Hypertriton: $\Lambda \leftrightarrow \Sigma$ conversion and tensor forces

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The separable potential equations that describe the hypertriton when ΛN-ΣN coupling and noncentral NN and YN forces are included, are formulated. Numerical solution of the equations for various potential models shows that Λ-Σ conversion in the YN interaction plays a significant role, even in the lightly bound ${}^3_\Lambda$H. When the ΣN channel is formally eliminated, the dispersive energy dependence of the resulting ΛN effective interaction is repulsive, whereas the resulting ΛNN three-body force is attractive. The contribution of the ΛN tensor force is shown to depend upon the inclusion of the NN tensor force and the relative sign of the ${}^3S_1$-${}^3D_1$ NN and ΛN tensor coupling. Also, a model which supports a ΣN bound state in the continuum appears to severely overbind the ${}^3_\Lambda$H system, indicating that such a phenomenon is not present in the $K^- d \rightarrow \Lambda N \pi$ reaction.

I. INTRODUCTION

The deuteron plays an important role in conventional, nonstrange nuclear physics by constraining our models of the nucleon-nucleon force. The precision with which we can measure properties of a bound system far exceeds that possible in measurements of the NN scattering amplitudes. Because neither the ΛN nor the ΣN (spin triplet or spin singlet) interactions possess sufficient strength to support a bound state,[1] it is the hypertriton (${}^3_\Lambda$H) that plays the important role of the deuteron in hypernuclear physics. It is the ground state of the ΛNN system ($J^\pi = \frac{1}{2}^+$, $T=0$) that must be used to constrain our models of the hyperon-nucleon (YN) force. The sparse data base for ΛN and ΣN scattering and reactions[2-5] is inadequate to fully determine the YN interaction.[6] Evidence for charge symmetry breaking in the ground state of the ${}^4_\Lambda$He-${}^4_\Lambda$H isodoublet[7] and the existence of a spin-flip excited state in that $A=4$ system[8] also provide important constraints on our modeling of the YN force.[9] However, an improvement in the precision of our knowledge of the ${}^3_\Lambda$H binding energy would significantly improve our constraints on YN potential models.

Exact equation calculations have played an important role in elucidating novel points of physics not readily apparent from simple effective two-body formulations of few-body problems.[10,11] The binding energy of a two-body system decreases as the effective range of the interaction becomes smaller, whereas that of the corresponding three-body and four-body systems increases. [This was the essence of the variational argument made by Thomas to show that the nuclear force must have a finite (nonzero) range or the triton would collapse to a point.[12,13]] It is this property of few-body equation calculations that appears to permit us to reconcile the charge symmetry breaking exhibited by the low energy Λp and Λn scattering parameters (the scattering lengths and effective ranges) and the Λ-separation energy difference observed in the $A=4$ isodoublet ground states.[9,14] We explore here, within a separable potential model framework, the importance of ΛN-ΣN coupling (Λ-Σ conversion) and tensor force effects in the ${}^3_\Lambda$H system.

The hypertriton is loosely bound, having a Λ-separation energy of only[7]

$$B_\Lambda({}^3_\Lambda\text{H}) = B({}^3_\Lambda\text{H}) - B({}^2\text{H}) \simeq 0.13 \pm 0.05 \text{ MeV} .$$

Thus, one expects this molecularlike system to be most sensitive to the long range aspects of the ΛN interaction. However, because the Λ ($T=0$) and N ($T=1/2$) cannot exchange a $T=1$ pion and conserve isospin, there is no one-pion-exchange mechanism contributing in first order to the ΛN interaction. The longest range components of the potential are due to the exchange of two pions or one kaon. The shorter range $\bar{K}$-exchange potential does admit a tensor force component. However, it is largely canceled by that of the K^*-exchange potential. (In contrast, the π-exchange and ρ-exchange tensor force contributions to the NN interaction do not cancel so completely, because the π and ρ masses are very different.) Therefore, tensor force effects in the ΛN interaction are anticipated to be somewhat smaller than those found in the NN interaction.[15-20]

On the other hand, ΛN-ΣN coupling effects are expected to be much more important in hypernuclear physics than are NN-$N\Delta$ coupling effects in nonstrange nuclear physics. The $m_\Lambda - m_\Sigma$ mass difference is only some 75 MeV, and the width of the Σ is small compared to that of the Δ, because the $\Sigma\pi$ channel lies below the K^--p threshold. Formally eliminating the Σ channel from the problem leads to an energy dependence in the resulting effective ΛN interaction and to ΛNN three-body forces.

Both of these effects are also the subject of current interest in the nonstrange sector.[21–24]

Few ${}^3_\Lambda$H calculations for models that include ΛN-ΣN coupling have been published. The study of Schick and Toepfer[25] illustrated the binding enhancement that occurs when channel coupling is included. Since the work of Dabrowski and Fedorynska,[26] based upon the simple Wycech[27] model, improved separable potential representations of the YN interaction,[28] which model the meson theoretic one-boson-exchange potentials of the Nijmegen group,[16] have appeared. We are unaware of any detailed studies of ${}^3_\Lambda$H involving $\Lambda N({}^3S_1$-${}^3D_1)$ tensor forces. We wish to report here results of new separable potential three-body calculations[29] that pertain to the following: (1) The dispersion (energy dependence) that results from embedding the ΛN-ΣN potential in a three-body system and reduces the ${}^3_\Lambda$H binding energy; (2) the three-body force effect that is due to coupling ΣNN states to ΛNN states and increases the ${}^3_\Lambda$H binding energy; (3) the $\Lambda N({}^3S_1$-${}^3D_1)$ tensor force effect, although expected to be an order of magnitude smaller than that due to the $NN({}^3S_1$-${}^3D_1)$ tensor force, has an overall contribution that depends on the presence of the $NN({}^3S_1$-${}^3D_1)$ tensor interaction and the relative sign of the tensor coupling in the NN and ΛN potentials. We also show how the binding energy of ${}^3_\Lambda$H can be utilized to constrain the sign of the coupling in the ΛN-ΣN channels. Although we can solve the ${}^3_\Lambda$H equations when the ΛN-ΣN coupling potentials contain tensor terms, no such separable potential exists in the literature, and our model results are incomplete in that sense.

In what follows we formulate the required three-body equations, which are equivalent to those of Faddeev[30] in their separable potential formulation, in Sec. II. Details of the kernel of the Faddeev equations are given in Appendix B while in Appendix A we consider the change in kinematics due to the Λ,Σ mass difference. The hyperon-nucleon interaction models employed and numerical results for the ${}^3_\Lambda$H system are presented in Sec. III. We summarize our results and conclusions in Sec. IV.

II. FADDEEV EQUATIONS FOR THE *YNN* SYSTEM

We consider the problem of a three-body system in which one or more of the particles has one or more energy or mass eigenstates. In particular, we want to write the equations for the YNN system where $Y=\Lambda,\Sigma$. With the resultant equations we will be able to examine the role of the ΛN-ΣN coupling in the binding energy of the hypertriton. In particular, we would like to examine the questions raised in the Introduction regarding this coupling between the ΛN and ΣN channels. Here we formulate the Faddeev equations for the YNN system in order to illustrate how the effects are isolated and their contributions determined.

A. The two-body interaction

To reduce the computational problem for this system, we restrict our results to separable NN and YN interactions. In this way, we reduce the dimensionality of the integral equations after partial wave expansion from two to one. This in turn allows us to include the tensor interaction in both the NN and YN potentials, yet constrain the computational complexity to a minimum.

We partial wave expand our two-body potentials in momentum space, as

$$\langle \mathbf{p}|V|\mathbf{p}'\rangle = \sum_{\substack{nll'\\ m_j m_t}} \langle \hat{\mathbf{p}}|lnm_jm_t\rangle V^n_{ll'}(p,p')\langle m_tm_jnl'|\hat{\mathbf{p}}'\rangle \,, \tag{1}$$

where $n=\{Sjt\}$ stands for the total spin S, the total angular momentum j, and the total isospin t, of the two-body system. Here, l is the orbital angular momentum. The state $\langle \hat{\mathbf{p}}|nlm_jm_t\rangle$ can then be written as

$$\langle \hat{\mathbf{p}}|lnm_jm_t\rangle = \sum_{\substack{m_{s_1}m_{s_2}m_l\\ m_Sm_{\tau_1}m_{\tau_2}}} (s_1m_{s_1}s_2m_{s_2}|Sm_S)(lm_lSm_S|jm_j)(\tau_1m_{\tau_1}\tau_2m_{\tau_2}|tm_t)Y_{lm_l}(\hat{\mathbf{p}})|s_1m_{s_1};s_2m_{s_2}\rangle|\tau_1m_{\tau_1};\tau_2m_{\tau_2}\rangle \,. \tag{2}$$

Here, $\{s_i,m_{s_i}\}$ and $\{\tau_i,m_{\tau_i}\}$ are the spin and its projection and isospin and its projection for the ith particle. For separable potentials in momentum space we can write

$$V^n_{ll'}(p,p') = g_{nl}(p)C^n_{ll'}g_{nl'}(p') = \langle p|g_{nl}\rangle C^n_{ll'}\langle g_{nl'}|p'\rangle \,. \tag{3a}$$

Since we are dealing with coupled channels for $l\neq l'$, we can write the above potential in matrix form as

$$V^n(p,p') = \langle p|V|p'\rangle \,, \tag{3b}$$

where

$$V=|g_n\rangle C_n\langle g_n| \tag{3c}$$

with

$$[C_n]_{ll'}=C^n_{ll'}, \quad [|g_n\rangle]_{ll'}=\delta_{ll'}|g_{nl}\rangle \,. \tag{4}$$

This potential includes the coupling due to the tensor force by admitting $C^n_{ll'}\neq 0$ for $l\neq l'$. To include the coupling between the ΛN and ΣN channels, we must generalize the above interaction to include this coupling. This is most simply achieved by replacing l by $\{r_1,r_2,l\}\equiv\ell$, where r_i is the quantum number that specifies the mass eigenstate of particle i. The corresponding scattering amplitude has the same partial wave expansion given in Eq. (1) with $V^n(p,p')\to t^n(p,p';E)$, and $t^n(E)$ given by

$$t^n(E)=|g_n\rangle\tau^n(E)\langle g_n| \ , \tag{5}$$

$$\tau^n(E)=[C_n^{-1}-G_0(E)]^{-1} \tag{6a}$$

$$=C_n[I-G_0(E)C_n]^{-1} \ , \tag{6b}$$

where

$$[G_0(E)]_{\ell\ell'}=[\langle g|G_0(E)|g\rangle]_{\ell\ell'}$$
$$=\delta_{\ell\ell'}\langle g_{n\ell}|G_\ell(E)|g_{n\ell}\rangle \ . \tag{7}$$

To properly include the ΛN and ΣN thresholds, we must include the rest mass of the particles in the energy E. Thus we have

$$\langle p|G_\ell(E)|p\rangle=\begin{cases}\left[E-\dfrac{p^2}{m_N}-2m_N\right]^{-1} & \text{for the } N\text{-}N \text{ system}\\[2ex] \left[E-\dfrac{p^2}{2\mu_{NY}}-m_N-m_Y\right]^{-1} & \text{for the } N\text{-}Y \text{ system} \ ,\end{cases} \tag{8}$$

where the reduced mass of the YN system is $\mu_{NY}=m_Nm_Y/(m_N+m_Y)$. For some separable potentials, e.g., the Yamaguchi[31] potential in the ${}^3S_1$-${}^3D_1$ NN channel, the strength matrix C_n is singular, and we must use Eq. (6b) to calculate $\tau^n(E)$.

In this way, we are able to evaluate the amplitude for NN scattering, for rank-one potentials, including the coupling that results from a tensor force. On the other hand, for the Y-N system, we can include both the tensor interaction and the coupling between the $N\Lambda$ and $N\Sigma$ channels. In the event that we have both $N\Lambda$-$N\Sigma$ coupling and a tensor interaction, the scattering amplitude $t_n(E)$ and $\tau^n(E)$ are 4×4 matrices. Although the above analysis is for a rank-one separable potential in which one of the particles can be in one of two states, the above procedure can be extended to higher rank potentials and the case where both particles can be in any one of a number of mass eigenstates. In the more general case the dimensionality of the matrices increases. If we have tensor coupling and both particles can be in one of two states, then we would be dealing with 8×8 matrices.

Because the problem of interest is the three-body YNN system, we must embed the above two-body amplitude in a three-body Hilbert space. Introducing the spectator particle notation, we can write the matrix element for the two-particle $(\alpha\beta)$ amplitude in the three-body Hilbert space, $T_\gamma(E)$, in terms of the two-particle amplitude in the two-body Hilbert space as

$$\langle \mathbf{q}_\gamma;\mathbf{p}_\gamma|T_\gamma(E)|\mathbf{p}'_\gamma\cdot\mathbf{q}'_\gamma\rangle=\delta(\mathbf{q}_\gamma-\mathbf{q}'_\gamma)\langle\mathbf{p}_\gamma|t_\gamma(E-\epsilon_\gamma)|\mathbf{p}'_\gamma\rangle \ , \tag{9}$$

where ϵ_γ is the energy of the spectator particle including its rest mass m_γ (see Appendix A). In Eq. (9) we have not included the spin and isospin quantum numbers; thus, the equation is an operator equation in the spin-isospin Hilbert space of the three-particle system.

B. The three-body equations

In this section, we would like to present a generalization of the Alt-Grassberger-Sandhas (AGS) equations which include the additional degrees of freedom that each particle can be in one or more mass eigenstate. In this way, we will be able to write the equation for the YNN ($Y=\Lambda,\Sigma$) system, where one of the particles can be either a Λ or Σ. The Alt-Grassberger-Sandhas (AGS) equations[32] for the three-body system are given by

$$U_{\alpha\beta}(E)=\bar{\delta}_{\alpha\beta}G_0^{-1}(E)+\sum_\gamma\bar{\delta}_{\alpha\gamma}T_\gamma(E)G_0(E)U_{\gamma\beta}(E) \ , \tag{10}$$

where $\bar{\delta}_{\alpha\beta}=1-\delta_{\alpha\beta}$, $T_\gamma(E)$ is the scattering amplitude for the $(\alpha\beta)$ pair of particles in the three-body Hilbert space, and $G_0(E)$ is the three-body Green's function

$$G_0(E)=(E-H_0)^{-1} \ . \tag{11}$$

Here, H_0 is the Hamiltonian for the three noninteracting particles, and because we include the possibility of the individual particle being excited, this free Hamiltonian is of the form

$$H_0=\sum_{\beta=1}^{3}\left[m_\beta+\frac{k_\beta^2}{2m_\beta}\right]=\sum_{\beta=1}^{3}m_\beta+\frac{p_\alpha^2}{2\mu_\alpha}+\frac{q_\alpha^2}{2\nu_\alpha} \ , \tag{12}$$

where

$$\mathbf{p}_\alpha=\frac{m_\gamma\mathbf{k}_\beta-m_\beta\mathbf{k}_\gamma}{m_\gamma+m_\beta}, \quad \mathbf{q}_\alpha=\frac{m_\alpha(\mathbf{k}_\beta+\mathbf{k}_\gamma)-(m_\beta+m_\gamma)\mathbf{k}_\alpha}{m_\alpha+m_\beta+m_\gamma} \tag{13}$$

and

$$\mu_\alpha=\frac{m_\beta m_\gamma}{m_\beta+m_\gamma}\equiv\mu_{\beta\gamma}, \quad \nu_\alpha=\frac{m_\alpha(m_\beta+m_\gamma)}{m_\alpha+m_\beta+m_\gamma} \ . \tag{14}$$

The second half of Eq. (12) is the free Hamiltonian in the three-body center of mass. Here, the masses of the particles are operators, and their values will depend on the state on which the Hamiltonian H_0 is acting, i.e., the eigenstates of H_0 are not only labeled by the momenta k_α, spin s_α, and isospin τ_α of the particles, but also by their masses or excitation energy. This implies that the eigenstates of H_0 are of the form

$$|(r_1\mathbf{k}_1)(r_2\mathbf{k}_2)(r_3\mathbf{k}_3)\rangle = |\phi^{r_1r_2r_3}_{\mathbf{p}_\alpha\mathbf{q}_\alpha}\rangle \;, \tag{15}$$

where $r_\alpha = \{s_\alpha, \tau_\alpha, m_\alpha\}$ labels the internal quantum numbers of particle α.

To reduce the AGS equations to a set of coupled one-dimensional integral equations, we introduce separable two-body amplitudes for both the NN and YN interactions. But first, we need to write the two-body amplitudes in the three-body Hilbert space in terms of the corresponding amplitudes in the two-body space. In this case, the analogous result to that in Eq. (9) is given by

$$\langle \phi^{r_1r_2r_3}_{\mathbf{q}_\alpha\mathbf{p}_\alpha} | T_\alpha(E) | \phi^{r_1'r_2'r_3'}_{\mathbf{p}_\alpha'\mathbf{q}_\alpha'}\rangle = \delta(\mathbf{q}_\alpha - \mathbf{q}_\alpha')\delta_{r_\alpha' r_\alpha} \langle \mathbf{p}_\alpha; r_\beta r_\gamma | t_\alpha(E-\epsilon_\alpha) | r_\beta' r_\gamma'; \mathbf{p}_\alpha'\rangle \;. \tag{16}$$

In operator form, we can write the above expression for $T_\alpha(E)$ as

$$T_\alpha(E) = \sum_{r_\alpha} \int d\mathbf{q}_\alpha |r_\alpha \mathbf{q}_\alpha\rangle t_\alpha[E-\epsilon_\alpha(q_\alpha)]\langle \mathbf{q}_\alpha r_\alpha| \;, \tag{17}$$

where $\epsilon_\alpha(q_\alpha)$ is the spectator particle kinetic energy including rest mass (see Appendix A). We now introduce the following angular momentum and isospin coupling scheme

$$\mathbf{S}_\alpha = \mathbf{s}_\beta + \mathbf{s}_\gamma, \quad \mathbf{j}_\alpha = \mathbf{l}_\alpha + \mathbf{S}_\alpha, \quad \mathbf{t}_\alpha = \boldsymbol{\tau}_\beta + \boldsymbol{\tau}_\gamma \;,$$
$$\boldsymbol{\mathcal{S}}_\alpha = \mathbf{j}_\alpha + \mathbf{s}_\alpha, \quad \mathbf{J} = \boldsymbol{\mathcal{L}}_\alpha + \mathbf{S}_\alpha, \quad \mathbf{T} = \mathbf{t}_\alpha + \boldsymbol{\tau}_\alpha \;, \tag{18}$$

where $\mathcal{S}_\alpha$ is the channel spin and $\mathcal{L}_\alpha$ is the orbital angular momentum of the spectator particle α relative to the pair $(\beta\gamma)$. Making use of the completeness of the eigenstates of the total angular momentum and isospin of the three-particle system in the above coupling scheme, we can write

$$T_\alpha(E) = \sum_{\substack{J M_J \\ T M_T}} \sum_{\substack{K_\alpha r_\alpha \\ \ell_\alpha \ell_\alpha'}} \int_0^\infty dq_\alpha q_\alpha^2 |q_\alpha r_\alpha \ell_\alpha K_\alpha J M_J T M_T\rangle t^{n_\alpha}_{\ell_\alpha \ell_\alpha'}[E-\epsilon_\alpha(q_\alpha)]\langle M_T T M_J J K_\alpha \ell_\alpha' r_\alpha q_\alpha| \;, \tag{19}$$

where $K_\alpha = \{n_\alpha, \mathcal{S}_\alpha, \mathcal{L}_\alpha\}$. Here, as in the two-body case, one writes $\ell_\alpha = \{r_\beta, r_\gamma, l_\alpha\}$ when the particles can be in more than one mass eigenstate with different spin and isospin. If we now assume a separable approximation for the two-body amplitude as in Eq. (5), we can write the two-body amplitude in the three-body Hilbert space as

$$T_\alpha(E) = \sum_{\substack{J M_J \\ T M_T}} \sum_{K_\alpha r_\alpha} \int_0^\infty dq_\alpha q_\alpha^2 |g_n; q_\alpha r_\alpha K_\alpha J M_J T M_T\rangle \tau^{n_\alpha}[E-\epsilon_\alpha(q_\alpha)]\langle M_T T M_J J K_\alpha r_\alpha q_\alpha; g_{n_\alpha}| \;, \tag{20}$$

where, as in the two-body case,

$$|g_{n_\alpha}; q_\alpha r_\alpha K_\alpha J M_J T M_T\rangle \equiv |g_{n_\alpha}; q_\alpha Q_\alpha J M_J T M_T\rangle \tag{21}$$

is a diagonal matrix. With this result at hand, we can write the AGS equations as a set of coupled one-dimensional integral equations. To achieve this, we first multiply the AGS equations [Eq. (10)] by the Green's function from both the right and the left. We then take matrix elements of the resultant equations using the states given in Eq. (21). Making use of the fact that our amplitudes are diagonal in total angular momentum and isospin, we obtain a set of coupled equations for the partial wave amplitude which are of the form

$$X^{JT}_{Q_\alpha;Q_\beta}(q_\alpha,q_\beta;E^+) = Z^{JT}_{Q_\alpha;Q_\beta}(q_\alpha,q_\beta;E^+) + \sum_{Q_\gamma Q_\delta} \int_0^\infty dq_\gamma q_\gamma^2 Z^{JT}_{Q_\alpha;Q_\gamma}(q_\alpha,q_\gamma;E^+)\tau^{n_\gamma}_{Q_\gamma Q_\delta}[E-\epsilon_\gamma(q_\gamma)]X^{JT}_{Q_\gamma;Q_\beta}(q_\delta,q_\beta;E^+) \;, \tag{22}$$

where $Q_\alpha = \{r_\alpha, K_\alpha\} = \{r_\alpha, \mathcal{S}_\alpha, j_\alpha, t_\alpha, S_\alpha, \mathcal{L}_\alpha\}$ are the quantum numbers that label the different three-body channels for a given total angular momentum J and isospin T. The Born term is given as the matrix element of the free three-particle Green's function, i.e.,

$$Z^{JT}_{Q_\alpha;Q_\beta}(q_\alpha,q_\beta;E^+) = \bar{\delta}_{\alpha\beta}\langle TJQ_\alpha q_\alpha; g_{n_\alpha}|G_0(E^+)|g_{n_\beta}; q_\beta Q_\beta JT\rangle \;. \tag{23}$$

In Appendix B, we present an explicit expression for this Born term in terms of the form factors of the separable potential. At this stage we would only like to point out that, because it corresponds to the exchange of particle γ, we expect $Z^{JT}_{Q_\alpha,Q_\beta}$ to be diagonal in r_α ($\alpha=1,2,3$), i.e., the mass, spin, and isospin of the particles do not change. This means that if we list our three-body channels so that the label corresponding to the internal degrees of freedom changes most slowly, then the matrix $Z^{JT}_{Q_\alpha,Q_\beta}$ is block diagonal. Each block along the block diagonal matrix corresponds to one choice for the masses of the three particles. On the other hand, the matrix τ^{n_α} couples the two-body channels that have different mass eigenstates, and in this way it will couple the different block diagonal matrices in $Z^{JT}_{Q_\alpha,Q_\beta}$. For example, if we turn off the coupling between the ΛN and ΣN channels, then certain nondiagonal elements of the matrix τ^{n_α} are set equal to zero, and the three-body problem for the YNN system decouples into two three-body problems, one for the ΛNN system and the other for the ΣNN system. We will illustrate this result later when we consider the antisymmetrized equations.

C. Antisymmetry in the hypertriton

Let us consider the reaction in which a hyperon Y is incident upon the deuteron. Since the system has two identical nucleons, antisymmetrizing our amplitude with respect to the two nucleons reduces the number of equations we need to solve. The possible reactions we have are

$$Y+d \to Y+d$$
$$\to N+(NY) .$$

Taking particles 1 and 2 to be the nucleons and particle 3 to be the hyperon, we can write the AGS equations for the above reaction, assuming separable potentials, as

$$X_{3,3}=Z_{3,1}\tau_1 X_{1,3}+Z_{3,2}\tau_2 X_{2,3} , \tag{24a}$$

$$X_{1,3}=Z_{1,3}+Z_{1,2}\tau_2 X_{2,3}+Z_{1,3}\tau_3 X_{3,3} , \tag{24b}$$

$$X_{2,3}=Z_{2,3}+Z_{2,1}\tau_1 X_{1,3}+Z_{2,3}\tau_3 X_{3,3} . \tag{24c}$$

In writing the above equations, we have concentrated on the particle labels at the expense of the channel labels we discussed in Sec. II B. This is motivated by the fact that we want to write a set of equations for the antisymmetrized amplitudes. In particular, we would like to write the input Born amplitudes $Z_{\alpha,\beta}$ in terms of the cyclic Born amplitudes $Z^c_{\alpha,\beta}$, which are defined as

$$Z^c_{\alpha,\beta}=\langle(\beta\gamma)\alpha|G_0(E)|(\gamma\alpha)\beta\rangle , \tag{25}$$

where γ is the exchanged particle.

For the reactions under consideration, we can express the two asymptotic states in terms of the following antisymmetric states:

$$|d,Y\rangle=|(NN)Y\rangle_{AS}=|(12)3\rangle \tag{26a}$$

for the $Y+d$ channel; while for the $N+(NY)$ channel we have

$$|(NY)N\rangle_{AS}=1/\sqrt{2}\{|(23)1\rangle-|(13)2\rangle\} . \tag{26b}$$

To carry out the antisymmetrization we make use of the symmetry of the Born amplitudes under permutation. In particular, we use the fact that

$$\begin{aligned} Z_{3,1}&=\langle(12)3|G_0(E)|(23)1\rangle \\ &=\langle(21)3|G_0(E)|(13)2\rangle \\ &=-\langle(12)3|G_0(E)|(13)2\rangle=-Z_{3,2} . \end{aligned} \tag{27a}$$

Similarly, we have

$$Z_{1,3}=(Z_{3,1})^\dagger \tag{27b}$$

and

$$\begin{aligned} Z_{1,2}&=\langle(23)1|G_0(E)|(13)2\rangle \\ &=\langle(13)2|G_0(E)|(23)1\rangle=Z_{2,1} . \end{aligned} \tag{27c}$$

We now make use of the above results to write the antisymmetrized single-nucleon-exchange amplitude as

$$\begin{aligned} Z^{AS}_{Y,N}&={}_{AS}\langle(NN)Y|G_0(E)|(NY)N\rangle_{AS} \\ &=1/\sqrt{2}\{Z_{3,1}-Z_{3,2}\} \\ &=\sqrt{2}Z^c_{3,1} , \end{aligned} \tag{28}$$

where in the last line we have used the fact that $Z_{3,2}=-Z_{3,1}$. Similarly, the antisymmetrized hyperon-exchange amplitude is given by

$$\begin{aligned} Z^{AS}_{N,N}&={}_{AS}\langle(NY)N|G_0(E)|(NY)N\rangle_{AS} \\ &=-\tfrac{1}{2}\{Z_{2,1}+Z_{1,2}\}=-Z_{1,2} . \end{aligned} \tag{29}$$

Using the above results for the antisymmetrized one-particle-exchange amplitude, we can write Eq. (24a) as

$$\begin{aligned} X_{3,3}&=Z_{3,1}\{\tau_1 X_{1,3}-\tau_2 X_{2,3}\} \\ &=(1/\sqrt{2})Z^{AS}_{Y,N}\tau^N\{X_{1,3}-X_{2,3}\} , \end{aligned} \tag{30}$$

where we have taken $\tau_1=\tau_2\equiv\tau^N$. We now can define the antisymmetrized amplitudes for the two reactions under consideration as

$$X^{AS}_{Y,Y}\equiv X_{3,3} \tag{31a}$$

and

$$X^{AS}_{N,Y}\equiv 1/\sqrt{2}\{X_{1,3}-X_{2,3}\} . \tag{31b}$$

This allows us to write Eq. (30) as

$$X^{AS}_{Y,Y}=Z^{AS}_{Y,N}\tau^N X^{AS}_{N,Y} . \tag{32}$$

Using the above definition of the antisymmetrized amplitudes, we can combine Eqs. (24b) and (24c) to get

$$\begin{aligned} X^{AS}_{N,Y}&=1/\sqrt{2}(Z_{1,3}-Z_{2,3})+1/\sqrt{2}(Z_{1,2}\tau_2 X_{2,3}-Z_{2,1}\tau_1 X_{1,3})+1/\sqrt{2}(Z_{1,3}\tau_3 X_{3,3}-Z_{2,3}\tau_3 X_{3,3}) \\ &=Z^{AS}_{N,Y}+Z^{AS}_{N,N}\tau^N X^{AS}_{N,Y}+Z^{AS}_{N,Y}\tau^Y X^{AS}_{Y,Y} , \end{aligned} \tag{33}$$

where $\tau^Y=\tau_3$. In writing the last line in Eq. (33), we have made use of the symmetry of the one-particle-exchange amplitudes, and the definition of the antisymmetrized amplitudes. We now can combine Eqs. (32) and (33) into a single matrix equation of the form

$$\begin{pmatrix} X_{Y,Y} \\ X_{N,Y} \end{pmatrix}=\begin{pmatrix} 0 \\ Z_{N,Y} \end{pmatrix}+\begin{pmatrix} 0 & Z_{Y,N} \\ Z_{N,Y} & Z_{N,N} \end{pmatrix}\begin{pmatrix} \tau^Y & 0 \\ 0 & \tau^N \end{pmatrix}\begin{pmatrix} X_{Y,Y} \\ X_{N,Y} \end{pmatrix} . \tag{34}$$

In writing the above equation, we have dropped the superscript that refers to the antisymmetry on the grounds that the amplitudes are now labeled by the physical channels and not the particle number. The input to these equations are the antisymmetrized one-particle-exchange amplitudes, and these are of two kinds: the nucleon-exchange amplitude which is given by

$$Z_{Y,N}=\sqrt{2}Z_{3,1}=\sqrt{2}\langle(12)3|G_0(E)|(23)1\rangle \tag{35a}$$

and

$$Z_{N,Y}=Z_{Y,N}\ . \tag{35b}$$

The other input amplitude corresponds to the exchange of a hyperon, and is given by

$$Z_{N,N}=-Z_{1,2}=-(-1)^R\langle(23)1|G_0(E)|(31)2\rangle\ , \tag{35c}$$

where the phase R is the result of permuting particles 1 and 3 in the ket, and is given by $R=s_1+s_3-S_2+\tau_1+\tau_3-t_2+l_2$.

D. The role of the effective three-body force

In Eq. (34) we have a set of two coupled equations for the YNN system. If we take the hyperon Y to be in one of two states, i.e., $Y=\Lambda,\Sigma$, and include the coupling between the ΛN and ΣN two-body channels, then the number of equations before partial wave expansion is four. These four equations can be written in matrix form as

$$\begin{bmatrix}X_\Lambda\\X_\Sigma\end{bmatrix}=\begin{bmatrix}Z_\Lambda\\0\end{bmatrix}+\begin{bmatrix}Z_\Lambda & 0\\0 & Z_\Sigma\end{bmatrix}\begin{bmatrix}\tau_{\Lambda\Lambda} & \tau_{\Lambda\Sigma}\\\tau_{\Sigma\Lambda} & \tau_{\Sigma\Sigma}\end{bmatrix}\begin{bmatrix}X_\Lambda\\X_\Sigma\end{bmatrix}, \tag{36a}$$

where

$$Z_Y=\begin{bmatrix}0 & Z_{Y,N}\\Z_{N,Y} & Z_{N,N}\end{bmatrix}\quad\text{for } Y=\Lambda,\Sigma\ , \tag{36b}$$

and

$$\tau_{YY}=\begin{bmatrix}\tau_{YY}^{Y} & 0\\0 & \tau_{YY}^{N}\end{bmatrix}\quad\text{for } Y=\Lambda,\Sigma,\quad \tau_{\Lambda\Sigma}=\begin{bmatrix}0 & 0\\0 & \tau_{\Lambda\Sigma}^{N}\end{bmatrix}. \tag{36c}$$

We now observe that for $\tau_{\Lambda\Sigma}^N=0$, there is no coupling between $N\Lambda$ and $N\Sigma$ channels. After partial wave expansion, this basic matrix structure is preserved, but the size of the matrix is determined by the number of three-body channels for a given (J,T).

We now turn to the question of an effective three-body force in the ΛNN space. In the presence of the ΛN-ΣN coupling, the two-body YN amplitude $T_i(E)$ $(i=1,2)$, is a solution of a coupled channel problem. There are two classes of diagrams that result from this coupling. These are illustrated in Figs. 1 and 2, where the shaded ellipse is the two-body potential. In Fig. 2 we have diagrams that generate the two-body ΛN amplitude including the coupling to the ΣN channel. On the other hand, the diagram in Fig. 1, may be considered as a three-body force in the ΛNN space. If we iterate the AGS equations, we get terms of the form

$$\langle\cdots|G_0T_1(E)G_0T_2(E)G_0|\cdots\rangle\ . \tag{37}$$

Assuming the separability of the amplitudes, this contribution gives two kinds of terms described in the following.

(i) Those that are normally present in the ΛNN problem in the absence of coupling to the ΣN channel. These are of the form

$$Z_{\dots,N}\tau_{\Lambda\Lambda}^{N}Z_{N,N}\tau_{\Lambda\Lambda}^{N}Z_{N},\dots\ , \tag{38}$$

which are diagrammatically illustrated in Fig. 3 with $Y=\Lambda$. Here the open ellipses correspond to the YN amplitude.

(ii) The second class of diagrams are only included if the coupling between the ΛN and the ΣN channels, at the two-body level, is included. These are of the form

$$Z_{\dots,N}\tau_{\Lambda\Sigma}^{N}Z_{N,N}\tau_{\Sigma\Lambda}^{N}Z_{N},\dots\ . \tag{39}$$

This corresponds to the diagrams in Fig. 3 with $Y=\Sigma$, and can be considered as a three-body force contribution.

To determine the contribution due to this effective three-body force, we need to compare the results of the full calculation with the results when excluding the three-body force. To achieve the latter, we include the coupling between the $N\Lambda$ and $N\Sigma$ channel when calculating τ^N. However, when calculating the kernel of our integral equation, we take τ^N to be diagonal, i.e., $\tau_{\Lambda\Sigma}^N=0=\tau_{\Sigma\Lambda}^N$. Then the contribution from the diagram in

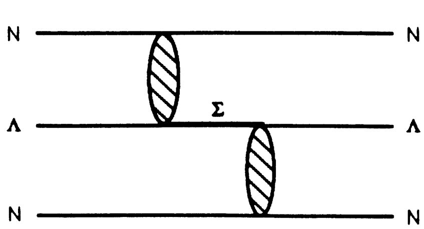

FIG. 1. Diagrammatic representation of an effective three-body force in the ΛNN space.

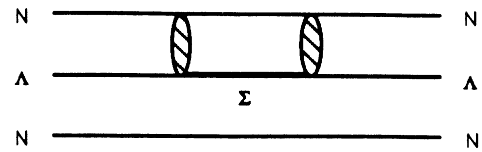

FIG. 2. Contribution to the YN amplitude due to coupling between the ΛN and ΣN channels.

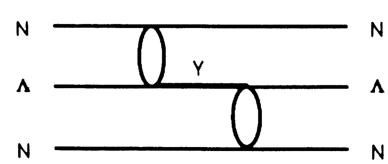

FIG. 3. Contribution to the three-body amplitude with intermediate state of a hyperon Y. For $Y=\Lambda$ this diagram is a standard contribution to the binding energy of ${}^3_\Lambda$H, while for $Y=\Sigma$ it corresponds to an effective three-body force contribution to the binding energy.

Fig. 3 with $Y=\Sigma$ is not included, but the coupling between the ΛN and ΣN is included in the calculation of the two-body amplitudes. In this approximation, the kernel of our integral equation is block diagonal, with one block corresponding to the ΛNN problem, while the second block corresponds to the ΣNN problem. In this way, we are able to determine the magnitude of this effective three-body force contribution to the binding energy of the hypertriton.

III. NUMERICAL RESULTS

In order to verify that our equations do indeed yield the correct ${}^3_\Lambda$H binding energy, we repeated the calculation of Dabrowski and Fedorynska.[26] The parameters of their YN potentials are listed in Table I. The more conventional Yamaguchi separable potential form[33,34] is

$$V_{ij}(\mathbf{p},\mathbf{p}')=\frac{-\lambda_{ij}}{2\mu_{ij}}g_i(\mathbf{p})g_j(\mathbf{p}')\ , \tag{40}$$

where the form factor for s waves is given by

$$g_i(\mathbf{p})=(p^2+\beta_i^2)^{-1}\ ; \tag{41}$$

the appropriate conversion from Dabrowksi and Fedorynska is

$$\lambda_{ij}=-\lambda_{ij}^{\rm DF}\frac{\beta_i^2\beta_j^2}{2\pi^2}\ . \tag{42}$$

The further conversion required to obtain our interaction strengths is

$$C_{ij}=-4\pi\lambda_{ij}/2\sqrt{\mu_{iN}\mu_{jN}}\ , \tag{43}$$

where the masses of the nucleon and hyperons are taken to be $m_N=939$ MeV, $m_\Lambda=1115$ MeV, and $m_\Sigma=1192$ MeV. Hypertriton binding energies for this model are quoted in Table II along with the corresponding Λ separation energies. (The NN spin-triplet parameters of Ref. 26 correspond to a deuteron binding energy of 2.42 MeV and scattering length of $a_t=5.38$ fm, and the spin-singlet low energy scattering parameters are $a_s=-23.7$ fm, $r_s=2.5$ fm.) The agreement between our Λ-separation energies (Table II) and those of Ref. 26 for the full model (row 1) and the one-channel equivalent model (row 3), in which one neglects the ΣN channel but uses one-channel ΛN potentials that have the same scattering lengths and effective ranges as the full two-channel model, is exact. We have also checked our equations in the one-channel approximation by reproducing the results of Gibson and Lehman[34] for ΛN model parameters of Herndon and Tang.[35] The results of the second row of Table II, in which the ΛN-ΣN coupling has been neglected ($\lambda_{XN}=0$) in the two-channel potential model, are included to illustrate the fact that ΛN-ΣN conversion enhances the attraction of the YN force in both the two-body and three-body problems. This is a consequence of the well-known effect in a classical coupled oscillator system: When the oscillators are coupled, they are "pushed" apart in frequency. In our case the ΛN interaction is more attractive after it is coupled to the ΣN system. The increased binding is also anticipated from the fact that, if one expands the Hilbert space, then a variational bound on the binding energy should increase in magnitude.

TABLE II. ${}^3_\Lambda$H binding energies (in MeV) for the Dabrowski and Fedorynska (Ref. 26) separable potential model parameters listed in Table I.

Model	$B({}^3_\Lambda\text{H})$	$B_\Lambda({}^3_\Lambda\text{H})$
Two channel	2.86	0.63
Two channel ($\lambda_{XN}=0$)	2.35	0.12
One channel	2.72	0.47

A. ΛN-ΣN coupling and ΛNN forces

In order to explore the questions raised in the Introduction about (1) the dispersive energy dependence of a coupled-channel potential embedded in a three-body problem and (2) the effective three-body force that results when the ΣN channel is formally eliminated, we have used the Stepien-Rudzka and Wycech separable potential model[28] that was designed to parametrize the Nijmegen[16] model F. (Schick and Toepfer[25] found a strong dependence of these effects upon the strength of the ΛN-ΣN coupling; thus, we use a more physical model.) The potential parameters are listed in Table III along with those

TABLE I. Separable potential parameters from Ref. 26 for the YN force. The numbers in parentheses are the strengths converted to the potential form defined in Eqs. (40) and (42). The β are in units of fm^{-1}, the $\lambda^{\rm DF}$ in fm, and the (λ) in fm^{-3}. The C_{ij} are the values of our strength parameters in units of fm^{-2}.

	$S=0$	$S=1$
$\lambda_{\Lambda N}$	−0.90 (0.1384)	−0.552 (0.1833)
λ_{XN}	0	±0.884 (∓0.2935)
$\lambda_{\Sigma N}$		−1.53 (0.5080)
$\beta_{\Lambda N}$	1.32	1.60
$\beta_{\Sigma N}$		1.60
$C_{\Lambda N}$	−0.336 78	−0.446 18
C_{XN}		±0.703 68
$C_{\Sigma N}$		−1.199 35

TABLE III. YN coupled-channel separable potential parameters for the model of Ref. 28. The units of β are fm^{-1} and of λ are fm. The equivalent single-channel strengths (fm^{-3}) and ranges (fm^{-1}) are those of Ref. 9 and correspond in sign to the definition in Eq. (40). Note that the strengths from Ref. 28 are related to those of Ref. 9 and the C_{ij} (fm^{-2}) of this paper by Eqs. (42) and (43).

	$S=1$		$S=0$	
$\lambda_{\Lambda N}$	−0.529 8	0.326 2	−0.725 1	0.095 2
λ_{XN}	+0.677 7		−1.097	
$\lambda_{\Sigma N}$	−0.987 1		+0.891 6	
β_Λ	1.60	1.725 1	1.18	1.201 1
β_Σ	2.00		1.44	
$C_{\Lambda N}$	−0.428 24	−0.792 03	−0.173 39	−0.232 21
C_{XN}	+0.842 89		−0.384 71	
$C_{\Sigma N}$	−1.889 13		+0.458 56	

of the equivalent one-channel potentials. The NN potential parameters for a central force $^3S_1$ model 0% and the Phillips 4% deuteron D-state $^3S_1$-$^3D_1$ tensor force model[36] are given in Table IV. The investigation was performed with both to ensure that our conclusions are unaffected by the presence of a noncentral NN force.

For the tensor force model, the form factors in Eq. (40) have the form

$$g(\mathbf{p})=g_c(p)+(s_{ij}(\hat{\mathbf{p}})/\sqrt{8})g_T(p) , \tag{44}$$

where

$$s_{ij}(\hat{\mathbf{p}})=3(\boldsymbol{\sigma}_i\cdot\hat{\mathbf{p}})(\boldsymbol{\sigma}_j\cdot\hat{\mathbf{p}})-(\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j) , \tag{45a}$$

$$g_c(p)=(p^2+\beta^2)^{-1} , \tag{45b}$$

$$g_T(p)=-\xi p^2(p^2+\beta_T^2)^{-2} . \tag{45c}$$

Because the separable potentials of Ref. 28 were parametrized to fit the low energy ΛN scattering parameters of the Nijmegen model, the signs of the coupling potentials were not uniquely determined. As was first observed by Dabrowski and Fedorynska,[26] we confirm that choosing the relative sign of the ΛN-ΣN coupling terms in the $^1S_0$ and $^3S_1$ channels to be opposite increases the $^3_\Lambda$H binding energy. In the case of the model of Ref. 28, we find the hypertriton to be unbound, if the signs of the ΛN-ΣN coupling potentials are assumed to be the same for $^1S_0$ and $^3S_1$. Thus, we list the parameters in Table III for the Stepien-Rudzka and Wycech model for a specific choice of signs for the coupling strengths.

Beginning with the full ΛN-ΣN coupled-channel model, we obtain a $^3_\Lambda$H binding energy of 2.46 (2.63) MeV using a $^3S_1$-$^3D_1$ ($^3S_1$) NN spin triplet force (see Table V). Note that the noncentral force effect in the NN channel is significant, when one is attempting to reproduce $B_\Lambda(^3_\Lambda\text{H})$ quantitatively. For the $^3S_1$-$^3D_1$ NN force we obtain $B_\Lambda(^3_\Lambda\text{H})=0.24$ MeV compared to 0.40 MeV for the $^3S_1$ NN force model. Replacing the ΛN-ΣN two-channel potentials by their equivalent one-channel potentials reduces the $^3_\Lambda$H binding energy from 2.46 to 2.29 MeV when the NN force is a $^3S_1$-$^3D_1$ model and from 2.63 to 2.37 MeV for the $^3S_1$ NN central-force model. Clearly, including explicit ΛN-ΣN coupling increases the binding energy of the three-body system. (Both potentials models have the same scattering lengths and effective ranges.) However, to understand the role of the energy dependence of the effective energy-dependent ΛN force

$$V^{\text{eff}}_{\Lambda N}=V_{\Lambda N}+|V_{XN}|^2/(E-H_{\Sigma N}) \tag{46}$$

when the ΣN channel is formally eliminated (see Fig. 2) as well as the role of true three-body forces, we have turned off the ΣNN diagram (Fig. 1) as described in Sec. II. The resulting $^3_\Lambda$H binding energy of 2.25 (2.31) MeV for the $^3S_1$-$^3D_1$ ($^3S_1$) NN force model is smaller than the one-channel, static ΛN potential approximation result of 2.29 (2.37) MeV. Thus, we verify that the dispersive energy dependence of the ΛN-ΣN interaction leads to a reduction in the $^3_\Lambda$H binding energy. (Again, both two-body potentials have the same scattering length and effective range, while the true three-body force terms have been removed from the $^3_\Lambda$H calculation in the case of the ΛN-ΣN coupled-channel model.) Restated, a static approximation to the energy-dependent effective interaction defined by Eq. (46) binds the hypertriton more than the effective interaction itself. (A similar result has been shown to hold for an NN-$N\Delta$ force model in the triton.[24])

At the same time, a comparison of the no ΣNN channel calculation with the full ΛN-ΣN coupled-channel calculation shows that the true three-body force effect is attractive and much larger than the dispersive energy-dependence effect. The binding is 2.46 (2.63) MeV for the full model compared to 2.25 (2.31) MeV for the effective, energy-dependent ΛN force model. Because both of these models overbind the hypertriton ($B_\Lambda=0.24$ and 0.40 MeV compared to the experimental 0.13±0.05 MeV), one is led to ask whether model calculations employing the best available meson theoretic YN potential models will show that the $^3_\Lambda$H system is bound only because there is a ΛNN three-body force. [A 7% D-state $^3S_1$-$^3D_1$ NN model yields $B_\Lambda(^3_\Lambda\text{H})\simeq 0.14$ MeV.]

Finally, we note that the theoretical three-body force that arises here in the hypertritron due to the formal elimination of the ΣN channel is attractive. This is to be contrasted with the phenomenological result in the model calculations of Bodmer *et al.*[37] for $A=4,5$ hypernuclei, where one finds a repulsive three-body force. This likely results from many-body effects combined with ΛN-ΣN coupling. For example, the excitation energies of the alpha core states in $^5_\Lambda$He (the even parity $T=1$, $S=0$ excited states lie more than 40 MeV up in the spectrum) are larger than the 2 MeV separation between the d^* system

TABLE IV. NN separable potential parameters from Refs. 10 and 36. The units of β are fm^{-1} and those of λ are fm^{-3}. The $C_{ll'}$ are our strength parameters (fm^{-2}): $C_{00}=-(4\pi/2\mu)\lambda$, $C_{02}=+(4\pi/2\mu)\lambda\xi$, and $C_{22}=-(4\pi/2\mu)\xi^2\lambda^2$.

	$S=1$		$S=0$
P_D (%)	0	4	
λ	0.381 5	0.243 1	0.149 3
β	1.405 6	1.313 4	1.164 8
ξ		1.689 4	
β_T		1.528 3	
$B(^2\text{H})$	2.225	2.225	
C_{00}	−1.007 4	−0.641 9	−0.394 29
C_{02}		+1.084 9	
C_{22}		−1.832 0	

TABLE V. $^3_\Lambda$H binding energies and Λ-separation energies (in MeV) for the separable potential model parameters listed in Table III.

	$(^3S_1$-$^3D_1)_{NN}$		$(^3S_1)_{NN}$	
	$B(^3_\Lambda\text{H})$	$B_\Lambda(^3_\Lambda\text{H})$	$B(^3_\Lambda\text{H})$	$B_\Lambda(^3_\Lambda\text{H})$
Full model	2.460	0.24	2.626	0.40
No ΣNN channel	2.253	0.028	2.305	0.079
One channel ΛN	2.292	0.067	2.368	0.14

TABLE VI. ΛN spin-triplet separable potential parameters for central and tensor forces having a scattering length of -2.06 fm and an effective range of 3.18 fm, along with parameters for the spin-singlet interaction. The $C_{ll'}$ are our strength parameters as defined in Table IV.

	$^3S_1$	$^3S_1$-$^3D_1$	$^1S_0$
λ (fm^{-3})	0.1581	0.1133	0.1086
β (fm^{-1})	1.385	1.362	1.266
ξ		7.700	
β_T (fm^{-1})		3.250	
C_{00} (fm^{-2})	-0.3846	-0.2756	-0.2642
C_{02} (fm^{-2})		$+2.1220$	
C_{22} (fm^{-2})		-16.3395	

and the deuteron and will strongly suppress ΛN-ΣN coupling effects. That is, ΛN-ΣN coupled-channel potentials can yield less binding in heavier systems (and nuclear matter) than their corresponding one-channel ΛN effective potentials. This is the essence of the $^4_\Lambda$He-$^4_\Lambda$H analysis of Ref. 9, where it is argued that suppression of ΛN-ΣN coupling due to such effects lowers the 0^+ and 1^+ state binding relative to simple ΛN effective interaction models. The implication of these conflicting three-body force results is that any phenomenological ΛNN force will depend strongly upon the spin-isospin quantum numbers of the nuclear core states.

B. Noncentral force effects

We have seen above that utilizing a $^3S_1$-$^3D_1$ NN spin-triplet potential model significantly reduces the hypertriton binding energy. What we wish to explore here is the comparative size, and the overall contribution of the tensor force in the NN and the ΛN interaction to the binding (Λ-separation) energy of the hypertriton. For that purpose, and in the absence of a separable tensor force potential for the ΛN-ΣN system, we have used the one-channel ΛN potential parameters listed in Table VI.

In Table VII, we present our results for the binding energy of the hypertriton for the four model calculations. Since the effect of the ΛN tensor force is expected to be small, we have ensured that the calculation of the binding energy is accurate to the number of significant figures reported in the tables. This required the use of 24 point Gauss quadratures in all our integrals. Using the central force ($^3S_1$) potential for both the NN and ΛN spin-triplet interaction, we obtain a binding (Λ-separation) energy of 2.300 MeV (0.074 MeV). Replacing the $^3S_1$ NN potential with a 4% D-state deuteron $^3S_1$-$^3D_1$ tensor force potential,[36] we find the binding (Λ-separation) energy to be 2.252 MeV (0.027 MeV). In contrast, if we replace the $^3S_1$ ΛN potential with a $^3S_1$-$^3D_1$ model[9] fit to the same scattering length and effective range, we find a binding (Λ-separation) energy of 2.296 MeV (0.070 MeV). That is, the reduction in the $^3_\Lambda$H binding energy due to the tensor force nature of the ΛN interaction is an order of magnitude smaller than that due to the tensor force nature of the NN interaction. On the basis that the contributions of the tensor forces are, in general, small and can be treated in perturbation theory, we would expect the final binding (Λ-separation) energy, when the tensor force is included in both the NN and the ΛN interactions, to be $\approx$2.248 MeV ($\approx$0.023 MeV). However, when including the tensor force in both interactions, we find the binding (Λ-separation) energy to be 2.260 MeV (0.035 MeV). In other words, the inclusion of the ΛN tensor force, when the NN tensor force is already present, produces additional attraction rather than enhancing the repulsion. This nonperturbative effect is similar to the inclusion of the coupling to the ΣN channel, i.e., the three-body force, discussed in Sec. II. In fact, a careful examination of the origin of this additional attraction reveals that on inclusion of the tensor force in both the NN and ΛN interaction, we get additional coupling between the $(NN)\Lambda$ and $(\Lambda N)N$ three-body channels in the Z matrix. This, in effect, leads to diagrams similar to that in Fig. 1, where now we have the $(NN)\Lambda$ channel with the NN tensor force leading to an NN relative angular momentum $l=2$. The new elements of the Z matrix, resulting from including the tensor interaction in both the NN and ΛN, couple the $(NN)\Lambda$ channel to the $(\Lambda N)N$ channel with the ΛN in relative orbital angular momentum $l=2$. Hence the contribution of the NN tensor force can be substantial even though its contribution in perturbation theory is small.

Another effect that arises from including the tensor force in both the NN and ΛN interactions is the sensitivity of the hypertriton binding energy to the sign of the tensor interaction C_{02}. This again is illustrated in Table VII, where we give the binding (Λ-separation) energy when the sign of C_{02} is changed. Here we observe that this change in binding energy is only present if the tensor interaction is included in both the NN and ΛN potential. In the last row in Table VII we have the result for the binding (Λ-separation) energy when we change C_{02} for

TABLE VII. $^3_\Lambda$H binding energy $B(^3_\Lambda\text{H})$, and Λ-separation energy $B_\Lambda(^3_\Lambda\text{H})$, in MeV for various combinations of central and tensor NN and ΛN potentials. Included in the table are the result of reversing the sign of the tensor interaction C_{02}. For the case when the tensor force is included in both interactions, we have reversed the sign of the tensor force in the ΛN potential.

NN		ΛN		$B(^3_\Lambda\text{H})$	$B_\Lambda(^3_\Lambda\text{H})$	$B(^3_\Lambda\text{H})$	$B_\Lambda(^3_\Lambda\text{H})$
$^3S_1$	$^3S_1$-$^3D_1$	$^3S_1$	$^3S_1$-$^3D_1$	$C_{02}>0$		$C_{02}<0$	
X		X		2.300	0.074	2.300	0.074
	X	X		2.252	0.027	2.252	0.027
X			X	2.296	0.070	2.295	0.071
	X		X	2.260	0.035	2.329	0.103

TABLE VIII. ${}^{3}_{\Lambda}$H binding and Λ-separation energies in MeV for the combination of central and tensor NN and ΛN potentials indicated. The ΛN spin-singlet potential was given the unphysically large strength of 0.1581 fm^{-3} in this case.

NN		ΛN		$B({}^{3}_{\Lambda}\mathrm{H})$	$B_{\Lambda}({}^{3}_{\Lambda}\mathrm{H})$
${}^{3}S_1$	${}^{3}S_1$-${}^{3}D_1$	${}^{3}S_1$	${}^{3}S_1$-${}^{3}D_1$	(MeV)	(MeV)
X		X		3.925	1.699
	X	X		3.540	1.315
X			X	3.905	1.679
	X		X	3.536	1.311

the ΛN potential. If C_{02} in both the NN and ΛN are changed, the results for the binding (Λ-separation) energy are 2.254 MeV (0.029 MeV). On the other hand, if we change the sign of C_{02} for the NN interaction only, we find that the binding (Λ-separation) energy is 2.344 MeV (0.119 MeV). Thus the hypertriton could be used to check the relative sign of the tensor force in the NN and ΛN interactions, the sign of the ${}^{3}S_1$-${}^{3}D_1$ mixing parameter ϵ_1. The magnitude of the ΛN contribution is determined by the relative sign of the tensor interactions in the NN and ΛN potentials. Existing models imply that the relative signs are the same; that is $\epsilon_1(\Lambda N)/\epsilon_1(NN)>0$ for low energy scattering.

Because the Λ-separation energies in this model study are so small [$B({}^{2}\mathrm{H})\approx 2.226$ MeV and 2.225 MeV for the ${}^{3}S_1$ and the ${}^{3}S_1$-${}^{3}D_1$ models, respectively], we performed a pseudomodel study in which the strength of the ΛN spin-singlet potential was arbitrarily set to the spin-triplet value of 0.1581. That is, the NN parameters are unchanged as are the spin-triplet ΛN parameters, but the model hypertriton becomes strongly bound in terms of the Λ-separation energy, $B({}^{3}_{\Lambda}\mathrm{H})$-$B({}^{2}\mathrm{H})$. Our results are presented in Table VIII and confirm the general features discussed above for the more realistic force model. The tensor force effect due to the NN interaction is again an order of magnitude larger than that due to the ΛN interaction. However, in this case the effect of the ΛN tensor interaction (reducing the binding) is almost completely cancelled by the binding enhancement due to the extra coupling between the $(NN)\Lambda$ and $(\Lambda N)N$ channels.

Based on these two model studies, we conclude that the effect of the ΛN tensor force depends on the inclusion of the tensor force in both the NN and ΛN potential and the relative sign of the tensor force in the two interactions. Furthermore, including tensor coupling in the ΛN-ΣN force is needed before a detailed picture is obtained.

C. ΣN bound state in the ΛN continuum

The $K^{-}d\to\Lambda N\pi$ reaction shows a "cusp" structure as one crosses the ΣN threshold just as one would expect.[38–40] The presence or absence of a ΣN bound state [i.e., whether the $V_{\Sigma N}$ component of the YN interaction is sufficiently attractive to support a bound state in the absence of ΛN-ΣN coupling ($V_{XN}=0$)], has been shown by Toker, Gal, and Eisenberg[41] to affect the shape of the structure of the cross section as one crosses the ΣN threshold. This phenomena has also been explored in detail by Dalitz and co-workers.[42] We have used the potential models A and B of Ref. 41 in model calculations of the ${}^{3}_{\Lambda}$H binding energy to investigate the effect of a ΣN bound state on that observable.

In our model study we have used the ${}^{3}S_1$-${}^{3}D_1$ and ${}^{1}S_0$ NN potential parameters of Table IV along with the ΛN ${}^{1}S_0$ parameter of Table III. Toker, Gal, and Eisenberg needed only to model the ΛN ${}^{3}S_1$ interaction for their $K^{-}d\to\Lambda N\pi$ study. We obtained a ${}^{3}_{\Lambda}$H binding energy of 2.41 MeV for model A (no ΣN bound state) and a binding energy of 2.84 MeV for model B (ΣN bound state in the continuum). Thus, we conclude that, for the Toker-Gal-Eisenberg parametrizations of the ΛN-ΣN ${}^{3}S_1$ interaction, the existence of a ΣN bound state in the continuum would severely overbind the hypertriton.

IV. CONCLUSIONS

We have formulated the separable potential Faddeev equations that describe ${}^{3}_{\Lambda}$H allowing for tensor forces and ΛN-ΣN coupling. Our model investigation has demonstrated the following. (1) The dispersive energy dependence that results from embedding the coupled-channel ΛN-ΣN two-body potential in a three-body system is repulsive and reduces the ${}^{3}_{\Lambda}$H binding energy. (2) The true three-body force due to coupling ΣNN states to the ΛNN state is attractive and increases the ${}^{3}_{\Lambda}$H binding energy. (3) The inclusion of tensor coupling in both the NN and ΛN channels produces an effect similar to that arising from the coupling between ΛN and ΣN in that it leads to a dispersive contribution and a "three-body force" contribution. As a result the $\Lambda N({}^{3}S_1$-${}^{3}D_1)$ contribution to the hypertriton binding energy depends sensitively on the presence of the NN tensor force and the relative sign of the tensor coupling in the NN and ΛN interactions. That is, $B({}^{3}_{\Lambda}\mathrm{H})$ depends upon the relative sign of the ${}^{3}S_1$-${}^{3}D_1$ mixing parameter ϵ_1 for the NN and ΛN interactions. Clearly, a complete analysis of the tensor force effect will require the inclusion of tensor coupling in the coupled ΛN-ΣN interaction channel, which is under investigation. Finally, the existence of a ΣN bound state in the continuum would appear to lead to severe overbinding of the hypertriton, and is therefore not supported by the existing data.

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APPENDIX A: DEFINITION OF SPECTATOR ENERGY

To some readers it may not be apparent what the definition of the spectator particle's energy in Eq. (9) must be. Because of the mass difference between the Λ and Σ hyperon, we have two different two-body unitarity cuts, both of which must be included in the two-body NY amplitudes. This can be most simply achieved by employing relativistic kinematics where the mass of the particle is part of the kinetic energy. On the other hand, the binding energy of the hypertriton is basically a nonrelativistic problem, and the potential used for the description of the N-Y interaction is a nonrelativistic potential. Thus to maintain the thresholds for $N\Lambda$ and $N\Sigma$ scattering within a nonrelativistic formulation, we must include the mass of the particle with its kinetic energy [see Eq. (12)]. The three-body Green's function for the NNY system is then given by

$$G_0(E)=\left[E-2m_N-m_Y-\frac{k_1^2}{2m_N}-\frac{k_2^2}{2m_N}-\frac{k_3^2}{2m_Y}\right]^{-1},$$

where particles 1 and 2 are the nucleons and particle 3 is the hyperon. For the case when nucleon 2 is the spectator, this Green's function can be written as

$$G_0(E)=\left[E-\left[m_N+\frac{k_2^2}{2m_N}\right]-\frac{k_1^2}{2m_N}-\frac{k_3^2}{2m_Y}-m_N-m_Y\right]^{-1} \quad \text{(A1)}$$

which can be considered as the two-body Green's function for particle 1 and 3 at the shifted energy of

$$E-(m_N+k_2^2/2m_N)\equiv E-\epsilon_2(k_2)\ ,$$

i.e., the total energy minus the kinetic energy of the spectator. In the three-body center of mass this Green's function takes the form

$$G_0(E)=\left[E-\epsilon_2(q_2)-\frac{q_2^2}{2(m_N+m_Y)}-\frac{p_2^2}{2\mu_2}-m_N-m_Y\right]^{-1}. \quad \text{(A2)}$$

This differs from the two-body free Green's function commonly encountered, by the fact that (i) the energy is shifted by the spectator particle's energy, i.e., $E\rightarrow E-\epsilon_2(q_2)$; (ii) the two-body Green's function is at a center-of-mass momentum $\mathbf{q}_2$, and to that extent it includes the energy of the pair's center-of-mass motion; (iii) the kinetic energy of the nucleon spectator does not in any way depend on the hyperon mass. These conditions are exactly what we expected from a relativistic theory where the Green's function is given by

$$G_0(E)=\left[E-(k_1^2+m_N^2)^{1/2}-(k_2^2+m_N^2)^{1/2}-(k_3^2+m_Y^2)^{1/2}\right]^{-1}. \quad \text{(A3)}$$

With these definitions for the three-particle Green's function we can write the N-Y amplitude in the three-body Hilbert space

$$\langle k_1k_2k_3|t_{13}(E)|k_1'k_2'k_3'\rangle=\delta(\mathbf{q}_2-\mathbf{q}_2')\langle \mathbf{p}_2|t_{13}[E-\epsilon_2(q_2)]|\mathbf{p}_2'\rangle\ , \quad \text{(A4)}$$

where $\epsilon_2(q_2)=m_N+q_2^2/2m_N$ in the three-body center of mass. For the relativistic case we would have instead $\epsilon_2(q_2)=(q_2^2+m_N^2)^{1/2}$.

APPENDIX B: THE KERNEL OF THE FADDEEV EQUATION

In this appendix, we present an explicit expression for the one-particle-exchange amplitude. In deriving this result, we have assumed cyclic labeling. In this way we can use these amplitudes in Eqs. (35a) and (35c). Because the one-particle-exchange amplitudes do not change the spin, isospin, and mass eigenstates of the three particles, we take these quantities to be fixed in our expressions. For a more detailed derivation of the results presented in this appendix, the reader is referred to the work of Stingl and Rinat,[43] or Afnan and Thomas.[44] We have for the one-particle-exchange amplitude, after partial wave expansion, that

$$Z^{JT}_{Q_\alpha,Q_\beta}(q_\alpha,q_\beta;E)=q_\alpha^{l_\beta}q_\beta^{l_\alpha}\sum_L\left[\sum_{a=0}^{l_\alpha}\sum_{b=0}^{l_\beta}A^{L,a,b}_{Q_\alpha,Q_\beta}\left[\frac{q_\alpha}{q_\beta}\right]^{a-b}p_\alpha^a p_\beta^b\right]\Gamma^L_{n_\alpha n_\beta}(q_\alpha,q_\beta;E)\ , \quad \text{(B1)}$$

where Q_α labels the three-body channels and includes the orbital angular momentum of the interacting pair, while n_α labels the two-body channels. The function $\Gamma^L_{n_\alpha,n_\beta}$ is given by

$$\Gamma^L_{n_\alpha,n_\beta}(q_\alpha,q_\beta;E)=\frac{1}{2}\int_{-1}^{+1}dx\frac{p_\alpha^{-l_\alpha}g_{n_\alpha}(p_\alpha)g_{n_\beta}(p_\beta)p_\beta^{-l_\beta}}{E-2m_N-m_Y-q_\alpha^2/2m_\alpha-q_\beta^2/2m_\beta-(\mathbf{q}_\alpha+\mathbf{q}_\beta)^2/2m_\gamma}P_L(x)\ . \quad \text{(B2)}$$

Here, we have

$$x=\hat{\mathbf{q}}_\alpha\cdot\hat{\mathbf{q}}_\beta \ , \tag{B3}$$

$$\mathbf{p}_\alpha=-\mathbf{q}_\beta-\rho_\alpha\mathbf{q}_\alpha, \quad \mathbf{p}_\beta=\mathbf{q}_\alpha+\rho_\beta\mathbf{q}_\beta \ , \tag{B4}$$

$$\rho_\alpha=\frac{m_\beta}{m_\beta+m_\gamma}, \quad \rho_\beta=\frac{m_\alpha}{m_\alpha+m_\gamma} \ . \tag{B5}$$

The coefficients $A^{L,a,b}_{Q_\alpha,Q_\beta}$ in Eq. (B1) are given in terms of $3J$, $6J$, $9J$, and $12J$ symbols as

$$\begin{aligned} A^{L,a,b}_{Q_\alpha,Q_\beta} = {} & (-)^R \hat{t}_\alpha\hat{t}_\beta\hat{l}_\alpha\hat{l}_\beta\hat{S}_\alpha\hat{S}_\beta\hat{j}_\alpha\hat{j}_\beta\hat{\mathcal{S}}_\alpha\hat{\mathcal{S}}_\beta\hat{\mathcal{L}}_\alpha\hat{\mathcal{L}}_\beta\hat{L}\,\hat{L} \begin{Bmatrix} \tau_\beta & \tau_\gamma & t_\alpha \\ \tau_\alpha & T & t_\beta \end{Bmatrix} \left[\frac{(2l_\alpha+l)!(2l_\beta+l)!}{(2a)!(2l_\beta-2b)!(2l_\alpha-2a)!(2b)!}\right]^{1/2} \\ & \times \sum_f \sum_{\Lambda\Lambda'} (\hat{f}\,\hat{\Lambda}\,\hat{\Lambda}')^2 \begin{Bmatrix} \mathcal{S}_\alpha & \mathcal{S}_\beta & f \\ \mathcal{L}_\beta & \mathcal{L}_\alpha & J \end{Bmatrix} \begin{Bmatrix} s_\alpha & \mathcal{S}_\alpha & \mathcal{S}_\beta & s_\beta \\ j_\alpha & f & j_\beta & s_\gamma \\ S_\alpha & l_\alpha & l_\beta & S_\beta \end{Bmatrix} \\ & \times \begin{bmatrix} \Lambda' & L & \mathcal{L}_\beta \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Lambda & L & \mathcal{L}_\alpha \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} l_\alpha-a & b & \Lambda' \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} a & l_\beta-b & \Lambda \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \mathcal{L}_\alpha & \mathcal{L}_\beta & f \\ \Lambda' & \Lambda & L \end{Bmatrix} \begin{Bmatrix} l_\alpha & l_\beta & f \\ a & l_\beta-b & \Lambda \\ l_\alpha-a & b & \Lambda' \end{Bmatrix} , \end{aligned} \tag{B6}$$

where $\hat{a}\equiv(2a+1)^{1/2}$ and the phase R is given by

$$\begin{aligned} R = {} & -J+\mathcal{L}_\alpha+\mathcal{L}_\beta+\mathcal{S}_\alpha+\mathcal{S}_\beta+j_\alpha+j_\beta-s_\alpha+S_\beta \\ & +l_\alpha+\tau_\gamma+\tau_\alpha-t_\beta+2T+L \ . \end{aligned} \tag{B7}$$

The $12J$ symbol in Eq. (B6) is that defined by Ord-Smith.[45]

To minimize the computational problem it is important to make use of the symmetry of the one-particle-exchange amplitude. This is given by

$$Z^{JT}_{Q_\alpha,Q_\beta}(q_\alpha,q_\beta;E)=Z^{JT}_{Q_\beta,Q_\alpha}(q_\beta,q_\alpha;E) \ . \tag{B8}$$

In addition, we can make use of the symmetry of $\Gamma^L_{n_\alpha,n_\beta}$ which is given by

$$\Gamma^L_{n_\alpha,n_\beta}(q_\alpha,q_\beta;E)=\Gamma^L_{n_\beta,n_\alpha}(q_\beta,q_\alpha;E) \ . \tag{B9}$$

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