Pion-nucleus scattering around the (3,3) resonance

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Elastic scattering of $\pi^{\pm}$ on ^{28}Si, ^{40}Ar, $^{40,48}Ca$, ^{90}Zr, and ^{208}Pb at energies around the (3,3) resonance is studied within the framework of the strong absorption model of Frahn and Venter. The parameters thus obtained are used in the analysis of the inelastic scattering of pions leading to the lowest 2^+ state in ^{28}Si. A reasonably good account of the scattering processes (elastic and inelastic) is given by the simple model.

I. INTRODUCTION

With the availability of pion beam facilitates at the LAMPF, SIN, CERN, TRIUMF, and other laboratories, the pion-nucleus scattering has been the subject of a number of investigations. Theoretical studies on the problem proceeded along the following general lines. One is the "zero range approximation" for the pion-nucleon interaction with the advantage of allowing one to work in the coordinate space. Another is the "momentum space approach" that requires the construction of an optical model potential for the pion nucleus in the momentum space. The other is the "Δ-hole model," which assumes that the dominant mode of interaction of pions with nucleus is to excite a nucleon to a Δ, with a corresponding hole in the nucleon state.

In the region of the (3,3) resonance, the pion-nucleon interaction is strongly absorptive and the pion pulse thus have a short mean free path (0.5 fm or so) in nuclear matter.[1] This is evidenced by the clear diffraction oscillation in the angular distribution exhibited by the pion-nucleus elastic scattering. This feature of surface absorption is similar to that observed for the composite particles and is commensurate with the strong absorption model of Frahn and Venter[2] (abbreviated as SAM). The model is generalization of the earlier diffraction models and is intermediate between the phase shift analysis at low energy and the dispersion relation at high energy and seeks its justification eventually in the latter.

The present work was undertaken in an attempt to see how far the pion-nucleus scattering around and below the (3,3) resonance can be described within the framework of the SAM without evoking the degrees of freedom associated with mesons and the excited states of nucleons. A similar study based on the phenomenological diffraction model, which is a simplified version of the SAM in fact, has recently been done by Chowdhury and Guo[3] for the scattering of antiprotons from nuclei.

II. THE SAM FORMALISM

A. Elastic scattering

The main line of approach is given below; details are in Ref. 2. The model starts with a direct parametrization of the scattering function η_l as given by

$$\eta_l \exp(-2i\sigma_l) = g(t) + i\mu\, dg(t)/dt \ . \tag{1}$$

Here σ_l is the usual Coulomb phase shift and $g(t)$ is a continuous monotonic function of angular momentum $t(=l+\frac{1}{2})$. The function $g(t)$ is characterized by a cutoff or critical angular momentum T and a diffuseness parameter Δ, with the requirement that its derivative should have a simple Fourier transform.[2] The parameter μ, more accurately $\mu/4\Delta$, is a measure of the real nuclear phase shift. A convenient form for $g(t)$ is the Woods-Saxon form, namely

$$g(t) = \{1+\exp[(T-t)/\Delta]\}^{-1} \ . \tag{2}$$

It is clear from relations (1) and (2) that the real part of the scattering function changes from small values at small l to unity at high l, with a rapid transition around the critical value T, while the imaginary part is surface peaked.

The amplitude for elastic scattering of a spin-zero projectile from a spinless target nucleus is given by the well known expression

$$f(\theta) = f_c(\theta) + (2ik)^{-1} \times \sum_{l=0}^{\infty} (2l+1)[\eta_l \exp(-2i\sigma_l) - 1] \times \exp(2i\delta_l) P_l(\cos\theta) \ , \tag{3}$$

where $f_c(\theta)$ is the Coulomb scattering amplitude and other symbols carry their usual meaning

Because of the SAM assumption, $\Delta/T \ll 1$, the dg/dt approximates the delta function $\delta(t-T)$. When $P_l(\cos\theta)$ is replaced by the leading term in its asymptotic expansion, the main contribution to the sum over l in (3) comes from the vicinity of $t=T$. The sum over l can be replaced by an integration by means of the Poisson summation formula; this introduces the Fourier transforms of dg/dt, namely

$$F(\Delta\theta)=\int_{-\infty}^{\infty} dg/dt \exp[-i(t-T)\theta]dt \ ,$$

which, because of (2), becomes

$$F(\Delta\theta)=(\pi\Delta\theta)/\sinh(\pi\Delta\theta) \ . \quad (4)$$

It is convenient to distinguish between the two regions of the Coulomb parameter n, namely $n \ll 1$ and $n \gg (2\pi)^{-1}$.

For $n \ll 1$, the scattering amplitude (3) then becomes[2]

$$f(\theta)=f_c(\theta)+f_n(\theta) \ ,$$

where

$$f_n(\theta)=\frac{T}{k}\left[\frac{\theta}{\sin\theta}\right]^{1/2} F(\Delta\theta) \times\left[i\frac{J_1(T\theta)}{\theta}-\left[\frac{2n}{T\theta^2}-\mu\right]J_0(T\theta)\right] .$$

For $n \gg (2\pi)^{-1}$, the scattering amplitude is

$$f(\theta)=f_c(\theta)+f_n^-(\theta), \quad \theta\le\theta_c \ \text{(Coulomb region)} ,$$
$$=f_n^+(\theta), \quad \theta>\theta_c \ \text{(diffraction region)} ,$$

where $\theta_c=2\arctan(n/T)$ is the critical angle, and

$$f_n^{\pm}(\theta)=\frac{i}{k}(T/2\pi\sin\theta)^{1/2}\exp(i\chi) \times\{A^{\pm}F[(\theta-\theta_c)]\exp-i(T\theta-\pi/4) -BF[\Delta(\theta+\theta_c)]\exp i(T\theta-\pi/4)\} \ , \quad (5)$$

with

$$A^{\pm}=\pm G(\pm u)(T/2\sin\theta_c)^{1/2}-\mu \ ,$$

$$B=(\theta+\theta_c)^{-1}+\mu \ ,$$

and $F[\Delta(\theta\pm\theta_c)]$ is the form factor as given by (4).

The function χ and $G(u)$ appearing in (5) are given by

$$\chi=T\theta_c-2n\ln\sin(\theta_c/2)+2\sigma_0$$

and

$$G(u)=\pi^{1/2}\exp i(u^2+\pi/4)\operatorname{erfc}[u\exp(i\pi/4)] \times\left\{1-u\left[\frac{\sin\theta_c}{2T}\right]\left[\frac{1}{\sin\theta_c}+(1+i\tfrac{2}{3}u^2)\cot(\theta_c/2)\right]\right\}+\left[\frac{\sin\theta_c}{2T}\right]^{1/2}\left[\frac{1}{\sin\theta_c}+(1+iu^2)\tfrac{2}{3}\cot(\theta_c/2)\right] ,$$

with

$$u=(T/2\sin\theta_c)^{1/2}(\theta-\theta_c)$$

and

$$\operatorname{erfc}(z)=2\pi^{-1/2}\int \exp(-\tau^2)d\tau \ .$$

The SAM formalism thus leads to a closed expression for elastic scattering cross section over all angles excluding large angles in terms of three adjustable parameters T, Δ, and μ.

B. Inelastic scattering

The formalism developed for elastic scattering can, under conditions of strong absorption, be easily extended to include the inelastic processes as well.[4] In conjunction with the Austern-Blair relation,[5] the amplitude for the inelastic scattering is expressed in terms of the elastic scattering matrix. This leads again to a closed expression for the inelastic cross section for various multipole modes of collective excitation, as given in Ref. 4. The procedure is outlined below.

The amplitude for the inelastic scattering *via* the single excitation of collective nuclear states of multipole order L is given by

$$f_{LM}(\theta)=\frac{i}{2}(2L+1)^{1/2}c_L\sum_{l,l'} i^{l-l'}[(2l'+1)^{1/2}\exp i[(\sigma_l+\sigma_{l'})]\partial\eta_{\bar{l}}/\partial\bar{l}\,\langle l'L00|l0\rangle\langle l'L,-MM|l0\rangle Y_{l'}^{-M}(\theta,0) \ , \quad (6)$$

where $\bar{l}=(l+l')/2$ and $\eta_{\bar{l}}$ is given in (1).

The $\partial n_{\bar{l}}/\partial\bar{l}$, under SAM conditions, is confined to a narrow range of $\bar{l}$ values around the cutoff angular momentum l_0, $T=l_0+\frac{1}{2}$. Since the summation in (6) is only over l and l' for which $|l-l'|$ is small, one can approximate the Coulomb phases by

$$\sigma_l+\sigma_{l'}\approx 2\sigma_{\bar{l}}\approx 2\sigma_T+(t-T)\theta_c \ .$$

The scattering amplitude (6) then becomes

$$f_{LM}(\theta)=\frac{i}{2}(2L+1)^{1/2}c_L\exp(2i\sigma_T)\sum_{l,l'}(2l'+1)^{1/2}P(\bar{l})\langle l'L00|l0\rangle\langle l'L,-MM|L0\rangle Y_{l'}^{-M}(\theta,0)\ ,$$

where

$$P(\bar{l})=\{\exp[i(t-T)\theta_c]\}\partial n_{\bar{l}}/\partial\bar{l}\ .$$

The Taylor expansion of $P(\bar{l})$ about l' gives the above scattering amplitude as

$$f_{LM}(\theta)=\tfrac{1}{2}i^{L+1}(2L+1)^{1/2}c_L\exp(2i\sigma_T)\times\sum_{r=0}^{\infty}\sum_{l'=0}^{\infty}(2l'+1)^{1/2}\times P^r(l')C_{LM}^r(l')Y_{l'}^M(\theta,0)\ , \tag{7}$$

where $P^r(l')$ is the rth derivative of $P(l')$ at l', and

$$C_{LM}^r(l')=\sum_l i^{l-l'-L}\frac{1}{r!}\left[\frac{l-l'}{2}\right]^r\times\langle l'L00|l0\rangle\langle l'L,-MM|l0\rangle\ .$$

Because of the localization of $P^r(l')$ the summation over l' in (7) is over the range $l'-L$ to $l'+L$. To a good approximation $C_{LM}^r(l')\approx C_{LM}^r$ and

$$C_{LM}^r=\sum_{\lambda=-L}^{L}i^{\lambda-L}\frac{1}{r!}(\lambda/2)^r\langle l_0L00|(l_0+\lambda),0\rangle\times\langle l_0L,-MM|(l_0+\lambda),0\rangle$$

are real coefficients and are excluded from the summation over l' in (7).

Next, using an asymptotic expression for $Y_{l'}^{-M}(\theta,0)$ in terms of the Bessel function $J_{|M|}(z)$ and converting the summation over l' to an integration by means of the Poisson sum formula, one can express

$$\sum_{l'}(2l'+1)^{1/2}P^r(l')Y_{l'}^{-M}(\theta,0)$$

appearing in (7) as

$$\sum_{l'=0}^{\infty}(2l'+1)^{1/2}P^r(l')Y_{l'}^{-M}(\theta,0)=(-)^{(M-|M|)/2}\pi^{-1/2}(-i\theta)^{r+1}(T/2\theta)\times(\theta/\sin\theta)^{1/2}\{[H_+ +(-)^{r+1}H_-]J_{|M|-1}(T\theta)+i[H_+ -(-)^{r+1}H_-]J_{|M|}(T\theta)\}\ ,$$

where

$$H_\pm=[1+\mu(\theta_c\pm\theta)]F[\Delta(\theta_c\pm\theta)]$$

and $[\Delta(\theta_c\pm\theta)]$ is given by (4).

The amplitude (7) for the inelastic scattering then becomes

$$\begin{aligned}f_{LM}(\theta)=&c_L(-)^{(M-|M|)/2}i^{L+1}(2L+1)^{1/2}[\exp(2i\sigma_T)](T/4\pi)^{1/2})(\theta/\sin\theta)^{1/2}\\&\times\{(H_-+H_+)[\alpha_{LM}(\theta)J_{|M|}(T\theta)-\beta_{LM}(\theta)J_{|M|-1}(T\theta)]\\&+i(H_--H_+)[\alpha_{LM}(\theta)J_{|M|-1}(T\theta)+\beta_{LM}(\theta)J_{|M|}(T\theta)]\}\ ,\end{aligned} \tag{8}$$

with

$$\alpha_{LM}(\theta)+i\beta_{LM}(\theta)=\sum_{\lambda=-L}^{L}i^{\lambda-L}\exp(i\lambda\theta/2)\langle l_0L00|(l_0+\lambda),0\rangle\langle l_0L,-MM|(l_0+\lambda),0\rangle\ .$$

A further simplification is made through the asymptotic expressions for the Clebsch-Gordan coefficients in terms of the rotation matrices.

A closed expression is thus obtained for inelastic scattering;[4] the ingredients are the SAM parameters T, Δ, and μ, describing the elastic scattering, and the only free parameter is the deformation length δ_L $[=(2L+1)^{1/2}c_L]$ scaling the inelastic scattering cross section.

III. RESULTS AND DISCUSSION

A. Elastic scattering

Angular distribution data[6–8] for the elastic scattering of $\pi^\pm$ on ^{28}Si, ^{40}Ar, $^{40,48}Ca$, ^{90}Zr, and ^{208}Pb were analyzed with the SAM formalism. Results are summarized in Tables I and II and some typical fits to data are illustrated in Figs. 1–6. Uncertainties in the values of T and Δ are about 5%, while those in μ are about 20% (μ is not a sensitive parameter either). The SAM parameters are uniquely given by an analysis and no combination of the parameters other than the ones shown could be found giving another minimum in the least squares value.

A reasonably good fit was obtained, usually up to about 90° or so. A somewhat inferior fit at the lower energies is not surprising, as then pions begin to propagate deeper in the nucleus, clearly not conforming to the SAM conditions of surface absorption. At still lower energies the pion-nucleon interaction is so weak that the nucleus may even appear almost transparent to the incident

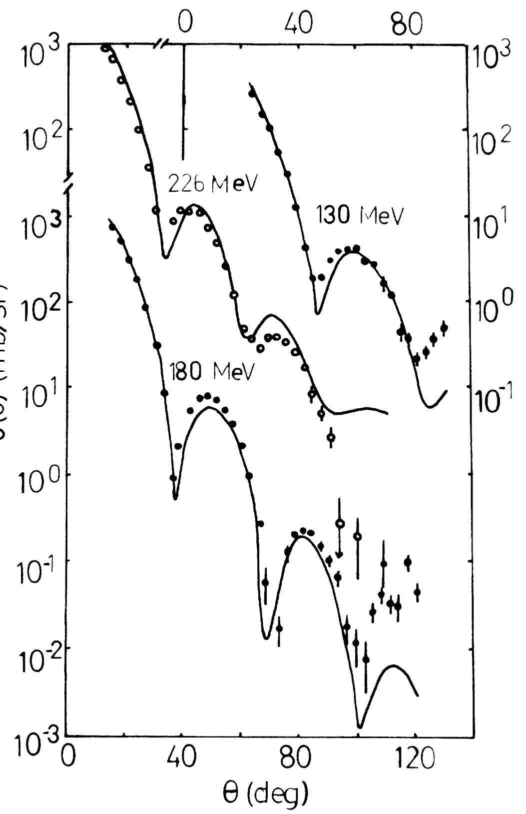

FIG. 1. SAM analysis of elastic scattering of π^+ from ^{28}Si.

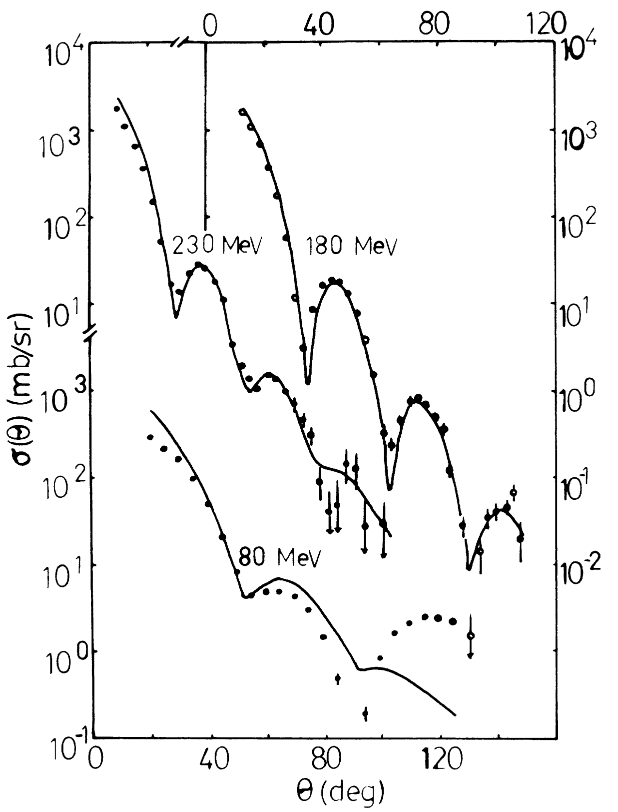

FIG. 3. SAM analysis of elastic scattering of π^+ from ^{40}Ca

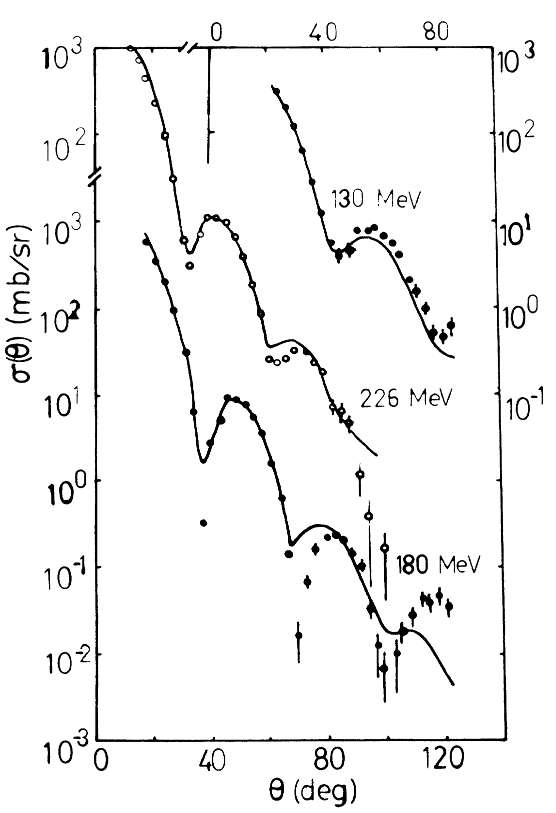

FIG. 2. SAM analysis of elastic scattering of π^- from ^{28}Si.

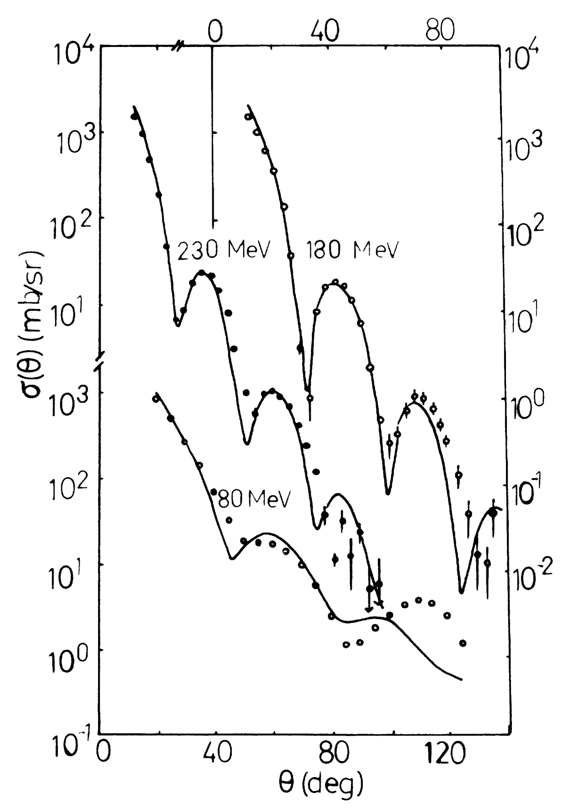

FIG. 4. SAM analysis of elastic scattering of π^- from ^{40}Ca.

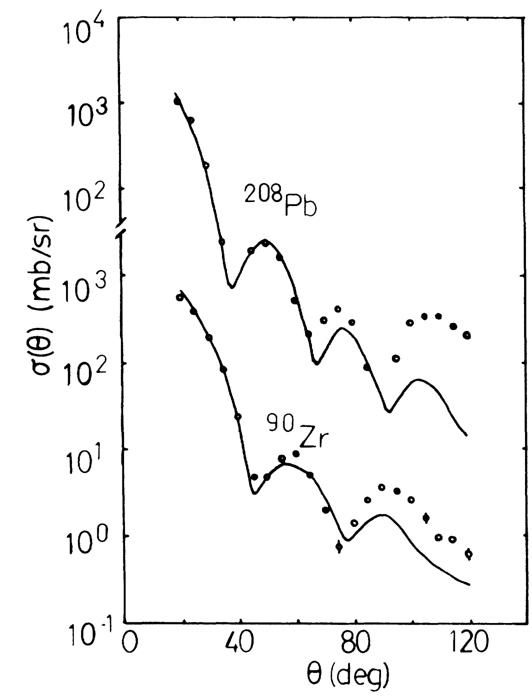

FIG. 5. SAM analysis of elastic scattering of 80 MeV π^+ from ^{90}Zr and ^{208}Pb.

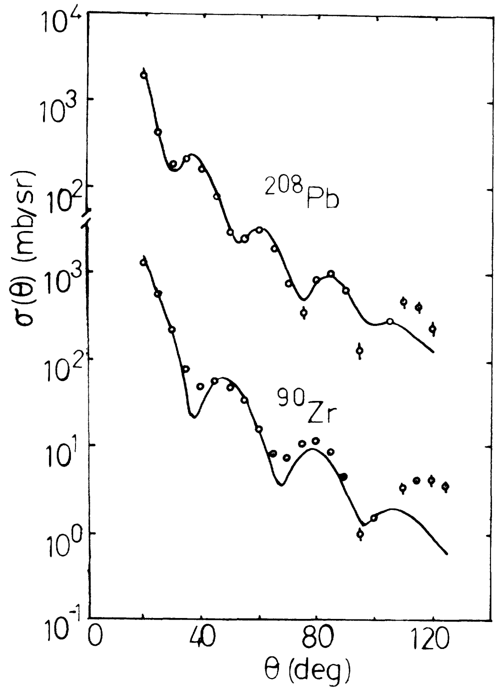

FIG. 6. SAM analysis of elastic scattering of 80 MeV π^- from ^{90}Zr and ^{208}Pb.

pions.[8]

Under conditions of strong absorption, the SAM parameters have a simple geometrical interpretation. The interaction radius R and the surface diffuseness d of nuclei are given from T and Δ through the well-known relations

$$T = kR((1-2n/kR)^{1/2} ,$$
$$\Delta = kd(1-n/kR)(1-2n/kR)^{-1/2} , \quad (9)$$

where n and k are, respectively, the Coulomb parameter and the wave number. The values of R and d are summarized in Tables I and II.

B. Inelastic scattering

Angular distributions of $\pi^\pm$ of energies 130, 180, and 226 MeV inelastically scattered to the 2_1^+ state ($E_x = 1.78$ MeV) in ^{28}Si (Preedom *et al.*[6]) were analyzed using the SAM parameters in Tables I and II. A satisfactory fit to the data could be obtained (Figs. 7 and 8) thus giving confidence in the SAM parameters and the model itself in describing the pion-nucleus scattering.

The quadrupole deformation parameters thus obtained are shown in Table III. It is seen that the deformation parameters given by the π^+ and π^- inelastic scattering agree to each other within errors. Also included in the table for a comparison are the deformation parameter adopted by Raman *et al.*[9] and also that given by the in-

TABLE I. SAM parameters for π^+.

Nucleus	Energy (MeV)	T	Δ	$\mu/4\Delta$	R (fm)	d (fm)
^{28}Si	130	4.70	0.80	0.14	4.99	0.84
^{28}Si	180	5.80	0.95	0.03	5.21	0.84
^{28}Si	226	6.40	0.85	0.12	5.12	0.67
^{40}Ar	180	6.60	1.00	0.10	5.93	0.89
^{40}Ca	80	4.10	0.60	0.21	5.63	0.80
^{40}Ca	130	5.50	0.90	0.11	5.85	0.94
^{40}Ca	180	6.50	0.90	0.04	5.85	0.79
^{40}Ca	230	7.50	0.95	0.11	5.95	0.75
^{48}Ca	130	5.70	0.85	0.16	6.06	0.89
^{48}Ca	180	6.70	0.85	0.04	6.02	0.75
^{48}Ca	230	7.70	1.10	0.13	6.10	0.86
^{90}Zr	80	5.20	0.65	0.15	7.27	0.86
^{208}Pb	80	6.20	0.65	0.15	8.99	0.86

TABLE II. SAM parameters for π^-.

Nucleus	Energy (MeV)	T	Δ	$\mu/4\Delta$	R (fm)	d (fm)
^{28}Si	130	5.00	0.80	0.14	5.15	0.84
^{28}Si	180	5.90	0.90	0.10	5.19	0.80
^{28}Si	226	6.60	1.00	0.11	5.19	0.78
^{40}Ar	180	6.80	0.80	0.03	5.96	0.71
^{40}Ca	80	4.85	0.50	0.18	6.27	0.66
^{40}Ca	130	5.90	0.85	0.15	6.04	0.89
^{40}Ca	180	6.80	0.95	0.03	5.95	0.84
^{40}Ca	230	7.70	1.05	0.05	5.98	0.82
^{48}Ca	130	6.20	0.70	0.20	6.36	0.84
^{48}Ca	180	7.10	0.95	0.04	6.21	0.84
^{48}Ca	230	8.00	0.90	0.08	6.21	0.71
^{90}Zr	80	6.00	0.50	0.18	7.61	0.66
^{208}Pb	80	7.45	0.50	0.05	9.21	0.66

elastic scattering of protons,[10] by way of example. The SAM values are systematically somewhat higher than these values. The β_2 values found by Preedom *et al.* from the distorted wave impulse approximation of their inelastic pion data on ^{28}Si (the same data as analyzed by us), on the other hand, are consistent with those given by other probes.

The controversy[11,12] as to whether or not the deformation parameter for pions should be larger than those by other probes is not over. The present work, limited of course to ^{28}Si, seems to be in agreement with similar higher β_2 values found for pions over protons and other probes in case of ^{24}Mg, ^{90}Zr, and ^{118}Sn.[12,13]

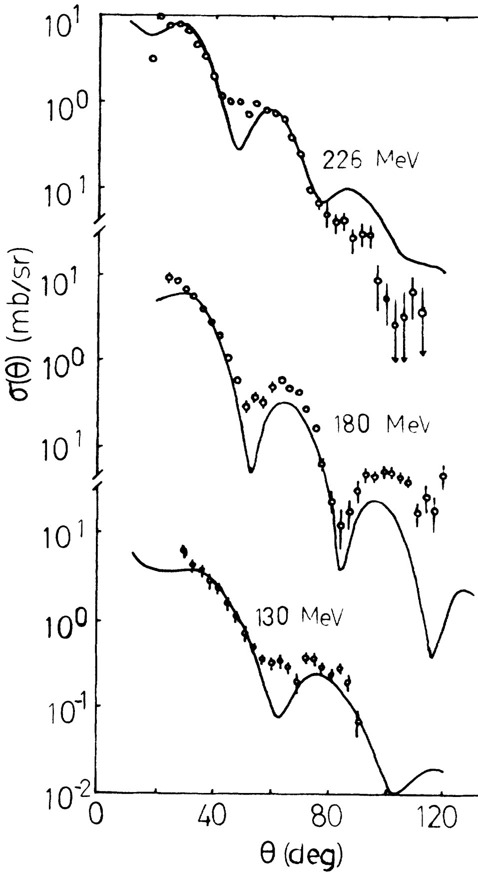

FIG. 7. SAM analysis of the inelastic scattering of π^+ from ^{28}Si leading to the lowest 2^+ state.

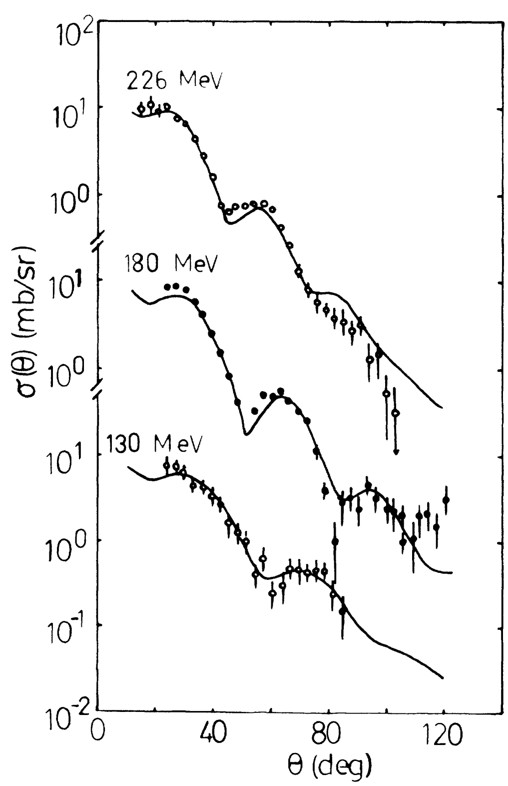

FIG. 8. SAM analysis of the inelastic scattering of π^- from ^{28}Si leading to the lowest 2^+ state.

TABLE III. The β_2 values leading to the 2^+ state ($E_x = 1.78$ MeV) in ^{28}Si.

Deformation parameter β_2			
$(\pi^+,\pi^{+\prime})$	$(\pi^-,\pi^{-\prime})$	d	(p,p')
0.56±0.06[a]	0.54±0.06[a]	0.41	0.40[e]
0.49±0.06[b]	0.46±0.05[b]		
0.40±0.06[c]	0.45±0.05[c]		

[a]At the pion energy 130 MeV.
[b]At the pion energy 180 MeV.
[c]At the pion energy 226 MeV.
[d]Adopted by Raman *et al.* (Ref. 9).
[e]Reference 10.

IV. CONCLUSION

The pion-nucleus scattering around the (3,3) resonance to a good approximation can be accounted for by the strong absorption model. Effects arising from higher order processes, like meson degrees of freedom and those from antinucleons and the excited states of nucleons, probably show up past the second diffraction peak (or higher in some nuclei). The strong absorption model of Frahn and Venter, for that matter any diffraction model, deals with an effective geometry of the interaction. Since only a few target mass numbers are covered in the present study, although over a wide range ($A = 28-208$), a comparison of the geometrical parameters of the reaction given by π^+ and π^- between themselves and with those given by other hadronic probes was not attempted. A detailed SAM analysis of the elastic scattering of pions from many more nuclei at about the same energy is undertaken for such a comparison.

The quadrupole deformation parameter of ^{28}Si, the only nucleus studied here through the inelastic scattering of pions, is found to be somewhat larger than that given by other probes. The β_2 values given by π^+ and π^-, however, agree to each other within error, as perhaps expected for a $T=0$ nucleus. Again SAM analysis of inelastic scattering on several nuclei is necessary before a meaningful comparison is made with other probes, as well as between π^+ and π^- themselves. The density distributions of protons and neutrons are expected to be different in size or shape or both in particular for $N \neq Z$ nuclei. Since the π^+ and π^- near the (3,3) resonance are known to interact differently with protons and neutrons, it would be interesting to see, as suggested by Bohr and Mottelson,[14] whether or not the β_2 values given by π^+ and π^- are different from each other, and in the present context to see if the SAM parameters, obtained from the corresponding elastic scattering, are sensitive enough to display this effect. Work on this is under way as mentioned above.

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[1]P. Hecking, Phys. Lett. **103B**, 401 (1981).
[2]W. E. Frahn, *Fundamentals in Nuclear Theory* (International Atomic Energy Agency, Vienna, 1967), p. 1; W. E. Frahn, and R. H. Venter, Ann. Phys. (N.Y.) **24**, 243 (1963).
[3]D. C. Chowdhury and T. Guo, Phys. Rev. C **39**, 1883 (1989).
[4]J. M. Potgieter and W. E. Frahn, Nucl. Phys. **80**, 434 (1966).
[5]N. Austern and J. S. Blair, Ann. Phys. (N.Y.) **33**, 15 (1965).
[6]B. M. Preedom, R. Corfu, J.-P. Egger, P. Gretillat, C. Lunke, J. Piffaretti, E. Schwarz, J. Jansen, and C. Perrin, Nucl. Phys. **A326**, 385 (1979).
[7]P. Gretillat, J.-P. Egger, J.-F. Germond, C. Lunke, E. Schwarz, C. Perrin, and B. M. Preedom, Nucl. Phys. **A364**, 270 (1981).
[8]M. J. Leitch, R. L. Burman, R. Carlini, S. Dam, V. Sandberg, M. Blecher, K. Gotow, R. Ng, A. Auble, F. E. Bertrand, E. E. Gross, F. E. Obenshain, J. Wu, G. S. Blanpied, B. M. Preedom, B. G. Ritchie, W. Bertozzi, M. V. Hynes, M. A. Kovash, and R. P. Redwine, Phys. Rev. C **29**, 561 (1984).
[9]S. Raman, C. H. Malarkey, W. T. Milner, C. W. Nestor, Jr., and P. H. Stelson, At. Data Nucl. Data Tables **36**, 1 (1987).
[10]R. de Swiniarski, H. E. Conzett, C. R. Lamontage, B. Frois, and R. J. Slobodrian, Can. J. Phys. **51**, 1293 (1973).
[11]G. W. Edwards and E. Rost, Phys. Rev. Lett. **26**, 785 (1971).
[12]C. A. Wiedner, J. A. Nolen, Jr., W. Saathoff, R. E. Tribble, J. Bolger, J. Zichy, K. Stricker, H. McManus, and J. A. Carr, Phys. Lett. **78B**, 26 (1978).
[13]J. L. Ullmann, P. W. F. Alons, B. L. Clausen, J. J. Kraushaar, J. H. Mitchell, R. J. Peterson, R. A. Ristinen, R. L. Boudrie, N. S. P. King, C. H. Morris, J. N. Kundson, and F. E. Gibson, Phys. Rev. C **35**, 1093 (1987).
[14]A. Bohr and B. Mottelson, *Nuclear Structure* (Benjamin, New York, 1975), Vol. I, p. 334 and Vol. II, p. 137.