Electrodisintegration of the deuteron in nuclear reactions

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We estimate that the electrodisintegration of the deuteron contributes at least 25% of the total reaction cross section when the deuteron interacts with a nucleus of charge $Z \gtrsim 40$ at a kinetic energy lower than 50 MeV. The neglect of this breakup channel in deuteron-nucleus optical potentials might lead to unrealistic predictions for the mass and energy dependence of the total deuteron reaction cross sections.

There is much interest in the mass and energy dependence of deuteron reaction cross sections. Because reaction cross sections have only been measured at energies between 20 and 25 MeV (Refs. 1 and 2), the desired energy dependence must be calculated theoretically. Predictions are often obtained through the use of the imaginary part of deuteron-optical potentials determined from fits to deuteron-nucleus differential elastic scattering cross sections at various energies. However, in all optical model analyses of which we are aware, the imaginary part of the deuteron-optical potential has been implicitly assumed to arise only from nuclear processes.[3] Accordingly, the range of this absorptive potential has been constrained to values roughly compatible with the range of the nuclear force. In this Brief Report, we show that for nuclei having a large charge, the electric breakup process of the deuteron can be a considerable fraction of the expected total deuteron reaction cross section at low energies. The electrodisintegration process cannot be fully simulated by nuclear absorption mechanism because it is so long ranged. Consequently, the use of the imaginary part of the deuteron-nucleus optical potential determined without the consideration of electrodisintegration may lead to unrealistic predictions for the mass and energy dependence of deuteron reaction cross sections. The electric disintegration of the deuteron was considered as early as 1935 by Oppenheimer,[4] by Dancoff[5] in 1947, and by Mullin and Guth[6] in 1951. However, as we shall see, these early attempts are unreliable. More recently, Kleinfeller *et al.*[7] have been interested in the electrodisintegration process in the context of heavy-ion reactions. In the following, we evaluate the deuteron electrodisintegration cross sections in the absence of nuclear breakup processes in order to compare it with nuclear reaction processes.

We consider the case where the only interaction between the deuteron and the target nucleus is the Coulomb interaction V_c. The Hamiltonian of the system is

$$H = K_d + K_A + h_d + h_A + V_c \ , \tag{1}$$

where $K_d + h_d \equiv (K_{\rm rel} + V_{np}) + K_{d(\rm c.m.)} \equiv H_{\rm rel} + K_{d(\rm c.m.)}$. The V_{np} is the interaction between the neutron and the proton of the deuteron, h_A is the Hamiltonian of the target nucleus (denoted as A), and $K_{\rm rel}$ and $K_{d(\rm c.m.)}$ are, respectively, the kinetic energy operators for the relative and center-of-mass motions of the np pair. Because $K_{d(\rm c.m.)} + K_A = K_{d-A} + K_{CM}$, where K_{d-A} is the kinetic energy operator for the relative deuteron-nucleus motion and K_{CM} is the kinetic energy operator for the motion of the entire system, the dynamically relevant Hamiltonian is

$$H = (H_{\rm rel} + K_{d-A}) + V_c \ . \tag{2}$$

In writing Eq. (2), we have suppressed K_{CM} and h_A. If we label the continuum and bound eigenstates of $H_{\rm rel}$ by χ_s and D, respectively, then these states satisfy the following equations: $H_{\rm rel}|\chi_s\rangle = E_s|\chi_s\rangle$; and $H_{\rm rel}|D\rangle = E_B|D\rangle$. The Hamiltonian of Eq. (2) leads to the following electrodisintegration matrix element:

$$\tau_{fi} = \langle \mathbf{P}'; \chi_s^{(-)} | V_c | \mathbf{P}; D \rangle \ , \tag{3}$$

where $\mathbf{P}'$ and $\mathbf{P}$ are the c.m. momenta of the final and initial np pairs and

$$V_c = \sum_j \frac{e^2}{|\mathbf{r}_p - \mathbf{r}_j|} \ , \tag{4}$$

with $\mathbf{r}_j$ being the position vector of the jth proton in the target nucleus, and $\mathbf{r}_p$ that of the proton in the deuteron. If we denote, respectively, the position vectors of the c.m. of the deuteron and the relative distance between the n and p in the deuteron by $\mathbf{r}_d$ and $\mathbf{x}(\equiv \mathbf{r}_n - \mathbf{r}_p)$, then we have $\mathbf{r}_p = \mathbf{r}_d - \mathbf{x}/2$ and V_c may be expanded as

$$V_c = e^2 \sum_j \left[\frac{1}{|\mathbf{r}_d - \mathbf{r}_j|} + \frac{(\mathbf{r}_d - \mathbf{r}_j)\cdot \mathbf{x}/2}{|\mathbf{r}_d - \mathbf{r}_j|^3} + \cdots \right] . \tag{5}$$

We observe that if we use the first term of Eq. (5) (the so-called monopole term) alone in Eq. (3), we find that the matrix element τ_{fi} vanishes identically since it becomes

$$\tau_{fi} \simeq \langle \mathbf{P}' | \sum_j \frac{e^2}{|\mathbf{r}_d - \mathbf{r}_j|} | \mathbf{P} \rangle \langle \chi_s^{(-)} | D \rangle \ . \tag{6}$$

The second factor on the right-hand side in Eq. (6) vanishes through orthogonality. Obviously, the Born approximation to τ_{fi} is a dangerous approximation, because we lose the orthogonality property that suppresses the monopole part of the Coulomb potential. The lack of orthogonality accounts for the large values for deuteron electrodisintegration obtained in the earliest work (see below).[3] The importance of the role of orthogonality has been discussed by many authors and particularly stressed by Eisenberg, Noble, and Weber.[8]

Thus, we seek a formulation in which the monopole term is separated out. We, therefore, introduce the auxiliary interaction $\bar{V}_C$ defined by $\langle D | V_c | D \rangle$. Clearly, $\bar{V}_c$ does not depend on $\mathbf{x}$. We reorganized the Hamiltonian as

$$H = (H_{\rm rel} + H_{d-A}) + U \equiv (H_0) + U \ , \tag{7}$$

where $H_{d-A} \equiv K_{d-A} + \bar{V}_c$ and $U = V_c - \bar{V}_c$. The eigenstates of H_{d-A} are the Coulomb scattering states of the deuteron. We denote them as ϕ_c. They are solutions of the equation $H_{d-A}|\phi_c\rangle = E_c|\phi_c\rangle$. The matrix element of the electric disintegration for the deuteron is then given by

$$M_{fi} = \langle \Psi^{(-)}_{\mathbf{k}'_n \nu'_n; \mathbf{k}'_p \nu'_p; \mathbf{k}'_A \nu'_A} | U | \phi^{(+)}_{c, \mathbf{k}_d}; \nu_d D; \mathbf{k}_A \nu_A \rangle \ , \tag{8}$$

where Ψ is the eigenstate of H and ν's are the third components of the spins. The $\mathbf{k}'_n$, $\mathbf{k}'_p$, and $\mathbf{k}'_A$ are the momenta of the neutron, the proton, and the nucleus in the final state. The $\mathbf{k}_d$ and $\mathbf{k}_A$ are the initial momenta of the deuteron and the nucleus. In the c.m. frame of the deuteron-nucleus system, we have $\mathbf{k}'_n + \mathbf{k}'_p + \mathbf{k}'_A = 0$ and $\mathbf{k}_d = -\mathbf{k}_A \equiv \mathbf{k}_0$. The total electric disintegration cross section is given by

$$\sigma = \frac{(2\pi)^4}{(2J_d+1)(2J_A+1)v_{in}} \int d\mathbf{k}'_n d\mathbf{k}'_p d\mathbf{k}'_A \delta(\mathbf{k}'_n + \mathbf{k}'_p + \mathbf{k}'_A) \delta[E_n(\mathbf{k}'_n) + E_p(\mathbf{k}'_p) + E_A(\mathbf{k}'_A) - W] \sum_{\nu'_n \nu'_p \nu_d \nu'_A \nu_A} |M_{fi}|^2 \ , \tag{9}$$

where $J_d (=1)$ and J_A are the spins of the deuteron and the nucleus. The quantity

$$W \equiv E_d(\mathbf{k}_0) + E_A(\mathbf{k}_0) = \sqrt{k_0^2 + m_d^2} + \sqrt{k_0^2 + m_A^2}$$

is the total c.m. energy of the system, and $v_{in} \equiv kW/(E_d E_A)$ is the velocity of the incident deuteron. Although the presence of Ψ^- and ϕ_c^+ in M_{fi} affects the angular distributions of the outgoing particles, we can obtain a reasonable first estimate of the total disintegration cross section by replacing them with the corresponding plane-wave states (Born approximation). The electrodisintegration matrix element is then given by

$$M_{fi} \simeq \langle \mathbf{k}'_n, \nu'_n; \mathbf{k}'_p, \nu'_p; \mathbf{k}'_A | (V_c - \bar{V}_c) | \mathbf{k}_0, \nu_d; -\mathbf{k}_0, \nu_A \rangle \ . \tag{10}$$

Using Eq. (10) in Eq. (9), we obtain

$$\sigma = \frac{(2\pi)^5 E_d(k_0) E_A(k_0)}{k_0 W} \int dE'_n dE'_p d\cos\theta_n d\phi_{p\hat{n}} E'_n E'_p (W - E'_n - E'_p) |\hat{V}_c(\mathbf{Q}) F_{ch}(\mathbf{Q})|^2 \times \left[[u(q) - F_d(Q/2) u(q')]^2/(4\pi) + \tfrac{1}{5} \sum_m |Y_m^2(\hat{q}) w(q) - F_d(Q/2) Y_m^2(\hat{q}') w(q')|^2 \right] , \tag{11}$$

where $\mathbf{Q} \equiv \mathbf{k}'_n + \mathbf{k}'_p - \mathbf{k}_0$ is the momentum transferred to the np pair, and $\mathbf{q} \equiv \mathbf{k}'_n - \mathbf{k}_0/2$ and $\mathbf{q}' \equiv (\mathbf{k}'_n - \mathbf{k}'_p)/2$. The variable $\phi_{p\hat{n}}$ denotes the azimuthal angle of $\mathbf{k}'_p$ when $\mathbf{k}'_n$ is taken as the polar axis. The $\hat{V}_c$ is the Fourier transform of V_c and F_{ch} is the nucleus charge form factor normalized to one. The u, w, and F_d are, respectively, the momentum-space s-and d-state radial wave functions, and the form factor of the deutron. The terms depending on q and q' arise, respectively, from the V_c and $\bar{V}_c$ in Eq. (10).

The calculated energy dependences for three representative nuclei (^{208}Pb, ^{58}Ni, and ^{13}Al) are given in Fig. 1. In our calculations, we used the deuteron wave function of the Paris Group (Lacombe *et al.*[9]) and experimental nuclear charge densities.[10] We see that in the region of 25 MeV the electrodistintegration cross section of the deuteron, as induced by the Coulomb field of ^{208}Pb is $\sim$1500 mb or about 65% of the observed total deuteron reaction cross section,[1,2] and remains at the level of half barn at $\sim$100 MeV. Because the Coulomb interaction is proportional to Z, the electric disintegration cross section decreases rapidly as the charge of the target nucleus decreases. In the case of ^{58}Ni and ^{13}Al, electrodisintegration represents only $\sim$18% and $\sim$7% of the observed total deuteron reaction cross section at 25 MeV. We found from our calculations that the electrodisintegration contributes to at least 25% of the total deuteron reaction cross section for targets with $Z \gtrsim 40$ in the energy region of 25 MeV. Since the interaction range, the mass/charge and energy dependence of the electric disintegration are all different from those of the nuclear processes, we believe that in the analysis of deuteron elastic scattering from nuclei with atomic number greater than that of zirconium, the electrodisintegration of the deuteron must be included explicitly.

We have mentioned that it is necessary to separate out the monopole term of the Coulomb potential in order to

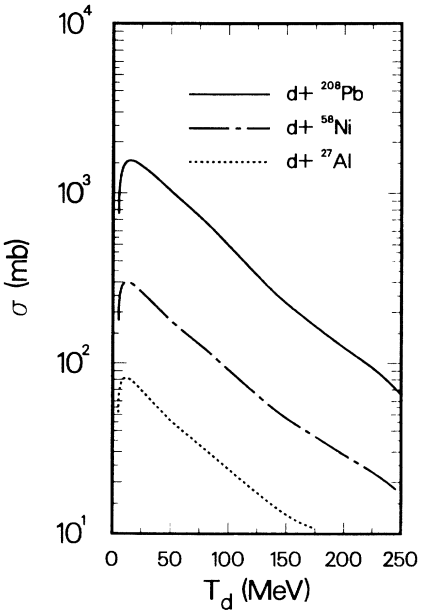

FIG. 1. Energy dependences of the calculated total deuteron electrodisintegration cross sections for ^{208}Pb, ^{58}Ni, and ^{13}Al.

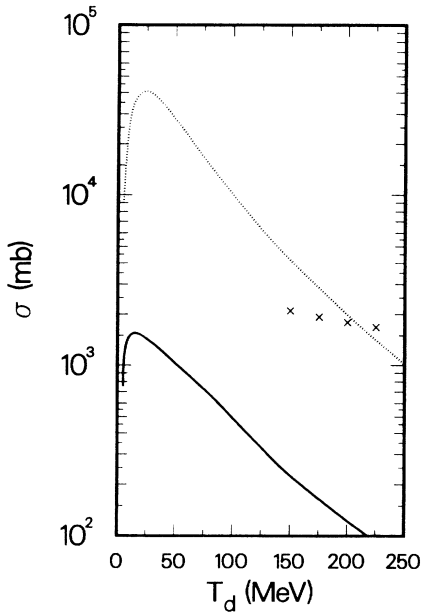

FIG. 2. Energy dependences of the total deuteron electrodisintegration cross sections for ^{208}Pb calculated with the Oppenheimer formula (×), the Born approximation to the matrix element τ (dotted curve), and the matrix element M (solid curve).

render sensible Born approximation. In Fig. 2 we compare the electrodisintegration cross sections for ^{208}Pb calculated with the use of the high-energy ($E \gtrsim 100$ MeV) Oppenheimer formula,[4] and the first Born approximation to the matrix elements τ_{fi} of Eq. (3), and M_{fi} of Eq. (10). An inspection of Fig. 2 shows that the Oppenheimer high-energy approximate formula and the τ_{fi} (both having the monopole contribution of V_c included) give cross sections of similar order of magnitude at energies between 100 and 200 MeV. However, these cross sections are much higher than the cross sections given by M_{fi}, indicating how misleading the Born approximation can be if one does not remove the monopole term. Although the use of plane waves for M_{fi} may not greatly affect the calculated total disintegration cross section, it will certainly modify the momentum spectrum of each individual outgoing particle. In particular, one can anticipate important influences on spin observables. Calculations with the use of the states χ_s and ϕ_c are anticipated.

In summary, the results presented here may significantly influence the interpretation of the observed deuteron reaction cross sections as well as the use of the existing deuteron-optical potentials to make predictions for the mass and energy dependence of the deuteron reaction cross sections.

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