

Charge form factors of the three- and four-body nuclei

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The charge form factors of ${}^3\text{H}$, ${}^3\text{He}$, and ${}^4\text{He}$ are calculated using the Monte Carlo method and variational ground-state wave functions obtained for the Argonne two-nucleon and Urbana-VII three-nucleon interactions. The model for the charge density operator contains the two-body exchange contributions of longest range. With some spread due to the uncertainty in the electromagnetic form factors of the nucleon the calculated charge form factors are in good agreement with the empirical values over the whole experimentally covered range of momentum transfer.

I. INTRODUCTION

We have recently shown that an excellent description of the measured magnetic form factors of ${}^3\text{H}$ and ${}^3\text{He}$ can be obtained by using the Argonne v_{14} potential¹ and the Urbana-VII three-nucleon interaction² for the construction of the wave functions and the associated exchange current density operators.³ We here continue this investigation to study the charge form factors of these nuclei and of the related nucleus ${}^4\text{He}$ with the same interaction model.

The charge form factors of ${}^3\text{He}$ and ${}^4\text{He}$ have been measured up to momentum-transfer values of 8.6 fm^{-1} ($1.7 \text{ GeV}/c$).⁴ Their structure at high values of momentum transfer is in qualitative agreement with calculations performed with nonrelativistic wave functions, with or without two-body effects.^{5,6} We shall here show that, with the Argonne v_{14} potential model and the usual two-body corrections that correspond to the meson-exchange mechanisms of longest range, we obtain charge form factors for these nuclei that are in essentially quantitative agreement with the empirical values (within the uncertainty limits set by the nucleon electromagnetic form factors), except for a remaining small discrepancy just after the first zero in the charge form factor of the α particle. This result has several implications. The first and most important one is the remarkable success of the nonrelativistic description even at the very large values of momentum transfer considered ($1.7 \text{ GeV}/c$). The second is the indication that the model for the exchange charge density is better than one *a priori* should expect. The third is the remarkable quality of the Argonne v_{14} interaction, with which it now appears possible to explain all the elastic electromagnetic structure functions of the bound three- and four-nucleon systems.

The structure of the exchange charge operator, in contrast to that of the exchange current operator, is associated with several uncertainty factors. While main parts of

the exchange current are linked to the form of the nucleon-nucleon interaction through the continuity equation, the most important exchange charge density operators are model dependent and may be viewed as relativistic corrections. Until a systematic method for a simultaneous nonrelativistic reduction of both the interaction and the electromagnetic current operator is developed, the definite form of the exchange charge operators remains uncertain, and one has to rely on perturbation theory. We shall here consider the exchange charge operators associated with pion, ρ -meson and ω -meson exchanges. The form for the pion exchange charge density is essentially that first introduced in Ref. 7. The ρ -meson exchange charge operator is that derived in Ref. 8, whereas the ω -meson exchange charge operator is the one first obtained in Ref. 9. In addition we consider the $\rho\pi\gamma$ and $\omega\pi\gamma$ exchange current mechanisms in which the virtual photon couples to the pion-vector-meson vertex. The former one of these exchange charge mechanisms is better established, as it can be linked, within the framework of the topological soliton or Skyrme model,^{10,11} to the chiral anomaly.

In the construction of the meson-exchange charge density operators, the short-range (or high momentum) behavior of the meson-nucleon vertices represents a factor of uncertainty. To avoid the need for *ad hoc* cutoff parameters at the meson-nucleon vertices we shall here construct the dressed pion and ρ -meson propagators directly from the nucleon-nucleon interaction by the method developed in Ref. 12. While this does not greatly change the numerical results from those obtained with free meson propagators with vertex form factors, it reduces the model dependence of the calculation. We shall also in the description of the exchange charge operators review their derivation in more detail than usual, as different versions of these operators have appeared in the literature. Note that there are no explicit Δ_{33} components in the wave function used. In the literature the

contributions from such components are often viewed as exchange charge effects.^{5,6}

This paper is divided into six sections. In Sec. II we briefly illustrate the method of constructing the wave functions, and discuss the form factors obtained in the impulse approximation. In Sec. III we describe the two-body exchange contributions to the charge density operator. In Secs. IV and V we present the results for the charge form factors of ^3H , ^3He , and ^4He . Finally, Sec. VI contains a concluding discussion.

II. THE CHARGE FORM FACTORS IN THE IMPULSE APPROXIMATION

In the impulse approximation the charge form factors are obtained as the appropriate nuclear matrix elements of the single-nucleon charge operator, for which we use the expression¹³

$$\rho(\mathbf{P}, \mathbf{q}) = \left[1 - \frac{q^2}{8m^2} \right] \frac{1}{2} [G_E^S(q) + G_E^V(q)\tau_z] - i \frac{\boldsymbol{\sigma} \cdot \mathbf{q} \times \mathbf{P}}{8m^2} \frac{1}{2} \{ [G_E^S(q) - 2G_M^S(q)] + [G_E^V(q) - 2G_M^V(q)]\tau_z \}. \quad (2.1)$$

Here \mathbf{q} is the momentum transfer to the nucleon, \mathbf{P} the sum of the initial and final nucleon momenta, and m the nucleon mass. The first term in the charge density operator is the electric form factor with the Darwin-Foldy correction and the second is the so-called "spin-orbit" term. The latter represents only a small correction, as will be shown explicitly below.

The nuclear wave functions required for the matrix elements of the charge density operator (2.1) are constructed by the variational Monte Carlo method¹⁴ from a Hamiltonian containing the Argonne v_{14} potential,¹ and the Urbana model VII three-nucleon interaction² (this last term takes into account excitation of intermediate Δ_{33} resonance configurations). These wave functions give binding energies, charge radii and asymptotic D - to S -state ratios in the d - p and d - n channels of ^3He and ^3H , and in the d - d channel of the α particle, which are quite close to the empirical values. Their accuracy has been further tested by direct comparison with results obtained with exact Faddeev¹⁵ and Green's Function Monte Carlo¹⁶ wave functions, as, for example, those for the two-body correlation functions,^{17,18} and the longitudinal energy-weighted sum rule.¹⁹ In Ref. 3 we have recently shown that by using the Argonne v_{14} model to construct the wave functions and the exchange current density operator one obtains magnetic form factors of ^3H and ^3He that are in excellent agreement with the empirical values over the whole measured range of momentum-transfer values.

The evaluation of the nuclear matrix elements with the variational wave functions is carried out by the Monte Carlo method developed in Refs. 3 and 14, without any approximation. This method is very convenient in that it treats numerically both the spin and isospin structure of the integrands (in addition to their spatial dependence),

and as a consequence the evaluation of the exchange charge operators described in the following section is no more complicated than the evaluation of the matrix elements of the single-nucleon operators.

III. THE EXCHANGE CHARGE DENSITY OPERATOR

The exchange charge density operators can be divided into two classes. The first is formed of the effective operators that represent non-nucleonic degrees of freedom, as e.g., nucleon-antinucleon pairs⁷ or nucleon resonances,^{20,21} and which arise when those degrees of freedom are eliminated from the state vector. The second are genuine dynamical exchange charge effects that appear even in a description, that would include the explicit non-nucleonic excitations in the state vector. In a description based on meson-exchange mechanisms these involve electromagnetic transition couplings between different mesons. The proper forms of the former operators depend on the method of eliminating the non-nucleonic degrees of freedom, and therefore evaluating their matrix elements with the usual nonrelativistic nuclear wave functions represents only the first approximation to a systematic reduction.²² We shall first consider the exchange charge operators of this class, to which belongs the longest range pion exchange charge operator.

We first consider the pion exchange charge operator that is associated with the "seagull" type Feynman diagrams in Fig. 1. To obtain the corresponding exchange charge operator, one may begin by considering the low-energy limit of the relativistic Born diagram in Fig. 2. If this is evaluated with the usual pseudovector pion-nucleon coupling

$$\mathcal{L} = i \frac{f_\pi}{m_\pi} \bar{\psi} \gamma_5 \gamma_\mu \partial_\mu \phi \cdot \tau \psi, \quad (3.1)$$

where ψ is the isodoublet nucleon field, ϕ the isovector pion field, f_π the pion-nucleon pseudovector coupling constant ($f_\pi^2/4\pi \simeq .081$) and m_π the pion mass, one obtains the charge operator²³

$$\rho = \frac{1}{2} [G_E^S(q) + G_E^V(q)\tau_{1,z}] \frac{1}{E_{in} - E} v_\pi(\mathbf{k}_2) + \frac{f_\pi^2}{2mm_\pi^2} \frac{1}{2} [F_1^S(q)\tau_1 \cdot \tau_2 + F_1^V(q)\tau_{2,z}] \frac{\boldsymbol{\sigma}_1 \cdot \mathbf{q} \boldsymbol{\sigma}_2 \cdot \mathbf{k}_2}{m_\pi^2 + k_2^2} + O(E_{in} - E). \quad (3.2)$$

Here \mathbf{q} is the momentum transfer to the nucleus and \mathbf{k}_2 the momentum transferred by the pion to the second nu-



FIG. 1. Pion-exchange contributions to the nuclear charge density operator.



FIG. 2. Meson-exchange diagrams that involve the Born terms in the relativistic amplitudes for photoproduction of virtual mesons.

cleon (Fig. 2). In (3.2) $v_\pi(\mathbf{k})$ is the pion-exchange potential (in momentum space).

The first term in the expression (3.2) contains the nonrelativistic intermediate state Green's function and the one-pion-exchange potential. It is therefore contained in the bound state matrix elements of the single-nucleon charge operator (i.e., in the impulse approximation). The second term represents a part of the seagull diagram in Fig. 1(a), and thus should be taken into account as an exchange charge density operator. The symmetrized version of the pion exchange charge operator that is obtained by combining the nonsingular nonvanishing seagull terms from both diagrams in Fig. 2 and those with the nucleon lines exchanged is

$$\rho_\pi = \frac{f_\pi^2}{2mm_\pi^2} \left\{ [F_1^S(q)\tau_1 \cdot \tau_2 + F_1^V(q)\tau_{2,z}] \frac{\boldsymbol{\sigma}_1 \cdot \mathbf{q} \boldsymbol{\sigma}_2 \cdot \mathbf{k}_2}{m_\pi^2 + k_2^2} + [F_1^S(q)\tau_1 \cdot \tau_2 + F_1^V(q)\tau_{1,z}] \frac{\boldsymbol{\sigma}_1 \cdot \mathbf{k}_1 \boldsymbol{\sigma}_2 \cdot \mathbf{q}}{m_\pi^2 + k_1^2} \right\}. \quad (3.3)$$

Here the momentum variables \mathbf{k}_1 , \mathbf{k}_2 , and \mathbf{q} are defined as $\mathbf{q} = \mathbf{k}_1 + \mathbf{k}_2$.

The exchange charge operator (3.3) was first obtained by Kloet and Tjon,⁷ who considered the nonrelativistic reduction of component in the pion-photoproduction amplitude that contains an intermediate nucleon-antinucleon pair, evaluated with the pseudoscalar pion-nucleon coupling. No such pair terms appear here as we use the pseudovector coupling, in which case the exchange charge operator appears as a seagull-type diagram. Note that in the original paper⁷ there is an additional correction to the electromagnetic form factor of the nucleon in Eq. (3.3) that involves the Pauli form factor. This is a consequence of the use of the pseudoscalar pion-nucleon coupling, which is inconsistent with the requirement of chiral symmetry.²⁴ In the (proper) pseudovector coupling model the anomalous Pauli term contributes a term that is smaller by a factor $(v/c)^2$ than the main term (3.3).

The exchange charge operator (3.3) represents a relativistic correction that arises in the nonrelativistic reduction of the Born term in the pion nucleon photoproduction amplitude. The form of the operator does of course depend on the method of carrying out the nonrelativistic reduction, and hence its use with conventional nonrelativistic wave functions is based more on phenomenological success than on solid theoretical argument.

The effect of the pion-exchange charge operator (3.3) is enhanced by the similar operator that is associated with ρ -meson exchange. The ρ -meson exchange charge operator can be derived in the same way as the pion exchange charge operator by considering the nonrelativistic reduction of the virtual ρ -meson photoproduction amplitudes in two-body diagrams of the form in Fig. 2, and eliminating the singular term that represents an iteration of the wave function. The complete form of the resulting operator is⁸

$$\rho_\rho = \frac{g_\rho^2(1+\kappa)^2}{8m^3} \times \left\{ [F_1^S(q)\tau_1 \cdot \tau_2 + F_1^V(q)\tau_{2,z}] \frac{(\boldsymbol{\sigma}_1 \times \mathbf{q}) \cdot (\boldsymbol{\sigma}_2 \times \mathbf{k}_2)}{m_\rho^2 + k_2^2} + [F_1^S(q)\tau_1 \cdot \tau_2 + F_1^V(q)\tau_{1,z}] \frac{(\boldsymbol{\sigma}_2 \times \mathbf{q}) \cdot (\boldsymbol{\sigma}_1 \times \mathbf{k}_1)}{m_\rho^2 + k_1^2} \right\}. \quad (3.4)$$

Here m_ρ is the ρ -meson mass, g_ρ the ρNN vector coupling constant and κ the ρNN tensor coupling constant [$g_\rho^2/4\pi = 0.55$ and $\kappa = 6.6$ (Ref. 25)].

The operator (3.4) may be viewed as a seagull type exchange operator (Fig. 1) or as an effective nonrelativistic representation of nucleon-antinucleon pair configurations that have been eliminated from the state vector. The diagrammatic interpretation is mainly a question of convention, which depends on the organization of the nonrelativistic reduction. To the order considered the operator is unique, except for some nonlocal corrections that are expected to be small.⁸

The pion and ρ -meson exchange charge operators (3.3) and (3.4) contain coupling constants and bare meson propagators, which are usually modified by *ad hoc* vertex form factors in order to take into account the finite extent of the nucleons. We shall here avoid that uncertainty by constructing them directly from the nucleon-nucleon interaction model (the Argonne v_{14} potential) using the method developed in Ref. 12. This implies replacing the pion- and ρ -meson propagators in Eqs. (3.3) and (3.4) by the Fourier transforms $v^{\sigma\tau}(k)$ and $v^{i\tau}(k)$ of the isospin dependent spin-spin and tensor components of the interaction model³ as

$$\frac{f_\pi^2}{3m_\pi^2} \frac{1}{m_\pi^2 + k^2} \rightarrow V_{PS}(k) = \frac{1}{3}[2v^{i\tau}(k) - v^{\sigma\tau}(k)], \quad (3.5a)$$

$$-\frac{g_\rho^2(1+\kappa)^2}{12m^2} \frac{1}{m_\rho^2 + k^2} \rightarrow V_V(k) = \frac{1}{3}[v^{i\tau}(k) + v^{\sigma\tau}(k)]. \quad (3.5b)$$

The replacements (3.5) are the ones required for the construction of an exchange current operator that satisfies the continuity equation with the interaction model. We here apply the replacement to the exchange charge operators as the generalized meson propagators constructed in this way do take into account the nucleon structure in a way that is consistent with the nucleon-nucleon interac-

tion. An additional reason for using the construction (3.5) is that it has been shown to lead to predictions for the magnetic form factors of the trinucleons that are in excellent agreement with the empirical data.³

The $T=1$ PS and V exchanges provide the largest contribution to the charge operator, and fortunately contain

$$\rho_\omega = \frac{g_\omega^2}{8m^3} \left[[F_1^S(q)\tau_1 \cdot \tau_2 + F_1^V(q)\tau_{1,z}] \frac{(\boldsymbol{\sigma}_1 \times \mathbf{q}) \cdot (\boldsymbol{\sigma}_2 \times \mathbf{k}_2)}{m_\omega^2 + k_2^2} + [F_1^S(q)\tau_1 \cdot \tau_2 + F_1^V(q)\tau_{2,z}] \frac{(\boldsymbol{\sigma}_2 \times \mathbf{q}) \cdot (\boldsymbol{\sigma}_1 \times \mathbf{k}_1)}{m_\omega^2 + k_1^2} \right]. \quad (3.6)$$

This operator can be derived in the same way as the ρ -meson exchange charge operator (3.4) above, and may be viewed as a seagull type exchange operator of the type in Fig. 1 with ω instead of π exchange. For the ωNN coupling constant g_ω we use the value 14.6 suggested by boson-exchange models for the nucleon-nucleon interaction.²⁶

All the exchange charge operators above belong to the first class of exchange operators, and appear as nonvanishing nonsingular seagull terms in the nonrelativistic reduction of the virtual photoproduction amplitudes for the exchanged mesons. The exchange charge operators that correspond to the $\rho\pi\gamma$ and $\omega\pi\gamma$ couplings shown in Fig. 3 belong to the (second) class of genuine dynamical exchange operators, which are associated with transverse four-vector currents. The $\rho\pi\gamma$ exchange charge operator that corresponds to the diagram in Fig. 3 has the form^{9,27}

$$\rho_{\rho\pi} = -\frac{f_\pi g_{\rho\pi\gamma} g_\rho (1+\kappa)}{2m_\pi m_\rho m} \tau_1 \cdot \tau_2 \times G_\omega(q) \left\{ \frac{\boldsymbol{\sigma}_1 \cdot \mathbf{k}_1 (\boldsymbol{\sigma}_2 \times \mathbf{k}_2) \cdot (\mathbf{k}_1 \times \mathbf{k}_2)}{(m_\pi^2 + k_1^2)(m_\rho^2 + k_2^2)} - \frac{\boldsymbol{\sigma}_2 \cdot \mathbf{k}_2 (\boldsymbol{\sigma}_1 \times \mathbf{k}_1) \cdot (\mathbf{k}_1 \times \mathbf{k}_2)}{(m_\pi^2 + k_2^2)(m_\rho^2 + k_1^2)} \right\}. \quad (3.7)$$

Here $g_{\rho\pi\gamma}$ is the $\rho\pi\gamma$ coupling strength for which we shall use the value 0.4.²⁸ In line with the usual vector meson dominance phenomenology we shall describe the electromagnetic transition form factor of the $\rho\pi\gamma$ vertex as a ω -meson pole term

$$G_\omega(q) = \frac{1}{1+q^2/m_\omega^2}. \quad (3.8)$$

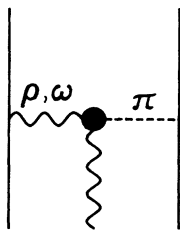


FIG. 3. The $\rho\pi\gamma$ and $\omega\pi\gamma$ terms in the two-body charge density operator.

no adjustable parameter. The other contributions which have been considered, namely those associated with the ω , $\rho\pi\gamma$, and $\omega\pi\gamma$ mechanisms, are relatively smaller, and we use empirical coupling constants and vertex form factors to calculate them. The ω -meson exchange charge operator is given by

The corresponding $\omega\pi\gamma$ exchange charge operator has the form

$$\rho_{\omega\pi} = -\frac{f_\pi g_{\omega\pi\gamma} g_\omega}{2m_\pi m_\omega m} G_\rho(q) \times \left\{ \frac{\boldsymbol{\sigma}_1 \cdot \mathbf{k}_1 (\boldsymbol{\sigma}_2 \times \mathbf{k}_2) \cdot (\mathbf{k}_1 \times \mathbf{k}_2)}{(m_\pi^2 + k_1^2)(m_\omega^2 + k_2^2)} \tau_{1,z} - \frac{\boldsymbol{\sigma}_2 \cdot \mathbf{k}_2 (\boldsymbol{\sigma}_1 \times \mathbf{k}_1) \cdot (\mathbf{k}_1 \times \mathbf{k}_2)}{(m_\pi^2 + k_2^2)(m_\omega^2 + k_1^2)} \tau_{2,z} \right\}. \quad (3.9)$$

Here $g_{\omega\pi\gamma}$ is the $\omega\pi\gamma$ coupling strength. We shall take $g_{\omega\pi\gamma}$ to have the value 0.68,²⁹ and assume that the electromagnetic vertex form factor $G_\rho(q)$ has the form of a ρ -meson pole term

$$G_\rho(q) = \frac{1}{1+q^2/m_\rho^2}. \quad (3.10)$$

The derivation of the $\rho\pi\gamma$ and $\omega\pi\gamma$ exchange charge operators is straightforward, given the transition current matrix elements $\langle V(k') | J_\mu(0) | \pi(k) \rangle$, with $V = \rho, \omega$. The general form of these current matrix elements is given, e.g., in Ref. 29. More recently it has been shown that the isoscalar $\rho\pi\gamma$ exchange charge operator can also be derived from the anomalous baryon current that carries the baryon charge in the topological soliton (or Skyrme) model.^{10,11} This derivation, which is independent of the detailed form of the effective chiral Lagrangian in the soliton model, links the $\rho\pi\gamma$ exchange current operator to the chiral anomaly and makes its strength, if not the detailed form of the expression (3.7), model independent.

In the ω , $\rho\pi\gamma$, and $\omega\pi\gamma$ exchange charge operators the meson-nucleon vertices have been taken to be pointlike. We shall take into account the finite extent of the nucleon by modifying the free meson propagators in the preceding expressions by introducing high momentum cutoff factors of the conventional monopole form. The pion propagator in the $\rho\pi\gamma$ and $\omega\pi\gamma$ exchange charge operators (3.7) and (3.9) are thus multiplied by a factor $(\Lambda_\pi^2 - m_\pi^2)/(\Lambda_\pi^2 + k^2)$. The vector meson propagators in the expressions (3.6), (3.7), and (3.9) are multiplied by corresponding monopole factors with Λ_π replaced by Λ_V and m_π by the corresponding vector meson mass. The mass scales of the cutoff factors do of course represent free parameters. We shall here use the values $\Lambda_\pi = 1.2$ GeV and $\Lambda_V = 2$ GeV, which have been found to be reasonable in studies of the reaction $\pi^+ d \rightarrow pp$, which is dominated by

two-body mechanisms.³⁰ The ω , $\rho\pi\gamma$, and $\omega\pi\gamma$ charge operators give relatively small contributions to the charge form factors, and the results are not very sensitive to the precise values of the Λ 's when these are taken to be larger than 1 GeV.

In addition to these two-body exchange charge density operators one may also consider analogous three- and four-body exchange charge density operators. The most obvious such higher rank charge operators, which involve rescattering of the exchanged pion and ρ mesons off intermediate nucleons, have been studied in Ref. 8. The conclusion in that work was that once both pion and ρ -meson exchange mechanisms are taken into account, the net contributions of the three- and four-body exchange charge operators to the charge form factors of the bound three- and four-nucleon systems are small. The reason for this relative insignificance is the strong cancellations between the pion- and ρ -meson exchange terms.

IV. THE CHARGE FORM FACTORS OF ${}^3\text{H}$ AND ${}^3\text{He}$

In Figs. 4 and 5 we show the calculated and measured charge form factors of ${}^3\text{H}$ and ${}^3\text{He}$. The data points in the figures are obtained from Refs. 4 and 31–34. The theoretical results are obtained with the parametrization

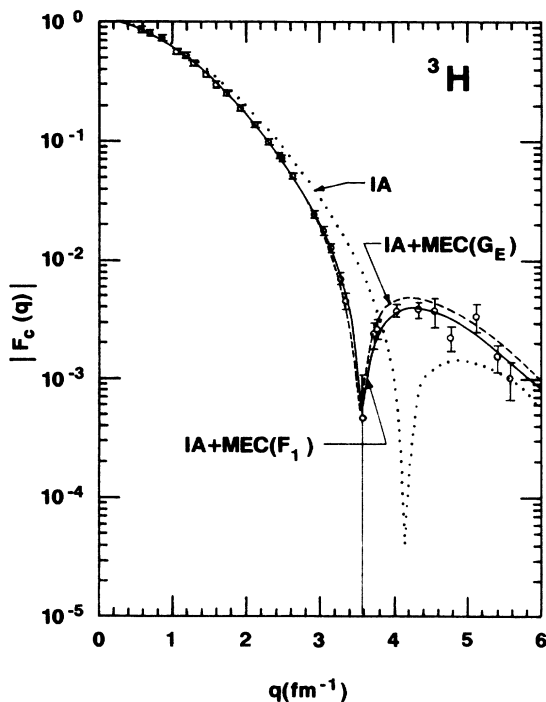


FIG. 4. The charge form factor of ${}^3\text{H}$, as function of the four-momentum transfer, in the impulse approximation (IA) and with inclusion of the two-body exchange charge operators, and of the Darwin-Foldy and spin-orbit corrections to the single-nucleon charge operator [IA+MEC(F_1)]; the results have been obtained with the Iachello-Jackson-Lande parametrization of the nucleon electromagnetic form factors. The curve labeled IA+MEC(G_E) has been obtained by using the Sachs form factor G_E (instead of F_1) in the PS , V , and ω charge operators.

for the nucleon electromagnetic form factors given in Ref. 35 ("5 parameter dipole fit"). The three-body wave functions used in the matrix elements of the charge density operator are those obtained from the Argonne v_{14} nucleon-nucleon and Urbana-VII three-nucleon interactions. The calculated form factors for both nuclei are in excellent agreement with the experimental data, except at the highest values above 7 fm^{-1} , where the form factor of ${}^3\text{He}$ is slightly underpredicted. The important role of the exchange charge density contributions above 3 fm^{-1} is evident, consistently with what was found in earlier studies.^{36,37} The small discrepancy at the very high values of momentum transfer in the case of ${}^3\text{He}$ (Fig. 5) may be due to a number of causes: missing relativistic corrections, three-body exchange charge density effects and finally the uncertainties in the electromagnetic form factors of the nucleon. Note that in Figs. 4 and 5 we also display the charge form factors calculated by replacing in Eqs. (3.3)–(3.6) the isoscalar and isovector combinations of the F_1 Dirac nucleon form factor with the corresponding expressions for the G_E Sachs parametrization. In contrast to what was found for the magnetic form factors of the three-body nuclei,³ the difference in the results obtained by using F_1 or G_E is not significant, due to the predominantly isoscalar character of the PS , V , and ω charge contributions.

The theoretical uncertainty caused by the lack of precise knowledge of the electromagnetic form factor of the nucleon is illustrated in Figs. 6 and 7, where the predicted charge form factors of ${}^3\text{H}$ and ${}^3\text{He}$ are given for a set of parametrizations of the electromagnetic form factors

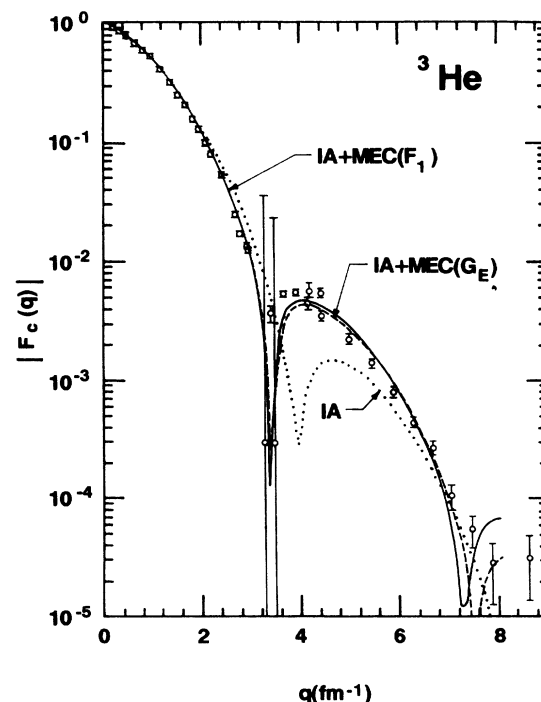


FIG. 5. Same as in Fig. 4, but of ${}^3\text{He}$.

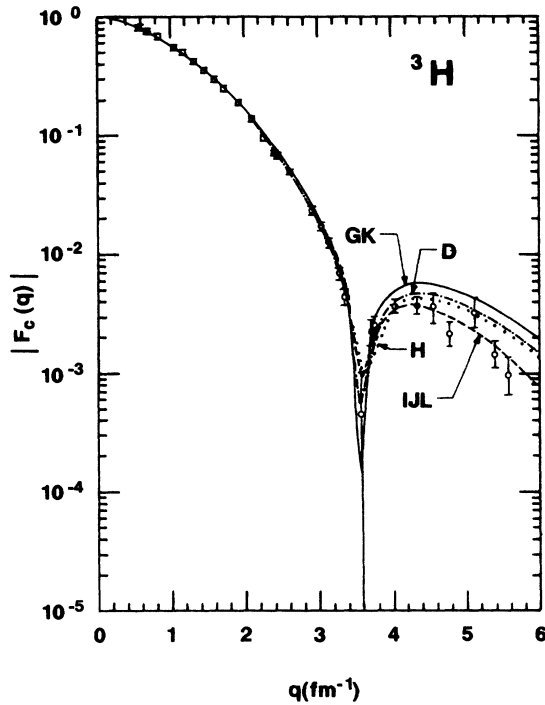


FIG. 6. The charge form factor of ${}^3\text{H}$, as function of the four-momentum transfer, obtained with different parametrizations of the nucleon electromagnetic form factors: dipole (D), Gari-Krumpelmann (GK), Höhler *et al.* (H) and Iachello-Jackson-Lande (IJL). All curves include the meson exchange contributions, and the Darwin-Foldy and spin-orbit corrections to the single-nucleon charge operator. The Dirac form factor F_1 is used in the PS , V , and ω charge operators.

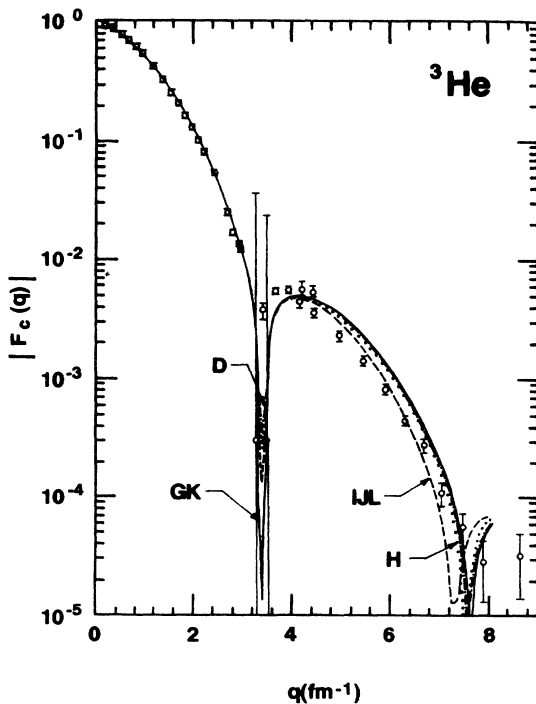


FIG. 7. Same as in Fig. 6, but of ${}^3\text{He}$.

of the nucleon. The results in Figs. 6 and 7 have been obtained (as in Figs. 4 and 5) with the form factor parametrization of Ref. 35 Iachello-Jackson-Lande (IJL) and the parametrizations of Ref. 38 Höhler (H) and Ref. 39 Gari-Krumpelmann (GK). In addition we show the results obtained with the simple dipole form for all the electromagnetic form factors of the nucleon [dipole (D)]. Note that F_1 is used in Eqs. (3.3)–(3.6). The differences among the different form factor parametrizations appear in the charge form factor of the trinucleons only above 4 fm^{-1} . If one considers both the ${}^3\text{H}$ and ${}^3\text{He}$ charge form factors simultaneously the best overall agreement with the empirical values is obtained with the IJL form factor.³⁵ The fact that when using this form factor parametrization there is a slight underprediction above 7 fm^{-1} is in our view more likely to be due to missing three-body exchange charge density corrections and/or missing nonobvious relativistic corrections than a reflection on the quality of the nucleon form factor parametrization.

For completeness we in Figs. 8 and 9 show the individual contributions from the different components of the charge density operator to the isoscalar and isovector combinations of the charge form factors of ${}^3\text{H}$ and ${}^3\text{He}$. The results reveal that at low and intermediate values of momentum transfer the (generalized) pion- and ρ -meson exchange charge operators (3.3) and (3.4) [with the re-

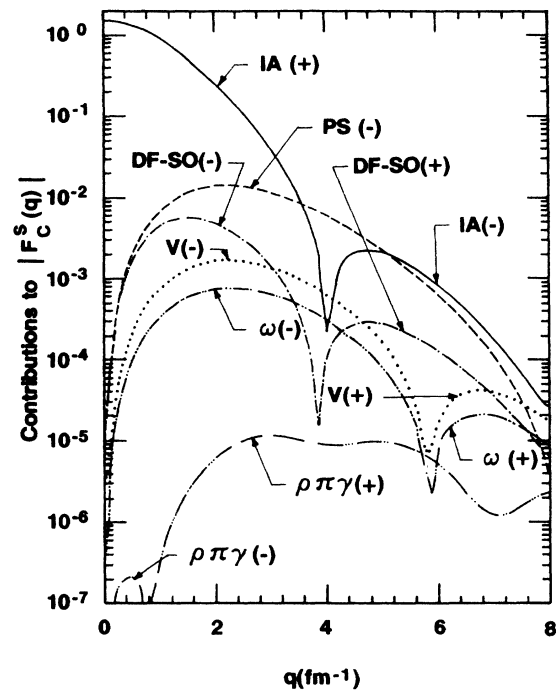


FIG. 8. The individual contributions to the isoscalar combination of the ${}^3\text{He}$ and ${}^3\text{H}$ charge form factors, as functions of the four-momentum transfer, obtained with the Iachello-Jackson-Lande parametrization of the nucleon electromagnetic form factors. The contributions due to the PS , V , ω , and $\rho\pi\gamma$ exchange charge operators, and the Darwin-Foldy and spin-orbit corrections to the single-nucleon charge operator (DF-SO) are displayed along with the impulse approximation (IA).

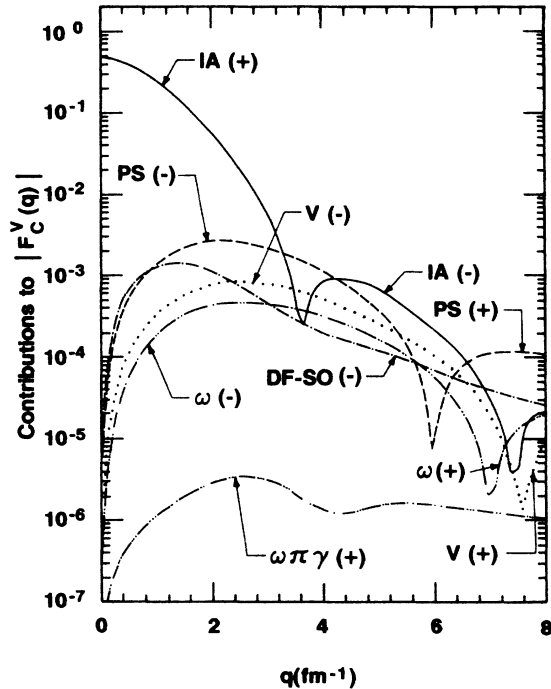


FIG. 9. The individual contributions to the isovector combination of the ${}^3\text{He}$ and ${}^3\text{H}$ charge form factors, as functions of the four-momentum transfer, obtained with the Iachello-Jackson-Lande parametrization of the nucleon electromagnetic form factors. The contributions due to the PS , V , ω , and $\omega\pi\gamma$ exchange charge operators, and the Darwin-Foldy and spin-orbit corrections to the single-nucleon charge operator (DF-SO) are displayed along with the impulse approximation (IA).

placements (3.5)] are by far the most important two-body terms. It should be noted here that these terms do not contain in the present approach any adjustable parameters. The $1/m^2$ corrections to the single-nucleon charge operator (the Darwin-Foldy and spin-orbit terms labeled collectively as DF-SO in Figs. 8 and 9) also give a significant contribution. The $\rho\pi\gamma$ (isoscalar) and $\omega\pi\gamma$ (isovector) exchange charge operators (3.7) and (3.9) become important only at very large values of momentum transfer.

The contributions associated with the $\rho\pi\gamma$ and $\omega\pi\gamma$ mechanisms are much smaller than those obtained in Ref. 6. However, they appear to be very sensitive to the short-range and D -state components in the wave function; indeed, we have verified that these contributions would be significantly larger if the matrix elements involving the D -state component are left out. Finally, it is worth noting, as already mentioned, the dominant isoscalar nature of the exchange contributions to the charge form factors of the three-nucleon isodoublet.

V. THE CHARGE FORM FACTOR OF ${}^4\text{He}$

The charge form factor of the α particle, obtained by the same methods as the charge form factors of the trin-

cleons in the previous section, is shown in Fig. 10. We have used the variational four-body wave function developed in Ref. 2 for the Argonne-Urbana interactions, and the IJL parametrization of the nucleon electromagnetic form factors.³⁵ The predicted charge form factor is in good agreement with the experimental data,^{4,40} except in the region of the diffraction maximum between 3.5 and 4 fm^{-1} . It should be noted, however, that the empirical values shown in the figure do not fall on a smooth curve in this region, which suggests the presence of a normalization discrepancy. The good fit to the data at high momentum transfer is rather remarkable.

In Fig. 11 we show the predictions as obtained with different parametrizations for the electromagnetic form factors of the nucleon. In this case the spread between the results obtained with the different form factor parametrizations is smaller than the corresponding spread in the case of the three-body form factors (Figs. 6 and 7). In Fig. 12 we finally show the explicit contributions from the different two-body exchange mechanisms. The numerically most important exchange charge density operators are again the (generalized) pion and ρ -meson exchange operators (3.3–3.5). The isoscalar $\rho\pi\gamma$ exchange charge density operator (3.7) gives noticeable a contribution only at the highest values of momentum transfer

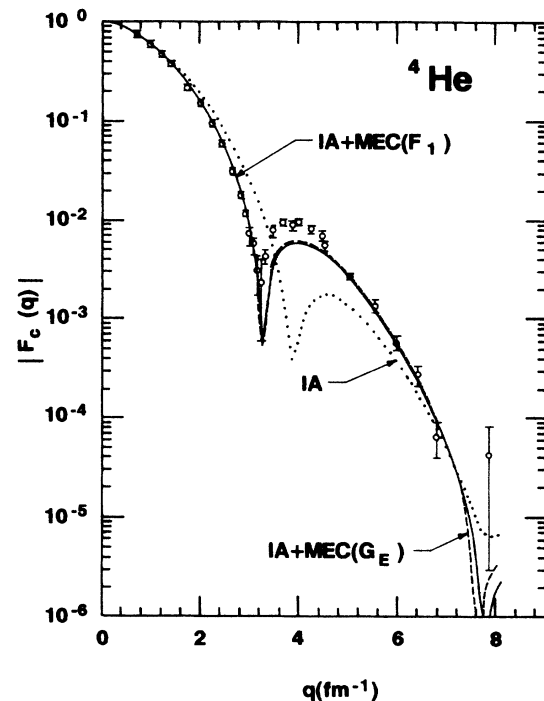


FIG. 10. The charge form factor of ${}^4\text{He}$, as function of the four-momentum transfer, in the impulse approximation (IA) and with inclusion of the two-body exchange charge operators, and of the Darwin-Foldy and spin-orbit corrections to the single-nucleon charge operator [IA+MEC(F_1)]; the results have been obtained with the Iachello-Jackson-Lande parametrization of the electromagnetic nucleon form factors. The curve labeled IA+MEC(G_E) has been obtained by using the Sachs form factor G_E (instead of F_1) in the PS , V , and ω charge operators.

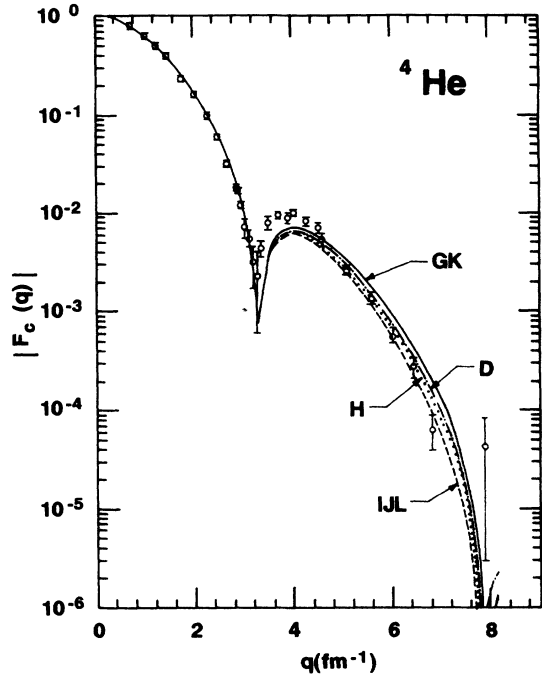


FIG. 11. The charge form factor of ${}^4\text{He}$, as function of the four-momentum transfer, obtained with different parametrizations of the nucleon electromagnetic form factors: dipole (D), Gari-Krümpelmann (GK), Höhler *et al.* (H), and Iachello-Jackson-Lande (IJL). All curves include the meson-exchange contributions, and the Darwin-Foldy and spin-orbit corrections to the single-nucleon charge operator. The Dirac form factor F_1 is used in the PS , V , and ω charge operators.

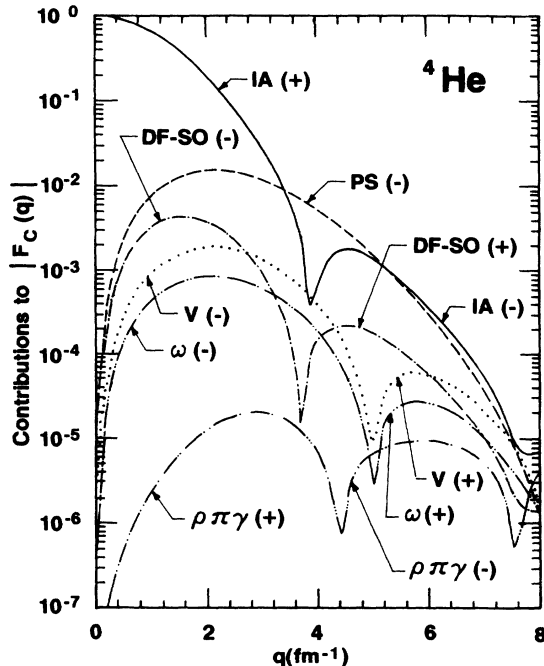


FIG. 12. The individual contributions to the ${}^4\text{He}$ charge form factor, as functions of the four-momentum transfer, obtained with the Iachello-Jackson-Lande parametrization of the nucleon electromagnetic form factors. The contributions due to the PS , V , ω , and $\rho\pi\gamma$ exchange charge operators, and the Darwin-Foldy and spin-orbit corrections to the single-nucleon charge operator (DF-SO) are displayed along with the impulse approximation (IA).

considered. As in the case of the trinucleons, the contribution of the $\rho\pi\gamma$ exchange charge density operator is much smaller than what was found in previous work with other wave function models.⁴¹

VI. DISCUSSION

The goals of this work were to analyze the charge form factors of the bound three- and four-nucleon systems with the variational wave functions that are obtained by using the Argonne v_{14} two-nucleon and Urbana-VII three-nucleon interactions. The results reveal the well-known impossibility of explaining the empirical charge form factors at intermediate values of momentum transfer without including contributions from two-body exchange mechanisms.^{3,5,6,36,37,42} With inclusion of the contributions from the two-body exchange charge density operators the results obtained here are, however, in remarkable agreement with the experimental data. The main theoretical uncertainty appears to be the spread among the form factors predicted with different parametrizations of the nucleon electromagnetic form factors.

This study of the charge form factors and the recent one of the ${}^3\text{H}$ and ${}^3\text{He}$ magnetic form factors³ show that it is possible to obtain remarkably good predictions for all the elastic electromagnetic structure functions of the bound three- and four-nucleon systems by using the Argonne v_{14} model for the nucleon-nucleon interaction, once the irreducible two-body exchange current effects are included. We note here that similar calculations using the Urbana v_{14} two-nucleon interaction were not in agreement with the measured magnetic form factors.³ It should be pointed out however that the theoretical basis for the exchange current density operators that contribute to the magnetic form factors of the trinucleons is much stronger than that for the presently considered exchange charge density operators, the most important of which are of the same order of magnitude as typical relativistic corrections.

The qualitatively most important result of this investigation is, however, the fact that it is possible to explain the elastic form factors of the three- and four-nucleon systems very well within the conventional quantum-mechanical framework based on nonrelativistic nucleon wave functions, once the non-nucleonic degrees of freedom are taken into account in terms of effective two-body exchange current and charge density operators.

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