Thermal decay rate of multidimensional fission under a nonlinear coupling

N. Takigawa
Department of Physics, Tohoku University, 980 Sendai, Japan

M. Abe
Department of Basic Science, Ishinomaki, Senshu University, 986 Ishinomaki, Japan

(Received 30 January 1990)

We study the effects of coupling between the fission coordinate and bosonic intrinsic excitations on the fission width by using a path integral method. We assume a general coupling form factor and discuss how the decay width at high temperature changes from that of the extended Kramers formula recently obtained in the bilinear coupling model. Applying the results to two different types of bosonic spectra we show a possibility to represent the effects of nonlinearity of the coupling in terms of a suitable renormalization of the friction and the mass for the fission coordinate.

The decay of a metastable system is a fundamental problem in many fields of physics. An important recent progress in this subject is the development of the path integral approach to Langer's prescription,[1–5] where the decay width of a metastable state is obtained by calculating the imaginary part of the free energy. This method can naturally describe the change of the decay mechanism from the quantum tunneling to the classical thermal hopping as the temperature is raised.[5] In addition, it can easily handle the decay in multidimensional systems such as the macroscopic quantum tunneling discussed by Leggett and others.[4,5] In the high temperature regime, the Kramers formula for the fission width was modified with this method by including a quantum correction and a memory effect in the dissipation factor.[5,6]

The studies so far made are, however, limited to the case of a bilinear coupling, i.e., to the case where the coupling Hamiltonian is linear with respect to both the coordinates of the oscillators, which represent the environments, and the decay coordinate. In the case of nuclear fission, however, the linear coupling form factor is clearly unrealistic, because the interaction between fission fragments should vanish when they are far apart. The aim of this work is thus to discuss what happens when the coupling Hamiltonian is not a linear, but a general function of the decay coordinate.

$$L=\frac{M}{2}\dot{q}^2-V(q)+\sum_i\frac{m_i}{2}(\dot{q}_i^2-\omega_i^2q_i^2)-\sum_i c_iq_if(q), \tag{1}$$

where q and q_i are the fission coordinate and the coordinates of the environmental oscillators, respectively. The last term describes the coupling between them, where the coupling form factor $f(q)$ is assumed to be a general function of the decay coordinate. Note that Eq. (1) does not contain the counter term that has been introduced in Refs. 4 and 5 in order to cancel the static potential renormalization.

We calculate the partition function by using a path integral technique to determine the imaginary part of the free energy. By eliminating the environmental degrees of freedom,[7] the partition function at the temperature $T=1/\beta$ can be expressed only by the fission coordinate $q(\tau)$ as

$$Z(\beta)=\int dq\int\mathcal{D}q\,\exp[-S_{\rm eff}(q)/\hbar]\;, \tag{2}$$

where the effective action $S_{\rm eff}(q)$ is given by

$$S_{\rm eff}(q)=\int_0^{\hbar\beta}d\tau\left[\frac{M}{2}\dot{q}^2+V_{\rm eff}(q)\right]+\frac{1}{2}\int_0^{\hbar\beta}d\tau\int_0^{\hbar\beta}d\tau'\kappa(\tau-\tau')f(q(\tau))f(q(\tau'))\;. \tag{3}$$

In Eq. (3), $V_{\rm eff}(q)$ is the renormalized potential $V(q)+\Delta V(q)$ with

$$\Delta V(q)=-\sum_i\frac{c_i^2}{2m_i\omega_i^2}[f(q)]^2\;. \tag{4}$$

The influence kernel $\kappa(\tau)$ is given by

$$\kappa(\tau)=\sum_i\left[\frac{c_i^2}{m_i\omega_i^2}{:}\delta(\tau){:}-\frac{c_i^2}{2m_i\omega_i}\frac{\cosh[\omega_i(|\tau|-\frac{1}{2}\hbar\beta)]}{\sinh(\frac{1}{2}\hbar\omega_i\beta)}\right], \tag{5}$$

where $:\delta(\tau):$ is a generalized delta function with a period $\hbar\beta$.[5] Equation (4) shows that the renormalization of the fission barrier is very sensitive to the properties of the coupling form factor. Both cases could occur, where $V_{\rm eff}$ has a lower or a higher barrier than that of the bare potential $V(q)$. Figure 1 illustrates these cases. The upper figure corresponds to the situation where $f(q)$ is peaked in the internal region, so that the fission barrier is raised. The lower figure shows, on the other hand, the case where $f(q)$ is localized around the barrier region, and the fission barrier is lowered.

We evaluate the path integral in Eq. (2) in the saddle point approximation with respecting the periodic boundary condition $q(0)=q(\hbar\beta)=q$. We particularly consider

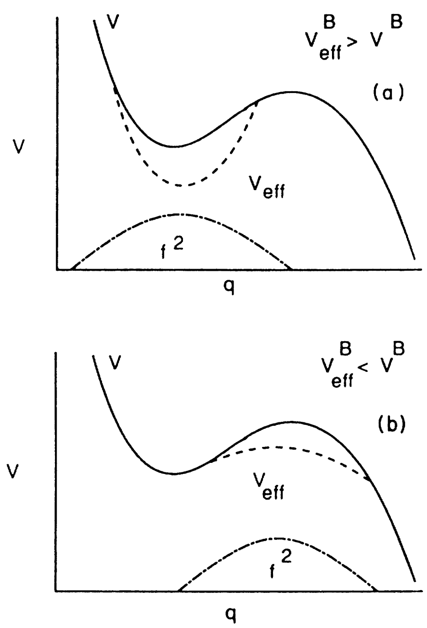

FIG. 1. The change of the fission barrier due to a nonlinear coupling to environmental oscillators. The solid and the dashed lines are the bare and the effective fission potentials, respectively. The dot-dashed line is the squared coupling form factor.

the high temperature regime, i.e., when $T > T_c$, T_c being the critical temperature above which the thermally activated decay dominates. It is of the order of 100–200 keV in the case of nuclear fission, since the curvature of the fission barrier is typically 1 MeV.[8] It is straightforward to generalize the procedure described in Ref. 5 to the case of the nonlinear coupling. Following Ref. 3, the decay width is given by the imaginary part of the free energy as

$$\Gamma(T)=-\frac{2}{\hbar}\frac{T_c}{T}\operatorname{Im}F \ . \tag{6}$$

Putting the critical temperature $T_c=\hbar\omega_R/2\pi$, we finally obtain

$$\Gamma(T)=\left[\frac{\omega_0}{2\pi}e^{-\beta V_b}\right]\left[\frac{\omega_R}{\omega_b}\right]f_Q \ , \tag{7}$$

where

$$f_Q=\prod_{n=1}^{\infty}\frac{\nu_n^2+\omega_0^2+\frac{1}{M}\hat{K}(\nu_n)\left[\frac{df}{dq}\right]_{q_0}^2}{\nu_n^2-\omega_b^2+\frac{1}{M}\hat{K}(\nu_n)\left[\frac{df}{dq}\right]_{q_b}^2} \ . \tag{8}$$

In these equations, ω_0 is the curvature parameter of V_{eff} at q_0, where V_{eff} takes a minimum corresponding to the compound nucleus, and ω_b is that at the barrier position q_b. The first factor on the right-hand side of Eq. (7) is the fission width of the transition state theory.[9] The second factor ω_R/ω_b corresponds to the friction factor originally introduced by Kramers,[6] but, as will be discussed later, it is generalized by taking a memory effect into account (see also Ref. 5). The f_Q is a factor, which originates from the quantum fluctuation of the path around the trivial classical trajectories $q_{\text{cl}}(\tau)=q_0$ and $q_{\text{cl}}(\tau)=q_b$. The $\nu_n=2\pi n/\hbar\beta$ are the Matsubara frequencies for environmental bosons, while $\hat{K}(\nu)$ is the Laplace transform of $\kappa(\tau)$ and is given by

$$\hat{K}(\nu)=\sum_i\frac{c_i^2}{m_i\omega_i^2}\frac{\nu^2}{\omega_i^2+\nu^2} \ . \tag{9}$$

Let us now compare the present results with those for the case of the bilinear coupling model. The most evident change can be seen in the quantum correction factor f_Q. It reflects the properties of the coupling form factor both at the fission barrier and at the potential minimum. As can be seen in Eq. (4), the effects of nonlinearity of the coupling Hamiltonian are explicit in the potential renormalization and also affect ω_0 and ω_b. Other effects appear in the critical temperature T_c and in the dissipation factor ω_R/ω_b. To see this more explicitly, let us study ω_R, which is the lowest positive solution of

$$\omega_R^2-\omega_b^2+\frac{1}{M}\hat{K}(\omega_R)\left[\frac{df}{dq}\right]_{q_b}^2=0 \ . \tag{10}$$

This is the condition that the denominator of Eq. (8) vanishes when ν_1 is replaced by ω_R. Equation (10) clearly shows that the critical temperature is influenced by the property of the coupling form factor at the potential barrier. In order to see the physical implication of Eq. (10), let us relate $\hat{K}$ to the retarded friction function γ appearing in the classical equation of motion.[10,11] It is defined by

$$M\ddot{q}=-\frac{d}{dq}V_{\text{eff}}(q)-\int_0^{t-t_0}d\tau\,\gamma(t,\tau)\dot{q}(t-\tau) \ , \tag{11}$$

and reads

$$\gamma(t,\tau)=\zeta(\tau)\left[\frac{d}{dq(t)}f(q(t))\right]\left[\frac{d}{dq(t-\tau)}f(q(t-\tau))\right] , \tag{12}$$

with

$$\zeta(\tau)=\sum_i\frac{c_i^2}{m_i\omega_i^2}\cos(\omega_i\tau) \ . \tag{13}$$

One can easily confirm that

$$\hat{K}(\nu)=\nu\hat{\zeta}(\nu) \ , \tag{14}$$

where $\hat{\zeta}(\nu)$ is the Laplace transform of $\zeta(\tau)$. Equations (10) and (14) suggest that the effects of nonlinearity to the critical temperature and to the dissipation factor can be effectively taken into account without changing their formulas if one suitably rescales the friction of the bilinear coupling model. The properly renormalized friction in that prescription is given by

$$\hat{\zeta}_{\text{eff}}(\nu)=\hat{\zeta}(\nu)\left[\frac{df}{dq}\right]_{q_b}^2 \ . \tag{15}$$

Contrary to Eq. (12), the effective friction $\hat{\zeta}_{\text{eff}}$, which governs the decay width, does not involve the retardation effect as for the form factor $f(q)$. In other words, the dependence of friction on $f(q)$ can be separated from the retardation effect. This is consistent with the fact that the quantity $\hat{\zeta}(\nu)$, which enters Eq. (10) through Eq. (14), becomes independent of ν and can be interpreted as a friction in the simple sense of the original Kramers work only when the memory effect can be ignored.[5]

We now explore our results by considering two frequency distributions that have been considered in Refs. 4 and 12. Caldeira and Leggett considered the limit when the frequencies ω_i are distributed continuously. They introduced a spectral function

$$\frac{\pi}{2}\sum_i \frac{c_i^2}{m_i\omega_i}\delta(\omega-\omega_i)\approx\eta\omega \ , \tag{16}$$

with a friction constant η. In this case,

$$\hat{K}(\nu)=\eta\nu \ . \tag{17}$$

Therefore,

$$\left[\frac{\omega_R}{\omega_b}\right]=(1+\alpha_b^2)^{1/2}-\alpha_b \ , \tag{18}$$

with

$$\alpha_b=\frac{\eta}{2M\omega_b}\left[\frac{df}{dq}\right]_{q_b}^2 . \tag{19}$$

Equation (19) confirms our previous assertion that the effects of the nonlinearity of the coupling form factor are equivalent to rescaling the friction coefficient as long as the critical temperature and the dissipation factor ω_R/ω_b are concerned.

The second spectrum was that used by Brink and Canto in order to study a simple model for induced fission. They replaced the sum over the oscillators with an integral over ω by introducing a function $\rho(\omega)$, i.e.,

$$\sum_i \frac{c_i^2}{m_i\omega_i^2}\left[\frac{\omega_i^2}{\omega_i^2+\Lambda^2}\right]\to\int_0^\infty d\omega\,\rho(\omega)\left[\frac{\omega^2}{\omega^2+\Lambda^2}\right] . \tag{20}$$

They have assumed that the spectrum of the oscillators is sufficiently dense and the coupling strength is slowly varying with frequencies. We simplify their model by assuming

$$\rho(\omega)=\begin{cases}\rho_0 & \text{for } \omega\le\omega_{\max} \ ;\\ 0 & \text{for } \omega>\omega_{\max} \ .\end{cases} \tag{21}$$

In this model,

$$\hat{\zeta}(\nu)=\rho_0\cot^{-1}\left[\frac{\nu}{\omega_{\max}}\right] . \tag{22}$$

If we assume, similarly to Ref. 12, that $\rho(\omega)$ extends to high frequency modes, i.e., $\omega_{\max}/\nu\gg1$ for physically relevant values of ν, then we have

$$\hat{K}(\nu)\approx\eta\nu-\Delta M\cdot\nu^2 \ , \tag{23}$$

with

$$\eta=\frac{\pi}{2}\rho_0, \quad \Delta M=\frac{\rho_0}{\omega_{\max}} \ . \tag{24}$$

One can easily confirm that η and ΔM thus introduced are the friction and the mass correction, respectively, discussed in Ref. 12 based on the multidimensional diffusion approach of Shang and Weidenmueller.[13] Note that the zeroth order term, which appeared in Ref. 12, does not exist in our formalism because of the absence of the counter term.

Inserting Eq. (23) into (10), we obtain

$$\left[\frac{\omega_R}{\omega_b}\right]=\frac{1}{1+x}\{[(1+x)+\alpha_b^2]^{1/2}-\alpha_b\} \ , \tag{25}$$

where α_b is related to η by the same equation as Eq. (19), while x is defined by

$$x=-\frac{\Delta M}{M}\left[\frac{df}{dq}\right]_{q_b}^2 . \tag{26}$$

Equation (26) shows that the nonlinear effect to the mass correction can also be treated by a suitable renormalization. A striking difference of the present model from that of Ref. 4 is that the mass correction could introduce a significant change of the critical temperature and therefore of the dissipation factor in the Kramers formula.

We conclude this paper by summarizing our results. We have used a path integral method to study how the coupling of the fission coordinate to bosonic intrinsic excitations affects the fission width. The prescription by Langer and Affleck was used to determine the decay width. We thus extended the well-known Kramers formula by including a memory effect in the dissipation factor and by multiplying a quantum correction factor. Our formula shows that the effects of nonlinearity of the coupling form factor can be taken into account by suitably renormalizing the friction coefficient and the effective mass in the corresponding formulas for the bilinear coupling model. This is the case for the critical temperature and for the dissipation factor in the Kramers formula. A similar renormalization cannot, however, be applied to dealing with the quantum correction factor, because it requires the value of df/dq both at the potential minimum and at the fission barrier. The potential renormalization is another factor which requires an explicit treatment of the nonlinearity. We explicitly kept the potential renormalization without introducing the counter term. This is because the potential renormalization is considered[14] to be an origin of the large enhancement of the fusion cross section in heavy ion collisions at sub-barrier energies, which are, in a sense, inverse processes of fission.

We have concentrated in this work on the high temperature regime. It is not so difficult to extend our results also to the low temperature regime, though one has to numerically deal with the classical bounce trajectory. This problem will be reported in a separate paper.

[1] J. S. Langer, Ann. Phys. (N.Y.) **41**, 374 (1967).
[2] G. Callan and S. Coleman, Phys. Rev. D **16**, 1762 (1977).
[3] I. K. Affleck, Phys. Rev. Lett. **46**, 388 (1981).
[4] A. O. Caldeira and A. J. Leggett, Phys. Rev. Lett. **46**, 211 (1981); Ann. Phys. (N.Y.) **149**, 374 (1983); **153**, 445(E) (1984).
[5] H. Grabert, P. Olschowski, and U. Weiss, Phys. Rev. B **36**, 1931 (1987).
[6] H. A. Kramers, Physica **7**, 284 (1940).
[7] R. P. Feynman, *Statistical Mechanics* (Benjamin, London, 1982), pp. 81–83.
[8] R. Vandenbosch and J. R. Huizenga, *Nuclear Fission* (Academic, New York, 1973).
[9] N. Bohr and J. A. Wheeler, Phys. Rev. **36**, 426 (1939).
[10] A. B. Balantekin and N. Takigawa, Ann. Phys. (N.Y.) **160**, 441 (1985).
[11] K. Nishinohara and N. Takigawa, Z. Phys. A **324**, 139 (1986).
[12] D. M. Brink and L. F. Canto, J. Phys. G **12**, L147 (1986).
[13] Z. J. Shang and H. A. Weidenmueller, Phys. Rev. C **28**, 2190 (1983).
[14] N. Takigawa and G. F. Bertsch, Phys. Rev. C **29**, 2358 (1984).