Quantum chaos for exact and broken K quantum number in the interacting-boson model

V. Paar and D. Vorkapić
Prirodoslovno-matematički fakultet, University of Zagreb, 41000 Zagreb, Yugoslavia
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We show that the exact K quantum number in the SU(3) limit of the interacting-boson model has a strong effect on the fluctuation properties: Pure sequences of a single K value have Δ_3 statistic close to the Gaussian overlap ensemble prediction while mixed sequences with combined K values are removed far from the Gaussian overlap ensemble because of K degeneracy, exceeding even the Poisson-ensemble prediction. Applying the extended interacting boson model to the realistic case of ^{164}Er nucleus, we have demonstrated that weak breaking of the K quantum number introduces a very rapid change in fluctuation properties towards those of a pure sequence.

In a recent Letter[1] the analysis of the energy spectrum of atomic nucleus ^{26}Al gave the evidence that fluctuation properties of mixed-isospin sequence are compatible to a single Gaussian overlap ensemble[2] (GOE) sequence. More precisely, it was shown that mixed sequence with combined $T=0$ and $T=1$ data set agrees with GOE, although the fluctuation properties in the case of exact isospin quantum number would be very different from those of GOE, close to the Poisson-ensemble[2] prediction. This was a first experimental indication that weak breaking of isospin symmetry introduces a rapid change in the fluctuation properties, in accordance with recent theoretical investigations.[3]

Certain statistical properties of a quantum spectrum that corresponds to a classically chaotic system agree with those expected for random matrices, and for a quantum spectrum that corresponds to a classically regular system, agree with Poisson-ensemble prediction.[2,4–11] On the basis of this conjecture resulting from model investigations, it follows that broken isospin quantum number does not affect chaotic behavior. On the other hand, the investigation of fluctuation properties of the shell-model energy spectrum in ^{24}Mg indicates that breaking of isospin for the states calculated in shell model is weaker, so that a mixed-isospin sequence can be distinguished from a GOE sequence.

In this paper we present results of the first investigation of effects due to exactly conserved and broken K quantum number on fluctuation properties of the energy spectrum. K quantum number characterizes rotational states in heavy nuclei; here we employ the SU(3) dynamical symmetry of the interacting-boson model (IBM),[12,13] which provides a suitable framework for algebraic description of rotational states. It should be noted that the IBM has some new features and therefore appears as an interesting case to test ideas about quantum chaos. Namely, the IBM has a microscopic basis, it successfully describes nuclear phenomenology accounting for collectivity, and is characterized by compact group structure.[13]

Before discussing breaking of K quantum number, it is advantageous to consider the case of complete K quantum-number conservation. We note that such a line of approach is an analogy to considerations[14] of isospin-symmetry breaking in compound-nucleus reactions. In the SU(3) limit of the IBM, the K quantum number is exactly conserved.[13] In accordance, in the first step of our considerations we shall assume that states of rotational nuclei have pure K quantum number. All the states with angular momentum $J=0$ have $K=0$. Therefore, the set of $J=0$ levels presents a pure sequence for the fluctuation analysis. On the other hand, the $J=2$ states have two possible values of K: $K=0$ and 2. Therefore, the set of all $J=2$ levels presents a mixed sequence, combined of two subsets of states with $(J=2,K=0)$ and $(J=2,K=2)$. In a similar way, the $J=4$ states can have three possible values of K: $K=0$, 2, and 4. Therefore, the set of all $J=4$ levels presents a mixed sequence consisting of three subsets of states with $(J=4,K=0)$, $(J=4,K=2)$, and $(J=4,K=4)$.

Unfolding of the energy spectrum and statistical analysis are performed as follows. Having a theoretical spectrum, first we determine the corresponding level density. We employ two different methods of unfolding the spectrum.

The first method, referred to as constant temperature (CT) method, is defined in the following way. The staircase function $N(E)$ (cumulative number of levels up to energy E for each set of quantum numbers) is fitted by the constant temperature (CT) formula of the constant temperature Fermi gas model.[15] In this way we obtain a rather good fit to the IBM energy spectra. However, in the upper part the IBM staircase function $N(E)$ falls below the CT prediction. This is an effect of truncation of the boson space. This upper part of the spectrum, affected by truncation, is deleted from the statistical analysis. To this end we employ the numerical criterion for truncation energy. Having a set of energy levels $E_1,E_2,E_3,\ldots,E_n$, in a diagram presenting the cumulative number of levels versus energy we draw a line through each pair of points $(E_1,1)$ and $(E_s,N(E_s))$, where $s=n/2,n/2+1,\ldots,n$. If there is s for which all points $(E_t,N(E_t))$ for $t>s$ lie below the line defined above, the energy E_s is taken as truncation energy. Thus, for each set of n quantum numbers we have a set of levels

for statistical analysis $E_1,E_2,E_3,\ldots,E_m$ $(m \leq n)$ and the smooth level density $\rho_{CT}(E)$ which corresponds to the CT fit. In the next step, the spectrum $E_1,E_2,\ldots,E_n$ is mapped into the unfolded spectrum $E'_1,E'_2,\ldots,E'_m$ by

$$E'_1=0$$

and

$$E'_{i+1}-E'_i=(E_{i+1}-E_i)\rho_{CT}(E_i)\ . \tag{1}$$

The second method of unfolding, referred to as the French-Wong (FW) method, is defined in the following way. For each set of quantum numbers, the spectrum $E_1,E_2,\ldots,E_n$ is mapped into the unfolded spectrum $E'_1,E'_2,\ldots,E'_n$ by using

$$E'_1=0$$

and

$$E'_{i+1}-E'_i=(E_{i+1}-E_i)\rho_{FW}(E_i)\ . \tag{2}$$

Here $\rho_{FW}(E_i)$ denotes the local level density according to the French-Wong prescription from Ref. 16.

Next, in the calculation of Δ_3 statistic we employ the method of overlapping intervals from Refs. 17 and 18, using the level density calculated in the previous step. For a given L the energy interval (E'_1,E'_m) or (E'_1,E'_n) is divided into overlapping intervals

$$(E'_1,E'_1+L),(E'_1+d,E'_1+d+L),(E'_1+2d,E'_1+2d+L),\ldots, \tag{3}$$

where d is some fraction of the mean level spacing. Here we employ the value $d=0.5$, i.e., the energy shift d is equal to a half of the mean spacing. (We have tested that a further decrease of d does not influence the fluctuation analysis.) For a given L, we calculate the corresponding Δ_3 statistic in each interval according to Ref. 6. By averaging over all intervals we obtain $\bar{\Delta}_3(L)$, which presents a fluctuation measure of the long range order in nuclear spectrum.

In Figs. 1(a)–1(d) we present the calculated Δ_3 statistic with constant temperature density for $J=0$ levels and for mixed and pure sequences of $J=2$ states. We obtain similar results also by using the French-Wong prescription[16] for level density. Figures 1(a) and 1(b) show Δ_3 statistic for all levels with $J=0$ and $J=2$, respectively. The $J=0$ levels present a pure sequence, while the $J=2$ levels are a mixed sequence. Accordingly, Δ_3 statistic for $J=0$ states is rather close to the GOE prediction, while for all $J=2$ states it is removed far from the GOE prediction, overshooting even the Poisson-ensemble prediction. Therefore, the latter pattern will be referred to as "supraregular." In Figs. 1(c) and 1(d) we display Δ_3 statistic calculated for pure sequences $(J=2,K=0)$ and $(J=2,K=2)$, respectively. In these two cases the Δ_3 pattern is similar to the case of the $J=0$ pure sequence. Thus, it is seen that the "supraregular" pattern of Δ_3 statistic for $J=2$ spectrum is due to K quantum number. In the SU(3) limit of IBM the eigenvalues do not depend on K quantum number and therefore K degeneracy appears, which shows up in the fluctuation properties of mixed sequences of levels. A similar effect appears for $J=4$ states too. The Δ_3 statistic for pure sequences $(J=4,K=0)$, $(J=4,K=2)$, and $(J=4,K=4)$ are rather close to the GOE sequence, while the mixed sequence of all $J=4$ levels exhibits a "supraregular" pattern.

Thus, in the case of exactly conserved K quantum number, the fluctuation properties are as follows: Each pure sequence of levels (with single values of both J and K) exhibits a fluctuation pattern which is close to GOE prediction, while each mixed sequence of levels (with a single value of J and several possible values of K) exhibits "supraregular" behavior due to K degeneracy.

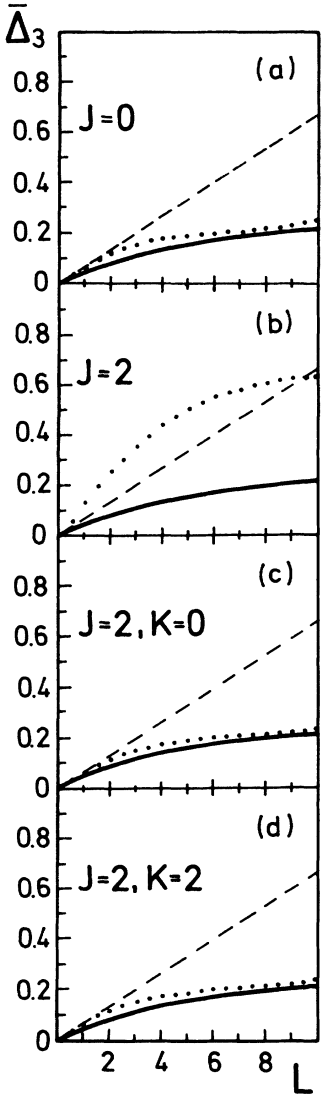

FIG. 1. Δ_3 statistic for energy levels in the SU(3) limit of IBM with parameters $\alpha=0.015$, $\beta=0.004$, $N=20$: (a) $J=0$ levels, (b) $J=2$ levels, (c) $J=2,K=0$ levels, and (d) $J=2,K=2$ levels. For comparison, we present the Poisson-ensemble (dotted line) and Gaussian overlap ensemble prediction (full line).

Let us now consider the fluctuation properties in the case when K quantum number is broken. To this end, it is suitable to use the extended version[19] of the interacting boson model, with the Hamiltonian

$$H = H_{\mathrm{IBM}}[\mathrm{SU}(3)] + \gamma O_l^0 \tag{4}$$

The first term denotes the standard IBM Hamiltonian in SU(3) limit[13] which conserves the K quantum number. The term O_l^0 is the scalar shift operator[19] which breaks the K quantum number without breaking SU(3) dynamical symmetry. It has nonvanishing off-diagonal matrix elements $\langle JK|O_l^0|JK\pm 2\rangle$ and therefore mixes the states of β and γ bands with identical angular momentum.[19] The quantity γ in Eq. (4) denotes the mixing parameter.

In the extended IBM the effect of breaking K quantum number on the energy spectrum was illustrated for several Gd and Er even-even nuclei. The energy spectra of these nuclei were fitted in the IBM and in the extended IBM; it was shown[19] that the theoretical results obtained by breaking K degeneracy are in better agreement with experimental data. In this work we investigate the corresponding fluctuation properties. In Fig. 2 we present Δ_3 statistic of the $J=2$ energy spectrum calculated in extended IBM for several values of mixing parameter γ, the other parameters being taken from the calculation[19] for the nucleus ^{164}Er. The Δ_3 curve for $\gamma=0$ corresponds to the IBM energy spectrum, with exact conservation of K quantum number. The Δ_3 curve for $\gamma=0.107$ corresponds to the extended IBM energy spectrum of ^{164}Er from Ref. 19, with weakly broken K quantum number.

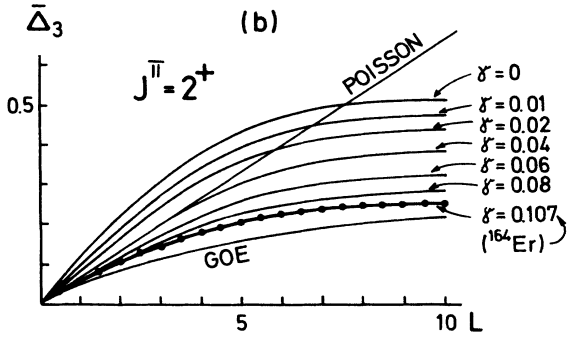

FIG. 2. Δ_3 statistic for energy levels in extended version (Ref. 19) of IBM with several values of mixing parameter γ. The Δ_3 statistic with mixing parameter $\gamma=0$ corresponds to IBM energy spectrum, with exact K quantum number, and Δ_3 statistics with $\gamma=0.107$ to the calculated (Ref. 19) energy spectrum of nucleus ^{164}Er, with breaking of K quantum number.

Several transitional cases, with intermediate values of mixing parameter γ, are also presented. It is seen how a small breaking of K quantum number introduces a very rapid change of fluctuation properties, with a shift towards those of a pure sequence. This result is consistent with general expectations due to Dyson[20] and Pandey.[21] In this connection we also note the important results of recent Letter[22] that a broken dynamical symmetry leads to nonintegrability; here the K quantum number is broken, but the SU(3) dynamical symmetry is not violated because the shift operator O_l^0 is built from the SU(3) generators.

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