Electromagnetic properties of some positive parity dipole states described in terms of quadrupole and octupole interacting bosons

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The first three positive parity dipole states predicted by a phenomenological quadrupole-octupole boson Hamiltonian are extensively studied. Their coupling to the neighboring positive and negative parity states, due to the $M1$ and $E\lambda$ ($\lambda=1,3$) transitions, respectively, are considered. Special attention is paid to the lowest two states which are of collective $M1$ nature. The signature which distinguishes them from the $M1$ state describing the scissors mode is also discussed.

I. INTRODUCTION

The low-lying collective $M1$ state, known as scissors mode, first discovered in high-resolution inelastic electron scattering[1] and immediately after confirmed in nuclear resonance fluorescence experiments,[2] has been observed by now in most deformed nuclei from ^{46}Ti to ^{238}U chiefly by means of (e,e') experiments.[3] In rare earth nuclei the mode is moderately fragmented with a mean excitation energy of $E \simeq 3$ MeV, is strongly coupled to the ground state by the orbital component of the $M1$ operator with a total strength ranging from $B(M1)\uparrow \simeq 2\mu_N^2$ to $B(M1)\uparrow \simeq 3.5\mu_N^2$ and is characterized in a distinctive way by the (e,e') $M1$ form factor.

The state has been studied in a variety of theoretical approaches of phenomenological as well as microscopic nature.[4]

Among the phenomenological descriptions quoted here[5–9] we mention the two rotor model[5] which provides the geometrical picture of the mode. In such a model indeed, the $M1$ state is obtained as a scissorslike oscillation of axially deformed proton versus neutron fluids around an axis perpendicular to their symmetry axes. Given its semiclassical nature, the two-rotor model (TRM) overestimates the energy and especially the $M1$ strength. It gives, however, a complete though semiquantitative characterization of the mode and reproduces correctly its $M1$ form factor up to moderately high-momentum transfer.

A more quantitative phenomenological description is provided by the interacting boson approximation (IBM2),[6] where the mode is described as a state of mixed symmetry with respect to the exchange of proton and neutron valence bosons. The model accounts quite satisfactorily for the $M1$ properties, specially the $M1$ form factor, but is not able to predict the energy of the mode.

Another phenomenological description which gives satisfactory results is the so-called generalized coherent state model (GCSM).[9] In this approach the $M1$ state is described as a dipole excitation, built out of two quadrupole bosons (one for protons and one for neutrons), of an axially symmetric coherent state. The model accounts realistically for both the $M1$ properties and the energy. This is obtained as the mean value of an effective Hamiltonian whose parameters are fixed by fitting some selected levels of the ground, beta, and gamma bands.

The phenomenological approaches, by their own nature, cannot account for the observed fragmentation of the mode. They can at most predict a splitting as a result of a deviation of a nuclear shape from axial symmetry.[10] A complete detailed description of the properties of the mode can be attained only within microscopic schemes.

In medium light nuclei the mode has been described successfully within a simplified shell-model scheme.[11] In heavy deformed nuclei the only feasible microscopic approaches are in practice of random phase approximation (RPA) type. A considerable number of the RPA calculations has been indeed devoted to the study of the mode. Only some of them are quoted here.[12–20] Although all supporting the orbital nature of the mode, they tend to overestimate the total strength and its spreading, and, which is more disturbing, are far from converging towards a unique result. The reason of such a confusion is to be attached mainly to the fact that the different RPA approaches use different and incomplete Hamiltonians, which affect differently the magnetic properties of the mode. These are in fact quite sensitive[21] to the detailed form of the Hamiltonian. If most of the relevant components of the nuclear Hamiltonian are included and its spherical symmetry, broken in intrinsic RPA, restored, the RPA description of the mode greatly improves.[19,20] Concerning the fragmentation, it is worth noticing that this can be reduced by going beyond RPA.[22]

From the brief analysis presented above it is clear that the mode lends itself to be described with more or less success within a variety of approaches which may strongly differ from each other on a technical as well as on a conceptual ground. The reason for such a paradox is to be found in the fact that the scissors correlation suggested by the semiclassical TRM has a simple form and fits therefore easily into different and apparently uncorrelated schemes.

Given the simplicity of the mechanism of excitation of the mode and, by contrast, the richness in content of the approaches used for its description, one is led to suspect that states other than the scissors mode may be collectively excited via an $M1$ transition.

In order to explore the possible occurrence of new collective $M1$ states we developed in this paper a model based on a Hamiltonian of interacting quadrupole and octupole bosons, where no distinction is made between protons and neutrons.[23] We will show that some collective magnetic dipole states emerge. Though differing from the scissors mode, these states lend themselves to a geometrical picture borrowed from the TRM, which was the inspiring source of the present model.

In the intrinsic frame, indeed, the equilibrium shape of the nucleus described by the quadrupole octupole Hamiltonian is a superposition of two possible shapes, an octupole (with three lobes in a plane cut) and a quadrupole shape (an ellypse in the same plane).

The symmetry axis of an octupole lobe may be assumed to coincide with that of the quadrupole ellipsoid. The equilibrium shape may then look like that produced by two intersecting ellipsoids. Suppose now that the two axes oscillate in the same plane against each other and that the whole system rotates around a ζ axis belonging to the plane of the axes. Such a state is expected to have a large $B(M1)\uparrow$ value since the protons describe closed orbits of large magnetic moments. The picture is similar to that provided by the TRM. Here, however, the two fluids are of the same nature, the shapes are different and the angle between the two symmetry axes is not small, but large. The magnitude of this angle should be decisive for the excitation energy of the mode. One expects that the collective $M1$ states described by the present formalism lie at a different energy than the scissors mode and be related to the neighboring negative-parity states by noticeable $E1$ and $E3$ transitions. The existence of such a mode should in fact reflect a zero point octupole oscillation of large amplitude in the ground state, consistently with the picture of the nuclear shape sketched above. If furthermore any of the states described in this paper had a nonnegligible overlap with the scissors mode, this would be a measure of the influence of the octupole degrees of freedom on that state.

We stress once more that in the present approach the $M1$ collective states are constructed without making distinction between protons and neutrons.

The quadrupole octupole interaction was treated by one of us (A.A.R.) both microscopically[24–28] and phenomenologically.[29,30] Here we concentrate ourselves only on those features which are closely related to the dipole states.

The model Hamiltonian is defined in Sec. II, where its matrix elements in a restricted quadrupole octupole coupled basis are also presented. The reduced probabilities for $M1,E1,E3$ transitions characterizing the 1^+ states are treated in Sec. III. The microscopic content of the model is briefly sketched in Sec. IV. The applications are presented in Sec. V and the final conclusions are drawn in Sec. VI.

II. THE MODEL HAMILTONIAN

In a series of publications we formulated (A.A.R.) a phenomenological model[31,32] in order to describe the electric properties of the lowest three major bands (i.e., ground, beta, gamma) in terms of the collective motion of the quadrupole degrees of freedom. The model was named "the coherent states model" (CSM) since the model functions for the member states of the three bands are obtained by projecting out of three orthogonal deformed states which are elementary excitations of an axially symmetric coherent state. These states are constructed so as the following requirements be fulfilled: (i) They depend on a real parameter d which simulates the deformation, (ii) the projected states are also orthogonal, (iii) in the vibrational limit ($d\rightarrow 0$) the projected states are required to describe the semiempirical Sakai-Sheline[33] scheme which accounts for several selection rules concerning the $E2$ interband transitions, (iv) in the rotation limit (d large) the results of the liquid drop model in the large deformation regime should be recovered. In this limit the $E2$ transitions satisfy the Alaga rule.[34]

The states which satisfy these conditions are

$$\begin{aligned}
&\varphi_{JM}^{(g)}=N_J^{(g)}P_{MO}^J\psi_g,\quad \psi_g=\exp[d(b_{20}^\dagger-b_{20})]|0\rangle_2\ ,\\
&\varphi_{JM}^{(\gamma)}=N_J^{(\gamma)}P_{M2}^J\Omega_{\gamma,2}^\dagger\psi_g,\quad \Omega_{\gamma,2}^\dagger=(b_2^\dagger b_2^\dagger)_{22}+d\sqrt{2/7}b_{22}^\dagger\ ,\\
&\varphi_{JM}^{(\beta)}=N_J^{(\beta)}P_{MO}^J\Omega_\beta\psi_g\ ,\\
&\Omega_\beta^\dagger=(b_2^\dagger b_2^\dagger b_2^\dagger)_0+\frac{3d}{\sqrt{14}}(b_2^\dagger b_2^\dagger)_0-\frac{d^3}{\sqrt{70}}\ ,
\end{aligned}\tag{2.1}$$

where $b_{2m}^\dagger$ $(-2\le m\le 2)$ are quadrupole boson operators and $|0\rangle_2$ the corresponding vacuum state, $N_J^{(k)}$ are normalization factors and P_{MK}^J is the projection operator

$$P_{MK}^J=\frac{2J+1}{8\pi^2}\int D_{MK}^{J*}(\Omega)\hat{R}(\Omega)d\Omega\ . \tag{2.2}$$

Here we have denoted by $\hat{R}(\Omega)$ the rotation operator corresponding to the Euler angles Ω and by $D_{MK}^J(\Omega)$ the Wigner functions. (Throughout this paper we shall use the phase convection of Rose.[35])

The upper index of the projected states indicates that they are model functions for the states of the ground, gamma, and beta bands, respectively.

These states span a restricted collective space (r.c.s.) on which an effective Hamiltonian $H_{\rm CSM}$ acts. This is chosen so that the following partial decoupling conditions will be satisfied:

$$\langle\varphi_{JM}^{(i)}|H_{\rm CSM}|\varphi_{JM}^{(\beta)}\rangle=0,\quad i=g,\gamma\ . \tag{2.3}$$

The simplest solution for $H_{\rm CSM}$ is

$$H_{\rm CSM}=A_1(22\hat{N}_2+5\Omega_{\beta'}^\dagger\Omega_{\beta'})+A_2\hat{J}_2^2\ , \tag{2.4}$$

where

$$\Omega^\dagger_{\beta'}=(b_2^\dagger b_2^\dagger)_0-\frac{d^2}{\sqrt{5}} ,$$

$\hat{N}_2$ is the quadrupole boson number operator and $\hat{J}_2^2$ is the squared angular momentum. The lower label indicates that the quadrupole boson representation is used.

The Hamiltonian H_{CSM} can be implemented with some additional terms which do not violate the condition (2.3).[31] These terms affect only the energies of the beta band not considered here and therefore will be ignored for our purposes.

The Hamiltonian (2.4) was successfully used for describing both the excitation energies as well as the electric properties of a large number of nuclei. The CSM, indeed, provides a realistic parametrization of the collective properties of nuclei from three distinct symmetry [O(6),SU(3),SU(5)] regions.

Being based on the use of quadrupole collective degrees of freedom only, the model has some limitations:

(i) It is not able to describe the strong backbending which takes place at 12^+ in the even-even *Pt* isotopes where the interplay between the collective and the single-particle degrees of freedom seems to play an essential role.

(ii) It is not able to describe the $E3$ properties, which are mainly due to the octupole degrees of freedom.

(iii) It is not suitable for describing $M1$ states, since it is not possible to build up states of an angular momentum $J=1$ out of quadrupole bosons of the same nature.

In order to overcome the above limitations, several improving extensions have been elaborated.

In a first extension the detailed structure around and beyond the backbending region has been nicely described by coupling two quasiparticles to a collective CSM core.[36]

The magnetic properties of the ground, beta, and gamma bands as well as the structure of the $M1$ 1^+ state, were described within a generalized version of the CSM, known as GCSM.[9] In this new model one considers two quadrupole bosons $b^\dagger_{k\mu}$ $(k=\pi,\nu)$, describing, in the zeroth order, the independent harmonic vibrations of two liquid drops associated to the proton (π) and neutron (ν) systems, respectively.

A further extension[30] has consisted in considering quadrupole and octupole interacting bosons with no distinction between protons and neutrons. Due to the coupling of the two types of collective motion, it was possible to describe not only the positive parity bands—as in the original CSM—but also two negative-parity bands with $K^\pi=0^-,1^-$, respectively.

The Hamiltonian considered here consists also of interacting octupole $(b^\dagger_{3m})$ and quadrupole $(b^\dagger_{2m})$ bosons but is accordingly modified so as to allow for the emergence of $M1$ states.

Such a boson Hamiltonian has the form

$$H=H'_{\mathrm{CSM}}+A_3\hat{N}_3+A_4\{[(b_3^\dagger b_3^\dagger)_2+(b_3b_3)_2][b_2^\dagger+b_2]\}_0 + A_5[(b_3^\dagger b_3)_2(b_2^\dagger+b_2)]_0 , \quad (2.5)$$

where A_3 is the octupole boson number operator, H'_{CSM} is essentially H_{CSM} except for the term $\hat{J}_2^2$ replaced here with the squared total angular momentum operator whose expression in terms of quadrupole and octupole bosons is

$$\hat{J}^2=\sum_\mu \hat{J}_\mu\hat{J}_{-\mu}(-)^\mu,\quad \hat{J}_\mu=\hat{J}_{2,\mu}+\hat{J}_{3,\mu} , \quad (2.6)$$

$$\hat{J}_{k,\mu}=\sum_{m,m'}\langle km|\hat{J}_{k,\mu}|km'\rangle b^\dagger_{km}b_{km'},\quad k=2,3 . \quad (2.7)$$

Here we denoted by $|km\rangle$ the one boson state

$$|km\rangle=b^\dagger_{km}|0\rangle_k , \quad (2.8)$$

where $|0\rangle_k$ are the vacuum states of the two bosons:

$$b_{km}|0\rangle_k=0,\quad k=2,3 . \quad (2.9)$$

For what follows it is useful to introduce an additional notation

$$H'_{\mathrm{CSM}}=A_1H^{(1)}_{\mathrm{CSM}}+A_2\hat{J}^2 . \quad (2.10)$$

Before discussing the technical aspects concerning the quadrupole octupole coupled basis it is worth commenting on some qualitative features of the model Hamiltonian.

Such a Hamiltonian can be interpreted in terms of the quadrupole $(\alpha_{2\mu},\pi_{2\mu})$ and octupole $(\alpha_{3\mu},\pi_{3\mu})$ conjugate coordinates

$$\alpha_{\lambda\mu}=\frac{1}{k_\lambda\sqrt{2}}[b^\dagger_{\lambda\mu}+(-)^\mu b_{\lambda-\mu}] ,$$
$$\pi_{\lambda\mu}=\frac{ik_\lambda}{\sqrt{2}}[b^\dagger_{\lambda-\mu}(-)^\mu-b_{\lambda\mu}],\quad \lambda=2,3 \quad (2.11)$$

of a liquid drop surface described by the equation

$$R=R_0\left[1+\sum_{\lambda=2,3}\sum_{-\lambda\le\mu\le\lambda}\alpha_{\lambda\mu}Y_{\lambda-\mu}(-)^\mu\right] . \quad (2.12)$$

By quantizing the small surface oscillations around a spherical shape, one obtains indeed the harmonic part of H (i.e., $H_v=22A_1\hat{N}_2+A_3\hat{N}_3$). The nonquadratic terms do not have a classical counterpart. These terms are specific of the quantum-mechanical picture and account for the anharmonic effects due to the interaction between the quadrupole and octupole degrees of freedom. Such an interaction has been used by many authors[24,29,37] to study the negative-parity states of spherical nuclei.

This Hamiltonian can be justified microscopically. It can be obtained through a boson expansion technique from a many-body Hamiltonian including the quadrupole-quadrupole (q_2q_2) and octupole-octupole (q_3q_3) two-body interaction.[26–28] In this case, as we will see in Sec. IV, the coefficients $A_i(1\le i\le 5)$ appearing in the Hamiltonian (2.5) have a microscopic structure, depending on the single-particle orbits as well as on the strength parameters of the q_2q_2 and q_3q_3 interactions.

Here the microscopic structure of the boson $b^\dagger_{\lambda\mu}$ is not explicitly considered. The coefficients A_i are therefore free parameters.

We notice that the radius expansion (2.12) does not contain the monopole and the dipole terms. It can be shown[28] that the dipole and monopole coordinates can be expressed in terms of α_2 and α_3. By requiring indeed that the mass of the nucleus be conserved and that the coordinates of the center of mass (c.m.) be zero (which is equivalent to the condition of vanishing momentum for the c.m.) one obtains two equations relating the four collective coordinates. The solution of these equations gives the following results: (i) The leading term for α_{00} is a linear combination of $(\alpha_2\alpha_2)_{00}$ and $(\alpha_3\alpha_3)_{00}$; (ii) the leading term of α_{1M} is proportional to $(\alpha_3\alpha_2)_{1M}$. This shows that including the monopole and the dipole coordinates in the expansion (2.12) amounts to a sort of renormalization of the quadrupole-octupole Hamiltonian.

We can summarize by saying that our model Hamiltonian is a linear combination of four types of terms: (i) the harmonic component H_v which prevails over the remaining ones in the vibrational region; (ii) the squared angular momentum $\hat{J}^2$ which plays a dominant role in very deformed nuclei; (iii) the pure quadrupole term $\Omega^\dagger_{\beta'}\Omega_{\beta'}$ which covers the intermediate regions; (iv) a quadrupole octupole interaction term which was chosen of the simplest form.

It is worth comparing the Hamiltonian (2.5) with the one used in Ref. 30 to study the $K^\pi=0^-,1^-$ octupole bands. The quadrupole octupole coupling term adopted there was of fourth order in bosons and had vanishing m.e. between the states of the positive-parity bands. This property was required by the effectiveness condition satisfied by the model Hamiltonian. The coupling terms used here are chosen so as to induce strong octupole correlations into the ground state. For a noninteracting system indeed, in the vibrational limit, the lowest 1^+ state is a three boson state, namely, $[(b_3^\dagger b_3^\dagger)_2 b_2^\dagger]_{1M}|0\rangle$. Let us now suppose that such a state is also the dominant component of the wave function describing the coupled system. In order to obtain a noticeable $M1$ transition to the correlated ground state, this should have a large component $(b_3^\dagger b_3^\dagger)_0|0\rangle$. Such a correlation can be generated by a coupling term which does not conserve the number of octupole bosons. The second coupling terms gives, on the contrary, a diagonal contribution to the states with a nonvanishing number of octupole bosons. Its strength determines, in fact, the position of the band-head state of the negative-parity band.

Clearly, the Hamiltonian (2.5) is defined in the product space $S_3\otimes S_2$ where S_λ is the space of the λ-boson states.

In this paper we shall assume that the low-lying states, including the $M1$ state, are described by the eigenstates of H within the collective subspace spanned by the following boson states:

$$B=\{\varphi^{(g)}_{JM},\varphi^{(\gamma)}_{JM},\psi^{(g)}_{J_1J_2;JM},\psi^{(\gamma)}_{J_1J_2;JM},\psi^{(g)}_{J_1;JM},\psi^{(\gamma)}_{J_1;JM}\}\ , \tag{2.13}$$

where $\varphi^{(g)}_{JM},\varphi^{(\gamma)}_{JM}$ were already defined by Eq. (2.1) and

$$\psi^{(i)}_{J_1J_2;JM}=\frac{1}{\sqrt{2}}[(b_3^\dagger b_3^\dagger)_{J_1}|0\rangle_3\otimes\varphi^{(i)}_{J_2}]_{JM}\ , \tag{2.14}$$

$$\begin{aligned}\psi^{(i)}_{J_1,JM}&\equiv|i,(3J_1)JM\rangle\\&=[b_3^\dagger|0\rangle_3\otimes\varphi^{(i)}_{J_1}]_{JM},\quad i=g,\gamma\ .\end{aligned} \tag{2.15}$$

The states $\varphi^{(i)}_{JM}$ and $\psi^{(i)}_{J_1J_2;JM}$ $(i=g,\gamma)$ are used to study the positive-parity states while the negative-parity states are described within the subspace spanned by $\psi^{(i)}_{J_1,JM}$ $(i=g,\gamma)$.

The choice of the basis (2.13) was suggested by the description of the ground and lowest octupole bands given in Ref. 30. Indeed, the $K^\pi=0^-,1^-$ octupole bands were described there by averaging an effective Hamiltonian on the model states:

$$\varphi^{(-)}_{JM,n}=N_{J,n}P^J_{M,n}\psi_n,\quad n=0,1\ , \tag{2.16}$$

where

$$\psi_0=b_{30}^\dagger|0\rangle_3\otimes\psi_g,\quad \psi_1=[b_3^\dagger|0\rangle_3\otimes b_2^\dagger\psi_g]_{21}\ . \tag{2.17}$$

These functions can be written in a more suitable form:

$$\varphi^{(-)}_{JM,0}=N_{J,0}\sum_{J'}\frac{1}{N^{(g)}_{J'}}C^{J'3J}_{000}[b_3^\dagger|0\rangle_3\otimes\varphi^{(g)}_{J'}]_{JM}\ , \tag{2.18}$$

$$\varphi^{(-)}_{JM,1}=N_{J,1}\sum_{J'}C^{J'2J}_{-110}[(b_3^\dagger b_2^\dagger)_2|0\rangle_3\otimes\varphi^{(g)}_{J'}]_{JM}\ . \tag{2.19}$$

From Eqs. (2.18) and (2.19) it is clear that $\varphi^{(-)}_{JM,0}$ belongs to the subspace defined by Eq. (2.13). We may therefore interpret $\varphi^{(-)}_{JM,0}$ as being obtained by diagonalizing a model Hamiltonian in the basis $\{\psi^{(i)}_{J_1,JM}\}$. The states $\varphi^{(-)}_{JM,0}$ correspond to the second lowest eigenvalue of H in the basis (2.13). This statement will become more clear in Sec. V, where the numerical results will be presented.

The matrix elements of the Hamiltonian (2.5) in the basis states (2.13) are

$$\begin{aligned}
\langle\varphi^{(i)}_{JM}|H|\varphi^{(j)}_{JM}\rangle&=\langle\varphi^{(i)}_{JM}|H^{(1)}_{\rm CSM}|\varphi^{(j)}_{JM}\rangle+\delta_{ij}A_2J(J+1)\ ,\\
\langle\psi^{(i)}_{J_1J_2JM}|H|\psi^{(j)}_{J_1'J_2'JM}\rangle&=\delta_{J_1J_1'}\delta_{J_2J_2'}\{\langle\varphi^{(i)}_{J_2M_2}|H^{(1)}_{\rm CSM}|\varphi^{(j)}_{J_2M_2}\rangle+\delta_{ij}(2A_3+A_2J(J+1))\}\\
&\quad+2A_5(-)^{J_2+J}\hat{J}_1\hat{J}_1'\hat{J}_2W(J_1'J_2'J_1J_2;J2)W(33J_1J_1';23)\langle\varphi^{(i)}_{J_2}||b_2^\dagger+b_2||\varphi^{(j)}_{J_2'}\rangle\ ,\\
\langle\varphi^{(i)}_{JM}|H|\psi^{(j)}_{J_1J_2JM}\rangle&=A_4\sqrt{2/5}\langle\varphi^{(i)}_J||b_2^\dagger+b_2||\varphi^{(i)}_{J_2}\rangle\delta_{J_12}\ ,\\
\langle i(3J_2)JM|H|j(3J_2')JM\rangle&=\delta_{J_2J_2'}\{\langle\varphi^{(i)}_{J_2M_2}|H^{(1)}_{\rm CSM}|\varphi^{(j)}_{J_2M_2}\rangle+\delta_{ij}[A_3+A_2J(J+1)]\}\\
&\quad+A_5\hat{J}_2W(3J_2'3J_2;J2)\langle\varphi^{(i)}_{J_2}||b_2^\dagger+b_2||\varphi^{(j)}_{J_2'}\rangle(-)^{J+J_2+1},\quad i,j=g,\gamma\ ,
\end{aligned} \tag{2.20}$$

where $\hat{J}_k=\sqrt{2J_k+1}$ and $W(abcd,ef)$ are Racah coefficients.

The analytical expressions of the m.e. of $H^{(1)}_{\rm CSM}$ as well as the reduced m.e. of $b_2^\dagger+b_2$ were given in Ref. 31. (Throughout this paper we use the following convention for the reduced m.e. of a tensor operator $T_{\lambda\mu}$: $\langle jm|T_{\lambda\mu}|j'm'\rangle=C^{j'\lambda j}_{m'\mu m}\langle j||T_\lambda||j'\rangle$.)

In the next section we shall study the electromagnetic (e.m.) properties of some low-lying eigenstates of H. These are obtained through a diagonalization procedure and have the form

$$|0^+0\rangle=C\varphi^{(g)}_{00}+\sum_{J,k}B^{(k)}_J\psi^{(k)}_{JJ;00} , \tag{2.21}$$

$$|J^-M\rangle=\sum_{k=g,\gamma}\sum_{J'}D^{(k)}_{JJ'}(b_3^\dagger|0\rangle_3\otimes\varphi^{(k)}_{J'})_{JM} , \tag{2.22}$$

$$|1^+M\rangle=\sum_{\substack{k=g,\gamma\\ J,J_1}}A^{(k)}_{JJ_1}\psi^{(k)}_{JJ;1M} , \tag{2.23}$$

$$|2_i^+M\rangle=\sum_{k=g,\gamma}C^{(i)}_k\varphi^{(k)}_{2M}+\sum_{k=g,\gamma}\sum_{J_1,J_2}B^{(i)}_{k;J_1J_2}\psi^{(k)}_{J_1J_2;2M},\quad i=g,\gamma . \tag{2.24}$$

Conventionally, we say that the state $J^\pi=2^+$ belongs to the band k if the amplitude $C^{(k)}_k$ is the dominant one in the expansion (2.24).

III. THE e.m. TRANSITION PROBABILITIES

In this section we shall study the e.m. properties of the lowest 1^+ states described by the wave functions (2.23). We shall namely derive analytical expressions for the reduced transition probabilities relating the 1^+ states to the neighboring positive-parity (by $M1$) and negative-parity (by $E1$ and $E3$) states. By applying these formulas to some concrete case it is possible to point out two essential features of the lowest 1^+ states: (i) They are of magnetic nature; (ii) the influence of the octupole degrees of freedom on the structure of the 1^+ state is reflected into the magnitude of the $B(E1)$ and $B(E3)$ values associated to this state. As we have already mentioned, the $B(M1)$ value characterizes not only the 1^+ state but also constitutes a measure of the importance of the octupole boson correlations in the ground state.

To begin with let us analyze the $M1$ transition $0^+\to1^+$. The transition operator is defined by

$$\hat{M}_{1k}=\sqrt{3/4\pi}(g_2\hat{J}_{2k}+g_3\hat{J}_{3k})\mu_N , \tag{3.1}$$

where μ_N stands for the nuclear magneton and g_λ denotes the λ-pole gyromagnetic factor. Additional contribution to the $M1$ strength may come from the higher-order components of the transition operator

$$\hat{M}^{an}_{1k}=\sqrt{3/4\pi}\{g_{22}[(b_2^\dagger+b_2)\hat{J}_2]_{1k}+g_{23}[(b_2^\dagger+b_2)\hat{J}_3]_{1k}\} . \tag{3.2}$$

The influence of the g_{22} term on the magnetic properties of nuclei has been investigated in Ref. 38.

The transition operator $\hat{M}_{1k}$ as well as the corrective term $\hat{M}^{an}_{1k}$ can be obtained from the classical expression of the nuclear convection current and the electromagnetic field generated by the colliding electron during the (e,e') process.

The $M1$ transition operator describing the inelastic electron scattering is

$$T^m_{1M}(q)=\frac{1}{c}\int d\mathbf{r}\, j_1(qr)\mathbf{Y}^{1M}_{l1}\mathbf{J}_p , \tag{3.3}$$

where c is the light velocity, $j_1(qr)$ the spherical Bessel function, $\mathbf{Y}^{JM}_{l1}$ stands for the vector spherical harmonic and $\mathbf{J}_p$ denotes the proton convection current defined by means of the charge density ρ_p and the proton velocity field $\mathbf{v}$

$$\mathbf{J}_p=\rho_p(r)\mathbf{v} . \tag{3.4}$$

The integration domain in Eq. (3.3) is bounded by the surface (2.12). Assuming a potential structure for the velocity field of the liquid drop, one finds

$$\rho\mathbf{v}=\frac{3Z_e}{4\pi R_0}\sum_{\lambda=2,3}\sum_{-\lambda\le\mu\le\lambda}\frac{1}{\lambda}\dot{\alpha}^*_{\lambda\mu}\nabla\left[\left[\frac{r}{R_0}\right]^\lambda Y_{\lambda\mu}(\theta,\varphi)\right]\times\theta(R-r) , \tag{3.5}$$

where $\theta(R-r)$ is the Heaviside function and the overdot indicates the time derivative operation.

By means of Eqs. (3.4), (3.5), and (2.12) the function (3.3) can be expanded in terms of the collective coordinates $\alpha_{\lambda\mu}$. The leading term of such an expansion is:

$$T^m_{1M}=\frac{eZR_0}{c}\left[\frac{3}{4\pi}\right]^{3/2}j_1(qR_0)\sum_{\lambda=2,3}(2\lambda+1)(-)^{\lambda+1}\times W(\lambda\lambda11;1\lambda-1)\times(\dot{\alpha}_\lambda\alpha_\lambda)_{1M} . \tag{3.6}$$

This expression can be quantized by means of the boson representation (2.11) of α and by using the equation

$$\dot{\alpha}_{\lambda\mu}=\frac{1}{i\hbar}[H,\alpha_{\lambda\mu}] . \tag{3.7}$$

One obtains in this way a boson representation of T^m_{1M} whose leading terms are J_{2M} and J_{3M}:

$$T^m_{1M}=\frac{1}{i}\frac{eZR_0}{\hbar c}\left[\frac{3}{4\pi}\right]^{3/2}j_1(qR_0)\times\left[\frac{\sqrt{5}}{k_2^2}(11A_1+3A_2)(b_2^\dagger b_2)_{1M}+\frac{\sqrt{14}}{3k_3^2}(A_3+12A_2)(b_3^\dagger b_3)_{1M}\right] . \tag{3.8}$$

From Eqs. (3.1) and (3.8) one finds the following expression for the gyromagnetic factor g_λ:

$$g_\lambda = \frac{ZR_0^2}{4\pi}\frac{Mc^2}{(\hbar c)^2}\left[\frac{3}{k_2^2}(11A_1+3A_2)\delta_{\lambda 2} + \frac{1}{k_3^2}(A_3+12A_2)\delta_{\lambda 3}\right],$$

$$\lambda = 2,3 . \quad (3.9)$$

If one takes into account the bosonic quadratic terms in the right-hand side of Eq. (3.7) and then inserts the resulting $\dot{\alpha}$ in (3.6), one obtains the explicit expressions of the gyromagnetic factors g_{22} and g_{23}, which represent the first-order anharmonic correction to the $M1$ operator. Since we do not use the contribution of $\hat{M}^{an}_{1\mu}$ to the $B(M1)$ value we shall not give here the explicit expressions of g_{22} and g_{23}.

The reduced m.e. of T^m_{1M} between the ground and the 1^+ states, given by Eqs. (2.21) and (2.23), respectively, is

$$\langle 0^+ || T_1^m || 1^+ \rangle = \frac{1}{i} e j_1(qR_0)\mathcal{F} , \quad (3.10)$$

where

$$\mathcal{F} = \frac{3}{4\pi}\frac{\hbar c}{R_0 Mc^2}(g_2 - g_3) f , \quad (3.11)$$

and

$$f = \sum_{k=g,\gamma} [A_{22}^{(k)}B_2^{(k)} + \sqrt{10/3}A_{44}^{(k)}B_4^{(k)} + \sqrt{7}A_{66}^{(k)}B_6^{(k)}] . \quad (3.12)$$

The terms A and B are the expansion coefficients of the 1^+ and 0^+ states, respectively, as shown in Eqs. (2.23) and (2.21).

The $M1$ form factor is given by

$$|F_1^m(q)|^2 = \mathcal{F}^2[j_1(qR_0)]^2 . \quad (3.13)$$

The transition probability has the expression

$$B(M1;0^+\to 1^+) \equiv |\langle 0||M_1||1^+\rangle|^2 = 2\left[\frac{Mc}{\hbar}\right]^2 R_0^2\mathcal{F}^2\mu_N^2 . \quad (3.14)$$

Inserting the expression (3.11) of $\mathcal{F}$ into (3.14) one obtains

$$B(M1;0^+\to 1^+) = \frac{9}{2\pi} f^2 (g_2-g_3)^2\mu_N^2 . \quad (3.15)$$

The following simple relation between the $M1$ strength (3.15) and the $M1$ form factor (3.13) holds

$$|F_1^m(q_{ph})|^2 = \tfrac{2}{9} q_{ph}^2 B(M1;0_g^+\to 1^+) , \quad (3.16)$$

where we denoted by q_{ph} the photon point value of the transferred momentum, which, expressed in terms of the excitation energy of the 1^+ state, is given by

$$q_{ph} = \frac{E_{1^+} - E_{0^+}}{\hbar c} . \quad (3.17)$$

It is worthwhile noticing that the $B(M1)$ value (3.15) is proportional to $(g_2-g_3)^2$. This is reminiscent of the factor $(g_p-g_n)^2$ appearing in the expression of $B(M1)\uparrow$ obtained within the formalism dealing with distinct proton and neutron quadrupole bosons. The appearance of such a factor in this latter scheme is interpreted as reflecting a motion in opposition of phase of the proton—neutron subsystems which amount to say that the state is asymmetric against $p-n$ exchange. This is true only if 0_g^+ is a (p,n) symmetric state. Similarly the appearance of the factor g_2-g_3 in the expression (3.15) of $B(M1)\uparrow$ can be interpreted as the signature of a relative rotation of quadrupole versus octupole bosons.

We mention, for the sake of completeness, that the GCSM based on the proton-neutron picture studies the motion of a system with 10 degrees of freedom (six Euler angles and four deformation variables) while here we deal with 12 variables (six Euler angles and six deformations). Noticeable is also the fact that the IBA2 uses also 12 variables. The present model however uses only two types of bosons while the IBA2 uses four bosons, i.e., two d and two s bosons.

The $B(M1)\uparrow$ value (3.15) depends on two parameters k_2 and k_3 which define the relation between coordinates and boson representations by means of Eq. (2.11). These are fixed by fitting the gyromagnetic factor of the state 2_g^+ Eq. (2.24) and the reduced m.e. $\langle 2_\gamma^+||M_1||2_g^+\rangle$.

For a given positive-parity state, with $J\neq 0$, whose energy is the ith eigenvalue of the Hamiltonian (2.5), the gyromagnetic factor is defined by

$$g_c^{(i,J)} = \frac{1}{J}\langle iJJ|g_2\hat{J}_{2,0} + g_3\hat{J}_{3,0}|i,JJ\rangle , \quad (3.18)$$

where

$$|i,JM\rangle = \sum_k C_{k,J}^{(i)}\varphi_{JM}^{(k)} + \sum_k B_{k,J_1J_2J}^{(i)}\psi_{J_1J_2;JM}^{(k)} . \quad (3.19)$$

The final result for $g_c^{(i,J)}$ is

$$g_c^{(i,J)} = g_2 + (g_2-g_3)\sum_{i,k}(B_{k,J_1J_2J}^{(i)})^2 \times \sum_{M_1}\frac{M_1}{J}(C_{M_1(J-M_1)J}^{J_1J_2J})^2 . \quad (3.20)$$

In our numerical calculations we have put the gyromagnetic factor corresponding to the lowest 2_g^+ state equal to the empirical value which is very close to Z/A. It results from Eq. (3.20) that $g_c^{(i,J)}$ does not depend on J when $g_2=g_3$. The common value characterizes also the negative-parity states. This remark would suggest a straight procedure for determining (g_2-g_3). Let us, namely, suppose that the gyromagnetic factors $g_c^{(1,2)}$ and $g_c^{(1,4)}$ of the states 2_g^+ and 4_g^+ were known. We could then determine (g_2-g_3) by just fitting $(g_c^{(4)}-g_c^{(2)})$ to the experimental data. Unfortunately this is not the case and therefore we need a second equation to determine the two unknowns. This is obtained from[4]

$$\langle 2^+_\gamma||M_1||2^+_g\rangle=\sqrt{3/4\pi}\left\{\sqrt{6}g_2(C_g^{(g)}C_g^{(\gamma)}+C_\gamma^{(g)}C_\gamma^{(\gamma)})+\sqrt{5}\sum_{\substack{k_1=g,\gamma\\ J_1,J_2}}B^{(g)}_{k_1J_1J_2}B^{(\gamma)}_{k_1J_1J_2}[g_2\tilde{J}_2W(1J_22J_1;J_22)+g_3\tilde{J}_1W(1J_12J_2;J_12)]\right\}\mu_N\ , \qquad (3.21)$$

where

$$\tilde{J}=[J(J+1)(2J+1)]^{1/2} \qquad (3.22)$$

and $C_i^{(j)},B_{k,J_1J_2}^{(m)}$ are the expansion coefficients of the states 2_g^+ and 2_γ^+ [Eq. (2.24)] obtained by diagonalizing the Hamiltonian matrix given by the relations (2.20). [Our convention for the reduced m.e. ($\langle J||O||J'\rangle$) is different from that of Ref. 37 ($\langle J||O||J'\rangle_{(e)}$). The two are related by $\langle J||O||J'\rangle_{(e)}=(1/\hat{J})(\langle J||O||J'\rangle)$.]

In order to characterize completely the 1^+ states we compute the $E3$ and $E1$ transition probabilities as well as the $M1$ transition strengths. To this aim we shall assume for the $E3$ and $E1$ transition operators the forms

$$Q_{3\mu}=q_3[b^\dagger_{3\mu}+(-)^\mu b_{3-\mu}]\ , \qquad (3.23)$$

$$Q_{1\mu}=q_1\sum_{\mu_1,\mu_2}C^{321}_{\mu_1\mu_2\mu}[b^\dagger_{3\mu_1}+(-)^{\mu_1}b_{3-\mu_1}]\times[b^\dagger_{2\mu_2}+(-)^{\mu_2}b_{2-\mu_2}]\ . \qquad (3.24)$$

Using the expressions (2.23) and (2.22) for the states 1^+ and J^-, respectively, one obtains

$$B(E3;1^+\to J^-)\equiv|\langle 1||Q_3||J^-\rangle|^2=2(2J+1)q_3^2|\sum_{\substack{J'=\text{even}\\ k=g,\gamma}}\hat{J}'A^{(k)}_{J'J'}D^{(k)}_{JJ'}W(331J';J'J)|^2 \qquad (3.25)$$

$$B(E1;1^+\to J^-)\equiv|\langle 1||Q_1||J^-\rangle|^2=q_1^2|\sum A^{(k)}_{J_1J_2}D^{(k')}_{JJ'}S^{JJ'}_{J_1J_2}\langle\varphi^{(k)}_{J_2}|b_2^\dagger+b_2|\varphi^{(k')}_{J'}\rangle|^2 \qquad (3.26)$$

where $S^{JJ'}_{J_1J_2}$ is given in the Appendix.

We normalize the $B(E3)$ and $B(E1)$ values, respectively, to $B(E3;3_1^-\to 0_g^+)$ and $B(E1;1^-\to 0_g^+)$, whose transition probabilities are given in the Appendix.

For the $M1$ transitions $1^+\to 2_g^+$ and $1^+\to 2_\gamma^+$ we get the expressions

$$B(1^+\to 2_i^+)=|2\sum A^{(k)}_{J_1J_2}B^{(i)}_{k;J_1J_2}(-)^{J_2}\times[g_2\tilde{J}W(2J_21J_2;J_11)-g_3\tilde{J}_1W(2J_11J_1;J_21)]|^2\ ,\quad i=g,\gamma\ . \qquad (3.27)$$

IV. TOWARDS A MICROSCOPIC INTERPRETATION

As we have already mentioned, the model Hamiltonian can be derived by expanding a many-body Hamiltonian, having as two-body interaction the pairing plus 2^λ pole ($\lambda=2,3$) interaction, in terms of the QRPA bosons. This has been achieved a long time ago by one of us (A.A.R.) in a series of publications.[26-28]

In this section we want to point out the ground-state correlations which make possible the magnetic like excitations.

To this goal we shall consider a microscopic Hamiltonian describing a system of nucleons moving in a spherical shell-model potential and interacting among themselves through a 2^λ pole ($\lambda=2,3$) interaction. The single-particle space is restricted to three states $\{j_im_i\}$ ($i=0,1,2$) of parity $(-)^i$ and energies ϵ_i, which are ordered as

$$\epsilon_0<\epsilon_1<\epsilon_2\ . \qquad (4.1)$$

We suppose that j_2 is a particle state while j_0 and j_1 are hole states.

Following the procedure of Ref. 26 the schematic Hamiltonian can be easily expanded in terms of the particle-hole RPA bosons

$$B^\dagger_{\lambda m}=X_\lambda(c^\dagger_{j2}c_{j\lambda'})_{\lambda m}+Y_\lambda(c^\dagger_{j\lambda'}c_{j2})_{\lambda m}\ ,\quad \lambda=2,3,\quad \lambda'=\delta_{\lambda,3}\ . \qquad (4.2)$$

The quadrupole-octupole coupling term of leading order in bosons, is of the same form as that given by (2.5), with

$$\begin{aligned}A_4&=-\Gamma_2X_3Y_3[X_2+(-)^{j_2+j_0}Y_2]-\Gamma_3[X_3+(-)^{j_2+j_1}Y_3]\times[X_2X_3(-)^{j_1-j_2}+Y_2Y_3(-)^{j_2-j_0}]\ ,\\ A_5&=\Gamma_2[X_2+(-)^{j_2+j_0}Y_2](X_3^2+Y_3^2)+\Gamma_3[X_3+(-)^{j_1+j_2}Y_3]^2[X_2+(-)^{j_0+j_2}Y_2]\times(-)^{j_1-j_2}\ .\end{aligned} \qquad (4.3)$$

Here the following notations have been used

$$\begin{aligned}\Gamma_2&=14\sqrt{5}\chi_2g_2(j_2j_0)[g_2(j_1j_1)W(3j_13j_1;j_22)+g_2(j_2j_2)W(3j_13j_2;j_12)]\ ,\\ \Gamma_3&=14\sqrt{5}\chi_3g_3(j_1j_0)g_3(j_2j_1)W(3j_12j_0;j_23)\ ,\end{aligned} \qquad (4.4)$$

where χ_λ stands for the strength of the 2^λ-pole interaction and

$$g_\lambda(j,j') = -\frac{\hat{j}}{\hat{\lambda}} \langle j || r^\lambda Y_\lambda || j' \rangle \ . \tag{4.5}$$

From (4.4) one sees that the ground-state component $(B_3^\dagger B_3^\dagger B_2^\dagger)_0|0\rangle\rangle$ which may be excited by an $M1$ operator is determined by the two-body interaction relating the pairs of states (hh)-(ph) and (pp)-(ph) (p =particle, h =hole).

The transition operator has also a nice microscopic justification. Let us indeed confine the angular momentum operator

$$\hat{j}_\mu = -\sum_j (c_j^\dagger c_j)_{1\mu} \tilde{j} \ ,$$
$$\tilde{j} = \left[\frac{j(j+1)(2j+1)}{3} \right]^{1/2} \tag{4.6}$$

to the restricted space of single-particle states and expand its (h,h) and (p,p) components in terms of $B_\lambda^\dagger, B_\lambda$. The first-order expansion is

$$\hat{j}_\mu = \sum_{\lambda=2,3} \tilde{g}_\lambda (B_\lambda^\dagger B_\lambda)_{1\mu} \ , \tag{4.7}$$

where

$$\tilde{g}_\lambda = (2\lambda+1)(X_\lambda^2 + Y_\lambda^2)(-)^{j_{\lambda'}+j_2} \times [\tilde{j}_{\lambda'} W(\lambda j_{\lambda'} \lambda j_{\lambda'}; j_2 1) + \tilde{j}_2 W(\lambda j_2 \lambda j_2; j_{\lambda'} 1)] \ . \tag{4.8}$$

We see that due to the truncation of the single-particle space the angular momentum operator $\hat{j}_\mu$ is not identical to the angular momentum in the boson representation [i.e., that given by (2.6)]. This implies different g factors for the quadrupole and octupole components of the $M1$ transition operator.

As a matter of fact this microscopic expression is consistent with that derived in Sec. III by using the liquid drop model [see Eqs. (3.8) and (3.9)].

The method presented above assures the conservation of the commutation relations of $\hat{j}_\mu$ with the quadrupole and octupole ph, ph, and hh operators. The mutual commutators of $\hat{j}_\mu$, however, are not preserved. This is the price we pay for having truncated the boson space to the quadrupole and octupole states.

We may then conclude that the phenomenological expressions of both the model Hamiltonian and the $M1$ transition operator have a simple microscopic justification.

The quite schematic microscopic model discussed here is sufficient for our limited purpose of illustrating the microscopic content of the boson model presented in this paper.

For a realistic microscopic description of the nuclear properties predicted by the model, the many-body Hamiltonian must operate in a larger shell-model space. This is to be truncated in such a way that the Hamiltonian can account in an effective way for the coupling of the collective RPA bosons with the noncollective ones excluded from the space. Such a procedure leads to the formulation of a nonlinear eigenvalue equation.[28]

Dealing with deformed nuclei, in order to enforce the stability of the solutions against the space truncation one needs to express the bosons in terms of Nilsson single-particle orbits. This was done for the neutron-proton system in order to describe the scissors mode by a RPA dipole state.[22] By adopting a deformed basis, however, one runs into the difficult problem of projecting out states of good angular momentum.

Another major difficulty is represented by the fact that the boson image of the many-body fermion space obtained through the boson expansion contains many spurious components due to violation of the Pauli principle. One could obviate to it by adopting the boson expansion procedure of Marumori *et al.*[39] which accounts for the Pauli principle at each order of the expansion.

In summary a correct microscopic approach in deformed nuclei would require (i) to perform a first-order Marumori expansion in terms of RPA deformed bosons; (ii) to formulate a nonlinear eigenvalue equation in the collective boson space; (iii) to account for the contribution coming from the noncollective bosons through an effective Hamiltonian; (iv) to project out states of good angular momentum. Such a program is in practice untractable. In all numerical applications[26–28,40,41] at least one of the above requirements is missing.

An alternative microscopic scheme which provides a quite satisfactory interpretation of phenomenological boson models is represented by the so-called fermion dynamical symmetry model (FDSM).[42]

This is a truncated shell-model scheme originally developed with the purpose of providing for a shell-model group theoretically founded justification of the IBM.[43] It is in fact assumed, as in IBM, that the building blocks of all lying collective states are S and D fermion pairs. Unlike IBM, however, these pairs preserve entirely and explicitly their fermion structure. They are constructed by using a single-particle basis in which the single-particle angular momentum is decomposed into a pseudo-orbital angular momentum and a pseudospin. Such a basis ensures a maximal decoupling of the S-D space from the rest of the shell-model space.

The model has quite a far-reaching group structure. It exhibits all dynamical symmetries contained in IBM but at a fermionic level and predicts additional new dynamical symmetries. Being an algebraic approach, this scheme offers a relatively simple description of the low-lying collective properties without making the approximations induced by the fermion-boson mapping as in IBM and more in general in all boson models. In particular the Pauli principle is strictly accounted for. The importance of such a principle has been pointed out recently[44] by showing that the dynamical Pauli effects accounted for in FDSM are crucial for the shell-dependent saturation of the $E2$ strengths observed experimentally. With respect to RPA boson expansion, the FDSM approach, by preserving the Pauli principle, avoids from the beginning the problem of state redundancy and spuriosity. Being formulated directly in the laboratory frame, avoids also the problems of angular momentum projection. Moreover, it accounts for the coupling with the excluded shell-model states by simply using the pseudospin basis and an effective interaction.

It should be extremely useful to explore the possibility of adopting such a microscopic approach in order to describe the states predicted by the boson model presented here. The many-body Hamiltonian underlying this latter model, pairing plus 2^{λ}-pole interaction, has indeed the right structure for being treated in the microscopic FDSM. On the other hand our boson model treats positive- and negative-parity states as well as their electromagnetic couplings. It would be interesting to investigate if the FDSM approach could be extended to describe the same states with minimal implementations which do not alter its group structure in an essential way.

V. NUMERICAL RESULTS AND DISCUSSION

The formalism described in the previous sections was applied to the case of ^{238}U.

Let us first analyze the energies. The model Hamiltonian involves the five parameters A_i $(i=1,5)$ and d. These were determined as follows. For each value of d the energies of the states $2_g^+, 4_g^+, 2_\gamma^+, 1^-, 3^-$ were fitted to the corresponding experimental data. The parameter d was fixed so as to attain an overall agreement for the ground and the lowest negative-parity bands.

The results for the six parameters are given in Fig. 1, where we plotted also the predicted energies for the ground and the lowest $K^\pi=0^-$ bands. We give also for comparison the corresponding experimental data taken from Refs. 45 and 46. The energies of the first six positive-parity states of angular momentum one are given on a separate column.

Only the decay properties of the first three 1^+ states are analyzed. These are fully determined by the structure of the wave functions (2.23) and by the transition operators given by Eqs. (3.1), (3.23), and (3.24). The coefficients $A_{JJ}^{(k)}$ of these wave functions for $1 \le k \le 3$ are given in Table I. This contains also the structure coefficients $B_J^{(k)}$ of the ground state (2.21). The largest weight for 0_g^+ pertains to $\varphi_{00}^{(g)}$ and amounts to $C=0.818$.

It is worthwhile mentioning the following features of the amplitudes of the functions given in Table I.

(i) There is a relatively large contribution of the two octupole components to the 0_g^+ state due to the A_4 term of the Hamiltonian (2.5).

(ii) The first two 1^+ states are mainly determined by the coupling of the two octupole boson states to the quadrupole ground band states, while the 1_3^+ state has the two octupole boson excitations of the states φ_{JM}^{γ} as dominant components.

The $E1$ and $E3$ transition operators [(3.23) and (3.24)] involve the parameters q_1 and q_3, respectively. As already mentioned, these were fixed by normalizing $B(E3;3^- \rightarrow 0_g^+)$ and $B(E1;1^- \rightarrow 0_g^+)$ to unity.

The $M1$ transition operator depends on the parameters k_2 and k_3 which define the canonical transformation (2.11). These were determined by means of Eq. (3.20) (considered for the first 2^+ state) and Eq. (3.21), where we used the corresponding experimental values on the l.h.s. of the equations. For the gyromagnetic factor $g_c^{(1,2)}$, the value Z/A was assumed while the reduced m.e. of the

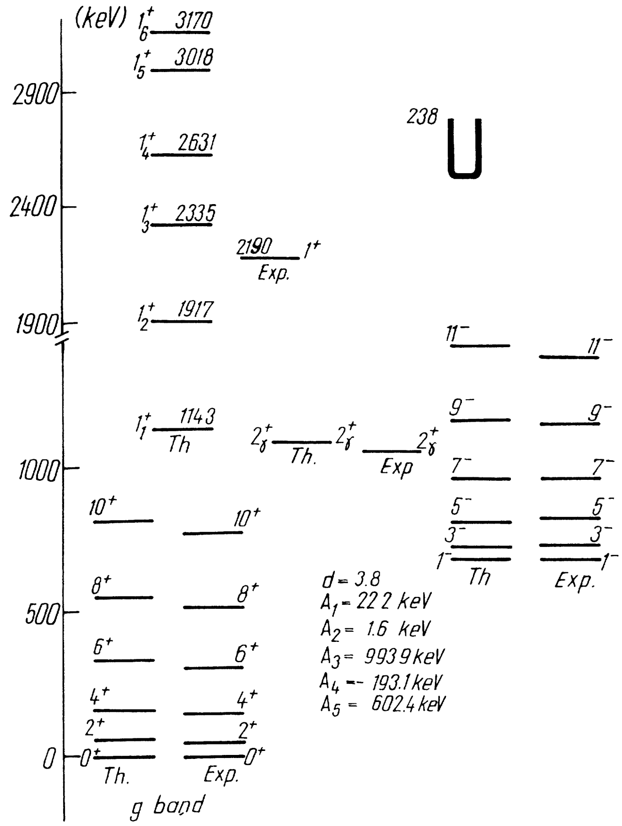

FIG. 1. The predicted (Th) and the experimental (Exp) energies for the ground and the $K^\pi=0^-$ bands are plotted for ^{238}U. Data are from Refs. 45 and 46. The predictions for the first six dipole states are also given. The scissors mode energy is taken from Ref. 3. The theoretical results are obtained by diagonalizing the Hamiltonian (2.5) with the set of parameters given at the bottom of this figure.

$M1$ transition $2_\gamma^+ \rightarrow 2_g^+$ was taken equal to $0.018\mu_N$. There are in fact no available data for this transition in the case of ^{238}U. The value mentioned above, however, is close to those reported in Ref. 47 for nuclei having a similar structure for the ground and gamma bands as ^{238}U. One obtains in this way the values

$$k_2=19.519, \quad k_3=4.344 . \tag{5.1}$$

TABLE I. The coefficients $B_J^{(k)}$ and $A_{JJ}^{(k)}$ defining the states 0_g^+ and 1_m^+ $(m=1,2,3)$ by means of Eqs. (2.21) and (2.23), respectively. They were obtained by diagonalizing the Hamiltonian (2.5) with the parameters A_i $(1 \le i \le 5)$ and d given in Fig. 1. Only those of magnitude larger than 0.1 are listed.

k	J	$B_J^{(k)}$ 0^+	$A_{JJ}^{(k)}$ 1_1^+	1_2^+	1_3^+
1	0	−0.283	0	0	0
1	2	0.439	0.257	−0.588	−0.113
1	4	−0.202	−0.475	0.585	0.103
1	6	0	0.833	0.484	0.162
2	2	0.1	0	0	0.434
2	4	0	0	0.115	−0.385
2	6	0	−0.12	−0.227	0.479

TABLE II. The $B(\mathcal{A}\lambda)$ values (for $\mathcal{A}\lambda$ equal to $M1$, $E1$ and $E3$) of the transitions $J_i^\pi \to J_f^\pi$ are listed. The strength q_1 (3.24) and q_3 (3.21) are fixed so that the reduced probabilities $B(E1;1^- \to 0^+)$ and $B(E3;3^- \to 0^+)$ are normalized to unity. The $B(M1)$ values are given in units of μ_N^2.

$B(\mathcal{A}\lambda;J_i^\pi \to J_f^{\pi'})$				
$\mathcal{A}\lambda$	$J_i^\pi \to J_f^{\pi'}$	$k=1$	$k=2$	$k=3$
	$0_g^+ \to 1_k^+$	1.120	0.601	0.093
$M1$	$1_k^+ \to 2_g^+$	0.184	0.111	0.015
	$1_k^+ \to 2_\gamma^+$	1.933	4.10^{-4}	0.312
	$1_k^+ \to 2^-$	0.322	0.450	0.026
$E3$	$1_k^+ \to 3^-$	0.807	0.043	4.10^{-4}
	$1_k^+ \to 4^-$	0.694	0.168	10^{-4}
$E1$	$1_k^+ \to 1^-$	3.348	0.694	0.092
	$1_k^+ \to 2^-$	16.77	7.023	0.112

The gyromagnetic factors were calculated by means of Eq. (3.9). The results are

$$g_2=0.371, \quad g_3=2.266 \ . \tag{5.2}$$

These values fully determine the $M1$ transition operator.

We note that the $M1$ strength of the $0_g^+ \to 1_k^+$ transitions depends only on the coefficients $A_{JJ'}^{(k)}$ with $J=J'$. This is not true for the $1^+ \to 2_i^+$ transitions with $i=g,\gamma$ [see Eq. (3.27)]. The absolute square values of $A_{JJ'}^{(k)}$ with $J \neq J'$ are however less than 0.1 for 1_1^+ and 1_2^+. In the case of 1_3^+ the data given in Table I should be implemented with

$$A_{43}^{(2)}=0.330, \quad A_{65}^{(2)}=-0.519 \ .$$

The numerical results concerning the $M1$ and $E\lambda$ ($\lambda=1,3$) transitions of the first three 1^+ states are collected in Table II.

It is worthwhile mentioning the collective nature of the $0_g^+ \to 1_k^+$ ($k=1,2$) and $1_1^+ \to 2_\gamma^+$ $M1$ transitions. Another good signature of the first excited 1_1^+ state is provided by its $E\lambda$ properties. The $B(E1)$ values of the $1_1^+ \to 1^-$ and $1_1^+ \to 2^-$ transitions are much larger than the strength of the $1^- \to 0^+$ transition.

One may then say that the 1_1^+ state is a very particular one which exhibits both magnetic and electric collective properties. Such a signature distinguishes it from the traditional scissors mode.

According to the result presented in Fig. 1, there are two 1^+ states whose energy lies close to the experimental energy (i.e., 2.19 MeV) of the scissors mode: these are 1_2^+ and 1_3^+. One may therefore expect some nonnegligible overlap between these states and the scissors mode. On the other hand, the 1_2^+ state has an $E1$ collective character.

We finally point out the splitting of the $M1$ strength among the first two (1_1^+ and 1_2^+) states. A similar splitting is predicted to occur also for the scissors mode in nuclei with a triaxial shape.[10]

VI. SUMMARY AND CONCLUSIONS

The main results achieved in the previous sections can be summarized as follows.

A schematic quadrupole-octupole boson Hamiltonian (2.5) is used to describe dipole states of positive parity. A special attention is paid to those states which can be collectively excited from the ground state through an $M1$ transition.

The inspiring source for this description was the TR model. Indeed, the picture of two interacting ellipsoids provided by TRM, can be simulated by a specific superposition of octupole and quadrupole nuclear shapes.

Unlike the formalisms used to describe the scissors mode, the present approach does not distinguish between proton and neutron fluids. The structure of the states investigated here is therefore different from the scissors like states, although some overlap between the two types of states is possible.

At the present stage one cannot say whether and how any of the states studied in Sec. V relates to the $M1$ states populated in an (e,e') experiment. Indeed, although the state 1_2^+ lies close to the experimental data (i.e., 2.19 MeV), its $M1$ strength is about $0.6\mu_N^2$, while the corresponding experimental value is $1.5\mu_N^2$. For a quantitative description of the interplay between this state and the scissors mode one should implement the Hamiltonian (2.5) with additional terms.

On the other hand, we were not interested in improving the description of the "traditional" $M1$ state but rather to study a new class of collective $M1$ states, namely, those which are built up by means of quadrupole-octupole excitations. This is why we did not make an extensive numerical analysis but restricted ourselves to a single nucleus, ^{238}U. The main results are

(i) the $M1$ strength is mainly distributed among the first two out of six dipole states.

(ii) These two dipole states (1_1^+ and 1_2^+) exhibit collective $M1$ properties.

(iii) The $B(M1)$ value for the transition $1_1^+ \to 2_\gamma^+$ is also quite large.

(iv) A good signature of the first $M1$ state is its large $B(E3)$ and $B(E1)$ values. The 1_2^+ state is also collectively coupled to the 1^- and 2^- states via $E1$ transition.

(v) Using the TRM language one may say that the quadrupole and octupole nuclear shapes move in a scissors fashion.

The results of this paper strongly suggest that the physics of low-lying magnetic collective excitations in complex nuclei has still many interesting features to be explored.

Experimental analysis of the $E\lambda$ ($\lambda=1,3$) properties of the collective $M1$ states lying in the energy region 1–2.5 MeV would greatly stimulate further studies on this subject.

APPENDIX

Here we shall list the explicit expressions of $S^{JJ'}_{J_1J_2}$ involved in (3.26) as well as those of the reduced probabilities for the $3^- \rightarrow 0_g^+$, and $1^- \rightarrow 0_g^+$ transitions:

$$
\begin{aligned}
S^{JJ'}_{J_1J_2} &= \sqrt{2}\hat{J}\hat{J}_1^2 \sum_{J''} \hat{J}'^2 \hat{J}'' W(331J_1;J_1J')W(3JJ_12;J_2J')W(J213;J'J'') , \\
B(E3;3_1^- \rightarrow 0_g^+) &= q_3^2 \left| CD_{30}^{(g)} - \frac{2}{\sqrt{7}} \sum_{J'=0,2,4,6} B_{J'}^{(g)} D_{3J'}^{(g)} - \frac{2}{\sqrt{7}} \sum_{J'=2,4,6} B_{J'}^{(\gamma)} D_{3J'}^{(\gamma)} \right|^2 , \\
B(E1;1_1^- \rightarrow 0_g^+) &= q_1^2 \Bigg| C \sum_k D_{12}^{(k)} \langle \varphi_2^{(k)} || b_2^\dagger + b_2 || \varphi_0^{(g)} \rangle \\
&\qquad - \sqrt{2} \sum_{J_1,J',k,k'} D_{1J'}^{(k)} B_{J_1}^{(k')} \hat{J}' W(33J'2;J_11) \langle \varphi_{J'}^{(k)} || b_2^\dagger + b_2 || \varphi_{J_1}^{(k')} \rangle \Bigg|^2 ,
\end{aligned}
\qquad \text{(A1)}
$$

where $\hat{J} = \sqrt{2J+1}$.

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