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Electroexcitation of the N^* (1440) in the relativistic constituent quark model

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Relativistic P_{11} wave functions are constructed that have 2S and 1S character in the static limit. Their constraint-free electromagnetic N-N^* transition form factors are calculated from a conserved current. A gauge condition at $q^2=0$ is met by a three-quark current. The 2S helicity amplitudes are too large compared with the sparse data, while the 1S results are an order too low. A mixture of the 1S and 2S states agrees with the electroproduction but misses the photon point.

I. INTRODUCTION TO THE RELATIVISTIC CONSTITUENT QUARK MODEL ON THE LIGHT CONE

Recently a basis of relativistic three-quark states for the nucleon,[1] the $\Delta_{33}(1232)$,[2] hyperons,[3] and the $S_{11}(1535)$,[4] has been constructed in the relativistic constituent quark model (RCQM), and their electromagnetic properties were successfully tested. It appears that relativistic effects account for the momentum transfer dependence up to nearly 1.5 GeV/c. Thereby the RCQM is removing a major flaw of the nonrelativistic quark model (NQM),[5] along with the improved axial-vector coupling $g_A(q^2)$ which, at $q^2=0$, was already achieved in bag models.[5]

In the RCQM nonstatic three-quark spin wave functions are obtained from the nonrelativistic case by means of free Melosh transformations of all quarks to the light cone[2,4] upon neglecting binding effects. This approximate treatment ensures conserved $\mathbf{J}^2$ and includes relativistic effects not only from the interacting quark as in bag models, but also from spectator quarks. The Melosh construction (of Appendix A of Ref. 4) yielding the nonstatic spin wave functions $\chi_0+\chi_2$ of Eqs. (1) and (6) effectively restores rotational invariance at least at low momentum transfer,[6] where the three independent symmetric spin invariants of the nucleon (N) and P_{11} state ($*$) at 1.44 GeV/c^2 are given by

$$\begin{aligned}
&\chi_{0N,*}=I_0(13,2)+I_0(23,1),I_0(12,3)=\bar{u}_{\lambda_1}\gamma_5 v_{\lambda_2}\bar{u}_{\lambda_3}u_{N,*}(P)\ ,\\
&I_1=-3\bar{u}_{\lambda_1}\gamma_\mu v_{\lambda_2}\bar{u}_{\lambda_3}\gamma^\mu\gamma_5 u_{N,*}(P)\ ,\\
&\chi_{2,N,*}=I_2(13,2)+I_2(23,1),I_2(12,3)=\bar{u}_{\lambda_1}\frac{\gamma\cdot P}{m_{N,*}}\gamma_5 v_{\lambda_2}\bar{u}_{\lambda_3}u_{N,*}\ .
\end{aligned} \tag{1}$$

Here we are working in the uds basis where quark 3 is the d quark in the proton or charged P_{11}. Starting from manifestly symmetric forms of these spin invariants [cf. Eqs. (11) and (28) of Ref. 1] that include isospin dependence, it is straightforward to show that the nonstatic spin wave functions χ are totally symmetric. The light-cone spinors $u_i=u(p_i)$ are solutions of the free Dirac equation with $p_i^2=m_q^2$, while $u_N(P)$ and $u_*(P')$ are the total momentum spinors of the nucleon and P_{11} state, respectively. The four-momentum of each particle of mass m is written as $p^\mu=[p^+=p_0+p_3,\ p^-=(m^2+\mathbf{p}_T^2)/p^+,\ \mathbf{p}_T]$, where the index 3 denotes an appropriate quantization axis and $\mathbf{p}_T=(p_1,p_2)$. The invariant quark momentum fractions $x_i=p_i^+/P^+$ with the total nucleon momentum $P=\sum_i p_i$ (for + and transverse components only) represent the longitudinal momentum components whose distribution can be measured in deep inelastic and elastic scattering at high energies. The x_i also serve to define the relative four-momentum variables for the three-body system

$$\begin{aligned}
&q_3=\frac{x_2p_1-x_1p_2}{x_2+x_1}\ ,\\
&Q_3=(x_1+x_2)p_3-x_3(p_1+p_2)\ ,
\end{aligned} \tag{2}$$

which are properly spacelike since $q_3^+=0=Q_3^+$ so that $q_3^2=-\mathbf{q}_{3T}^2$ and $Q_3^2=-\mathbf{Q}_{3T}^2$. Integrations over the quark relative momentum variables are three dimensional, rather than four dimensional, with

$$d\Gamma=\prod_{i=1}^{3}\frac{dx_i}{x_i}\delta\left(1-\sum_j x\right)dq_{3T}^2dQ_{3T}^2/(16\pi^3)^2 \tag{3}$$

as volume element in momentum space.

II. $P_{11}(1440)$ WAVE FUNCTIONS

The totally symmetric nucleon and P_{11} momentum distributions are taken as common relativistic Gaussians with $1S$ and $2S$ structures in the static limit, respectively, for ease of comparison with the NQM and its extensions,

$$\psi_N = N_o\phi_0\chi_N,\quad \psi_{*1S}=N_1\phi_0\chi_0,\quad \psi_{*2S}=N_2\phi_{2S}\chi_1\ , \tag{4}$$

$$\phi_0=\exp(-M_3^2/6\alpha^2),\quad \phi_{2S}=(M_3^2/6\alpha^2-C)\phi_0\ , \tag{5}$$

$$\begin{aligned}\chi_N&=\chi_{0N}+\chi_{2N},\quad \chi_1=\chi_{0*}+\chi_{2*}\ ,\\ \chi_0&=\chi_{0*}+C_2(\chi_{0*}+\chi_{2*})/2\ ,\end{aligned} \tag{6}$$

where

$$\begin{aligned}M_3^2&=-q_3^2\frac{1-x_3}{x_1x_2}-\frac{Q_3^2}{x_3(1-x_3)}+\sum_i\frac{m_q^2}{x_i}\\ &=\sum_{i=1}^{3}(m_q^2+\mathbf{k}_{iT}^2)/x_i\end{aligned} \tag{7}$$

is the totally symmetric invariant mass squared of the three-body system with $\mathbf{k}_{iT}=\mathbf{p}_{iT}-x_i\mathbf{P}_T$, $\mathbf{k}_{3T}=\mathbf{Q}_{3T}$, etc. The constant C and C_2 in Eqs. (5) and (6) will be specified below. The size parameter $\alpha=0.32$ GeV $\sim m_N/3$ and the effective quark mass $m_q=0.38$ GeV/c^2 are determined by the axial-vector form factor so that $g_A=1.25$; the nucleon magnetic moments become $\mu_p=2.68\ \mu_N$, $\mu_n=-1.63\ \mu_N$, which improve upon including pion cloud corrections.[7]

While the vector spin invariant I_1 in Eq. (1) cannot be appreciably present in the nucleon, because it would generate a pseudotensor term in its axial-vector current that is not seen in experiments,[8] it may appear in the P_{11} state. Nonetheless, we will not consider it here further. Already the presence of two independent spin invariants I_0 and I_2 of Eq. (1) provides for a new relativistic three-quark P_{11} Fock component χ_0 in Eq. (6) that is orthogonal to the nucleon with $1S$ behavior in the static limit.

In the static limit all momentum components are small compared to m_q so that $x_i\to m_q/m_N\sim\frac{1}{3}$, and the relativistic momentum variables q_3, Q_3 of Eq. (2) become $\mathbf{p}_\rho/\sqrt{2}=(\mathbf{p}_1-\mathbf{p}_2)/2$, $-\sqrt{2/3}\mathbf{p}_\lambda=(2\mathbf{p}_3-\mathbf{p}_1-\mathbf{p}_2)/3$, where $\mathbf{p}_\rho,\mathbf{p}_\lambda$ are the relative momentum variables conjugate to the conventional Jacobi coordinates[5] $\boldsymbol{\rho},\boldsymbol{\lambda}$ of the three-quark system. Similarly $M_3^2-9m_q^2\sim(\mathbf{p}_\rho^2+\mathbf{p}_\lambda^2)$, which is the main ingredient of the $2S$ wave function in momentum space in the NQM, viz.,

$$\phi_{2S}\sim[(\mathbf{p}_\rho^2+\mathbf{p}_\lambda^2)/\alpha^2-3]\exp[-(\mathbf{p}_\rho^2+\mathbf{p}_\lambda^2)/2\alpha^2]\ .$$

Therefore, we also consider the following polynomials in M_3^2 that have the proper $2S$ behavior in the static limit when $\alpha\sim m_q$:

(i) $(M_3^2/6\alpha^2-3)\sim\phi_{2S}$ for $\sum_i m_q^2/x_i\sim 9m_q^2$,

(ii) $\left[M_3^2-\sum_i m_q^2/x_i\right]\Big/6\alpha^2-1.5\sim\phi_{2S}$.

Moreover, orthogonality in the RCQM follows from the Weinberg equation of motion for a Hermitian interaction and is independent of the frame. When C in $(M_3^2/6\alpha^2-C)$ of Eq. (5) is determined from imposing orthogonality to the nucleon RCQM wave function, then $C=3.6634$ is obtained, which is close to the nonrelativistic case (i) where $C=3$. When C_2 in the relativistic P_{11} spin wave function $\chi_{0*}+C_2(\chi_{0*}+\chi_{2*})/2$ of Eq. (6) with $1S$ behavior in the static limit is determined from orthogonality to the nucleon, then $C_2=-1.0553$ is obtained. We note that from the Clebsch-Gordan construction of the nonstatic spin wave functions starting from the common nucleon and P_{11} expression in the NQM, it follows that χ_{0*} and χ_{2*} have the same static limit. These details are given in Ref. 1 and generalized to the $S_{11}(1535)$ in Ref. 4. Hence, in the static limit

$$\chi_0\sim-0.0553(\chi_{0*}+\chi_{2*})/2$$

of Eq. (6) has only about 5.5% overlap with the nucleon NQM wave function, and its $1S$ momentum distribution is orthogonal to that of the $2S$-P_{11} state.

III. ELECTROMAGNETIC N-N^* TRANSITION FORM FACTORS

The calculation of the electromagnetic N-N^* helicity amplitudes is similar to those for the nucleon[9] and the $S_{11}(1535)$.[4] In the Drell-Yan frame, where $q^+=0$, only the triangle diagram of Fig. 1 contributes and the current matrix element becomes

$$\langle N_*\lambda'|J^+|N\lambda\rangle=\sum_{j=1}^{3}\int d\Gamma\psi_*^\dagger(x_i'q_3'Q_3')\times\bar{u}_je_j\gamma^+x_j^{-1}u_j\psi_N(x_iq_3Q_3)\ , \tag{8}$$

where e_j is the charge of the struck quark and sums over the quark spin and isospin projections are implied. The evaluation of Eq. (8) uses some of the results, techniques, and symbolic codes of Ref. 4.

The N-N^* transition current matrix element can be parametrized in terms of two form factors F_{1*} and F_{2*}:

$$\langle N_*\lambda'|J^\mu|N\lambda\rangle=\bar{u}'_*\left[\frac{F_{1*}}{m_*+m_N}(q^2\gamma^\mu-\gamma\cdot qq^\mu)+iF_{2*}\sigma^{\mu\nu}q_\nu\right]u_N\ , \tag{9}$$

$$(N_*\lambda'|J^\mu|N\lambda\rangle=\bar{u}'_*\left[\frac{h_1}{Q^+}(P\cdot qq^\mu-q^2P^\mu)+\frac{h_3}{Q^+}im_*\varepsilon^{\mu\nu\rho\sigma}q_\nu P_\rho\gamma_\sigma\gamma_5\right]u_N\ , \tag{10}$$

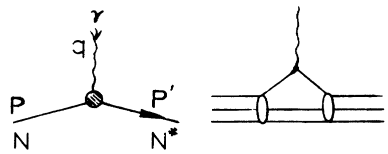

FIG. 1. Triangle diagram for the electroexcitation of the $N^*(1440)$ from the nucleon.

where $q=P'-P$ is the momentum transfer, $\varepsilon_{0123}=1$, and $Q^+=(m_*+m_N)^2-q^2$. The h_i are the constraint-free form factors;[10] it is easy to see that h_1 is the longitudinal and h_3 the transverse one.[10,4] The Gordon decomposition of the current

$$\bar{u}'_*\gamma^\mu u_N=\bar{u}'_*(P'^\mu+P^\mu+i\sigma^{\mu\nu}q_\nu)u_N/(m_*+m_N) \tag{11}$$

in conjunction with the identity

$$-i\varepsilon^{\mu\nu\rho\sigma}\gamma_\sigma\gamma_5=\gamma^\mu\gamma^\nu\gamma^\rho-g^{\mu\nu}\gamma^\rho+g^{\mu\rho}\gamma^\nu-g^{\nu\rho}\gamma^\mu$$

relates Eqs. (9) and (10) so that

$$\begin{aligned} h_1&=-2(F_{1*}+F_{2*}),\\ h_3&=2\left[1+\frac{m_N}{m_*}\right]\left[F_{2*}+\frac{q^2F_{1*}}{(m_*+m_N)^2}\right]. \end{aligned} \tag{12}$$

The extra q^μ term occurs in Eq. (9) because $m_*\neq m_N$, and it leads to the gauge condition in the $q^+=0$ frame

$$\langle N_*\uparrow|J^+|N\uparrow\rangle=\bar{u}'_{*\uparrow}F_{1*}q^2\gamma^+u_{N\uparrow}/(m_*+m_N)=0 \tag{13}$$

at $q^2=0$. It may be satisfied by including a many-body current, i.e., replacing the one-body current $e_q\gamma^\mu$ by e_qJ^μ with

$$J^\mu=\gamma^\mu-\frac{\gamma\cdot q}{q\cdot(P'+P)}(P'^\mu+P^\mu)\ , \tag{14}$$

because

$$0=q\cdot J=\tfrac{1}{2}(q^+J^-+q^-J^+)-\mathbf{q}_T\cdot\mathbf{J}_T=\tfrac{1}{2}q^-J^+$$

for $q^2=0$ and $q^+=0$, while

$$q^-=(m_*^2-m_N^2-q^2)/P^+\neq 0$$

even at $q^2=0$ since $m_*>m_N$. The normalizations N_i in Eq. (4) are determined from the charge at $q^2=0$ of the proton and charged P_{11} state.

Both transition form factors F_{i*} are obtained from J^+, the "good" component of J in the $q^+=0$ frame,

$$\begin{aligned} \langle N_*\uparrow|J^+|N\uparrow\rangle&=\frac{P^+}{\sqrt{m_*m_N}}\cdot\frac{F_{1*}q^2}{m_*+m_N}\ ,\\ \langle N_*\uparrow|J^+|N\downarrow\rangle&=-\frac{2F_{2*}(q_1-iq_2)}{\sqrt{m_*m_N}}P^+\ . \end{aligned} \tag{15}$$

The transverse and longitudinal helicity amplitudes $A_{1/2}$ and $S_{1/2}$ are then defined as[11]

$$\begin{aligned} A_{1/2}&=\left[\frac{2\pi\alpha}{K_W^*}\right]^{1/2}\langle N_*\uparrow|-L_{\rm em}|N\downarrow\rangle\ ,\\ S_{1/2}&=\left[\frac{2\pi\alpha}{K_W^*}\right]^{1/2}\frac{|\mathbf{q}_*|}{\sqrt{-q^2}}\langle N_*\uparrow|-L_{\rm em}|N\uparrow\rangle\ , \end{aligned} \tag{16}$$

where $L_{\rm em}=-J\cdot\varepsilon$ is the electromagnetic interaction, K_W^* is the energy of an equivalent real photon, and q^* the three-momentum transfer of the virtual photon, both in the isobar rest frame,

$$\begin{aligned} &K_W^*=(m_*^2-m_N^2)/m_*,\mathbf{q}_*{}^2=(m_*^2-m_N^2+q^2)^2/4m_*^2-q^2\ ,\\ &\alpha=\frac{e^2}{4\pi}=\frac{1}{137.036}\ . \end{aligned} \tag{17}$$

Combining this with Eq. (16) yields

$$\begin{aligned} A_{1/2}&=-\frac{q_Lm_Nh_3}{Q^+}\left[(E_L^*+m_*)\frac{2\pi\alpha m_*}{K_W^*}\right]^{1/2}\ ,\\ S_{1/2}&=\frac{q^*m_Nq_Lh_1}{Q^+}\left[\frac{\pi\alpha(E_L^*+m_*)}{K_W^*m_*}\right]^{1/2}\ , \end{aligned} \tag{18}$$

where

$$q_L=(m_*^2-m_N^2-q^2)/2m_N,\quad E_L^*=P'\cdot P/m_N$$

are the photon momentum and N^* energy in the laboratory, respectively.

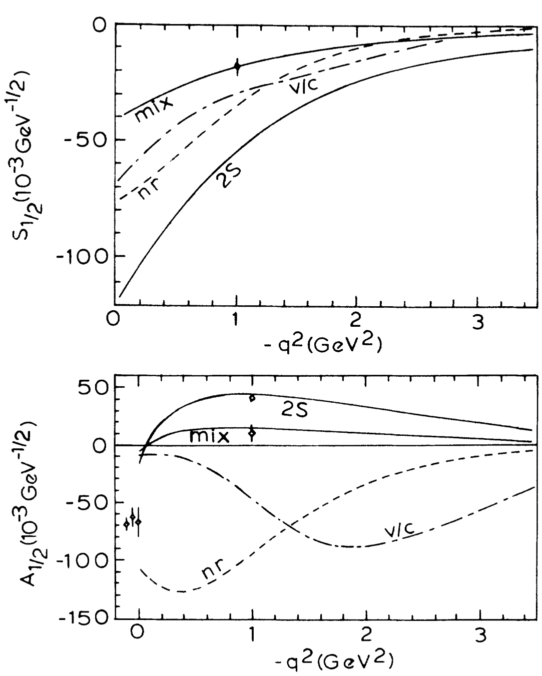

FIG. 2. The $A_{1/2}$ and $S_{1/2}$ helicity amplitudes of the charged $P_{11}(1440)$ resonance. The 2S state of the RCQM is the solid line marked 2S; the mixed 2S-1S is the other solid line. The NQM result is the dashed line marked nr, the dot-dashed line marked v/c is the NQM with relativistic corrections, both from Ref. 11. For the data points we refer to the compilations in Refs. 12 and 13.

IV. RESULTS AND DISCUSSION

One of the reasons for this study was to look into the Roper resonance in the framework of a relativistic quark model such as the RCQM that, with its possibly richer spectrum of nonstatic spin states than in the NQM, would allow for a split $N^*(1440)$. This intriguing possibility is left open here because it can only be decided on the basis of dynamical calculations which go beyond the scope of this paper and the RCQM. We report here electromagnetic responses of two P_{11} basis states that are orthogonal to the nucleon and have $2S$ and $1S$ behaviors in the static limit, respectively.

The RCQM has several advantages over nonrelativistic quark models that make it attractive if not yet realistic. So far it has only two parameters (or three if the strange quark is included): the three-quark core size $\alpha \sim m_N/3$ of the nucleon and other baryons and the common up-down constituent quark mass $m_q \sim m_N/3$, which reproduce the axial-vector[4] and electromagnetic form factors of the nucleon up to nearly 1.5 GeV/c momentum transfer,[9] but not G_E^n. It predicts similarly $\Delta_{33}(1232)$ and $S_{11}(1535)$ properties.[2,4] As a relativistic many-body framework on the light cone it is simpler than instant formulations because on-shell spinors may be used for internal quarks instead of interaction dependent propagators. Moreover, there are only three-dimensional integrations over internal particles, and the total momentum is properly treated.

From Fig. 2 it is clear that the rather model (and approximation) dependent transverse response $A_{1/2}$ misses the photon point, while the longitudinal helicity amplitude $S_{1/2}$ is a factor of 3 too large in magnitude, but is qualitatively similar to the nonrelativistic calculation and the results of a p/m_q expansion to second order[11] (denoted by v/c in Fig. 2), both of whose $A_{1/2}$ also miss the photon point. The case (i) in Sec. II is within 20% and (ii) is practically indistinguishable from the case with $C=3.6634$ in Eq. (5). The neutral $A_{1/2}$ and $S_{1/2}$ have the same sign as the proton case, but are smaller by factors of about 6 and 20, respectively, whereas those of the $1S$ wave function that is orthogonal to the nucleon have the opposite sign compared to the $2S$ case, and are about a factor of 50 smaller than the $2S$ results. Mixing the $1S$ and $2S$ wave functions, $0.34(2S)+0.94(1S)$, as shown in Fig. 2, would yield the correct magnitude for both helicity amplitudes, but needs to be justified by a dynamical calculation on the light cone. Such more realistic calculations are now called for, although the data are sparse and of poor quality, as this situation is expected to be remedied with the advent of a new generation of electron accelerators, ELSA, MAMI B, and CEBAF, in the near future.

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