Pairing effects in $N=82$ isotones

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(Received 24 July 1989)

The importance of pairing effects in the $N=82$ isotones is extensively investigated by studying the string of nuclei with A ranging from 135 to 151. The pairing Hamiltonian is treated by an equations-of-motion method which is strictly number conserving. The coupling strength is determined by an analysis of the properties of the even-mass isotones. The single-particle energies are extracted from the experimental spectra of ^{137}Cs and ^{145}Eu through use of the equations which give the energies of the seniority-one states in a way which is analogous to the inverse gap equation method. A detailed comparison of the calculated results with experimental data in even and odd nuclei provides firm evidence of the prominent role of proton pairing correlations in the 50–82 shell. It is found that non-negligible correlations are present in ^{146}Gd.

I. INTRODUCTION

In recent years $N=82$ isotones have been the subject of extensive study. It is now well recognized that most of the low-lying states of these nuclei can be interpreted as pure excitations of the valence protons outside the ^{132}Sn inert core. The interest in the $N=82$ region is related to the fact that it provides the opportunity to study the effect of adding protons to a doubly-magic core over a large number of nuclei (from $A=133$ to $A=151$). A special role in this region is played by the nucleus ^{146}Gd for which a doubly-magic character has been evidenced by several experimental studies.[1] Actually, many states in the surrounding nuclei can be described in terms of very simple shell-model configurations assuming ^{146}Gd as an inert core. It is therefore of great relevance to investigate in detail the shell-model structure of the $N=82$ isotones, and in particular to asses how good $Z=64$ is as a magic number.

Assuming that $N=82$ and $Z=50$ are closed inert cores, the valence protons can occupy the single-particle orbits $1g_{7/2}$, $2d_{5/2}$, $2d_{3/2}$, $3s_{1/2}$, and $1h_{11/2}$. This model space, however, is already too large to allow for a full shell-model calculation should one want to consider nuclei with more than six or eight protons outside the core. It is therefore necessary to resort to some truncation method to reduce the matrices to be diagonalized to manageable size. In this context, the seniority truncation seems to be quite appropriate since there is considerable evidence that only states with seniority $v \leq 4$ play a significant role in the description of the low-energy spectra. Extensive shell-model calculations along this line are currently being carried out.[2]

While a detailed description of the $N=82$ nuclei certainly requires these kinds of calculations, some features and trends can be interpreted in a much simpler way. In fact, in this region the pairing effects seem to be particularly pronounced. The role of pairing correlations in the ^{146}Gd region was studied by several authors in various theoretical contexts.[3–12] In particular, Chasman[3] carried out a calculation of proton excitations in the $N=82$ isotones with $Z=61$ through $Z=66$ assuming a pure pairing force as residual interaction and making use of the method of correlated quasiparticles.[13] His comparison between theory and experiment, although rather limited, confirmed the importance of correlations of this kind.

In the past several years we have developed[14–18] a number-conserving treatment of pairing Hamiltonians which has proved to be an advantageous alternative to the BCS theory. On the above grounds we decided to make use of our method to perform a detailed study of the pairing effects in the $N=82$ isotones. It is worthwhile to mention, however, that this study not only seemed interesting in its own right, but was also seen as a first step toward more realistic calculations. This stems from the fact that our treatment of pairing correlations resulted from the simplest practical application of an equations-of-motion formalism which we advocate as a particularly convenient framework to deal with shell-model problems. Actually, we are presently carrying out a study of the $N=82$ nuclei making use of this formalism properly specialized to treat a general shell-model Hamiltonian within the seniority scheme.[19]

This paper is organized as follows. In Sec. II we give a brief outline of our method. In Sec. III we describe how we have determined the pairing strength and the single-particle energies. In Sec. IV we compare the results of our study with the experimental data for the $N=82$ nuclei with A ranging from 135 to 151. Section V presents a summary of our conclusions.

II. OUTLINE OF THE METHOD

Since our approach is described in detail elsewhere, only a brief description is given below together with the main formulae we have used in our calculations. The same notation as that adopted in our previous papers[14–18] will be used throughout.

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Let us consider the pairing Hamiltonian in the form

$$H=\sum_j \varepsilon_j \hat{N}_j - \sum_{jj'} G_{jj'} A_j^\dagger A_{j'} \ . \quad (1)$$

The essence of the method is to treat the seniority-zero problem first. In this case, the wave function of an N-particle state, $|N,\beta\rangle$, is given the form

$$|N,\beta\rangle=\sum_{j\gamma} c_{j\beta\gamma}(N) A_j^\dagger |N-2,\gamma\rangle \ , \quad (2)$$

and use is made of the equations of motion for the zero-coupled pair operators $A_j^\dagger$. This leads to the eigenvalue equation

$$\sum_{j'\gamma'} M_{j\gamma j'\gamma'} X_{j'\beta\gamma'}(N)=E_\beta(N) X_{j\beta\gamma}(N) \ , \quad (3)$$

with

$$M_{j\gamma j'\gamma'}=[2\varepsilon_j+E_\gamma(N-2)]\delta_{jj'}\delta_{\gamma\gamma'} - G_{jj'}\Omega_j[\delta_{\gamma\gamma'}-2\rho_{j\gamma'\gamma}(N-2)] \ , \quad (4)$$

where

$$X_{j\beta\gamma}(N)=\langle N,\beta| A_j^\dagger |N-2,\gamma\rangle \ , \quad (5)$$

$$\rho_{j\gamma'\gamma}(N-2)=\langle N-2,\gamma'| a_{jm}^\dagger a_{jm} |N-2,\gamma\rangle \ , \quad (6)$$

and $\Omega_j=j+\frac{1}{2}$.

Equation (3) yields the energies E_β and the two-particle transfer amplitudes for the N-particle system once the solution of the $(N-2)$-particle system is known. This implies that the N-particle problem is to be solved through a chain calculation. Since we are considering seniority-zero states, this involves only even nuclei and has to be started from $N=0$.

The method outlined above can be applied at several levels of approximation depending on the number of core states $|N-2,\gamma\rangle$ which are included in the expansion (2). At the simplest stage of approximation, which we call first-order theory, only one core state, the ground state $|N-2,0\rangle$, is taken into account. In this case, and for constant pairing force, $G_{jj'}=G$, the eigenvalue problem (3) reduces to a simple dispersion relation.[15,17] The first-order theory, however, can describe with good accuracy only the ground state. Since one of the aims of the present work was to investigate the relevance of pairing correlations in the 0^+ excited states of the $N=82$ isotones, we have made use of a higher-order application of the theory to solve the seniority-zero problem. Some details about these calculations will be given in Sec. IV.

We only mention here that inherent in the above formalism is the use of the overcomplete set of basis vectors $A_j^\dagger|N-2,\gamma\rangle$. We refer the reader to Refs. 14 and 15 for a detailed description of the method of solution of Eq. (3) including the procedure for removing the redundant states at each step of the chain calculation.

Let us come now to the treatment of states of seniority $v=\sum_j v_j>0$ (v_j is the seniority of level j). We shall only give explicitly here the relevant equations of the first-order theory. In fact, as in the seniority-zero case, the lowest order of approximation suffices to describe with good accuracy the lowest state for each set of the quantum numbers v_j; and it is these states (for $v=1$ and $v=2$) which are considered in the present study.

The formalism is provided by the equations of motion for the $P^\dagger$ operators defined as

$$|N=v,L,K,J,M\rangle=P_{KJM}^\dagger(j_1^{v_1} j_2^{v_2} \ldots j_n^{v_n})|0\rangle \ , \quad (7)$$

where the symbol L stands for all the seniority quantum numbers v_j and K denotes the additional labels necessary to completely specify the states. In the first-order theory, the wave function for a seniority-v state with $(N+v)$ particles is written as

$$|N+v,L,K,J,M\rangle=c^L(N+v)P_{KJM}^\dagger(j_1^{v_1} j_2^{v_2} \ldots j_n^{v_n})|N,0\rangle \ . \quad (8)$$

This leads to the following equation for the energies of the seniority-v states (since for a given L states with different J and K are degenerate, we omit these labels):

$$E_L(N+v)=E_0(N)+\sum_j v_j\Gamma_j \ , \quad (9)$$

with

$$\Gamma_j=\varepsilon_j+\frac{X_j(N)\Delta_j(N)}{\Omega_j[1-\rho_j(N-2)]} \ , \quad (10)$$

where

$$\Delta_j(N)=\sum_{j'} G_{jj'} X_{j'}(N) \ , \quad (11)$$

and we have denoted by $X_j(N)$ and $\rho_j(N-2)$ the matrix elements $\langle N,0| A_j^\dagger |N-2,0\rangle$ and $\langle N-2,0| a_{jm}^\dagger a_{jm} |N-2,0\rangle$.

Equations (3) and (9) have been derived by the use of the equations of motion for creation operators, namely they result from the formulation of the theory in terms of particles. The equations corresponding to the hole formalism are of the same form and will not be given explicitly here. It is important, however, to point out that both formalisms are needed in practical applications. This is particularly true for the treatment of the states with $v>0$ in the first-order theory, wherein one has to use the appropriate formalism depending on the considered v_j and the number of valence particles. How to decide whether to use the particle or the hole formalism is discussed in detail in Ref. 15 for the seniority-zero case and in Ref. 18 for the seniority-one case (the criterion given in the latter may be easily generalized, however, so as to apply for $v>1$).

From Eqs. (9)–(11) it appears that the first-order solutions for $v>0$ are easily obtained using as input the results for the seniority-zero problem. Clearly, to produce the needed input it is only necessary to carry out a first-order seniority-zero calculation. As mentioned before, however, one has to go beyond that to describe seniority-zero excited states. Of course, in this case one will produce more accurate results for the ground-state properties, in particular for the occupation numbers and two-particle transfer amplitudes (see Refs. 15 and 17). In any case, once the seniority-zero solutions have been produced, the solutions for a given value of v can be obtained for each value of N separately. In other words, it is only the seniority-zero problem which requires a chain calculation involving all even nuclei up to the desired value of N. This feature of our number-conserving approach may be viewed as the counterpart of what occurs in the BCS theory, where the zero-quasiparticle ground state corresponds to an ensemble of nuclei with different even particle numbers. This analogy is further borne out by noting that expression (9) (as well as the corresponding expression in the hole formalism; see, e.g., Ref. 18) is formally similar to that giving the energies of the v-quasiparticle states in the BCS theory. Without going into further details, we only mention here that the BCS theory can be recovered from our first-order theory by relaxing particle-number conservation.[20]

The above discussion should have made it clear that our first-order theory retains the same attractive features of the BCS theory, while providing a much better approximation scheme.

III. SINGLE-PARTICLE ENERGIES AND PAIRING INTERACTION STRENGTH

In the present work we consider all the single-proton levels within the 50–82 shell. A most natural choice of the energy spacings between these levels would consist in taking them (at least for the lightest isotones) from the experimental spectrum of the single-proton valence nucleus ^{133}Sb. Actually, only the $g_{7/2}$, $d_{5/2}$, and $h_{11/2}$ states can be firmly associated with observed levels (the ground-state, the 0.962, and 2.793 MeV excited levels, respectively) in this nucleus.[21] So one has to partly rely on theoretical or empirical predictions anyway. As a matter of fact, a reasonable identification of the $d_{3/2}$ state with the observed level at 2.708 MeV is provided by the theoretical considerations put forward in Ref. 22, while the location of the $s_{1/2}$ level can be assessed from the empirical analysis of Ref. 23 (2.99 MeV).

To test the adequacy of this set of single-particle (s.p.) energies, we made use of it to calculate the seniority-zero, seniority-one, and seniority-two states for the $N=82$ isotones varying the pairing strength G (we consider a constant pairing force, $G_{jj'}=G$) within reasonable limits, namely 0.17–0.22 MeV (this is essentially the widest range according to most of the existing literature[3,4,7,9,24–26]). It turned out that the 0^+ excited states and the seniority-two states are much more sensitive to variations in the coupling strength G than the seniority-one states. This coincides with the findings of Chasman,[3] whose analysis, however, concerned only the seniority-one and -two states in a limited number of nuclei. Thus, it is essentially the properties of the even nuclei which determine the value of G. We found that the first excited 0^+ state in the nuclei considered (which can be reasonably assumed to be predominantly seniority-zero states) and some high-spin states, which can be identified as seniority-two states, are rather well reproduced by using $G=0.21$ MeV. The spectra of odd-A nuclei, however, are not in satisfactory agreement with the experiment, the discrepancy being in some cases larger than 0.4 MeV. The detailed comparison between the calculated and experimental spectra in the odd nuclei also reveals that the choice of a unique set of s.p. energies is too restrictive. This is not surprising, as there is clear evidence of non-negligible changes in the single-proton spectrum when approaching ^{146}Gd.

On the above grounds we came to the conclusion that, at least within the framework of the pairing model, the use of the "experimental" single-proton energies is not very appropriate. Indeed, this has not been the choice in most of the existing calculations. Actually, the problem of determining the single-particle spectrum for the $N=82$ nuclei has received a great deal of attention over the past ten years;[3,7,9,26–30] thus, several sets of single-proton energies are available in the literature. These have been obtained in various ways, a most simple one consisting in a BCS analysis of the low-lying levels of one or more odd-A nuclei. It should be noted that the values of the s.p. energies depend not only on the empirical data which have been analyzed, but also on the way they have been determined.

In this situation, therefore, we decided to look for our own solution to this problem. This has been done by making use of our formalism for $v=1$ in a way which comes close to the well-known inverse gap equation (IGE) method.[31] By this kind of procedure the pairing strength G and the s.p. energies may be determined simultaneously from the energies of the low-lying states in odd-mass nuclei and from the even-odd mass differences. We have found it more appropriate, however, to separate the determination of G from that of the s.p. energies. This is motivated by the fact that, as already mentioned, the properties of the even nuclei are more sensitive than the spectra of odd nuclei to changes in the pairing strength, while the situation is just the opposite for the ε_j. Actually, we have fixed the value of G by reproducing the energy (2.9 MeV) of the 10^+ state in ^{148}Dy, which can be identified as a pure seniority-two $(\pi h^2_{11/2})$ state. Our choice has been dictated by the fact that the theoretical energy of this state has turned out to be fairly insensitive to reasonable changes in the s.p. energies. This is not surprising as it may be viewed as a consequence of the "doubly-magic" character of ^{146}Gd.

Proceeding as described above, we found $G=0.21$ MeV. It should be noted that a smaller value would be obtained by making use of the even-odd mass differences. To further check the validity of our choice we calculated the properties of the even nuclei letting G vary from 0.17 to 0.22 MeV and using different sets of s.p. energies. It turned out that it is in no way possible to obtain a good overall agreement with experiment outside the interval 0.20–0.22 MeV. A value of G within these limits has been used in Ref. 7 and is consistent with the values obtained from the analysis of other mass regions.[25] The smaller value (0.17 MeV) resulting from the work of Chasman[3] is only due to the larger number of s.p. levels used.

We shall now describe briefly how the s.p. energies can be determined within the framework of our approach, once (as in our case) the pairing strength is known. In the first-order theory the energies of the seniority-one states are given by [see Eq. (9)]

$$E_j(N+1)=E_0(N)+\Gamma_j \ . \qquad (12)$$

We assume that for single closed-shell odd-A nuclei the low-lying states with angular momentum and parity corresponding to the valence s.p. orbits and having the largest single-particle spectroscopic factors are pure seniority-one states. If $E_{j'}^{\rm exp}(N+1)-E_j^{\rm exp}(N+1)$ is the experimental energy splitting between states with angular momentum j' and j, respectively, then from Eqs. (12) and (10) we have

$$\varepsilon_{j'}-\varepsilon_j=\frac{X_j(N)\Delta_j(N)}{\Omega_j[1-\rho_j(N-2)]}-\frac{X_{j'}(N)\Delta_{j'}(N)}{\Omega_{j'}[1-\rho_{j'}(N-2)]}+E_{j'}^{\rm exp}(N+1)-E_j^{\rm exp}(N+1) \ . \qquad (13)$$

By means of these equations a set of s.p. energies can be obtained for the chosen value of G in the following way. Using as starting input a reasonable set of ε_j (e.g., that taken from ^{133}Sb) the quantities Δ_j, X_j, and ρ_j are computed (as described in Sec. II) for the even nuclei with $N-2$ and N particles. This makes it possible, through use of Eq. (13), to produce a new set of s.p. energies, say ε_j', namely a new input for solving the $(N-2)$- and N-particle problem again. This procedure is iterated until convergence is reached. It should be mentioned that the convergence rate is rather high, the difference between two consecutive sets of s.p. energies being less than one percent in at most five iterations. For the sake of simplicity we have described here our procedure within the framework of the particle formalism. Obviously, this procedure holds true when making use of the hole formalism.

Clearly, by applying this procedure to each odd-A nucleus one can assess how the s.p. energies vary with A and then investigate the relevance of this variation to the properties of the even nuclei (this has been done, for instance, in Ref. 26 using the IGE method). We have not gone in this direction, however, since our aim has been to understand which genuine physical effects arise from pairing correlations in the $N=82$ region. As already pointed out earlier in this section, significant changes in the s.p. energies occur around $Z=64$. We have therefore determined two sets of ε_j, one for the nuclei with Z up to 61 and the other for the heavier ones.

The first set has been determined by fitting the excitation energies[32] of the low-lying levels in ^{137}Cs, whose interpretation as seniority-one states is supported by the experimental values of the one-particle spectroscopic factors reported in Ref. 33. The obtained values (in MeV) are as follows: $\varepsilon_{7/2}=0.0$, $\varepsilon_{5/2}=0.769$, $\varepsilon_{11/2}=2.350$, $\varepsilon_{3/2}=2.560$, and $\varepsilon_{1/2}=2.646$. The second set has been determined by using the excitation energies[34] of the low-lying states in ^{145}Eu, whose identification as seniority-one states is also well assessed.[33] The ε_j obtained from this analysis are $\varepsilon_{7/2}=0.0$, $\varepsilon_{5/2}=0.456$, $\varepsilon_{11/2}=2.418$, $\varepsilon_{1/2}=2.530$, and $\varepsilon_{3/2}=2.802$.

It should be noted that the main difference between these two sets of s.p. energies is the larger $d_{5/2}$-$h_{11/2}$ spacing in the second one. Clearly, this reflects the "doubly-magic" character of ^{146}Gd. How wide the s.p. gap at $Z=64$ is is still, however, a matter of discussion. We will come back to this point in Sec. V.

IV. RESULTS AND COMPARISON WITH EXPERIMENT

We present here the results of our study of the $N=82$ isotones. These results concern even and odd nuclei with A ranging from 135 to 151. All of the calculations were carried out by using the method described in Sec. II. For the sake of clarity we now summarize briefly the essentials of these calculations. The seniority-zero states were obtained from a 25th-order application of the theory which suffices to produce the various physical quantities with very good accuracy for all values of A. The properties of the states with seniority $v>0$ were then calculated by making use of the first-order theory, as discussed in detail in Sec. II. We considered a constant pairing force with a coupling strength $G=0.21$ MeV and used two sets of s.p. energies determined from the experimental spectra of ^{137}Cs and ^{145}Eu, respectively, by the procedure described in Sec. III, where the numerical values are to be found. The first set has been used in the calculation of the properties of the lightest even and odd nuclei up to and including $A=143$, while the second one has been employed for all the other nuclei considered.

A. Even-mass nuclei

1. 0^+ states

The first 0^+ excited states obtained from our calculations for all the $N=82$ isotones with A ranging from 136 to 150 are compared with the experimental ones[1,35–37] in Fig. 1. It appears that the behavior of the energy of these states as a function of A is remarkably well reproduced. The quantitative agreement with experiment may also be

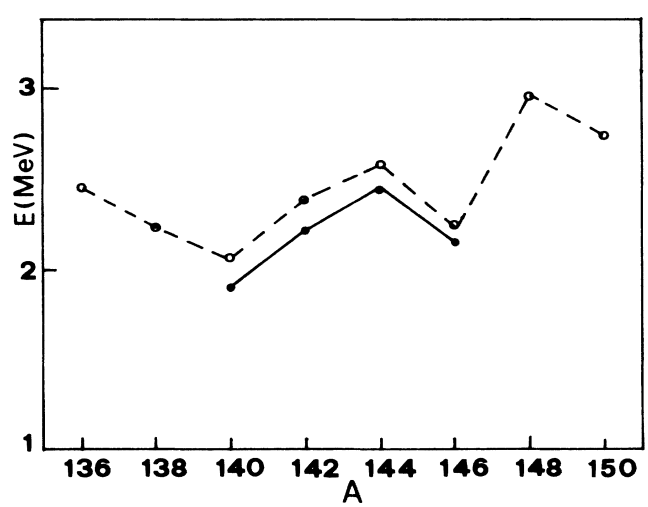

FIG. 1. Energy of the first 0^+ excited state in even $N=82$ isotones with A ranging from 136 to 150. The theoretical results are represented by open circles, while the experimental data by solid circles. The latter are taken for $A=140$, 142, 144, and 146 from Refs. 35, 36, 37, and 1, respectively.

TABLE I. Experimental (Ref. 33) and calculated occupation numbers for the ground state of the even-mass $N=82$ nuclei.

j \ A	136 Expt.	136 Calc.	138 Expt.	138 Calc.	140 Expt.	140 Calc.	142 Expt.	142 Calc.	144 Expt.	144 Calc.	146 Calc.	148 Calc.	150 Calc.
$\frac{7}{2}$	3.5 ± 0.4	2.90	4.3 ± 0.4	4.20	5.6 ± 0.3	5.28	$5.7^{+0.2}_{-0.4}$	6.03	6.3 ± 0.2	6.48	7.10	7.25	7.36
$\frac{5}{2}$	0.5 ± 0.2	0.65	0.7 ± 0.3	1.11	1.8 ± 0.2	1.72	$2.6^{+0.2}_{-0.3}$	2.58	$3.6^{+0.1}_{-0.2}$	4.03	4.95	5.17	5.32
$\frac{11}{2}$	0.0 ± 0.7	0.31	$1.0^{+1.0}_{-0.7}$	0.49	$0.6^{+0.6}_{-0.2}$	0.70	$1.3^{+0.6}_{-0.4}$	0.98	1.6 ± 0.3	1.06	1.40	2.62	3.89
$\frac{3}{2}$	0.0 ± 0.2	0.09	0.0 ± 0.2	0.14	0.0 ± 0.2	0.20	0.2 ± 0.1	0.28	0.3 ± 0.1	0.26	0.33	0.58	0.86
$\frac{1}{2}$	0.0 ± 0.2	0.04	0.0 ± 0.2	0.07	0.0 ± 0.2	0.09	0.2 ± 0.1	0.13	0.2 ± 0.1	0.16	0.21	0.38	0.57

considered satisfactory, the largest discrepancy being 0.19 MeV for ^{142}Nd. This indicates the importance of the seniority-zero components in the structure of the first 0^+ excited states in the $N=82$ region. It should be noted, however, that in four out of the eight nuclei considered there is no evidence thus far for 0^+ excited states. The detection of such levels would certainly be of great relevance in the light of the present considerations.

Concerning the observed higher-lying 0^+ states in ^{140}Ce, ^{142}Nd, ^{144}Sm, and ^{146}Gd, any attempt to identify them with those predicted by the theory could be misleading. In fact, there is evidence[1,9] that their description is beyond the scope of the present calculation.

In Table I we compare the calculated occupation numbers of the ground state with those obtained by Wildenthal, Newman, and Auble[33] from the analysis of the (^{3}He,d) and (d,^{3}He) reactions. Of course, such experimental data are only available for the stable $N=82$ isotones. We also report, however, our predictions for the three heavier isotones ^{146}Gd, ^{148}Dy, and ^{150}Er. We see that almost all of our values fall within the estimated uncertainties which are, however, very large in many cases. In this connection, the detailed discussion of the structure of the ground-state wave functions given in Ref. 33 is of particular relevance. Our results are essentially in line with the conclusions of the above authors. For instance, they point out that the apparent increase in the occupancy of the $h_{11/2}$ orbit, when passing from ^{140}Ce to ^{138}Ba, is not real and that the "true" occupation value for the latter nucleus should be close to 0.5; our value is 0.49. We shall not go further into this matter here, but refer the reader to Ref. 33 for details.

On the above grounds we may conclude that the present study provides clear evidence of the dominant role of pairing correlations in the ground states of the $N=82$ isotones.

2. *High-spin states*

In Table II we show for some even nuclei the calculated excitation energies of the seniority-two states arising from the configurations $g_{7/2}h_{11/2}$ and $h^2_{11/2}$. We compare our results with the experimental energies[1,37–41] of those states which can be uniquely associated with the maximally aligned states formed from the two above configurations, namely the 9^- and 10^+ states. Concerning the latter, the comparison is limited to ^{146}Gd and the two adjacent nuclei. In fact, the 10^+ states observed in the other isotones may not be of such a simple nature.[38–40] We see that our results are in very good agreement with the experiment in all of the cases considered.

B. Odd-mass nuclei

1. *Energy spectra*

In Figs. 2 and 3 the calculated energies of the various seniority-one states are compared with the experiment[42–48] for all of the odd nuclei with $135 \leq A \leq 151$, except, of course, ^{137}Cs and ^{145}Eu, whose levels have been used to determine the s.p. energies.

As regards the lightest nuclei (Fig. 2), there is in all of the low-energy experimental spectra more than one level with angular momentum and parity corresponding to one of the valence s.p. orbits. In these cases we identify the theoretical states with the experimental ones that are preferentially excited in one-nucleon transfer reactions.[33] From Fig. 2 it appears that all the experimental states are very well reproduced by our calculation, with the exception of the $\frac{3}{2}^+$ state in ^{141}Pr and ^{143}Pm. In fact, for these nuclei the calculated energies are about 0.25 MeV lower than the experimental ones; this produces an inversion of the $\frac{3}{2}^+$ and $\frac{1}{2}^+$ states with respect to the experimental spectra. This may be viewed as an indication that non-negligible components of higher seniority are present in these states. Indeed, the presence of such components is evidenced by the calculations of Refs. 49 and 50. It should be mentioned that these calculations[49] also predict a rather large contribution of components with $v>1$ to

TABLE II. Comparison of experimental and calculated energies (in MeV) of the 9^- and 10^+ states in the even-mass $N=82$ isotones (see text for comments). The experimental data for $A=138$, 140, 142, 144, 146, and 148 are taken from Refs. 38, 39, 40, 37, 1, and 41, respectively.

Configuration	$J^\pi_{\max}$	Nucleus	Expt.	Calc.
$g_{7/2}h_{11/2}$	9^-	^{138}Ba	3.63[a]	3.70
		^{140}Ce	3.49	3.55
		^{142}Nd	3.48	3.41
		^{144}Sm	3.46	3.42
		^{146}Gd	3.43	3.35
$h_{11/2}h_{11/2}$	10^+	^{144}Sm	4.22[b]	4.47
		^{146}Gd	3.86	3.86
		^{148}Dy	2.92	2.95

[a]Uncertain spin and parity assignment.
[b]No parity assignment.

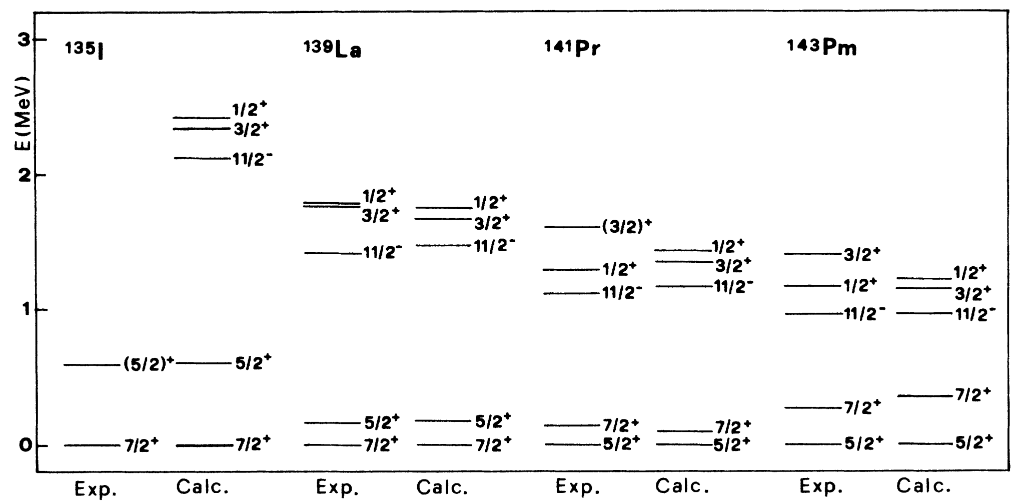

FIG. 2. Experimental and calculated levels of ^{135}I, ^{139}La, ^{141}Pr, and ^{143}Pm. The experimental data are taken for $A=135$, 139, 141, and 143 from Refs. 42, 43, 44, and 45, respectively.

the wave function of the $\frac{1}{2}^+$ state in ^{141}Pr. As a matter of fact, apart from the $\frac{3}{2}^+$ states in ^{141}Pr and ^{143}Pm, the largest discrepancy (0.13 MeV) between our results and the experimental ones occurs for this state. In this connection, it is to be noted that in ^{141}Pr two $\frac{1}{2}^+$ states, rather close in energy, are strongly excited in one-nucleon transfer reactions.[33] The state reported in Fig. 2 is the lowest one, which has the largest spectroscopic factor (0.61 vs 0.51).

The data available for the heavier nuclei (Fig. 3) are more scanty. In particular, owing to the instability of the neighboring even nuclei, no spectroscopic factors are known. In these nuclei for each value of the spin and parity of the s.p. valence orbits only one state has been observed in the low-energy spectrum (below 1.2 MeV); and it is these states which are reported in Fig. 3. The experimental situation requires, however, some further comments. The spin assignment $\frac{1}{2}^+$ to the ground state of ^{147}Tb is based on the recent study of Ref. 46. It should be mentioned that in an earlier work[51] the assignement $\frac{11}{2}^-$ was proposed. The level ordering[52] in the two heavier isotopes ^{149}Tb and ^{151}Tb may be viewed as a confirmation of the $\frac{1}{2}^+$ assignment. For the ground state of ^{151}Tm there is still ambiguity between $\frac{1}{2}^+$ and $\frac{11}{2}^-$ assignment. The energy systematics reported in Ref. 52 argues in favor of the $\frac{11}{2}^-$ assignment. We have therefore assumed the $\frac{11}{2}^-$ state as ground state in the experimental spectrum and placed the $\frac{1}{2}^+$ state at our calculated energy (0.045 MeV). It should be noted that this value is quite in agreement with the above-mentioned systematics.

From Fig. 3 we see that the calculated spectrum of ^{149}Ho is in quite good agreement with the experimental one. As regards ^{151}Tm, the energies of the $\frac{3}{2}^+$ and $\frac{7}{2}^+$ states relative to the $\frac{1}{2}^+$ state are well reproduced, while that of the $\frac{5}{2}^+$ level turns out to be rather larger than the experimental value (by 0.32 MeV). The structure of the theoretical spectrum of ^{147}Tb is similar to the one observed, but the ordering of the close-lying levels $\frac{1}{2}^+, \frac{11}{2}^-$ and $\frac{3}{2}^+, \frac{5}{2}^+$ is inverted.

The nucleus ^{147}Tb with one proton in excess to ^{146}Gd has been considered by some authors the principal source of information on the single-proton energies near $Z=64$.

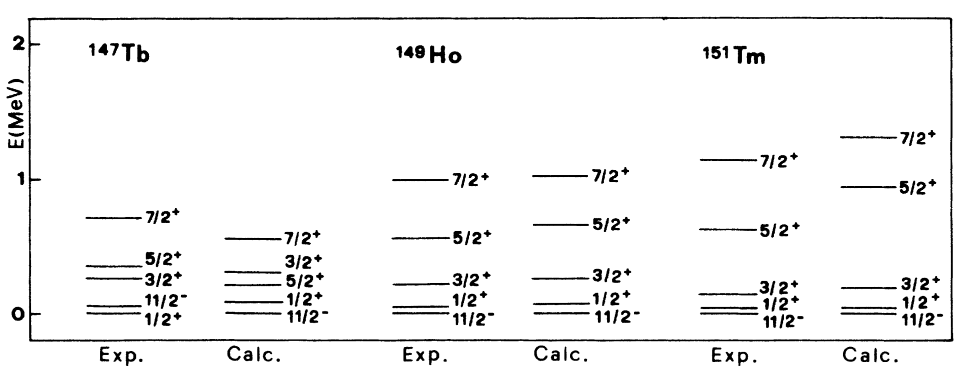

FIG. 3. Experimental and calculated levels of ^{147}Tb, ^{149}Ho, and ^{151}Tm. The experimental data are taken for $A=147$, 149, and 151 from Refs. 46, 47, and 48, respectively.

TABLE III. Experimental (Ref. 33) and calculated stripping spectroscopic factors [see Eqs (14) and (16)] for levels of odd-mass $N=82$ nuclei.

j \ A	135	137		139		141		143		145	
	Calc.	Expt.	Calc.	Expt.	Calc.	Expt.	Calc.	Expt.	Calc.	Expt.	Calc.
$\frac{7}{2}$	0.81	0.60	0.64	0.43	0.47	0.28	0.34	0.25	0.24	0.17	0.18
$\frac{5}{2}$	0.95	1.02	0.89	0.94	0.82	0.64	0.71	0.54	0.57	0.33	0.32
$\frac{11}{2}$	0.99	1.01	0.97	0.84	0.96	0.96	0.94	0.82	0.92	0.83	0.91
$\frac{3}{2}$	0.99	0.79	0.98	0.73	0.96	1.04	0.95	1.13	0.93	1.03	0.93
$\frac{1}{2}$	0.99	0.86	0.98	0.65	0.97	0.61	0.95	1.08	0.94	1.00	0.92

As already mentioned, the s.p. energies used to calculate the spectra presented in Fig. 3 have been determined from the experimental levels of ^{145}Eu, the main reason being the more complete information available for this nucleus. It seemed to us worthwhile, however, to investigate the outcome of the alternative choice of using s.p. energies obtained from ^{147}Tb. The main difference between these two sets of s.p. energies is the higher location of the $h_{11/2}$ level in the latter (this is about 0.2 MeV higher than that determined from ^{145}Eu) which causes it to lie above the $s_{1/2}$ level. It turns out that the use of the s.p. energies determined from ^{147}Tb results in a considerable deterioration of the agreement with the experimental data. In fact, the $\frac{11}{2}^-$ and $\frac{1}{2}^+$ states are inverted in all of the three nuclei, ^{145}Eu, ^{149}Ho, and ^{151}Tm. In addition, the $\frac{5}{2}^+$ and $\frac{7}{2}^+$ states in the two latter nuclei are well above the observed ones, the discrepancy ranging from about 0.24 to 0.55 MeV. This indicates that the low-energy states of ^{147}Tb are not adequately described as seniority-one states.

2. *Spectroscopic factors*

Within the framework of the formalism outlined in Sec. II the single-particle spectroscopic factors (see Ref. 18 for definitions) assume a very simple form. For the sake of completeness we give here the explicit expressions of these quantities.

In the particle formalism (see Sec. II) one has

$$S_j^2(N+1)=1-\rho_j(N) \tag{14}$$

and

$$T_j^2(N+1)=\frac{X_j^2(N+2)}{\Omega_j^2[1-\rho_j(N)]} \tag{15}$$

for stripping and pickup, respectively. In the hole formalism the corresponding expressions are

$$S_j^2(N+1)=\frac{X_j^2(N+2)}{\Omega_j^2\rho_j(N+2)}\ , \tag{16}$$

$$T_j^2(N+1)=\rho_j(N+2)\ . \tag{17}$$

In Tables III and IV we compare the calculated stripping and pickup spectroscopic factors with those of Ref. 33. These tables show a very good agreement between experiment and theory in practically all cases. It should be noted that this is particularly true for ^{145}Eu. This is a confirmation that this nucleus is quite a good source of information on the s.p. energies near $Z=64$.

V. SUMMARY AND CONCLUSIONS

We have reported here the results of a comprehensive study of the $N=82$ isotones aimed at assessing the role of proton pairing correlations in this region. In this study we have made use of a number-conserving approach to the pairing-force problem which we have ourselves developed in prior work.

Although the importance of pairing for the $N=82$ nuclei has long been recognized, we think that the results of the present work put it on a far more quantitative basis. In fact, a large number of experimental data are very well reproduced by our calculations. It should be emphasized that we have studied seventeen nuclei using only two sets of s.p. energies and a single value of the pairing strength. As illustrated in Sec. III, the determination of these quantities has been brought about through a detailed analysis of the experimental data over the whole $N=82$ region. The quality of our results is a clear confirmation

TABLE IV. Experimental (Ref. 33) and calculated pickup spectroscopic factors for levels of odd-mass $N=82$ nuclei. The theoretical values as given by Eqs. 15 and 17 are multiplied by $(2j+1)$.

j \ A	135		137		139		141		143	
	Expt.	Calc.	Expt.	Calc.	Expt.	Calc.	Expt.	Calc.	Expt.	Calc.
$\frac{7}{2}$	2.74	2.90	3.91	4.18	6.21	5.28	6.06	6.03	6.85	6.58
$\frac{5}{2}$	0.34	0.65	1.01	1.11	1.71	1.72	2.70	2.58	3.80	3.53
$\frac{11}{2}$		0.31		0.49	0.7	0.70	1.03	0.97	1.65	1.33
$\frac{3}{2}$		0.09		0.14		0.20		0.27	0.48	0.37
$\frac{1}{2}$		0.04		0.07		0.09	0.09	0.13	0.23	0.17

that the chosen values of the s.p. energies and pairing strength represent a most appropriate choice.

According to our calculations a relatively strong effect of proton pairing is present at $Z=64$, the occupation of the three s.p. levels above the gap being close to two protons. This means that ^{146}Gd is not really a doubly-magic nucleus.

Clearly, the calculated amount of pairing correlations in ^{146}Gd depends on both the intensity of the residual interaction and the magnitude of the energy gap in the s.p. spectrum. Concerning this latter quantity, some further comments are in order. In fact, after the first experimental works[53–55] pointing to the existence of a pronounced gap at $Z=64$, its size has been a matter of discussion, and different values are proposed in the literature, ranging from about 1.6 to 2.5 MeV. Our value (1.9 MeV) compared to those suggested by similar calculations[7,9,26,27] turns out to be somewhat smaller. This is essentially because we have related the $Z=64$ gap to the spectrum of ^{145}Eu, rather than to that of ^{147}Tb, as in the references quoted. Our choice is well justified by the findings discussed in Sec. IV B.

In conclusion, the success achieved by the present calculation reflects the simplicity of the $N=82$ nuclei in terms of shell model and indicates that most of the properties of these nuclei may be accurately described by using a more realistic interaction within the framework of the seniority scheme with a truncation at low seniority values. Along these lines we are currently exploiting our equations-of-motion formalism.[19]

This work was supported in part by the Italian Ministry of Education.

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