^{-1}_{i,\mu;j,\nu}C_{i,\mu;j,\nu}\ , \tag{6}$$

in doing which we absorb all multiplicative factors in the parameters $g_{i,\mu;j,\nu}$. As a first step towards the determination of TA's, consider the SCP's $C_{S,S;0,0}$ which may be written in terms of the real part of the bilinear amplitudes using Eqs. (2) and (4) as

$$C'_{S,S;0,0}=\sum_{m'_a,m'_b}\sum_{m_b=-1}^{0}\mathrm{Re}(F_{m'_a,m'_b;-1/2,m_b}F^{*}_{m'_a,m'_b;1/2,m_b+1}+F_{m'_a,m'_b;1/2,m_b}F^{*}_{m'_a,m'_b;-1/2,m_b+1})\ . \tag{7}$$

Explicitly,

$$C'_{S,S;0,0}=\mathrm{Re}(f_6f_4^*+f_{10}f_5^*+f_{12}f_6^*+f_3f_1^* +f_8f_2^*+f_{11}f_3^*+f_2f_3^*+f_7f_8^*+f_8f_{11}^* +f_4f_5^*+f_5f_{10}^*+f_6f_{10}^*)\ . \tag{8}$$

We therefore need a set of at least twelve independent combinations of the above bilinears (real parts) in order to determine each of them. Indeed, the set of SCP's

$$\mathcal{R}=\{C_{S,S;0,0},C_{L,L;0,0},C_{S,S;N,0},C_{L,L;N,0}\ , C_{S,S;0,N},C_{L,L;0,N},C_{S,SN;0,0},C_{L,LN;0,0}\ , C_{S,SN;N,0},C_{L,LN;N,0},C_{S,SN;0,N},C_{L,LN;0,N}\}$$

is one such combination. In $\mathcal{R}$, $C_{S,S;0,0}$, $C_{L,L;0,0}$, $C_{S,S;N,0}$, and $C_{L,L;N,0}$ have already been measured.[14] Note that the SCP's with $\mu=LN$ (SN) correspond to the initial deuteron being tensor polarized with the only nonzero component being LN (SN). We postpone the discussion on the possible means of extracting these observables from experimental measurements to the end. Interchanging $S\leftrightarrow L$ and $SN\leftrightarrow LN$ in the component μ in each of the SCP's $C_{i,\mu;j,\nu}$, we obtain a complement set, $\mathcal{J}$ of spin observables, the measurement of which would determine the imaginary part of each of the bilinears in Eq. (6). The observables in $\mathcal{J}$, viz., $C_{S,L;0,0}$ or A_{SL} and $C_{S,L;N,0}$ have also been measured. Thus $\mathcal{R}+i\mathcal{J}$ determines the complex bilinears in Eq. (6) completely (See Table I). Notice, however, that the TA's belonging to the matrices $\mathcal{A}$ and $\mathcal{B}$ in Eq. (1) form, respectively, two nonintersecting sets in the sense that no bilinear in Eq. (6) connects an amplitude from $\mathcal{A}$ to an amplitude belonging to $\mathcal{B}$. This would lead to undetermined relative phases between TA's $\in\mathcal{A}$ and TA's $\in\mathcal{B}$. This problem is, however, solved by measuring

$$\{C_{S,0;S,0},C_{L,0;L,0},C_{S,0;L,0}\ , C_{S,NN;S,0},C_{L,NN;L,0},C_{S,NN;L,0}\}$$

since

$$\mathrm{Re}f_{11}f_4^*=(C'_{S,0;S,0}+C'_{L,0;L,0} -C'_{S,NN;S,0}-C'_{L,NN;L,0})/6 \tag{9}$$

and

$$\mathrm{Im}f_{11}f_4^*=(2C'_{S,0;L,0}-C'_{S,NN;L,0}-C'_{L,NN;S,0})/6\ , \tag{10}$$

where the spin rotation parameters have already been measured.[4,13] Thus we have determined the twelve complex bilinears in Eq. (6) apart from $f_{11}f_4^*$. Finally measuring the unpolarized differential cross section Y_0, the analyzing powers[11,12] A_{N0} or $C_{N,0;0,0}$ and A_{0NN} or $C_{0,NN;00}$, the spin rotation parameter,[4,13] $C_{N,0;N,0}$, and the SCP's $C_{N,NN;0,0}$ and $C_{0,NN;N,0}$, $|f_4|^2$ can be evaluated to be

$$|f_4|^2=(Y_0+4C'_{N,0;0,0}-4C'_{0,NN;0,0}+3C'_{N,0;N,0} -2C'_{N,NN;0,0}-2C'_{0,NN;N,0})/24\ . \tag{11}$$

Choosing f_4 to be real and positive (an overall phase being arbitrary), the complex TA's, viz., f_{11}, f_6, and f_5, get determined since $f_{11}f_4^*$, $f_6f_4^*$, and $f_4f_5^*$ are known. Extending the argument to the rest of the bilinears the remaining amplitudes may similarly be determined without any ambiguities. Of course, many more equivalent sets of observables exist, which may be used for determining the TA's depending on the experimental feasibility.

Lastly we discuss how the spin parameters $C_{i,\mu;j,\nu}$ with $\mu=SN$ or LN may be measured by using magnetic fields to polarize the deuteron. When both the initial proton and deuteron are polarized, the yield Y is given by

$$Y=Y_0\left[1+\sum_{i,\mu}P_is_\mu C_{i,\mu;j,\nu}\right]\ , \tag{12}$$

where $C_{i,\mu;j,\nu}$ are the SCP's defined in Eq. (2) with $j,\nu=0$ and P_i and s_μ are, respectively, proportional to the proton and deuteron polarization parameters. Let the proton be polarized, say along **L** and the deuteron be polarized by applying a magnetic field, its direction defined by $\theta=\theta_0$; $0<\theta_0<\pi/2$ and $\varphi=0$ with respect to the normal **N** (remember, **N** coincides with the **Z** axis in our choice). Parity restricts the sum of the number of S plus the number of L components in $C_{i,\mu;j,\nu}$ to be even only and $\varphi=0$ implies that $\mu=S,SN$ do not contribute. Therefore the yield, say, Y_1, is given by

$$Y_1=Y_0(1+P_Ls_LC_{L,L;0,0}+P_Ls_{LN}C_{L,LN;0,0})\ . \tag{13}$$

Retaining the proton polarization along **L** and reversing the magnetic field direction (applied to the deuteron) to $\theta=\pi-\theta_0$, and $\varphi=\pi$, the yield Y_2 is obtained as

$$Y_2=Y_0(1-P_Ls_LC_{L,L;0,0}+P_Ls_{LN}C_{L,LN;0,0})\ . \tag{14}$$

Using Eqs. (13) and (14), we therefore obtain

$$P_Ls_{LN}C_{L,LN;0,0}=(Y_1+Y_2)/2Y_0-1 \tag{15}$$

TABLE I. Matrix determining the bilinears in terms of the spin correlation parameters (SCP's).

Bilinears \ SCP's	$C'_{L,L;0,0}$ ($C'_{L,S;0,0}$)	$C'_{S,S;0,0}$ ($C'_{S,L;0,0}$)	$C'_{L,L;N,0}$ ($C'_{L,S;N,0}$)	$C'_{S,S;N,0}$ ($C'_{S,L;N,0}$)	$C'_{L,LN;0,0}$ ($C'_{L,SN;0,0}$)	$C'_{S,SN;0,0}$ ($C'_{S,LN;0,0}$)	$C'_{L,LN;N,0}$ ($C'_{L,SN;N,0}$)	$C'_{S,SN;N,0}$ ($C'_{S,LN;N,0}$)	$C'_{L,L;0,N}$ ($C'_{L,S;0,N}$)	$C'_{S,S;0,N}$ ($C'_{S,L;0,N}$)	$C'_{L,LN;0,N}$ ($C'_{L,SN;0,N}$)	$C'_{S,SN;0,N}$ ($C'_{S,LN;0,N}$)
Re (Im) $8f_{10}f_5^*$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	1	-1	-1
Re (Im) $8f_{1/2}f_6^*$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	-1	-1	1	1
Re (Im) $8f_3f_1^*$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	1	1	1
Re (Im) $8f_8f_2^*$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	-1	-1	-1	-1
Re (Im) $8f_2f_3^*$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	1	-1	-1	1
Re (Im) $8f_7f_8^*$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	-1	1	1	-1
Re (Im) $8f_5f_9^*$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	1	-1	1	-1
Re (Im) $8f_6f_{10}^*$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	-1	1	-1	1
Re (Im) $8f_6f_4^*$	1	1	1	1	-1	-1	-1	-1				
Re (Im) $8f_{11}f_3^*$	1	1	-1	-1	1	1	-1	-1				
Re (Im) $8f_8f_{11}^*$	1	-1	-1	1	-1	1	1	-1				
Re (Im) $8f_4f_5^*$	1	-1	1	-1	1	-1	1	-1				

and

$$P_L s_L C_{L,L;0,0} = (Y_1 - Y_2)/2Y_0 \ . \quad (16)$$

On the other hand, with the initial polarization being prepared as previously described, if the N component of the proton polarization is measured, SCP's $C_{L,L;N,0}$ and $C_{L,LN;N,0}$ are obtained. Similarly measurement of the N component of the final deuteron vector polarization with the same initial preparations would give the observables $C_{L,L;0,N}$ and $C_{L,LN;0,N}$. Finally the choices $(\theta_0, \pi/2)$ and $(\pi-\theta_0, 3\pi/2)$, $0<\theta_0<\pi/2$, for the magnetic field direction (in order to polarize the deuteron) will yield the SCP's $C_{i,\mu;j,\nu}$ with $\mu = S, SN$.

I thank Professor J. Arvieux and Professor F. Sperisen for making available some of their reprints/preprints. I also thank Professor G. Ramachandran for his encouragement. I thank the University Grants Commission (India) for financial assistance.

[1]See J. Arvieux and J. M. Cameron, in *Advances in Nuclear Physics,* edited by J. W. Negele and Eric Vogt (Plenum, New York, 1987), Vol. 18, p. 151, for a comprehensive review.

[2]G. H. Berthold, A. Stadler, and H. Zankel, Phys. Rev. Lett. **61**, 1077 (1988).

[3]W. Tornow, J. Herdtweck, W. Arnold, and G. Mertens, Phys. Lett. B **203**, 341 (1988).

[4]Sun Tsu Shun *et al.*, Phys. Rev. C **31**, 515 (1985).

[5]M. Bleszynski *et al.*, Phys. Lett. **106B**, 42 (1981).

[6]H. Zankel and W. Plessas, Z. Phys. A **317**, 45 (1984).

[7]See, e.g., C. R. Howell *et al.*, Phys. Rev. Lett. **61**, 1565 (1988); H. Witala, T. Cornelius, and W. Glöcke, Few Body Syst. **3**, 123 (1988); Y. Koike and J. Haidenbauer, Nucl. Phys. **A463**, 365C (1987); D. L. Adams and M. Bleszynski, Phys. Lett. **150B**, 405 (1985); A. Boudard and M. Dillig, Phys. Rev. C **31**, 302 (1985); Proceedings of the VI International Symposium on Polarized Phenomena in Nuclear Physics, Osaka, 1985 [J. Phys. Soc. Jpn. **55**, Suppl., 854 (1986)]; *Three Body Force in Three Nucleon System,* Proceedings of the International Symposium, Washington, D.C., 1986 (Springer-Verlag, Berlin, 1986); *Intersections between Particle and Nuclear Physics (Lake Louise, Canada, 1986),* Proceedings of the Second Conference on the Intersections of Particle and Nuclear Physics, AIP Conf. Proc. No. 150, edited by Donald F. Geesaman (AIP, New York, 1986).

[8]F. Sperisen, J. Phys. Soc. Jpn. **55**, Suppl., 852 (1986); for a general discussion on polarization measurements and amplitude determination in *p-d* scattering, see M. J. Moravcsik and H. M. Sinky, Ann. Phys. (N.Y.) **171**, 205 (1986).

[9]J. Arvieux, A. Boudard, G. Gaillard, Nguyen Van Sen, G. Perrin, Ye Yanlin, and Y. Koike, Few Body Syst. **3**, 27 (1987); H. O. Klages, Nucl. Phys. **A463**, 353 (1987); F. Sperisen, W. Grübler, V. König, P. A. Schmelzbach, K. Elsener, B. Jenny, C. Schweizer, J. Ulbricht, and P. Doleschall, *ibid.* **A422**, 81 (1984); K. Hatanaka *et al.*, *ibid.* **A426**, 77 (1984); J. E. Simmons, *ibid.* **A416**, 553C (1984).

[10]J. Arvieux *et al.*, Nucl. Phys. **A431**, 613 (1984).

[11]M. Haji Saied *et al.*, Phys. Rev. C **36**, 2010 (1987).

[12]F. Irom *et al.*, Phys. Rev. C **28**, 2380 (1983).

[13]G. S. Weston, Los Alamos National Laboratory Report LA 10174-T, 1984; see also Ref. 4.

[14]D. L. Adams *et al.*, Nucl. Phys. **A480**, 530 (1988); G. Igo *et al.*, Phys. Rev. C **38**, 2777 (1988).

[15]The STF differs slightly from the conventional transverse frame (TF): $(\mathbf{x},\mathbf{y},\mathbf{z})$ ($\mathbf{z}$ chosen normal to the reaction plane and y axis chosen along $-\mathbf{p}_i$ and $-\mathbf{p}_f$) and may be obtained by a trivial rotation about the z axis through an angle $-\pi/2$ so that $\mathbf{x}\to\mathbf{Y}$, $\mathbf{y}\to-\mathbf{X}$, $\mathbf{z}\to\mathbf{Z}$. The transverse amplitudes in the STF are then related to the conventional ones (see, for example, Ref. 16) through

$$F_{m_a',m_b';m_a,m_b} = \exp[i\pi(m_a'+m_b'-m_a-m_b)/2] \times F^{TF}_{m_a',m_b';m_a,m_b} \ .$$

[16]A. Kotanski, Acta Phys. Pol. **30**, 629 (1966).

[17]In contrast to the case of helicity amplitudes where this advantage is lost.