Structure functions of nuclei in the "instant" form of dynamics

L. Heller
Theoretical Division, Los Alamos National Laboratory, Los Alamos, New Mexico 87545

A. W. Thomas
Department of Physics, University of Adelaide, Adelaide, South Australia 5000, Australia
(Received 14 December 1987)

Convolution formulas for the inelastic structure functions of a nucleus are derived using the "instant" form of relativistic particle dynamics, and are contrasted with other convolution approaches. Using only physical nucleon constituents and harmonic oscillator shell model wave functions, the magnitude of the dip in the European Muon Collaboration ratio in the intermediate range of x is reproduced. The calculations of this ratio fail to come back up towards unity as x decreases below 0.4, and possible reasons for this are discussed.

I. INTRODUCTION

Over the past few years a number of independent experiments[1,2] have confirmed the original observation of a systematic difference between the structure functions of heavy nuclei and deuterium,[3] known as the European Muon Collaboration (EMC) effect. Many exciting explanations have been offered, including dynamical rescaling,[4] bloating of the nucleon,[5] multiquark states,[6] partial deconfinement of quarks,[7] excess pions,[8–10] and so on. On the other hand, a number of groups have proposed that the conventional treatment of the Fermi motion may need to be modified to take into account the fact that nucleons are actually bound inside nuclei.[11–13] If nucleon binding requires a quark-level description to be properly understood, these two approaches may be two sides of the same coin. Nevertheless, the description of nuclei as nucleons bound by meson exchange forces has been undeniably successful and it seems reasonable to push such an approach to its limits before hailing the EMC effect as an indication of truly new physics.

There are essentially two treatments of binding and Fermi-motion corrections in the recent literature. Akulunichev *et al.*[11] and Dunne and Thomas[12] viewed the process as the collision between the virtual photon and an off-mass-shell nucleon. The purely kinematic correction is not sufficient to explain all of the EMC effect unless an ansatz for the structure function of the off-mass-shell nucleon is used.[12] Unfortunately, there does not seem to be any independent way to test this ansatz. Thus, its effect can only be viewed as an estimate of the uncertainty in the binding correction calculated this way. Since this can be as much as 40% of the EMC effect it is not very satisfactory.

The alternative approach used so far[10,14,15] relies on the "light-front" form of relativistic particle dynamics.[16] There the quantities $P^+=E+P_z$ and $\mathbf{P}_T$ are conserved at each step, but $P^-=E-P_z$ is only conserved overall. To understand the significance of this choice of dynamics it is necessary to couple it with the assumptions that go into the derivation. The two assumptions that are of immediate interest are the following: (i) The current operator of the nucleus consists of a sum of one-body operators (the currents of the hadronic constitutents), and (ii) there is no final-state interaction between the debris of the constituent that interacts with the current and the residual nucleus. The first assumption, which says that there are no two- (or more-) body currents, bears directly on the form of dynamics assumed. Suppose, for example, that this assumption is (approximately) valid in the laboratory frame in the "instant"[16] form of dynamics, in which the three-momentum $\mathbf{P}$ is conserved at each step but E is only conserved overall. If an interaction dependent unitary transformation is carried out that brings the generators of the Poincaré group from the instant form into the light-front form, then that same transformation applied to the current will produce two-body currents. It is also plausible that assumption (ii) will be more nearly correct in one form of dynamics than another. We do not have any *a priori* way of knowing in which form of dynamics these assumptions are better satisfied, but we are led to study the instant version because the wave functions of nuclei are conventionally given in that form.

It must be stated, however, that the one-body current assumption is not consistent with Lorentz invariance. In the instant form of relativistic particle dynamics all components of the boost generator contain interaction, and the commutator of these terms with a one-body current will produce two-body currents, in general. This point must be kept in mind in Sec. II, where the convolution formula for the structure tensor is derived. Although written in tensor notation, it is not a covariant equation. Our numerical calculations are performed in the laboratory frame.

A brief outline of the paper is as follows. In Sec. II we derive the convolution formula for deep-inelastic scattering from nuclei in the instant form. This is written in a more familiar form in Sec. III, and the behavior as a function of Q^2 is examined in detail. We also discuss the limits of the convolution formula there. The results are presented in Sec. IV, and in Sec. V we make some concluding remarks. In the present paper it is assumed that nucleons are the only constituents of the nucleus; a remark about the role of pions is made in the conclusion.

II. FORMAL DEVELOPMENT OF THE INSTANT FORM

A. Derivation of the convolution formula

We first outline the derivation, which parallels that in Ref. 15 but uses a different form of dynamics. We write down the definition of the Lorentz tensor $W_A^{\mu\nu}(q,p)$ that contains the product of matrix elements of the electromagnetic current describing a virtual photon of four-momentum q interacting with the target nucleus of four-momentum p, with $p^2=m_A^2$. The scaling variable x_A is defined by

$$x_A=\frac{Q^2}{2p\cdot q}\ , \tag{2.1}$$

where $Q^2=-q^2>0$; we also use the variable $x=m_A x_A/m_N$, which varies from zero to $\sim A$. Then we do the same for $W_N^{\mu\nu}(\tilde{q},k)$ when a photon of four-momentum $\tilde{q}$ strikes a free nucleon with four-momentum k, with $k^2=m_N^2$. Making use of one additional assumption discussed below leads to $W_A^{\mu\nu}$ being expressed as a three-dimensional integral with respect to $\mathbf{k}$ of an integrand in which the current matrix elements are precisely the same as the ones that enter $W_N^{\mu\nu}$. The final task is to relate $\tilde{q}$ and q in such a way that the overall energy-momentum-conserving δ functions in the two tensors are both satisfied.

From its definition

$$W_A^{\mu\nu}(q,p)=\sum_{\alpha,\beta}\int d^3p'd^3p''\delta(q+p-p'-p'')\times\langle A,\mathbf{p}|J^\mu(0)|\alpha,\mathbf{p}';\beta,\mathbf{p}''\rangle\times\langle\alpha,\mathbf{p}';\beta,\mathbf{p}''|J^\nu(0)|A,\mathbf{p}\rangle E_A(\mathbf{p})\ , \tag{2.2}$$

where we have made use of assumption (ii) to write the final state as two noninteracting clusters: the residual nucleus is in state α with three-momentum $\mathbf{p}'$, and the debris from the struck nucleon is in state β with three-momentum $\mathbf{p}''$. The states are normalized as

$$\langle A,\mathbf{p}|A,\mathbf{p}'\rangle=\delta(\mathbf{p}-\mathbf{p}')\ , \tag{2.3}$$

and the factor $E_A(\mathbf{p})=(m_A^2+\mathbf{p}^2)^{1/2}$ converts this to an invariant normalization.

To make use of the one-body current assumption (i), we introduce into Eq. (2.2) the identity operator in the form of a sum over a complete set of states. These are simultaneous eigenstates of the three-momentum operators of all A nucleons, and therefore also of the free Hamiltonian. The overlap of the target ground state $|A,\mathbf{p}\rangle$—which is an eigenstate of the total three-momentum and interacting Hamiltonian—with these states gives the momentum space wave function of the target, shown in Eq. (2.4). The δ function in that equation is indicative of the instant form of dynamics since the total three-momentum operator does not contain interactions. Equation (2.5) shows the same thing for the residual nucleus:

$$\delta\left[\mathbf{p}-\sum_{i=1}^{A}\mathbf{k}_i\right]\phi_{A,\mathbf{p}}(\mathbf{k}_1,\ldots,\mathbf{k}_A)=\langle\mathbf{k}_1,\ldots,\mathbf{k}_A|A,\mathbf{p}\rangle\ , \tag{2.4}$$

$$\delta\left[\mathbf{p}'-\sum_{i=2}^{A}\mathbf{k}_i\right]\phi_{\alpha,\mathbf{p}'}(\mathbf{k}_2,\ldots,\mathbf{k}_A)=\langle\mathbf{k}_2,\ldots,\mathbf{k}_A|\alpha,\mathbf{p}'\rangle\ . \tag{2.5}$$

We also need the one-particle overlap function

$$\psi_{\alpha,\mathbf{p}}(\mathbf{k})=\int d^3k_2\cdots d^3k_A\phi^*_{\alpha,\mathbf{p}-\mathbf{k}}(\mathbf{k}_2,\ldots,\mathbf{k}_A)\times\phi_{A,\mathbf{p}}(\mathbf{k},\mathbf{k}_2,\ldots,\mathbf{k}_A)\delta\left[\mathbf{p}-\mathbf{k}-\sum_{i=2}^{A}\mathbf{k}_i\right]\ . \tag{2.6}$$

Making the additional assumption (iii) that interference terms—in which the current J^μ interacts with one nucleon and J^ν with a different nucleon—can be neglected, and using Eqs. (2.4) and (2.5) in (2.2), yields

$$W_A^{\mu\nu}(q,p)=AE_A(\mathbf{p})\sum_\alpha\int d^3k\rho_{\alpha,\mathbf{p}}(\mathbf{k})\left\{\sum_\beta\int d^3p''\delta(\mathbf{q}+\mathbf{k}-\mathbf{p}'')\delta[q^0+E_A(\mathbf{p})-E_\alpha(\mathbf{p}-\mathbf{k})-E_\beta(\mathbf{p}'')]\times\langle\mathbf{k}|J^\mu(0)|\beta,\mathbf{p}''\rangle\langle\beta,\mathbf{p}''|J^\nu(0)|\mathbf{k}\rangle\right\}\ . \tag{2.7}$$

The overall energy- and momentum-conserving δ function from (2.2) has been written out explicitly in (2.7), and

$$\rho_{\alpha,\mathbf{p}}(\mathbf{k})\equiv|\psi_{\alpha,\mathbf{p}}(\mathbf{k})|^2 \tag{2.8}$$

satisfies the completeness relation

$$\sum_\alpha\int d^3k\rho_{\alpha,\mathbf{p}}(\mathbf{k})\equiv\int d^3k\rho_{\mathbf{p}}(\mathbf{k})=1\ . \tag{2.9}$$

As expected in this form of dynamics

$$\sum_{\alpha} \int d^3k\, \mathbf{k} \rho_{\alpha,\mathbf{p}}(\mathbf{k}) = \frac{1}{A}\mathbf{p} \ . \tag{2.10}$$

We would now like to relate the quantity within the large curly brackets in Eq. (2.7) to the structure functions of a *free,* i.e., *physical* nucleon; we will then take these quantities directly from experiments on free nucleons and thereby evaluate the structure functions of the nucleus. The way to establish this relation is to write down, *ab initio,* the definition of the structure tensor $W_N^{\mu\nu}(\tilde{q},k)$ for a free nucleon of four-momentum k struck by a photon of four-momentum $\tilde{q}$, with the latter quantities not yet specified. See Fig. 1 for the kinematics. When this is done the expression looks just like the quantity in the large curly brackets in Eq. (2.7), except that the momentum-conserving δ function is $\delta(\tilde{\mathbf{q}}+\mathbf{k}-\mathbf{p}'')$, and the energy-conserving δ function is

$$\delta[\tilde{q}^{\,0}+E_N(\mathbf{k})-E_\beta(\mathbf{p}'')] \ .$$

$E_N(\mathbf{k})$ is the energy of a free nucleon of momentum $\mathbf{k}$.

By making the identifications

$$\tilde{\mathbf{q}}=\mathbf{q}$$

and

$$\tilde{q}^{\,0}=q^0-\Delta \ , \tag{2.11}$$

where

$$\Delta \equiv \Delta_{\alpha,\mathbf{p}}(\mathbf{k}) = E_N(\mathbf{k})+E_\alpha(\mathbf{p}-\mathbf{k})-E_A(\mathbf{p}) \ , \tag{2.12}$$

it is seen that [to within the normalization factor $E_N(\mathbf{k})$] the quantity in large curly brackets in Eq. (2.7) is *precisely* the structure tensor of a free nucleon.

Since the target nucleus is stable, $\Delta > 0$, and in its rest frame Δ is just the sum of the binding energy B_α and the kinetic energies of the nucleon and the recoiling nucleus. (If one were doing time-ordered perturbation theory, Δ would be referred to as the amount by which the $N\alpha$ intermediate state is "off the energy shell." We are not doing perturbation theory here, however.) These relations lead to the convolution formulas below in which the structure functions of a free nucleon are evaluated at shifted values of the arguments.

Although the quantity Δ in Eq. (2.11) has a magnitude of only tens of MeV—since it consists of nuclear binding and kinetic energies—and q^0 is large compared to the nucleon mass in the deep-inelastic region, nevertheless Δ makes a non-negligible contribution to $\tilde{Q}^2$ as can be seen from Eq. (2.11), where

$$\begin{aligned}\tilde{Q}^2 \equiv \tilde{Q}^2_\alpha(\mathbf{k}) &= -\tilde{q}^{\,2} = -(q^0-\Delta)^2+\mathbf{q}^2 \\ &= Q^2+2q^0\Delta-\Delta^2 \\ &\cong Q^2\left[1+\frac{\Delta}{m_N x}\right] \ .\end{aligned} \tag{2.13}$$

The final line of Eq. (2.13) is written in the rest frame of the target, and neglects terms of relative order Q^{-2}. It shows that $\tilde{Q}^2$ exceeds Q^2 by a few percent, and the difference grows at small x.

Using the definition of the scaling variable for a free nucleon of momentum $\mathbf{k}$ struck by a photon $\tilde{q}$, $\tilde{x} \equiv \tilde{Q}^2/2k\cdot\tilde{q}$, and neglecting more terms of relative order Q^{-2},

$$\tilde{x} \equiv \tilde{x}_\alpha(\mathbf{k}) \simeq x\frac{m_N}{E_N(\mathbf{k})+k_Z}\left[1+\frac{\Delta}{m_N x}\right] , \tag{2.14}$$

in the coordinate system in the rest frame of the target in which $\mathbf{q}$ is chosen along the negative z axis. Equations (2.13) and (2.14) can be generalized to any frame in which $\mathbf{q}_\perp=0$ by replacing $m_N x = m_A x_A$ by $p^+ x_A$, provided that $|\mathbf{p}| \ll q^0$ and $|\mathbf{k}| \ll q^0$. Putting Eqs. (2.13) and (2.14) into (2.7), the structure tensor for the nucleus (per nucleon), $\overline{W}{}^{\mu\nu}_A(q,p)$, becomes

$$\overline{W}{}^{\mu\nu}_A(q,p) = \sum_\alpha \int d^3k\, \rho_{\alpha,\mathbf{p}}(\mathbf{k}) \frac{E_A(\mathbf{p})}{E_N(\mathbf{k})} W_N^{\mu\nu}(\tilde{q},k) \ . \tag{2.15}$$

As discussed in the Introduction, this convolution formula is not Lorentz covariant because of the one-body current assumption. [The equation in light-front dynamics that corresponds to the second part of Eq. (2.11) is $\tilde{q}_- = q_- - \Delta_-$, where Δ_- represents the amount of nonconservation of P_- at the $AN\alpha$ vertex. When this is multiplied by $\tilde{q}_+ = q_+$, which remains finite in the Bjorken limit, $\tilde{Q}^2$ and Q^2 differ at most by a finite amount[15] rather than a finite *percentage* as in Eq. (2.13). Consequently, although there is still an additive term involving Δ_- in the relation analogous to Eq. (2.14), it vanishes in the limit.]

To obtain the connection between the structure functions $F_{1,2}$ for the nucleus and the nucleon, we use

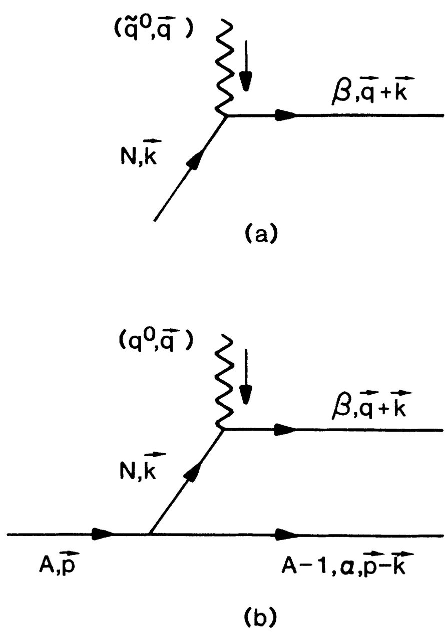

FIG. 1. Kinematics for the structure functions. The same matrix elements of the electromagnetic current occur in the nucleon case (a) as in the nuclear case (b).

$$W_N^{\mu\nu}=-\left[g^{\mu\nu}+\frac{\tilde{q}^{\mu}\tilde{q}^{\nu}}{\tilde{Q}^2}\right]F_{1,N}+\frac{\tilde{k}^{\mu}\tilde{k}^{\nu}}{k\cdot\tilde{q}}F_{2,N}\ , \quad (2.16)$$

where $\tilde{k}=k+(k\cdot\tilde{q}/\tilde{Q}^2)\tilde{q}$, and the corresponding formula for W_A with the replacements $\tilde{q}\to q$, $k\to p$, and $\tilde{k}\to\tilde{p}$. The inversion of the latter expression yields

$$F_{1,A}=-\frac{1}{2}\left[g_{\mu\nu}-\frac{\tilde{p}^{\mu}\tilde{p}^{\nu}}{\tilde{p}^2}\right]W_A^{\mu\nu} \quad (2.17)$$

and

$$F_{2,A}=-\frac{1}{2}\frac{p\cdot q}{\tilde{p}^2}\left[g_{\mu\nu}-\frac{3\tilde{p}_{\mu}\tilde{p}_{\nu}}{\tilde{p}^2}\right]W_A^{\mu\nu}\ , \quad (2.18)$$

and inserting (2.15) and (2.16) into (2.17) and (2.18) produces the desired connection. After some lengthy algebra we obtain for the nuclear structure functions per nucleon, $\bar{F}_{1,A}$ and $\bar{F}_{2,A}$,

$$\bar{F}_{1,A}(x,Q^2)=\sum_{\alpha}\int d^3k\frac{m_A}{E_N(\mathbf{k})}\rho_{\alpha}(\mathbf{k})F_{1,N}(\tilde{x},\tilde{Q}^2) \quad (2.19)$$

and

$$\bar{F}_{2,A}(x,Q^2)=\sum_{\alpha}\int d^3k\frac{m_N}{E_N(\mathbf{k})}\rho_{\alpha}(\mathbf{k})\frac{x}{\tilde{x}}F_{2,N}(\tilde{x},\tilde{Q}^2)\ , \quad (2.20)$$

where ρ_α without the $\mathbf{p}$ subscript means that it is evaluated in the rest frame of the target. The functions $F_{1,N}$ and $F_{2,N}$ appearing in Eqs. (2.19) and (2.20) are precisely the structure functions of a *free* nucleon, but at shifted values of the arguments. The relationship between the arguments of the two sets of structure functions is given in Eqs. (2.12)–(2.14).

Equations (2.19) and (2.20) are not strictly correct at small x because of the neglect of terms of order $(\Delta/m_N)^2$ which are really $(\Delta/m_N x)^2$. The exact equations (in the Bjorken limit) are given in the Appendix. We do not show them here because corrections in Δ^2 are of relativistic order and we eventually use a nonrelativistic nuclear density. Furthermore, if the nucleon structure functions obey the Callan-Gross relation (see below)—as in all our calculations—Eqs. (2.19) and (2.20) are correct.

To conclude this section on the formalism using instant dynamics, we make a remark concerning assumption (ii) from the Introduction, the neglect of final-state interactions. What is at issue here is the relative size of the time interval T between the action of the two electromagnetic currents and the mean free time for the debris from the struck nucleon to interact with the residual nucleus. In the space-time description of deep-inelastic scattering in the quark-parton model,[17] T and $m_N x$ are conjugate variables. From our estimate of the final-state interaction time we find that it is less than T for $x<0.1-0.3$. Not only is the kinematics in Eq. (2.7) altered (which would lead to a smearing of the nucleon structure function), but, in addition, the second current matrix element in that equation becomes significantly different from the first. In the small-x region, therefore, the neglect of final-state interactions may not be justified. Further difficulties with the small-x region are discussed at the end of Sec. III. It may be the case, therefore, that low-order moments of the structure functions (or quark distribution functions) cannot be reliably calculated in a convolution model since they involve integration over all x.

B. The Callan-Gross relationship

One may now easily prove from Eqs. (2.19) and (2.20) that if the nucleon structure functions obey the Callan-Gross relation, so do the nuclear structure functions. Suppose that indeed

$$F_{2,N}(\tilde{x},\tilde{Q}^2)=2\tilde{x}F_{1,N}(\tilde{x},\tilde{Q}^2)\ , \quad (2.21)$$

and substitute this into Eq. (2.20). Dividing both sides of the resulting equation by $2x$ yields

$$\frac{\bar{F}_{2,A}(x,Q^2)}{2x}=\sum_{\alpha}\int d^3k\frac{m_N}{E_N(\mathbf{k})}\rho_{\alpha}(\mathbf{k})F_{1,N}(\tilde{x},\tilde{Q}^2)\ . \quad (2.22)$$

From Eq. (2.19) this can be rewritten in the form

$$\bar{F}_{2,A}(x,Q^2)=2\frac{m_N x}{m_A}\bar{F}_{1,A}(x,Q^2)=2x_A\bar{F}_{1,A}(x,Q^2)\ . \quad (2.23)$$

Equation (2.23) is just the Callan-Gross relationship for a nuclear target.

III. DISCUSSION OF THE FORMAL RESULTS

A. The convolution formula

It is of some value to change one variable of integration in Eqs. (2.19) and (2.20) from k_z to $y\equiv k^+/m_N$. This will not only make the comparison with earlier work much simpler, but will also enable us to discuss quantum chromodynamics (QCD) evolution in a straightforward manner. Although we shall consider only Eq. (2.20) for $F_{2,A}(x,Q^2)$ in detail, exactly the same procedure can be applied to Eq. (2.19).

Let us rewrite Eq. (2.20) in the form

$$\bar{F}_{2,A}(x,Q^2)=\sum_{\alpha}\int dk_z\int d^2k_\perp\int dy\,\delta\left[y-\frac{E_N(\mathbf{k})+k_z}{m_N}\right]\frac{m_N}{E_N(\mathbf{k})}\rho_{\alpha}(\mathbf{k})\frac{x}{\tilde{x}}F_{2,N}(\tilde{x},\tilde{Q}^2)\ . \quad (3.1)$$

Recall, from Eqs. (2.12)–(2.14), that $\tilde{x}$ and $\tilde{Q}^2$ depend on $\mathbf{k}^2$, the square of the momentum of the struck nucleon. (We are working in the rest frame of the target.) In order to proceed we make one approximation, namely, to replace $\mathbf{k}^2$ wherever it appears [except in $\rho_\alpha(\mathbf{k})$] by its average value in state α, $\langle \mathbf{k}^2 \rangle_\alpha$. If we then define [cf. Eq. (2.12)]

$$\delta_\alpha = \frac{E_N(\langle \mathbf{k}^2 \rangle_\alpha) + E_\alpha(\langle \mathbf{k}^2 \rangle_\alpha) - m_A}{m_N} , \tag{3.2}$$

$$\langle E_N \rangle_\alpha = E_N(\langle \mathbf{k}^2 \rangle_\alpha) , \tag{3.3}$$

and realize, from Eq. (2.14), that

$$\tilde{x} = (x + \delta_\alpha)/y , \tag{3.4}$$

and interchange the order of integration in Eq. (3.1), it becomes

$$\bar{F}_{2,A}(x,Q^2) = \sum_\alpha \int dy \int dk_z \delta \left[y - \frac{\langle E_N \rangle_\alpha + k_z}{m_N} \right] \int d^2k_\perp \frac{m_N}{\langle E_N \rangle_\alpha} \rho_\alpha(\mathbf{k}) \frac{xy}{(x+\delta_\alpha)} F_{2N} \left[\frac{x+\delta_\alpha}{y}, Q^2 \left[1 + \frac{\delta_\alpha}{x} \right] \right] . \tag{3.5}$$

Because $\langle E_N \rangle_\alpha$ and δ_α are now constants, independent of k_z, the integral over k_z can be performed trivially, giving

$$\bar{F}_{2,A}(x,Q^2) = \sum_\alpha \int dy\, f_\alpha(y) \frac{m_N y x}{\langle E_N \rangle_\alpha (x+\delta_\alpha)} F_{2,N} \left[\frac{x+\delta_\alpha}{y}, Q^2 \left[1 + \frac{\delta_\alpha}{x} \right] \right] , \tag{3.6}$$

where

$$f_\alpha(y) = m_N \int d^2k_\perp \rho_\alpha(\mathbf{k}) |_{k_z = m_N y - \langle E_N \rangle_\alpha} . \tag{3.7}$$

This can be made even more concise by defining

$$\bar{f}_\alpha(y) = \frac{m_N}{\langle E_N \rangle_\alpha} y f_\alpha(y) , \tag{3.8}$$

in terms of which

$$\bar{F}_{2,A}(x,Q^2) = \sum_\alpha \left[1 + \frac{\delta_\alpha}{x} \right]^{-1} \int dy\, \bar{f}_\alpha(y) F_{2,N} \left[\frac{x+\delta_\alpha}{y}, Q^2 \left[1 + \frac{\delta_\alpha}{x} \right] \right] . \tag{3.9}$$

Except for the correction factor $(1+\delta_\alpha/x)$ which appears in three places in Eq. (3.9) this is the standard form of convolution for Fermi-motion corrections in deep-inelastic scattering from a nucleus. At this point we note that Eq. (3.7) for $f_\alpha(y)$ is identical to Eq. (2.20) of Dunne and Thomas[12] (DT), except that there the condition was $k_z = m_N y - m_i$. Thus, the analytic expressions obtained by DT for the harmonic oscillator model may be taken over here by simply replacing their m_i by $\langle E_N \rangle_\alpha$. For example, for the 0s state we would find

$$f_\alpha(y) \sim \exp[-(m_N y - \langle E_N \rangle_\alpha)^2 / m_N \omega] , \tag{3.10}$$

which peaks at $y = \langle E_N \rangle_\alpha / m_N > 1$. The peak in $f_\alpha(y)$ would also occur at $y > 1$ for all other states α.

In concluding this discussion we remind the reader that Eq. (3.6) [or Eq. (3.9)] is only approximate, because we replaced $\mathbf{k}^2$ by $\langle \mathbf{k}^2 \rangle_\alpha$. On the other hand, we have been able to check this approximation to some extent by instead replacing $\mathbf{k}^2$ by $(\langle \mathbf{k}_\perp^2 \rangle_\alpha + k_z^2)$ and performing the k_z integration completely. The difference between the two calculations for one nucleus was less than 1% for $x < 0.8$.

B. QCD evolution

By using the operator product expansion one has been able to prove (independent of the taret involved) that beyond some unspecified momentum scale the moments of structure functions like F_2 should evolve with Q^2 according to perturbative QCD. In first-order QCD this means that (for nonsinglet moments M_n)

$$\frac{M_n(Q^2)}{M_n(Q_0^2)} = \left[\frac{\alpha(Q^2)}{\alpha(Q_0^2)} \right]^{d_n} , \tag{3.11}$$

where d_n (>0) is the appropriate anomalous dimension.

Suppose that, as in Refs. 12 and 15 one has an expression of the form

$$F_A(x,Q^2) = \int dy\, \bar{f}(y) F_N \left[\frac{x}{y}, Q^2 \right] ; \tag{3.12}$$

then it follows directly from the definition of the moments

$$M_n^{(A,N)}(Q^2) = \int dx\, x^n F_{(A,N)}(x,Q^2) \tag{3.13}$$

and Eq. (3.12) that

$$M_n^A(Q^2) = \left[\int dy\, y^{n+1} \bar{f}(y) \right] M_n^N(Q^2) . \tag{3.14}$$

Since the term in large brackets is independent of Q^2,

$$\frac{M_n^A(Q^2)}{M_n^A(Q_0^2)} = \frac{M_n^N(Q^2)}{M_n^N(Q_0^2)} , \tag{3.15}$$

as it should.

If, one the other hand, we consider Eq. (3.9), the presence of the factor $(1+\delta_\alpha/x)$ appears to present a problem. In particular, the nth moment of the nuclear structure function will have lower moments of the nucleon mixed in. At high enough values of Q^2 the lower moment will dominate $(d_{n+1} > d_n)$ and the nth nuclear moment will evolve incorrectly with increasing Q^2. Although this observation presents a severe problem of principle for the present formulation of the problem, we believe the following comments should be taken into account. As discussed at the end of Sec. II, the impulse approximation for the nucleus is expected to break down somewhere in the small-x region. Thus, the formulas derived here should not be taken too seriously for x below some minimum value, $x_{\min}$, which we expect to be of order (0.1–0.3). Provided this limitation of the calculation is kept in mind we see no major problem with proceeding to use it. (We note that the small-x region is anyway subject to other corrections, such as contributions from the virtual mesons responsible for nuclear binding, and shadowing.)

IV. NUMERICAL CALCULATIONS

We have investigated the predictions of our model for deep-inelastic scattering on ^{4}He, ^{12}C, and ^{40}Ca. Following DT modified as described below Eq. (3.9), we use a simple harmonic oscillator model for the nuclear density. The separation energies and harmonic oscillator parameters were also taken from DT (Table I of Ref. 12), in order to simplify comparison between the two models. Only one other detail needs to be mentioned. The average value of $\mathbf{k}^2$ for each single-particle state enters the present calculation through $\langle E_N \rangle_\alpha$—see Eqs. (3.3) and (3.10). Rather than use the values given by the harmonic oscillator wave functions we have chosen to use values calculated for the eigenstates of a Woods-Saxon single-particle potential.[18] These are summarized in Table I. For the deuteron we prefer $\langle \mathbf{k}^2 \rangle_d^{1/2} = 140$ MeV/c, but we also show some results with 170 MeV/c in order to illustrate the sensitivity.

The nucleon structure function $F_{2,N}(\tilde{x}, \tilde{Q}^2)$ appearing

TABLE I. Square root of the average value of $\mathbf{k}^2$ (in MeV) used for each single-particle state. See Eqs. (3.3) and (3.10).

Single-particle state	^{4}He	^{12}C	^{40}Ca
0s	170	149	110
0p		194	156
1s			195
0d			180

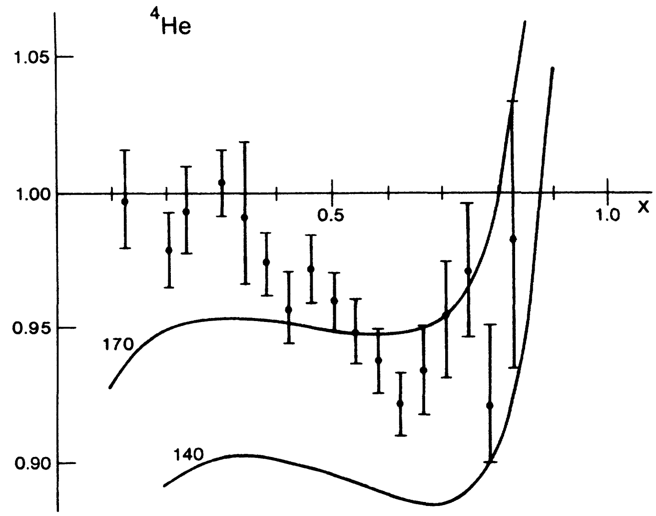

FIG. 2. The calculated EMC ratio of the structure function $\bar{F}_2$ for ^{4}He divided by that for deuterium vs x. The data are from Bodek *et al.* (Ref. 1). The sensitivity to our input is shown by the two theoretical curves for this ratio, which are labeled by the mean square momentum in deuterium, taken as 140 MeV/c (preferred) and 170 MeV/c.

in Eq. (3.9) is also the same as DT, who, in turn, took it from the analytic parametrization of Buras and Gaemers[19]—adjusted for three flavors. The valence quark distribution has the form

$$xV(x,Q^2) = \frac{3\Gamma(a+b)}{\Gamma(a)\Gamma(b)} x^a (1-x)^{b-1} , \tag{4.1}$$

with

$$a = 0.7 - 0.163t , \quad b = 3.6 + 0.741t , \tag{4.2}$$

and

$$t = \ln[\ln(Q^2/\Lambda^2)/\ln(Q_0^2/\Lambda^2)] . \tag{4.3}$$

Here we take $\Lambda = 0.3$ GeV and $Q_0^2 = 1.8$ GeV^2. For the sea quarks we use

$$xS(x) = \alpha \left[\frac{\alpha}{\beta} - 1 \right] (1-x)^{(\alpha/\beta - 2)} \tag{4.4}$$

with

$$\alpha = 0.2382 e^{-0.617t} + 0.3598 - 0.488 e^{-0.395t} , \quad \beta = 0.003\,23 e^{-1.2t} + 0.163 e^{-0.549t} - 0.157 e^{-0.593t} . \tag{4.5}$$

Finally, $F_{2,N}(x,Q^2)$ is related to $xV(x)$ and $xS(x)$ as

$$F_{2,N}(x,Q^2) = [5xV(x) + 4xS(x)]/18 \tag{4.6}$$

for an isoscalar target.

The results of our calculations are shown in Figs. 2–4. Clearly, while the large-x behavior is reasonable and the EMC ratio is less than unity at intermediate x, the shape of the prediction is quite different from the Stanford

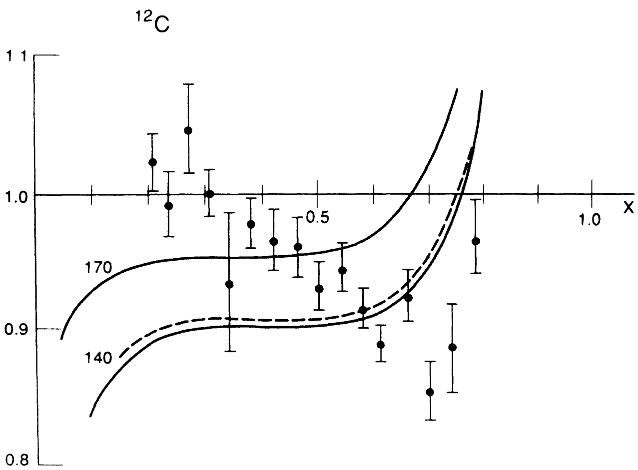

FIG. 3. The EMC ratio for ^{12}C—otherwise as for Fig. 2. The dashed curve shows the small effect of the explicit modification of the Q^2 argument of $F_{2,N}$ in Eq. (3.9).

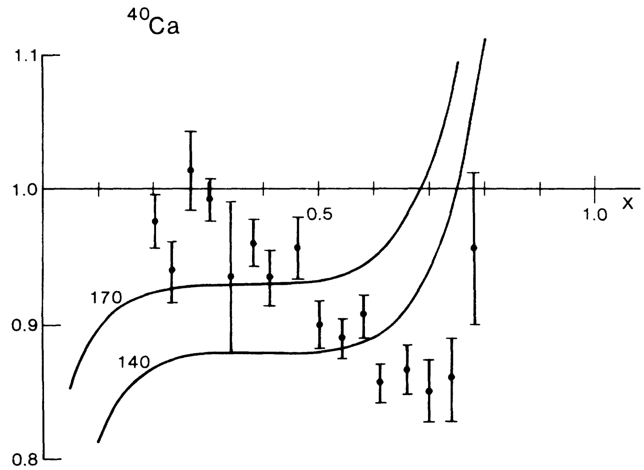

FIG. 4. The EMC ratio for ^{40}Ca— otherwise as for Fig. 2.

Linear Accelerator Center (SLAC) data. In particular, the factor $(1+\delta_\alpha/x)^{-1}$ in Eq. (3.9) is in large part responsible for the pronounced dip at small x. We have also tested the importance of the explicit appearance of $\tilde{Q}^2$ rather than Q^2 in Eq. (3.9). It is very small as shown by the dashed versus the solid curve labeled 140 for ^{12}C. (The labels are the values of $\langle \mathbf{k}^2 \rangle^{1/2}$ used for deuteron—recall that 140 MeV is our preferred value.)

Our calculation of the structure function of the deuteron itself is in fair agreement with the data for $x>0.3$, but it exhibits the same defect of falling below the data at smaller x, as for heavier nuclei. Using a nucleon-nucleon potential with a repulsive core gives an F_2^D that is very close to that obtained from a harmonic oscillator with $\langle \mathbf{k}^2 \rangle^{1/2}=140$ MeV. The contribution of the kinetic energy of the recoiling spectator nucleon to Δ in Eq. (2.12) is included in these calculations.

V. CONCLUSION

We have examined the predictions of the instant form of nuclear dynamics for deep-inelastic scattering from a variety of targets. The resulting convolution formulas involve the structure function of physical nucleons, which is obtained directly from experiment without any need for an off-mass-shell extrapolation. In the instant version of dynamics the arguments of the nucleon structure function $F_N(\tilde{Q}^2,\tilde{x})$ differ from the arguments of the nuclear structure function $F_A(Q^2,x)$ by amounts that depend explicitly on the *binding energy* as well as the kinetic energy of the nucleons in the nucleus. In particular, the change in the scaling variable x via Eqs. (2.14) and (2.12) is a distinctive feature of the instant version.

In each version of dynamics one must choose appropriate nuclear wave functions, and we have used traditional wave functions of the harmonic oscillator shell model type. The numerical results are somewhat disappointing for x less than about 0.4. There is some discussion at the end of Sec. II to the effect that the convolution formula may break down in this region, but it is not at all clear why it should be so bad. This is particularly mysterious if we compare with the results of Dunne and Thomas who, while allowing the nucleon to go off shell in a somewhat arbitrary way, nevertheless found quite reasonable values at small x—independent of the off-shell prescription. Certainly there needs to be a lot more work on the question of final-state interaction corrections in nuclear deep-inelastic scattering.

It is widely recognized that if a system is bound some of the momentum must be carried by the mesons responsible for the binding. This has led a number of authors to combine the binding and pionic "explanation" of the EMC effect.[10,12,13] There is a possibility that if this were done consistently within the instant form of dynamics, the dip at small x might be compensated to some extent. This is our next priority.

For the present, the EMC effect remains more of a mystery than ever. Even though there may be important new physics hidden in the data, it will not be possible to extract it until the binding corrections can be dealt with in a reliable manner.

During the course of this investigation we learned of other work using instant dynamics.[20,21] Equation (3) of Ref. 20 is the same as Eq. (2.15) of the present paper, *except* for the presence of $k^+=E_N(\mathbf{k})+k_z$ (called p_i^+) in place of $E_N(\mathbf{k})$ in the denominator. The factor of $E_A(\mathbf{p})/E_N(\mathbf{k})$ in Eq. (2.15) arose from the normalization of the wave functions, as given in Eqs. (2.8) and (2.9), together with the requirement that $W_A^{\mu\nu}$ transform like a Lorentz tensor. The step from $W^{\mu\nu}$ to the structure functions $F_{1,2}$ that are actually measured is simply given by the projection operators in Eqs. (2.17) and (2.18). Consequently, Eq. (21) in Ref. 20, which relates F_2^A to F_2^N, also differs from our Eq. (2.20) by a factor of $E_N(\mathbf{k})/k^+$, which can depart from unity by a significant amount.

ACKNOWLEDGMENTS

L.H. wants to thank M. Bolsterli, W. R. Gibbs, and E. Moniz for discussions; he also thanks F. Coester for a careful reading of the manuscript and suggestions for clarifying it. A.W.T. would like to acknowledge discussions with R.P. Bickerstaff. This work was supported in part by the Australian Research Grants Commission, the University of Adelaide, the U.S. Department of Energy, and the Australian-American Education Foundation, which granted L.H. a Fulbright Senior Scholar Award.

APPENDIX

As we remarked at the end of Sec. II A, the neglect of terms in $(\Delta/m_N x)^2$ is not justifiable at small x. It is possible to retain all the terms in $(\Delta/m_N x)$ which survive in the Bjorken limit. Instead of Eqs. (2.19) and (2.20) we find

$$\bar{F}_{1,A}(x,Q^2)=\sum_\alpha \int d^3k \frac{m_A}{E_N(\mathbf{k})}\rho_\alpha(\mathbf{k})\left\{\left[1-\frac{\Delta^2}{8m_N^2x^2(1+\Delta/m_N x)}\right]F_{1,N}(\tilde{x},\tilde{Q}^2)+\frac{\Delta^2}{8m_N^2x^2(1+\Delta/m_N x)}\frac{F_{2,N}(\tilde{x},\tilde{Q}^2)}{2\tilde{x}}\right\}, \tag{A1}$$

and

$$\bar{F}_{2,A}(x,Q^2)=\sum_\alpha \int d^3k \frac{m_N}{E_N(\mathbf{k})}\rho_\alpha(\mathbf{k})\left[\frac{x}{\tilde{x}}\frac{(1+\Delta/m_N x+3\Delta^2/8m_N^2x^2)}{(1+\Delta/m_N x)}F_{2,N}(\tilde{x},\tilde{Q}^2) -\frac{3}{4}\left[\frac{\Delta}{m_N x}\right]^2\frac{1}{(1+\Delta/m_N x)}xF_{1,N}(\tilde{x},\tilde{Q}^2)\right]. \tag{A2}$$

It is easily seen that (A1) and (A2) reduce to (2.19) and (2.20), respectively, when $(\Delta/m_N x)$ is small. A little more effort is required to check that if $F_{1,N}$ and $F_{2,N}$ satisfy the Callan-Gross relationship Eq. (2.21), Eqs. (A1) and (A2) reduce *exactly* to (2.19) and (2.20).

We are grateful to Dr. R. P. Bickerstaff for drawing to our attention our initial oversight concerning terms in $(\Delta/m_N x)^2$.

[1]A. Bodek *et al.*, Phys. Rev. Lett. **51**, 534 (1983).
[2]K. Rith, Inst. Phys. Conf. Ser. **86**, 395 (1987).
[3]J. J. Aubert *et al.*, Phys. Lett. **123B**, 275 (1983).
[4]F. E. Close *et al.*, Phys. Rev. D **31**, 1004 (1985).
[5]L. S. Celenza *et al.*, Phys. Rev. Lett. **53**, 892 (1984).
[6]A. Krzywicki *et al.*, Phys. Rev. D **14**, 152 (1976); C. E. Carlson and T. J. Havens, Phys. Rev. Lett. **51**, 261 (1983); S. Date, Prog. Theor. Phys. **70**, 1682 (1983).
[7]O. Nachtmann and H. J. Pirner, Z. Phys. C **1**, 227 (1984).
[8]C. H. Llewellyn Smith, Phys. Lett. **128B**, 107 (1983).
[9]M. Ericson and A. W. Thomas, Phys. Lett. **128B**, 112 (1983).
[10]E. L. Berger, F. Coester, and R. B. Wiringa, Phys. Rev. D **29**, 398 (1984); also see. E. L. Berger and F. Coester, Argonne Report No. ANL-HEP-PR-87-13, 1987.
[11]S. V. Akulinichev *et al.*, Phys. Rev. Lett. **55**, 2239 (1985).
[12]G. V. Dunne and A. W. Thomas, Phys. Rev. D **33**, 2061 (1986); Nucl. Phys. **A455**, 701 (1986).
[13]R. P. Bickerstaff and A. W. Thomas, Phys. Rev. D **35**, 108 (1987).
[14]E. L. Berger and F. Coester, Phys. Rev. D **32**, 1071 (1985).
[15]E. L. Berger and F. Coester, in *Workshop on Nuclear Chromodynamics, Quarks and Gluons in Particles and Nuclei,* edited by S. Brodsky and E. Moniz (World Scientific, Singapore, 1986), p. 255.
[16]P. A. M. Dirac, Rev. Mod. Phys. **21**, 392 (1949).
[17]R. L. Jaffe, in *Relativistc Dynamics and Quark-Nuclear Physics,* edited by M. B. Johnson and A. Picklesimer (Wiley, New York, 1986), p. 537.
[18]R. Barrett and D. F. Jackson, *Nuclear Sizes and Structure* (Oxford University Press, Oxford, 1977).
[19]A. Buras and K. Gaemers, Nucl. Phys. **B132**, 249 (1978).
[20]M. B. Johnson and J. Speth, Nucl. Phys. **A470**, 488 (1987).
[21]L. S. Kisslinger and M. B. Johnson, Phys. Rev. C **36**, 1081 (1987). This paper considers explicit quark effects on the structure functions using both instant dynamics and light-front dynamics. The calculations using instant dynamics contain the same normalization as in Ref. 20; the reason we think this is wrong is detailed in the text.