Investigating fundamental antiproton-nucleon interactions by means of \bar{p} -nucleus elastic scattering

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Antinucleon-nucleus elastic scattering is addressed from the point of view of the off-shell properties of the two-body \bar{p} -nucleon system. The energy dependence of the interaction is studied. Comparison with \bar{p} -¹²C elastic-scattering data at 50 MeV is satisfactory but shows very little sensitivity to the underlying potential. Scattering data at lower projectile energies should improve the situation.

I. INTRODUCTION

One of the most interesting subjects in modern physics is the interaction of matter and antimatter. Intensive work is proceeding on many facets of the antinucleonnucleon interaction.¹⁻³ Recent models involve antinucleon-nucleon exchange, direct quark rearrangement, meson exchange, and quark annihilation.⁴ Tests of these models involve detailed studies of the $\bar{p}p$ annihilation channels,⁵ cross sections, and spin observables. These tests provide essential information about on-shell properties of the $\bar{p}p$ system. Off-shell properties, such as the range of the interaction, cannot be inferred unambiguously from studies of the two-body system, however.

Within the framework of potential theory, the *t*-matrix of a two-body system is the solution of a Lippmann-Schwinger (LS) equation t = V + Vgt, where $g = (E - K_{\overline{p}} + K_N)^{-1}$ is the Green's function, $K_{\overline{p}}$ is the kinetic energy of the antiproton, and K_N is the kinetic energy of the nucleon. V is the fundamental antiprotonnucleon potential. It is inferred from deeper (e.g., fieldtheoretic) considerations. V is an object of fundamental theoretical significance; a major purpose of this paper is to determine whether experimental \overline{p} -nucleus elastic scattering can shed light on this fundamental quantity.

The off-shell properties of the antinucleon-nucleon interaction can, in principle be determined from experiments involving a antinucleon-nucleus system. One way to learn about the annihilation range, for example, would be to look for annihilation processes occurring on more than one nucleon.⁶ The analysis of such processes, while very interesting, involves ambiguities. For example, the pions produced in the annihilation of the antinucleon on a nucleon may be absorbed by neighboring ones. The resulting nucleon spectra may be difficult to distinguish from those produced by a true three-body annihilation.

The evaluation of V requires knowledge of both the onand off-shell values of the *t*-matrix. Free \bar{p} -nucleon processes probe only the on-shell portion of the *t*-matrix. We may learn about the off-shell part of the *t*-matrix through a study of many-body \bar{p} -nucleus processes.

The \bar{p} -nucleus optical potential is the expectation value of τ (the "t matrix" describing the interaction of an antiproton with a nucleon bound in a nucleus) with respect to the ground-state nuclear wave function. The operator τ is defined by $\tau = V + VGQ\tau$ where Q is a projection operator onto excited nuclear states and G is the \bar{p} nuclear Green's function

$$G = (E - K_{\overline{p}} - H_{\text{Nuc}})^{-1}$$

We will approximate H_{Nuc} , the nuclear Hamiltonian, by $K_N + U_{NC}$ where U_{NC} is the potential between the struck nucleon and the residual nucleus ("core"). U_{NC} accounts for nucleon binding corrections.

In solving the LS equation determining τ a complete set of nucleon energy eigenstates is inserted^{7,8} between G and Q. The "blocking" operator Q excludes the occupied nuclear levels. The resulting expression for the optical potential contains a sum over all of the nuclear levels [see Eq. (1)]. τ has been eliminated in Eq. (1) so that only V and the free *t*-matrix appear. The argument of *t* is the incident \bar{p} energy shifted by the energies of the nuclear levels; because of this off-shell values of *t* are needed in the evaluation of the optical potential. The action of Q is reflected in the replacement of *t* by V for the occupied levels in the optical potential. In this way the $\bar{p}p$ potential explicitly enters into the calculation of the optical potential.

The present work deals with finite nuclei, but the ideas are easily translated into the language of nuclear matter.

If the momentum transfer resulting from the collision is not sufficient to lift the struck nucleon above the Fermi sea, then the collision process is strongly modified; the scattering is largely Pauli blocked and the $\bar{p}p$ potential controls the scattering. For large momentum transfer the blocking is minimal and, roughly speaking, the two-body *t*-matrix controls the scattering.

In low-energy pion-nucleus scattering the two-body pion-nucleon potential is real. The replacement at low momentum transfer of the complex *t*-matrix by a real potential increases the transparency of the nuclear medium; it may be partly responsible for the weak pion-nuclear optical potential seen at low energies.⁹

In contrast, the $\bar{p}p$ potential has a larger imaginary part than the corresponding *t*-matrix. The resulting \bar{p} nucleus optical potential is even more opaque than a conventional optical potential.

Antiproton-nucleus scattering experiments on ${}^{12}C$ and ${}^{40}Ca$ have been performed at the Low Energy Antiproton Ring (LEAR).¹⁰ The differential cross sections show typical diffractive behavior. It is relatively straightforward to fit the data with phenomenological optical or black-sphere¹¹ models. However, optical-model calculations based upon the fundamental antinucleon-nucleon *t*-matrix, ${}^{12-15}$ while fairly successful, sometimes require that the imaginary parts be increased phenomenological ly¹² to obtain agreement with experiment. Medium corrections, including recoil, binding and blocking may

help to resolve this discrepancy. Reference 15 would seem to be the closest to the present work.

In Sec. II we review the formalism of the three-body optical potential; blocking, binding, and recoil corrections are emphasized. Section III gives a detailed analysis of the effect of the different medium corrections on the predicted differential cross sections of elastic \bar{p} -nucleus scattering. A comparison is made with existing data.

II. FORMULATION OF THE MODEL

We employ an optical potential, derived in Refs. 7 and 8, which embodies the concepts outlined in the Introduction. The derivation is based on a three-body model (projectile, struck nucleon, and core). The fundamental approximation made in the derivation is that the projectile mass is much smaller than the sum of the projectile and the nucleon mass. This approximation is less accurate for antiproton scattering than for pion scattering, for which the optical potential was originally derived. (The need for this approximation is somewhat ameliorated if the range of the interaction is short.) In order to extract precise values of basic parameters, a better theory may be needed, however the present theory should suffice to map out the most interesting qualitative features.

The optical potential is

$$\langle \mathbf{k}' | V(E) | \mathbf{k} \rangle = \sum_{A} \int \int d\mathbf{q} d\mathbf{q}' \phi_{A}(\mathbf{q}') \phi_{A}^{*}(\mathbf{q}) \sum_{A'} \widetilde{\phi}_{A'}(\mathbf{P}') \langle \mathbf{p}' | \vartheta^{AA'} | \mathbf{p} \rangle \widetilde{\phi}_{A'}^{*}(\mathbf{P}) , \qquad (1)$$

where

$$\langle \mathbf{p}' | \vartheta^{AA'} | \mathbf{p} \rangle = \begin{cases} \langle \mathbf{p}' | V_{\overline{NN}}(E + E_A - \widetilde{E}_{A'}) | \mathbf{p} \rangle & \text{if } A' \text{ is a state occupied in the target ,} \\ \langle \mathbf{p}' | t_{\overline{NN}}(E + E_A - \widetilde{E}_{A'}) | \mathbf{p} \rangle & \text{if } A' \text{ is a state not occupied in the target.} \end{cases}$$

The indices A = (n,l) and A' = (n',l') label the eigenstates of H_{Nuc} . See Eqs. (12) and (13) of Ref. 7 with the correction for frame-transformation effects given in Sec. II of Ref. 8. In this expression **k** and **k'** are the initial and final \bar{p} momenta, and

$$\mathbf{p} = -a\mathbf{q} + b\mathbf{k}$$

$$\mathbf{P} = \mathbf{q} + c\mathbf{k}$$
,

where

$$a = \frac{\omega_{\bar{p}}}{\omega_N + \omega_{\bar{p}}} \approx \frac{1}{2} ,$$

$$b = \frac{\omega_N(\omega_{\bar{p}} + \omega_N + \omega_C)}{(\omega_N + \omega_C)(\omega_{\bar{p}} + \omega_N)} \approx (A+1)/2A \approx \frac{1}{2} ,$$

$$c = \frac{\omega_C}{\omega_N + \omega_C} \approx (A-1)/A \approx 1 .$$

 $\omega_{\bar{p}}$, ω_N , and ω_C are the energies of the incident \bar{p} , stuck nucleon, and nuclear core (of mass number A-1) in the \bar{p} -nucleus center-of-mass frame.

The ϕ_A are single-particle solutions of the Schrödinger equation (with relativistic kinematics) for a nucleon in a central potential well, and $\tilde{\phi}_{A'}$ are single-particle states for a particle of twice the nucleon mass (i.e., the sum of the struck nucleon and projectile mass) in the same well. E_A and $\tilde{E}_{A'}$ are the corresponding energy eigenvalues. The simple form of the $\vartheta_{AA'}$'s given in Eq. (1) comes about because we have used an infinite square-well potential (of radius R) in the calculation of the intermediate nucleon-core spectrum. For this potential the wave functions ϕ_A and $\tilde{\phi}_{A'}$ are spherical Bessel functions: $j_l(k_{nl}r)$, where $k_{nl} = z_{nl}/R$, and z_{nl} are the zeros of the *l*th spherical Bessel function. The wave functions for an infinite square-well potential are independent of the mass of the particle.

The integration over q and q' in Eq. (1) would be greatly facilitated if we could take $a \approx 0$ because then $\mathbf{p} \approx b\mathbf{k}$, and the matrix element of ϑ is independent of both q and q'. This approximation is what is referred to as "neglecting recoil corrections." This approximation (although sometimes made) is expected to be poor for \overline{p} scattering because in this case $a \approx 0.5$. We will study the error introduced by omitting recoil corrections in the next section.

The approximate experimental resolution of the LEAR data is ± 2 degrees. We have averaged our calculated differential cross section over this angular range in those calculations that are compared with the data. In the region 10–15 degrees the differential cross section falls increasingly rapidly with angle, hence the averaging procedure enhances the cross section substantially. There is also a considerable filling in of the minimum (and even a slight shift in its position) from angle averaging.

The approximation technique introduced in Ref. 7, Sec. III is used to put the nonlocality inherent in Eq. (1) into a manageable form. We have used a spin-average *t*matrix and potential and have neglected any \bar{p} -nucleus spin-orbit interaction.

The derivation of the present optical potential is rigorously correct only for an energy-independent potential such as the Dover-Richard² potential. Our application to the Paris potential, ¹ which is energy dependent, constitutes an additional approximation.

For the calculations presented here we have used a shell-model density with the radius and depth of the well adjusted to reproduce the point nucleon distribution extracted from the experimental charge density. We assume that neutrons and protons have the same distribution.

III. THE PHYSICS OF THE CONNECTION BETWEEN THE $\bar{p}p$ AND THE $\bar{p}A$ INTERACTION

We now discuss the different elements of physics that relate the nuclear scattering amplitude to the fundamental interaction. Particular emphasis is placed on the features of the two-body interaction, which are not available from the two-body data.

A. Calculations without the effect of blocking

In pion-nucleus scattering recoil introduces an admixture of the *p*-wave amplitude to the *s*-wave one. The contribution is proportional to the ratio of the pion to nucleon mass, which is small. The effect is not negligible, however, because *p*-wave pion-nucleon interaction is larger than the *s*-wave interaction even at rather low energies; a small admixture still produces a large effect. In the $\bar{p}p$ case the *p*-wave amplitude is smaller than the *s*wave one, but the mass ratio is much larger (i.e., 1); again the effect is important. Recoil is usually neglected in the medium corrections to the optical potential in nuclear matter. Figure 1 shows that these corrections cause a significant shift of the minimum in the differential cross section. This is clearly a very important effect.

We next examine nucleon binding corrections. We



FIG. 1. Comparison of calculations using the free *t*-matrix, with and without recoil. The data is from Ref. 10.

temporarily omit the blocking operator Q to separate the effect of binding from that of blocking. If the levels are not blocked, the *t*-matrix replaces ϑ in Eq. (1), hence the $\bar{p}N$ potential does not directly enter into the calculation. The nuclear energy shifts $(E_A - \tilde{E}_{A'})$ present in Eq. (1) describe the excitation energy of the intermediate nuclear states. As discussed in Sec. II the states are chosen as eigenstates of H_{Nuc} . In the absence of the energy shifts the *t*-matrix could be factored out of the sum over A', the intermediate nuclear states. The resulting sum yields



FIG. 2. Calculations demonstrating the effect of energy shifts.

the nuclear form factor, and the $t\rho$, or "closure" optical potential is obtained. The importance of the binding corrections can be judged from Fig. 2 where a calculation with the use of the free *t*-matrix is compared with that using a "shifted" one for an energy of 46.8 MeV. Experimental data from LEAR is also plotted.¹⁰ It is worth emphasizing that we do not use a single energy shift, but instead perform the sum over a complete set of intermediate nuclear states in Eq. (1) to obtain an effective *t*matrix⁷ for each partial wave.

The off-shell dependence of the *t*-matrix may also play an important role in the scattering of a \overline{p} from a close pair of nucleons. The \overline{p} scattered wave emerging from one nucleon may reach the second nucleon while it is still being distorted by the first nucleon. In other words, the scattering occurs when the antiproton "sees" two nucleons at once. This is the classic off-shell scattering process. The relevant range in this "pair" scatter is that of the *t*-matrix, and it may be very different from the range of the potential, as we now discuss.

The half-off-shell t-matrix is defined by

$$t_{l,\overline{N}N}(k,q) = \int r^2 dr [V_{\overline{N}N}\psi_k]_l(r) j_l(qr)$$
⁽²⁾

and may be approximated by

$$t_{I\overline{N}N}(k,q) \simeq t_l^R(k) v_l^R(q) + t_l^I(k) v_l^I(q) .$$
(3)

 $t_l^c(k)$ are the real (c = R) and imaginary (c = I) parts of the on-shell *t*-matrix; the corresponding form factors are assumed to be

$$v_l^c(q) = (k^2 + \alpha_{c,l}^2) / (q^2 + \alpha_{c,l}^2) .$$
(4)

The ranges $\alpha_{c,l}$ are estimated from the q dependence of Eq. (2). Because the interaction is strongly absorbing, the wave function ψ is highly damped inside of some spatial region. We see from Eq. (2) that it is the range of the product $V\psi$ in coordinate space which determines the range of the off-shell t-matrix. If the potential is strongly absorbing the wave function may be very strongly absorbed, typically inside of 1 fm, and the range of the tmatrix may have little reference to the range of the potential. If the fundamental $\overline{p}p$ system were weakly interacting the wave function could be approximated by a plane wave in Eq. (2). In this case the momentum dependence (as well as the value) of the *t*-matrix would be given by the Fourier transform of V, and the range of the tmatrix would be the same as that of the potential. Calculations using the Paris potential show that those waves which are strongly interacting (the most important) display a range of the order of 1 fm^{-1} in momentum space, as expected from the preceding comments.

There is a second point that must be discussed concerning the range: the possible suppression of the onepion-exchange (OPE) contribution in elastic scattering from an isospin-zero nucleus. Since such a nucleus cannot emit a pion and remain in its ground state, it seems plausible that the long-range interaction coming from one pion exchange is not present. Because the OPE range is very long, the effective interaction range would then be reduced if OPE were omitted. In fact, we can argue that at the level of the $\bar{p}p$ t-matrix, the effect of OPE is actually present. For strongly interacting systems particle exchange is a concept valid only for a potential (or the kernel of a Bethe-Salpeter equation). If the interaction is strong, many iterations of the potential are necessary to accurately produce the *t*-matrix. In fact, for the isospin combination needed for the first order optical potential for an N = Z nucleus, $(3T_1 + T_0)/4$, and for the weakly scattered waves (for which the Born approximation holds and hence *t* is proportional to *V*) we observe significant cancellation between the two isospin components of the wave function for r > 1 fm. For the strongly interacting waves this cancellation, while still present, is minor. Figure 3 shows the sensitivity to the off-shell range and we see that, while the dependence is weak, the data prefers the longer range.

B. Calculations including blocking

We now restore the $\overline{p}N$ potential in the optical potential [Eq. (1)]. The very short-range behavior of the imaginary part of the Paris potential is not relevant to the fitting of the \overline{p} -nucleon scattering data, and its imaginary part is singular at the origin. The low-energy $\overline{p}A$ potential strength is roughly given by the volume integral of the $\overline{p}N$ potential. Upon comparison with potentials obtained for \overline{p} atoms it is immediately clear that this volume integral is much too large. Consequently, we have used a variant of the original Paris $\overline{p}N$ potential, which contains a cutoff at small values of r. We have multiplied the imaginary part of the Paris potential by the short-range cutoff function

$$f_C(r) = (1 - e^{-mr})^4$$

and recalculated χ^2 for $\overline{p}p$ scattering for various values of the mass parameter *m*. If *m* is two or three times the nucleon mass there is essentially no change in χ^2 from that given by the Paris potential. The volume integral, how-



FIG. 3. Differential cross sections showing the effects of the off-shell range of the *t*-matrix.

ever, is greatly reduced and is now consistent with the \bar{p} atom data. (This technique has already been applied¹⁶ to the atomic data. A complete survey of all available data would be valuable, but this is beyond the scope of the present work.) If the cutoff mass is further reduced to one nucleon mass a slight change in the χ^2 is observed. A value of the cutoff parameter less than about one nucleon mass begins to degrade the quality of fit to the two-body data. We have constructed optical potentials for m=1, 2, and 3 nucleon masses in order to investigate the model dependence of the potential. At 50 MeV the dependence is small, but see Fig. 6 for a low-energy result.

The importance of the "blocked" terms in Eq. (1) (for which the $\overline{p}N$ potential appears explicitly) depends on the incident energy. For elastic scattering the most important waves are those around l = kR, where k is the incident momentum and R is the nuclear radius. For \overline{p} nucleus scattering the very low partial waves are completely absorbed. Unfortunately, it is precisely these partial waves that are affected by the presence of the potential as we now see.

The operator Q prevents the occupation of the filled intermediate nuclear levels. On partial-wave analysis of Eq. (2) there are four relevant angular momenta: the angular momenta of the initial and intermediate nucleoncore states, the $\bar{p}N$ partial wave, and the $\bar{p}A$ partial wave. If for a given intermediate nucleon-core angular momentum these four angular momenta cannot be coupled to zero then the intermediate state does not contribute to the $\bar{p}A$ potential (see Clebsch-Gordan coefficients in Ref. 7). For the nucleus ¹²C, if only s and p waves contribute to the $\bar{p}N$ amplitude then only $l_{\bar{p}A} = 0,1,2$ can be blocked. Partial waves with l higher than 2 then will not have explicit contributions of V to the optical potential. For energies as low as 50 MeV, the first three partial waves are



FIG. 4. Calculations showing the effect of Pauli blocking on the recoiling nucleon at 46.8 MeV.



FIG. 5. Same as Fig. 4 except at 5-MeV incident kinetic energy.

essentially completely absorbed whether one uses the blocking operator Q or not. We conclude that at this energy the replacement of t by V is not expected to significantly affect the scattering cross section.

Figure 4 shows the differential cross section with and without blocking at 46.8 MeV for m equal to one nucleon mass. We see, as expected, that the effect is small at this energy.

Note that this result is different from that obtained



FIG. 6. Calculations with various assumptions about the potential used in the calculation of the Pauli effect.

with more classical strongly interacting projectiles such as protons or pions. For these cases the fundamental two-body interaction is governed at low energies by a real potential, while the *t*-matrix is complex. Thus the replacement of some of the terms in the summation of Eq. (1) of "t" by "V" normally reduces the absorptive content of the potential. Hence we are accustomed to associating the Pauli blocking with an increased transparency of the nucleus. For protons the effect is strong and persists even to 100-MeV incident energy. In the present case the replacement of t by V in the appropriate terms of Eq. (1) in fact often makes the \overline{p} -nucleus potential more absorptive.

ly annihilated so that little effect is seen. The secret to revealing the "potential dominated" partial waves is to *lower* the energy of the incident antiproton. If the energy is lowered sufficiently that these low partial waves become the principal contributors to the scattering and can be seen more readily. For ¹²C we require a value of k so that $kR \approx 1-2$, i.e., $K_{\overline{p}} \approx 3-13$ MeV. Figure 5 shows the effect of including the Pauli effect at a scattering energy of 5 MeV. We have now presented the data over the full angular range, but the momentum transfer spanned is only half that covered by the data at 46.8 MeV so the scattering theory can be con-

As already remarked, however, this additional absorption

occurs in partial waves that are already almost complete-

sidered to be of the same, or of greater accuracy. There is a noticeable sensitivity to the potential, even the relatively small angles of 30-40 degrees.

Figure 6 shows the effect of the variation of the $\overline{p}N$ potential. It is clear that features of the potential are much more discernable at this low energy than they were at 46.8 MeV. This is due to the fact that the "blocked" waves (essentially l=0, 1, and 2) are now the most important ones in describing the scattering.

In summary, we have found that antiproton-nucleus scattering is sensitive to the underlying $\overline{p}N$ potential V provided that the incident energy is very low, of the order of 5 MeV. There is little sensitivity to V at 46.8 MeV, where the experiment at LEAR was performed. At these higher energies other medium corrections, such as nucleon recoil, are of importance.

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