Bound states, resonances, and poles in low-energy $\bar{K}N$ interaction models

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The locations of the dynamic poles in the complex energy plane of the s-wave T matrix for the coupled $(\bar{K}N,\Sigma\pi)$ system are calculated. Investigated are a quark bag model and several potential models, including one that agrees with the strong interaction shift in kaonic hydrogen as well as scattering data. Each potential model is found to support either one or no poles, while the bag model is found to support two. The poles energies are found to be quite different from the energy of the $\Lambda^*(1405)$ determined by phenomenological analyses along the real energy axis.

I. INTRODUCTION

The antikaon-nucleon interaction at low energy is neither simple nor well understood. Not simple, since even at zero energy, K^-p couples to the open or nearly open, strangeness-1 channels:

$$K^-p \to K^-p \ , \tag{1}$$

$$K^-p \to \pi\Sigma + 100 \text{ MeV} \ , \tag{2}$$

$$K^-p \to \bar{K}^0 n - 5 \text{ MeV} \ , \tag{3}$$

$$K^-p \to \pi^0\Lambda^0 + 180 \text{ MeV} \ , \tag{4}$$

with a Λ^* (1405) resonance in the s-wave, isospin 0, $\Sigma\pi$ channel (2). Not well understood, since the scattering and reaction data, small in number and large in error, lead to uncertainties in the phenomenological models of the interaction.[1-5] In particular, there is still a question as to whether the Λ^* (1405) is a composite $\Sigma\pi$ resonance,[3] a $\bar{K}N$ quasibound (Gamow) state, an elementary three-quark state,[6] or some combination of these.[7,8]

A further puzzle is introduced by the three measurements[9-11] of the strong interaction shift in kaonic hydrogen all finding the $1S$ level to have more binding than provided by just the Coulomb force. This implies that the real part of the $\bar{K}N$ scattering amplitude f is *positive* at threshold—contradicting the K matrix analyses of scattering that find a *negative* real part. (The possibility of Coulomb-nuclear interference causing the sign change has now been ruled out.[12-14])

For a weakly attractive interaction, Ref starts out positive at threshold and reverses sign each time the interaction's strength increases to the point where it supports another subthreshold bound state. Accordingly, the sign and magnitude of Ref deduced from the shift in hydrogen is related to the existence and nature of subthreshold states. Yet since the present state of the atomic and scattering data leaves the connection between them rather tenuous, one view is that we should put off making the connection at present.[3,15]

Kumar and Nogami[16] take another view by proposing that below and above threshold data are compatible if the Λ^* (1405) is truly elementary (noncomposite), since then the $\bar{K}N$ amplitude can be positive at threshold after rapidly changing sign twice below threshold. Although Kumar and Nogami do not test their hypothesis against data, it does remain as a possibility. However, since the hypothesis is equivalent to a potential that vanishes just at the energy where the amplitude changes sign [a Castillejo-Dalitz-Dyson[17] (CDD) zero] and still manages to fit all data, it has been criticized as not making good physical sense.[2] Kiang *et al.*[18] have responded to that criticism by suggesting a two-channel model containing a noncomposite resonance (and CDD zero), but for which the assumption made in the traditional K matrix analysis—namely that the K^{-1} matrix is a linear function of energy—fails. If nature follows this suggestion, the hydrogen experiment could be correct—but the extrapolations of the K matrix analyses to the Λ^* (1405) would not.

Schnick and Landau[4,5] fitted parameters of a potential model to the reaction and scattering data for the channels (1)–(4), and in doing so found another means to make the above and below threshold data compatible. While some fits were just updates of Alberg, Henley, and Wilets[1] (AHW), the use of relativistic kinematics and less conventional starting values led to a conventional $\Sigma\pi$ resonance as well as a Ref_{k^-p} that agreed with the atomic data.

A more microscopic view of the $\bar{K}N$ interaction was developed by Veit *et al.*[7] In their view there is the internal dynamics of quarks within a bag making up the nucleon, and the external dynamics of a cloud of kaons coupling throughout the bag. Veit *et al.*[7] obtain respectable fits to the low, positive energy $\bar{K}N$ data after adjusting parameters, and conclude that the Λ^* is predominantly composite (a $\bar{K}N$ bound state) with only 14% of its strength coming from the elementary, bare quarks.

In the present paper we explore the classic question[20] of the relation between the properties of a resonance [the $\Lambda^*(1405)$] and the poles of the corresponding T matrix. Experimentally, physics is conducted along the real energy axis, and it is there that a resonance produces a characteristic signature in T. Conventionally (for theory), physical values of energy are those obtained by approaching the real axis of the first (physical) energy sheet from above, with bound state and resonance poles appearing as poles of T off the positive energy axis.[20] Convention and experiment concur that a pole in T on the second (unphysical) energy sheet *near* the real axis produces a narrow resonance along the real energy axis. Yet from a theoretical viewpoint, one of the best ways to characterize a model of $\bar{K}N$ scattering is by knowing the singularities, poles, and branch cuts of T or S in the complex energy plane. As a matter of convention,[20] we consider poles of T to be either resonant poles (second sheet) or bound state poles (first sheet)—even if they are not near the real energy axis. The conditions for which a pole on the second sheet may actually cause a resonance depend on a number of factors, such as the existence and proximity of other poles. Nevertheless, we search for poles even far from the real energy axis in an effort to understand better the sometimes puzzling behavior of T along the real line.

II. THEORY

A. Coupled-channels T matrices

We treat the s-wave $\bar{K}N$ interaction in a coupled-channels formalism using either the physical (charge) basis states of Eqs. (1)–(4), or the five isospin channels:

$$\bar{K}N, \Sigma\pi, \Lambda\pi, \quad (I=0,1) \ . \tag{5}$$

The dynamics arise from the coupled Lippmann-Schwinger equations

$$T(k',k;E)=V(k',k)+\frac{2}{\pi}\int_0^\infty dp\, p^2 V(k',p)G_E(p)T(p,k;E) \ , \tag{6}$$

where T, V, and G are matrices of the form

$$V\equiv\begin{bmatrix} V_{11} & V_{12} \\ V_{21} & V_{22}\end{bmatrix}, \quad T\equiv\begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22}\end{bmatrix} , \tag{7}$$

$$G_E=\begin{bmatrix} \dfrac{1}{E+i\epsilon-E_1(p)} & 0 \\ 0 & \dfrac{1}{E+i\epsilon+\Delta M_{12}-E_2(p)}\end{bmatrix} . \tag{8}$$

While 3–6 channels are used in the calculations, for simplicity we do not indicate them all here. The $i\epsilon$ procedure in the Green's function is used only for open channels to guarantee the correct outgoing-wave boundary conditions. For "relativistic" calculations, the channel energy is given by

$$E_i(p)=(p^2+m_i^2)^{1/2}+(p^2+M_i^2)^{1/2}-m_i-M_i \ , \tag{9}$$

and for nonrelativistic calculations by

$$E_i(p)=\frac{p^2}{2\mu_i} \ . \tag{10}$$

Here m_i, M_i, and μ_i are the particles' masses and reduced mass in channel i, and ΔM_{1i} is the energy released into channel i. Since some 100 MeV is released into the $\Sigma\pi$ channel, the pions are always relativistic and it is important to use relativistic propagators.

B. Resonance poles

If the T matrix satisfies the Lippmann-Schwinger equation (6), it will have "dynamic" or "moving" poles[20] when

$$\det(1-VG_E)=0 \ . \tag{11}$$

It will also have poles where the potential V does, yet since these "potential" or "fixed" poles arise from the model used for the potential, they do not reveal the dynamics of the interaction and we do not solve for them.[21]

Equation (11) is the same condition as that for the Schrödinger equation to have a bound state. We apply it here to a system with open, coupled channels (that is complex G_E) and accordingly determine the complex eigenenergy[13,22]

$$E=E_R-i\Gamma/2 \ . \tag{12}$$

Whereas this energy can be viewed as just the location of a pole of the T matrix, we prefer to consider it to be the energy of a "Gamow state" with lifetime Γ. As discussed in the Introduction, whether a pole is a "bound state," or "resonance," depends on its location in the energy plane—and the path it took to get there as absorption is introduced into the problem. Since we only report states with negative imaginary energies (positive lifetimes), they will always be in the lower half of the energy planes, and will be resonances if their home is on the second sheet, or bound states if their home is on the first.

C. Potentials

The potential models investigated were previously fit to the scattering and reactions data, and correspond to the coupled channels $(i,j)=1,2$, for each isospin state $I=0,1$. They are described by s-wave, separable potentials

$$V_{ij}^I(k',k)=\frac{4\pi\lambda_{ij}^I}{(k^2+\beta_I^2)(k'^2+\beta_I^2)} \ , \tag{13}$$

with the values of the λ and β given in Tables I and II.

Fits[5] $nr1$, $nr2$, and $nr3$ update and extend AHW's fit[1] $2C$, while $r1$ and $r2$ include relativistic kinematics. Fits $nr1$, $nr2$, $nr3$, and $r1$ show resonance behavior in the $\bar{K}N$ channel: a peak in $\mathrm{Im}f_0$ near 1405 MeV, with $\mathrm{Re}f_0$ changing sign from plus to minus as the energy passes through "resonance." We see an example of this in Fig. 1

TABLE I. Potential parameters, isospin 0.

Potential	β_0^{-1} (fm)	λ_0^{11} (MeV^2)	λ_0^{12} (MeV^2)	λ_0^{22} (MeV^2)
$nr1$	0.0235	-1.47×10^8	-7.72×10^6	-2.83×10^8
$nr2$	0.138	-2.61×10^4	-4.88×10^4	$+3.46\times10^4$
$nr3$	0.223	-1.82×10^5	-1.54×10^5	-4.36×10^5
AHW	0.180	-3.58×10^5	-1.23×10^5	-4.52×10^5
$r1$	0.261	-2.89×10^7	$+2.99\times10^7$	-3.13×10^7
$r2$	0.0962	-1.05×10^6	-1.71×10^5	-1.29×10^6

for potential $r1$. The T from potential $nr3$ in Fig. 2 is different from the others in that it does *not* appear to have resonant structure in the $\Sigma\pi$ channel. Yet it is still interesting to study its properties since it produces a satisfactory $\Sigma\pi$ mass spectrum for the reaction $\pi^- p \to \pi^\pm \Sigma^\mp K^0$ (where much of the spectrum's shape arises from phase space and channel openings[5]).

Compared to $r1$, relativistic fit $r2$ (Fig. 3) has a smaller T in the $I=0, \bar{K}N$ channel and a slightly larger one in $I=1$, so that their sum, the $K^- p$ amplitude, has a real part which does *not* change sign near 1405 MeV. Consequently, $r2$ is the only potential considered which agrees with the sign of the experimental strong interaction shift in kaonic hydrogen. As seen in Fig. 4, $r2$ still has a good resonance shape in the $I=0, \Sigma\pi$ channel (Re$f_{\Sigma\pi}$ changing sign near where Imf peaks).

Since these T matrix plots along the real axis do manifest a resonance, in the second column of Table III we list the resonance energy E_R and negative half width $\Gamma/2$ as deduced from plots of $k\,\text{Im}f_{\Sigma\pi}$ vs energy. They fall in the ranges $(-41 \le E_R \le -22)$ MeV, $(-41 \le -\Gamma/2 \le -27)$ MeV, with $nr3$ having no apparent resonant behavior in the $\Sigma\pi$ channel. In the third column of Table III we list the eigenenergy determined by solving (11). This "resonance pole" energy is seen to fall in the range $(-63 \le E_r \le -8)$ MeV, $(-39 \le E_i \le -14)$ MeV. In all cases the resonance pole energy is quite different from the resonance's energy and half width deduced along the real energy axis.

D. Cloudy bag model

We have reproduced the cloudy bag model calculations of Veit *et al.*[7] by solving the six-channel Lippmann-Schwinger equations (6) with the equivalent potential

$$V_{ij} = \langle i|H_c|j\rangle + \sum_{B_0} \langle i|H_s|B_0\rangle \frac{1}{E-M_0(B_0)} \langle B_0|H_s|j\rangle \ . \quad (14)$$

Here $|i\rangle$ and $|j\rangle$ are meson-baryon states, and $|B_0\rangle$ is a baryon bag state of mass $M_0(B_0)$. Note that the "potential" in (14) is nonlocal and energy dependent, with a fixed pole at $M_0 = 1630$ MeV, the bare mass of the Λ^*.

The $\bar{K}N$ and $\Sigma\pi$ f matrices calculated with the cloudy bag model (CBM) potential (parameter set A with $M_s=0$) are shown in Fig. 5. There appears to be a resonance signal in Im$f_{\bar{K}N}$—although somewhat less pronounced than those of the potential models (compare Figs. 4 and 5). By examination of their plots, Veit *et al.*[7] deduced the Λ^* (1405) properties given in the second column of Table III, $(E_R, -\Gamma/2) = (-22, -41)$ MeV. Yet we also see in Fig. 5 that Im$f_{\bar{K}N}$ peaks right near the 1432 MeV threshold, and that the real parts of both $f_{\bar{K}N}$ and $f_{\Sigma\pi}$ dip below zero and then rise again right near threshold. It thus appears that something is distorting the resonance in the CBM.

Our eigenenergy search with the CBM found two poles (third column of Table III). The first is at $(-95, -77)$ MeV, much lower in energy and wider that the 1405, and the second at $(+13, -12)$ MeV, right above threshold and very narrow. These poles are on the second (unphysical) sheet for the $\Sigma\pi$ channel, but on the first (physical) sheet for the $\bar{K}N$ channel. Correspondingly, in the $\bar{K}N$ channel they are unstable bound states, one of which can decay into the $\bar{K}N$ continuum, the other of which decays into the $\Sigma\pi$ continuum, and in the $\Sigma\pi$ channel they are resonances.

In Fig. 6 we present a three-dimensional visualization of the CBM poles on the first energy sheet. The near-threshold pole is quite near the cut on the real energy axis, and thereby affects the physical scattering amplitude the most. It also appears more strongly in the $\bar{K}N$ channel than the $\Sigma\pi$ one, is out of phase with the negative energy pole, and is not a standard Breit-Wigner resonance. (The pole at the bare mass, 1630 MeV, is still present—but much above threshold.)

TABLE II. Potential parameters, isospin 1.

Potential	β_1^{-1} (fm)	λ_1^{11} (MeV^2)
$nr1$	0.128	$(-7.37\times10^5 - 2.14\times10^5 i)$
$nr2$	0.293	$(-3.88\times10^4 - 4.25\times10^4 i)$
$nr3$	0.616	$(-1.72\times10^4 - 1.13\times10^4 i)$
AHW	0.500	$(-6.08\times10^3 - 4.42\times10^3 i)$
$r1$	0.223	$(-1.25\times10^5 - 2.02\times10^4 i)$
$r2$	0.0678	$(-2.75\times10^6 - 3.77\times10^4 i)$

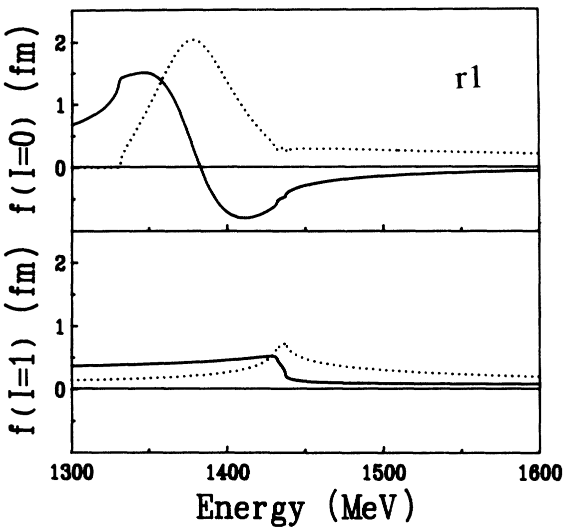

FIG. 1. Real and imaginary parts (solid and dotted curves) of the isospin 0 and isospin 1, $\bar{K}N$ elastic scattering amplitude for the relativistic potential $r1$. Potentials $nr1$ and $nr2$ produce similar resonance shapes (but with different magnitudes).

We note that the behavior of Ref *along the real energy axis* of the physical energy sheet in Fig. 6(a) agrees with the energy dependence shown in Fig. 5 (which is along the physical energy axis as approached from above), but that this is not true for Imf. This is a consequence of the unitarity cut beginning at threshold ($E=0$ in Fig. 6) and extending to infinity (the heavy arrow). While Ref is continuous across this cut, Imf is discontinuous (a consequence of the discontinuity in the imaginary part of the Fredholm determinant).

As a consequence of these two poles in the CBM's scattering amplitude, the peak in Im$f_{\bar{K}N}$ and the zero in and the Re$f_{\bar{K}N}$ is at a higher k than those in Im$f_{\Sigma\pi}$ and Re$f_{\Sigma\pi}$. In addition, Ref in both channels changes sign twice—as to be expected when there are two resonances present. This behavior of Ref is interesting and not without experimental consequences. Both the CBM and

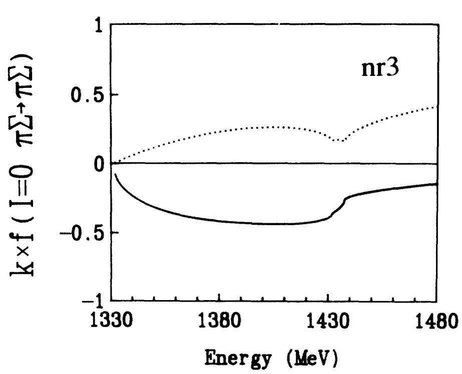

FIG. 2. Real and imaginary parts (solid and dotted curves) of the isospin 0, $\Sigma\pi$ elastic scattering amplitude times momentum for the nonrelativistic potential $nr3$. This is an exceptional case in which there is no clear resonance signal.

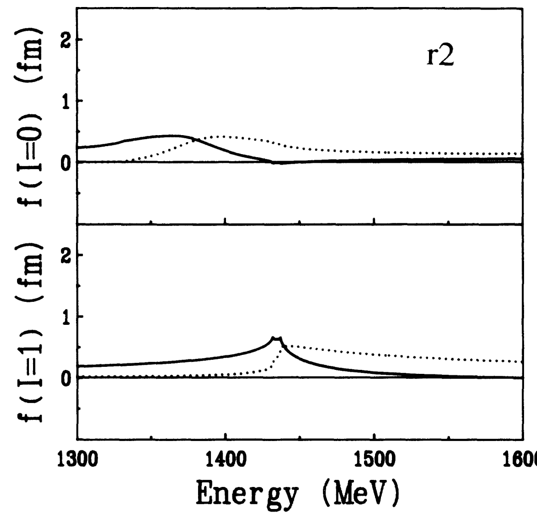

FIG. 3. Real and imaginary parts (solid and dotted curves) of the isospin 0 and isospin 1, $\bar{K}N$ elastic scattering amplitude for the relativistic potential $r2$. This potential, when combined with the Coulomb one, produces a strong interaction shift for kaonic hydrogen in agreement with experiment.

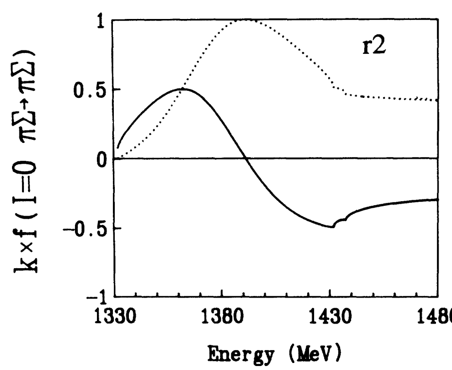

FIG. 4. Real and imaginary parts (solid and dotted curves) of the isospin 0, $\Sigma\pi$ elastic scattering amplitude times momentum for the relativistic potential $r2$. A strong resonance signal is manifest here even though not present in the $\bar{K}N$ channel, Fig. 3.

TABLE III. $I=0$ resonance parameters deduced from plots along real energy axis (second column) and resonance pole positions in complex energy plane (third column). The numerical uncertainty is approximately ±2 MeV.

Model	$(E_R, -\Gamma/2)$	(E_r, E_i) (MeV)
$nr1$	(−22, −27)	(−8, −14)
$nr2$	(−41, −35)	(−63, −32)
$nr3$	None	None
AHW	(−34, −32)	(−14, −25)
$r1$	(−35, −41)	(−58, −32)
$r2$	(−32, −35)	(−52, −39)
CBM	(−22, −41)	(−95, −77), (+13, −12)

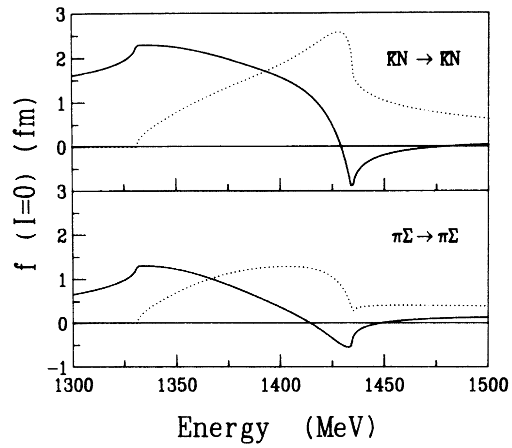

FIG. 5. Real and imaginary parts (solid and dotted curves) of the isospin 0, $\bar{K}N$ and $\Sigma\pi$ elastic scattering amplitude times momentum for cloudy quark bag model (CBM). Note that threshold is at 1432 MeV.

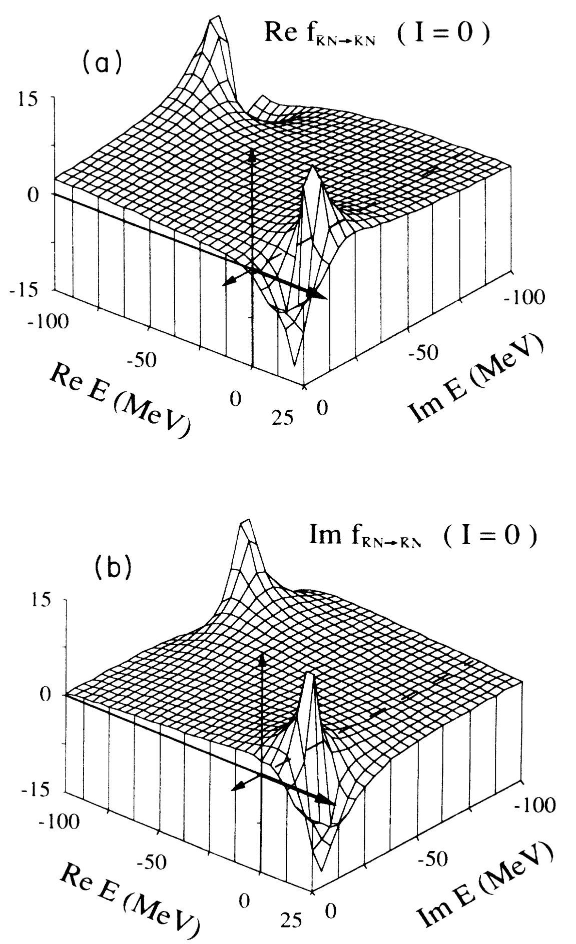

FIG. 6. (a) Real and (b) imaginary parts (solid and dashed curves) of the isospin 0, $\bar{K}N$ scattering amplitude for the CBM as a function of complex energy for the physical energy sheet (measured relative to the $\bar{K}N$ threshold). The heavy arrow indicates the start of the unitarity cut across which Imf is discontinuous. One pole is below threshold, the other above.

potential $r2$ (the one agreeing with kaonic hydrogen) are relativistic and fit nearly the same data, yet in the dip region around 1425 MeV their Ref's differ in sign. The hydrogen data suggest a positive sign.

In a follow-up study to Ref. 4, the authors of Ref. 23 worked with a potential model having similarities to the CBM. Yet they used nonrelativistic kinematics—known to be a bad approximation,[2,4,24,25] and fit to scattering data by placing a CDD pole into their potential at 1431 MeV. This additional pole is quite near threshold, quite far from the bag model's one at 1630 MeV, and leads to an additional sign change in Ref at the $\bar{K}N$ threshold. Although we have not examined this model numerically, previous studies lead us to suspect that the proper inclusion of relativistic kinematics would lead to different behavior for the amplitudes.

III. DISCUSSION

The results of Table III indicate that for each model with resonance behavior along the real energy axis, a pole in T exists. Yet for each case, the complex-energy pole is too far from the real energy axis for it to agree with the resonance's energy. This is not a surprise, since for poles not near the real energy axis, approximate analytic continuation is inaccurate. It also confirms studies by Afnan and Pierce[24] and Elsey and Afnan[25] who obtained even larger differences for the πNN system. As expected, the accepted[26] value of $(-27\pm5, -20\pm10)$ for the resonance energy reflects the behavior along the real axis.

Potential $nr3$ did not show a conventional resonance along the real energy axis, and no resonance pole was found. Of course a pole may exist in a region outside of that searched or may be of a nature for which the numerical search routine fails, but in either case we believe there is not a strong one near the real energy axis.

The cloudy bag model[7] predicted T matrices whose behavior along the real energy axis appeared to have a distorted or displaced Λ^* resonance (Imf having a broad maximum near threshold—rather than below, and Ref dipping below zero—but then coming right back up). This behavior becomes more understandable[8] after our finding in the $\bar{K}N$ channel of a bound state pole below threshold and *a second one* above threshold.

If a $\bar{K}$ forms bound states with individual nucleons, it should also be possible to have "hypernuclei" into which a $\bar{K}$ or Σ has entered a nucleus and converted a nucleon to a Λ^*. While a quantitative exploration of multinu-

TABLE IV. The bound state pole position in K^-p system with (C) and without (NC) the Coulomb interaction.

Model	$(E_r,E_i)_C$ (MeV)	$(E_r,E_i)_{NC}$ (MeV)
$nr1$	$(-10, -16)$	$(-7, -15)$
$nr2$	$(-62, -33)$	$(-61, -33)$
$nr3$	$(-10, -77)$	$(-9, -77)$
AHW	$(-13, -25)$	$(-11, -24)$
$r1$	$(-57, -32)$	$(-56, -33)$
$r2$	$(-53, -38)$	$(-52, -39)$

cleon bound states will be given by Fink *et al.*,[19] we have tested if bound states exist with a physical proton by searching for complex energy poles with charge basis states (the previous searches used isospin basis). In Table IV we see that the isospin 1 attraction is strong enough for K^-p strong bound states to exist (the K^-p potential is the average of $I=0$ and $I=1$ ones), strong enough in fact to yield a state for $nr3$ which did not have one for pure $I=0$. We also see in Table IV that the Coulomb force perturbs these levels slightly, although in most cases the perturbation is less than our numerical uncertainty of ± 2 MeV.

In closing, we note that there still appears to be physics to unravel in order to understand the low-energy $\bar{K}N$ interaction. Since none of these models is perfect, it seems prudent to keep an open mind about them all.

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