BRIEF REPORTS

Monopole strength and shape coexistence in the $A \simeq 100$ mass region

H. Mach,* M. Moszyński,† and R. L. Gill

Brookhaven National Laboratory, Upton, New York 11973

G. Molnár‡

Institut für Kernphysik, Kernforschungsanlage Jülich, D-5170 Jülich, Federal Republic of Germany

F. K. Wohn, J. A. Winger, and John C. Hill

Ames Laboratory, Iowa State University, Ames, Iowa 50011

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Half-lives of 5.60(15) and 1.58(4) ns were measured for the 0_2^+ levels in ^{100}Zr and ^{100}Mo, respectively, using a β-γ-γ fast timing method. Analysis of the revised $\rho(E0)$ systematics in the crucial $A \simeq 100$ region indicates that high values of $\rho(E0)$ do not by themselves imply strong configuration mixing between two coexisting states as presently assumed.

The abrupt onset of deformation in the $A \simeq 100$ mass region and the accompanying phenomena of coexistence and mixing of spherical and deformed configurations have been extensively studied in the past fifteen years. The strength $\rho^2(E0)$ of electric monopole transitions between 0^+ states has been investigated as a criterion for shape coexistence in neutron-rich Sr and Zr nuclei.[1-5] This idea was generalized in a recent paper,[6] in which the anomalously high values of $\rho^2(E0)$ for the $N=60$ isotones ^{98}Sr, ^{100}Zr, and ^{102}Mo were stated to be caused by strong mixing between a deformed ground state and a spherical first-excited 0^+ state.

The present Brief Report reports new lifetimes of excited 0^+ states that clarify conflicting results in the crucial $A \simeq 100$ region. Revised systematics of the $E0$ strength parameter, together with our recent analysis[7] of the structure of ^{98}Sr and ^{100}Zr, are used here to point out important differences between the various isotones with neutron numbers $N = 58$–60 in the shape transitional region and to modify the somewhat oversimplified picture given in Ref. 6.

Measurements were performed at the fission-product separator TRISTAN on line to the BNL High Flux Beam Reactor using a β-γ-γ fast timing technique with electronics optimized for low-energy γ rays.[8] Fast timing is derived from β-γ coincidences in a thin NE111A plastic scintillator for β rays and a BaF$_2$ crystal for γ rays, while a desired decay branch is selected from the mass-separated activity using a coincident γ ray observed in a Ge detector. β-γ time resolution (full width at half maximum—FWHM) ranged from 220 ps at 0.2 MeV to 100 ps at 1.3 MeV γ-ray energy. Details of the experi-

mental setup and present data analysis are discussed in Ref. 8. Lifetimes were obtained by the slope method, where the delayed time spectra were χ^2 fitted to an analytical function with a prompt response approximated by a Gaussian.[8] Due to the well-defined slope, the value of the lifetime is insensitive to the time delay brought by the indirect γ feeding, provided that such a delay is much smaller than the lifetime of interest, as is the case for the lifetimes reported here.

Figure 1 shows the χ^2 fits to the delayed time spectra, and the insets show the key γ-γ cascades. Low-spin β decays of ^{100}Y and ^{100}Nb strongly feed the 0_2^+ states. No significant time delay (< 15 ps) is introduced by a weak indirect γ feeding to the 0_2^+ states, which has been verified[8] using the centroid shift method. In the case of ^{100}Zr the most significant indirect feeding comes from the 879-keV 2_2^+ state[5] for which a half-life shorter than 10 ps was determined in the present work. The same limit ($T_{12} < 10$ ps) was determined for the half-life of the 1464-keV 2_3^+ level feeding the 0_2^+ state of ^{100}Mo. The latter result is consistent with the more precise value of 2.9(7) ps established[9] from the recoil distance Doppler method. The lifetimes deduced from the data of Fig. 1 were consistent with those obtained from the time spectra with the gates in the BaF$_2$ and Ge detectors reversed. [For the latter, no correction was needed for the $T_{1/2}$ of 13 ps (Ref. 10) for the 2_1^+ state on ^{100}Mo, while for ^{100}Zr, we held fixed the independently determined $T_{1/2}$ of 0.55(2) ns (Ref. 7) for the 2_1^+ state in ^{100}Zr.] A weighted average of these 0_2^+ half-life results is included in Table I.

Our results are compared with previous half-lives in
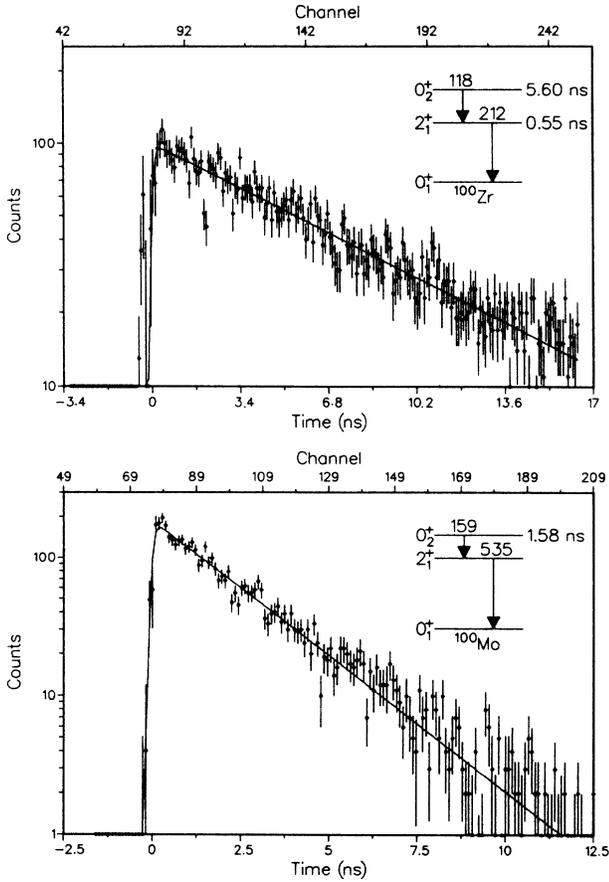
FIG. 1. Top: delayed time spectra for the 0_2^+ level in ^{100}Zr obtained with a 118-keV BaF_2 gate and a 212-keV Ge gate; the χ^2 fit curve is for $T_{1/2} = 5.60$ ns. Bottom: delayed time spectrum for the 0_2^+ level in ^{100}Mo selected with a 159-keV BaF_2 gate and a 535-keV Ge gate; χ^2 fit is for $T_{1/2} = 1.58$ ns.

the third column of Table I. Our new ^{100}Mo 0_2^+ half-life (obtained by the very reliable slope method) resolves a discrepancy between Ref. 14 and the earlier measurements of Refs. 10–12 in favor of the latter. [The techniques used were Coulomb excitation,[10] delayed coincidences[11] (with a poor time resolution), and recoil distance Doppler shifts.[12]] Our result strongly disagrees with the 3.0(1) ns of Ref. 14 obtained by a delayed coincidence method but with a time resolution an order of magnitude inferior to that of our measurement. The strong disagreement between Ref. 14 and the other studies implies a systematic error in the experiment of Ref. 14. In the case of ^{100}Zr, our new half-life does not support the earlier measurement of Ref. 2 that was also obtained by a delayed coincidence method but with a much inferior time resolution. [During revision of the present paper we became aware of more recent results: $T_{1/2} = 1.65(4)$ ns (Ref. 15) for ^{100}Mo, obtained by the same method used in our work, and $T_{1/2} = 5.36(23)$ ns (Ref. 16) for ^{100}Zr, obtained by the centroid shift method, both of which are in very good agreement with our results.]

Newly calculated values of the transition strength $\rho^2(E0)$ are included in the fifth column of Table I, together with the branching ratios used in the calculations. [The $\rho^2(E0)$ values were obtained using well-known formulas[17] and tabulated electronic factors.[18]] For ^{100}Mo the new $\rho^2(E0)$ value is twice as large as the latest reported value[14] due to the different $T_{1/2}$ values. For ^{100}Zr the new $T_{1/2}$ and branching ratio[5] values differ from previous data,[2] resulting in a net 40% reduction of the old $\rho^2(E0)$ value.[2] Since the new results affect the $\rho^2(E0)$ systematics of Ref. 6, we have replotted $\rho^2(E0,0_2^+ \rightarrow 0_1^+)$ in Fig. 2 for even-even nuclei in the $A \simeq 100$ region. Contrary to Fig. 4 of Ref. 6, the ^{100}Zr$_{60}$ point is no longer the largest one

TABLE I. $T_{1/2}$ and $E0$ results for 0_2^+ states in selected $N = 58-60$ isotones.

	E_{lev} (keV)	$T_{1/2}$ (ns)	Branching ratio	$\rho^2(E0)$	X_{211}	Ref.
^{100}Mo$_{58}$	695	1.58(4)		0.032(2)^a		This work
		1.7(2)	$I(E0)/I(E2)=0.110(8)$	0.042(6)		11
		1.46(50)				12
			$I_k(E0)/I_k(E2)=0.759(45)$	0.046(12)		13
		1.8(5)				10
		3.0(1)	$I_k(E0)/I_{tot}=0.0712(36)$	0.016(1)	0.0116(6)	14
		1.65(4)^d				15
^{102}Mo$_{60}$	698	0.028(11)				12
			$I_k(E0)/I_{tot}=0.00478(78)$	0.12(5)	0.062(1)	14
^{100}Zr$_{60}$	331	5.60(15)		0.092(20)^b	0.048(10)^b	This work
		3.37(30)	$I(E0)/I(E2)=1.05$	0.243(15)		2
			$I(E0)/I(E2)=0.47(10)$	0.16(3)		5
		5.36(23)^d				16
^{98}Sr$_{60}$	215	25(2)	$I_k(E0)/I_k(E2)=1.14(3)$	0.053(9)		3
					0.031(1)^c	3,7

^{a}Calculated with branching ratio from Ref. 14.
^{b}Calculated with branching ratio from Ref. 5.
^{c}Calculated with branching ratio from Ref. 3 and 2_1^+ half-life from Ref. 7.
^{d}Reported after the present paper was submitted; included during revision.

and the local minimum at ^{100}Mo$_{58}$ has disappeared. Also, we found the ^{98}Sr$_{60}$ value from Ref. 3 to be smaller than the incorrect value that was inadvertently used in Fig. 4 of Ref. 6.

Considering the $N=60$ isotones, for which the maximum $\rho^2(E0)$ values are attributed[6] to strong mixing of spherical and deformed configurations, it is clear from Fig. 2 that only the ^{100}Zr value, and perhaps the ^{102}Mo value, are similar to the large $E0$ strengths found in this mass region for $0_3^+ \to 0_2^+$ transitions. The difference between the ^{98}Sr and ^{100}Zr values is especially striking in view of the similar behavior of their 0_2^+ energies.[27] Moreover, according to our recent coexistence analysis,[7] these two isotones have very similar structures since each has 0_1^+ and 0_2^+ states whose properties indicate weak mixing $(11-14\%)$ of a strongly deformed $(\beta \sim 0.4)$ 0^+ state and a weakly deformed $(\beta \sim 0.2)$ 0^+ state. However, there is no contradiction in this, since even in the simplest model for shape coexistence the $E0$ strength depends not only on the mixing amplitudes but also on the difference between the mean square radii for the two coexisting shapes.[26,6]

To show this explicitly, let θ be the mixing angle (so that the two mixing amplitudes are given by $a = \sin\theta$ and $b = \cos\theta$) and let the two deformations be β_1 and β_2. Then, with the $E0$ operator expanded to second order in deformation,

$$\rho^2(E0) = (3Ze/4\pi)^2 a^2 b^2 (\beta_1^2 - \beta_2^2)^2 . \qquad (1)$$

This expression reduces to Eq. (2.15) of Ref. 6 for maximal mixing (i.e., $a^2 = b^2 = 0.50$, or 50% mixing) and when $\beta_1 \gg \beta_2$.

There are two interesting aspects of the $\rho^2(E0)$ systematics of Fig. 2 to consider. Firstly, it is easy to verify numerically that even the highest $\rho^2(E0)$ values in Fig. 1 are in fact rather moderate since Eq. (1) allows for much higher values of $\rho^2(E0)$ for the case where both strong mixing and large differences in the deformations are simultaneously present. Secondly, the moderate $\rho^2(E0)$ values of ~ 0.1 (in units of e^2) can be explained by nearly any mixing, from very weak to maximal. A $\rho^2(E0)$ of 0.1 (for $Z \simeq 40$) can occur for very weak mixing provided that the difference between the two deformations is large enough. To illustrate this, let $\beta_1 = 0.4$ and $\beta_2 = 0.1$, then a mixing of only 5% $(a^2 = 0.05)$ is enough to give a $\rho^2(E0)$ of 0.1. Maximal mixing also gives a $\rho^2(E0)$ of 0.1 for a smaller (but still quite significant) difference in the deformations, such as $\beta_1 = 0.30$ and $\beta_2 = 0.15$.

The present data on the $N=60$ isotones and the expression for $\rho^2(E0)$ thus clearly show that, contrary to the conclusions of Ref. 6, the strength of the $E0$ transition alone is not a unique indicator of the amount of configuration mixing. In fact, both $N=60$ isotones ^{98}Sr and ^{100}Zr, even though they have $\rho^2(E0)$ values much higher than the average in the region, are characterized by rather weak configuration mixing of 10% and 14%, respectively.[7]

Concerning the third $N=60$ isotone, ^{102}Mo, the large error in the $\rho^2(E0)$ value (due to the imprecise lifetime) prevents a reliable interpretation. However, the ratios
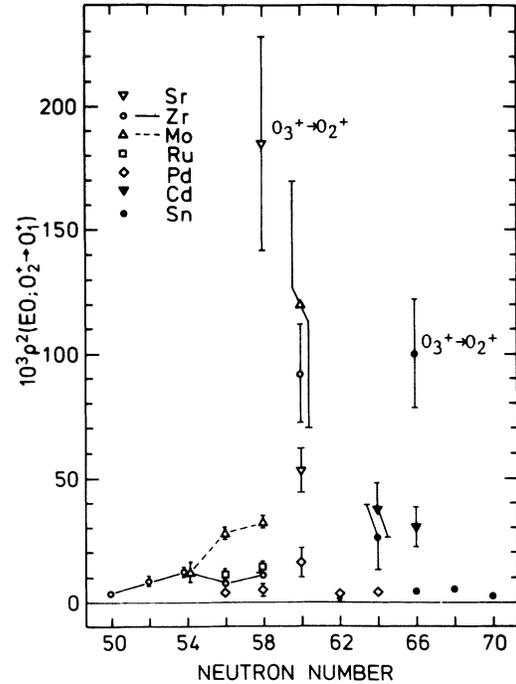
FIG. 2. Systematics of the $\rho^2(E0; 0_2^+ \to 0_1^+)$ strength for even-even nuclei in the $A \simeq 100$ mass region. Data are from Refs. 19 and 20 or as follows: $^{90-96}$Zr (Ref. 21); ^{102}Mo (Ref. 14); ^{102}Ru (Ref. 22); ^{102,104}Pd (Ref. 23); ^{108,110}Pd (Ref. 14); ^{112,114}Cd (Ref. 24); and the present work for ^{100}Mo and ^{100}Zr. The $0_3^+ \to 0_2^+$ values for ^{96}Sr (Ref. 25) and ^{112}Sn (Ref. 26) have also been added.

X_{211} [defined by $X_{211} = B(E0; 0_2^+ \to 0_1^+)/B(E2; 0_2^+ \to 2_1^+)$, with $B(E0; 0_2^+ \to 0_1^+) = e^2 R^4 \rho^2(E0; 0_2^+ \to 0_1^+)$] given in Table I are more precise than the $\rho^2(E0)$ values. For the data of Table I, the X_{211} ratios seem to correlate well with the monopole strengths $\rho^2(E0)$. The increase of the X_{211} value for ^{102}Mo with respect to that for ^{100}Mo thus suggests that the monopole strength in ^{102}Mo may well be as large as for the other two $N=60$ isotones.

Provided that the ^{102}Mo $E0$ strength is indeed large, there is a reasonable alternative to the interpretation[6] based on a coexisting "weakly deformed" state, since large X_{211} and $\rho^2(E0)$ values are typical for β vibrational bands in deformed nuclei.[20] It is thus possible that the 0_2^+ state of ^{102}Mo is a β bandhead instead of a coexisting weakly deformed state. Therefore the large X_{211} value of ^{102}Mo could be consistent with either of these two viewpoints. However, the β bandhead viewpoint is supported by IBA2 model calculations,[28] according to which rather strong configuration mixing takes place in ^{98}Mo and ^{100}Mo, whereas the two lowest 0^+ states of ^{102}Mo both belong to the same deformed configuration. At least, strong mixing between the lowest two 0^+ states of ^{98}Mo$_{56}$ and ^{100}Mo$_{58}$ is consistent with the observed $\rho^2(E0)$ values that are much larger than the corresponding value of ^{96}Zr$_{56}$ (see Fig. 2) for which this mixing is weak.[29] As for the ^{102}Mo case, however, more precise and complete data are needed to clarify the issue.

In conclusion, new half-lives for the 0_2^+ states of ^{100}Mo and ^{100}Zr yield values for the $E0$ strength parameter $\rho^2(E0;0_2^+ \to 0_1^+)$ that are substantially different from earlier data. Reanalysis of the systematics confirms that high values of $\rho^2(E0)$ do not necessarily mean strong configuration mixing. Instead it must be recognized that knowledge of the difference between the two deformations is required in order to use $\rho^2(E0)$ values as a viable indicator of the strength of the configuration mixing between the two coexisting shapes.

*Present address: The Studsvik Science Research Laboratory, S-61182, Nyköping, Sweden.

†Permanent address: Institute for Nuclear Studies, PL 05-400, Swierk-Otwock, Poland.

‡Permanent address: Institute of Isotopes, Budapest, H-1525 Hungary.

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