Separable nucleon-nucleon potential with delta isobar degrees of freedom

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(Received 6 September 1989)

A separable representation of the Argonne v_{28} potential is presented. The potential includes delta isobar degrees of freedom within a coupled channel formalism. The separable potential was developed mainly for applications to nuclear matter. A quite high degree of accuracy was then reached to fulfill the requirements set by such calculations. The expansion was also extended above the $N\Delta$ threshold to allow for the evaluation of the average nuclear potential felt by the delta inside nuclear matter.

I. INTRODUCTION

Phenomenological nucleon-nucleon potentials, which explicitly include the delta isobar degrees of freedom, have recently been introduced. The intermediate range part of the nucleon-nucleon potential has been described by the exchange of two pions (TPE). In such TPE processes, displayed in Fig. 1, nucleon resonances can be excited in the intermediate states, and these terms have been found to give a considerable contribution[1] to the intermediate range attraction. The virtual excitation of the Δ is also of considerable relevance in many-body systems where it produces two major effects. The first one, the Pauli effect, comes from the fact that the Pauli principle for the nucleons excludes certain $N\Delta$ intermediate states in the computation of the two-body reaction matrix inside the nuclear medium. The second one, the dispersion effect, is related to the difference in the denominators for the intermediate $N\Delta$ and $\Delta\Delta$ states, in the many-body system and two-body scattering between two isolated nucleons. These modifications of the two-body reaction matrix are absent if the medium range attraction is treated phenomenologically.

The intermediate excitations of the delta degrees of freedom have also the effect of introducing three-nucleon forces as illustrated in Fig. 2. These three-body forces can be derived in a natural way in the framework of the coupled channel formalism, where the delta is treated as an elementary particle and the delta components are generated by a generalized one-pion exchange (OPE) potential containing transition operators with explicit $\pi N\Delta$ couplings. Along these lines the Argonne group[2] has presented a nucleon-nucleon potential, the v_{28} potential, which explicitly introduces $N\Delta$ and $\Delta\Delta$ channels and gives an excellent fit of the deuteron properties and two-body scattering data up to an energy of 400 MeV. This appears as an extension of the works of Ref. 3. The Argonne v_{28} is modeled on a conventional nucleon-nucleon potential with standard basic operators, the Argonne v_{14}, by adding terms which contain generalized spin and isospin operators acting on the delta state $(\frac{3}{2},\frac{3}{2})$, in a total of 28 component operator potential. The possible processes are depicted in Fig. 3. Since the delta has spin and isospin $\frac{3}{2}$, this introduces many possible couplings between partial waves in each channel, as displayed in Table I.

More recently the Bonn group[4] has presented an extension of the previous Bonn meson-exchange nucleon-nucleon interaction, which includes $N\Delta$ and $\Delta\Delta$ box diagrams above pion production threshold, including the inelastic channels.

Unfortunately, the use of these potentials for the many-body system is quite involved because of the large number of components for each channel, even within simple approximations like the Brueckner scheme for nuclear matter. A possible way of overcoming this problem, can be the use of a separable form of the original potentials, which greatly simplifies the many-body equations. The techniques to achieve such representations have been extensively studied[5] in the last decade and have reached such a level of accuracy that they can be safely used in few or many-body calculations.

In this work we study a separable form of the Argonne v_{28} potential, which is specially deviced to be used in nuclear matter calculations. The method is described in Sec. II. In Sec. III, the results for the most relevant channels of the Argonne v_{28} potential, are presented and discussed individually. Section V is devoted to the conclusions.

II. THE MODEL

Following Adhikari and Sloan[6] a rank N separable expansion of a given interaction V, can be constructed according to the *ansatz*

$$V_N=\sum_{n,m=1}^{N} V|u_n\rangle D^{(N)}_{nm}\langle v_m| \ ,$$
$$(D^{-1})_{nm}=\langle v_m|u_n\rangle \quad n,m=1,N \ , \tag{1}$$

where $|v_n\rangle,|u_n\rangle$ are a set of given functions with the requirement that D is a nonsingular matrix. Then by construction

$$V_N|u_n\rangle=V|u_n\rangle \quad n=1,N \ ; \tag{2}$$

that is, the action on the basis function $|u_n\rangle$ of the separ-

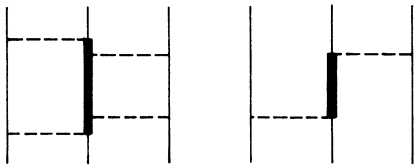

FIG. 1. Two pion exchange process contributing to the nucleon-nucleon interaction. The full, double, and dashed lines represent the nucleon, the delta and the pion, respectively.

able potential V_N equals the one of the original potential V.

Choosing the kets $|u_n\rangle$ as some selected eigenstates, bound or continuum, of the full Hamiltonian and $\langle v_n|=\langle u_n|V$, in order to make the potential symmetric, one recovers the Ernst, Shakin, and Thaler (EST) method.[7] From Eq. (2), it follows that in the EST method, the selected bound and scattering states are exactly reproduced by the separable potential V_N.

The alternative method developed by the authors, the Gamow separable expansion,[8] which was proved to be quite accurate for conventional realistic nucleon-nucleon potentials, encounters some difficulties with potentials which possess thresholds. In fact, it is numerically cumbersome to calculate Gamow vectors for coupled channels with thresholds like the search for resonances in a many-body problem.[9] The approximation scheme developed in Ref. 10 for calculating Gamow vectors cannot be applied in this case.

If the potential couples different partial waves, the radial Schrödinger equation for the two-body wave function may be written for a given channel λ,

$$-\frac{\hbar^2}{2\mu_\alpha}\frac{d^2}{dr^2}\Psi_\alpha^\lambda(r)+\sum_\beta V_{\alpha\beta}^\lambda\Psi_\beta^\lambda(r)+\frac{\hbar^2}{2\mu_\alpha}\frac{l_\alpha(l_\alpha+1)}{r^2}\Psi_\alpha^\lambda(r)=(E-s_\alpha^\lambda)\Psi_\alpha^\lambda(r) \quad (3)$$

s_α^λ being the threshold energy in the component α, equal to zero for the NN sector, 295 MeV for the $N\Delta$ and 590 MeV for the $\Delta\Delta$ one. More explicitly, the channel index λ and component index α denote the set of conserved quantum numbers $\{J_\lambda T_\lambda P_\lambda\}$ and the set of quantum numbers $\{l_\alpha S_\alpha \mu_\alpha\}$ respectively, with J_λ the total angular momentum, T_λ the total isotopic spin, P_λ the parity, l_α the relative orbital angular momentum, S_α the total spin and μ_α the reduced mass. The quantity V^λ is the two-body potential V projected into the channel λ.

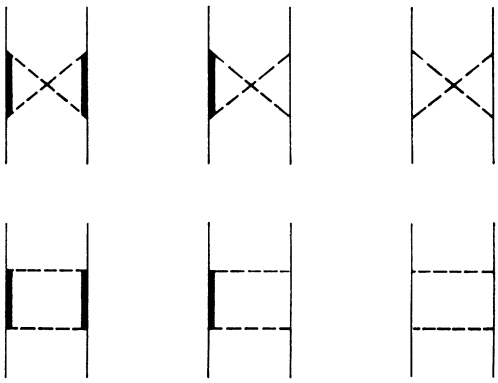

FIG. 2. Diagrams which contribute to the three-nucleon forces. Notation as in Fig. 1.

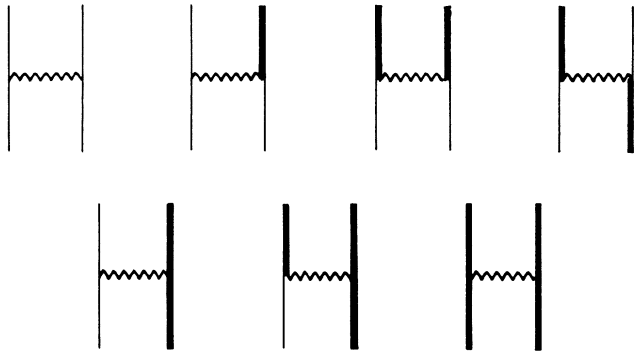

FIG. 3. Processes contained in the v_{28} model. The curly line represents the exchange of different mesons.

For a given positive energy E, the system of coupled differential Eqs. (3) has a set of linearly independent scattering state solutions. Their number equals the number of components α for which $E>s_\alpha^\lambda$; that is, the number of open channel components. Each one of these solutions can be specified by a set of linearly independent boundary conditions. The functions u_n, used to construct the form factors $V|u_n\rangle$ of the separable potential of Eq. (1) will then be chosen as these scattering states solutions. The deuteron wave function will also be included in the case of the $^3S_1-{}^3D_1$ channel.

In order to obtain an Hermitian and time-reversal invariant potential, real form facators are needed. The standing wave boundary conditions must then be used in solving Eq. (3). Furthermore, for the N open channels we specify each of the N linearly independent wave solutions, by assuming that only one of its components has an incoming free wave part $\phi_{l_\alpha}(r)$. The others will only have the spherical outgoing part.

In order to solve Eqs. (3), we transform them into a set of coupled integral equations. For the component α above threshold, $\alpha^>$, they read,

TABLE I. NN, $N\Delta$, and $\Delta\Delta$ partial waves ($^{2S+1}l_J$) considered in the separable expansion.

NN	$N\Delta$	$\Delta\Delta$
$^1S_0$	$^5D_0$	
$^1D_2$	$^5S_2{}^3D_2{}^5D_2{}^5G_2$	
$^3P_0$	$^3P_0$	$^3P_0{}^7F_0$
$^3P_1$	$^3P_1{}^5P_1{}^5F_1$	$^3P_1{}^7F_1$
$^3P_2-{}^3F_2$	$^3P_2{}^5P_2{}^3F_2{}^5F_2$	$^3P_2{}^7P_2{}^3F_2{}^7F_2{}^7H_2$
$^3S_1-{}^3D_1$		$^3S_1{}^3D_1{}^7D_1{}^7G_1$
$^3D_2$		$^3D_2{}^7D_2{}^7G_2$
$^1P_1$		$^1P_1{}^5P_1{}^5F_1$

$$\Psi^{\lambda}_{\alpha_i^>}(r)=\phi^{\lambda}_{l_i}(k_\alpha,r)\delta(\alpha,i)+\sum_\beta \int_0^{+\infty} dr' G^{(P)}_\alpha(k_\alpha;r,r')\frac{2\mu_\alpha}{\hbar^2}V^{\lambda}_{\alpha\beta}(r')\Psi^{\lambda}_{\beta i}(r') \ . \tag{4}$$

Similarly for the possible components α below threshold $\alpha^<$,

$$\Psi^{\lambda}_{\alpha_i^<}(r)=\sum_\beta \int_0^{+\infty} dr' G^{(+)}_\alpha(k_\alpha;r,r')\frac{2\mu_\alpha}{\hbar^2}V^{\lambda}_{\alpha\beta}(r')\Psi^{\lambda}_{\beta i}(r') \tag{5}$$

with the momentum $k_\alpha=[2\mu^\lambda_\alpha/\hbar^2(E-s^\lambda_\alpha)]^{1/2}$ for $E>s^\lambda_\alpha$ and the positive purely imaginary root otherwise. The index i specifies the boundary condition, that is the component α for which the incoming wave is different from zero. In writing Eq. (4) and Eq. (5) we followed the conventions of Ref. 11 for the normalizations, principal value and outgoing Green's functions G^P and $G^{(+)}$, respectively.

The choice of the Green's function $G^{(+)}$ and the definition of k_α guarantees that when a channel component is closed, the corresponding wave function component Ψ_α is decaying exponentially to zero for large radius.

The set of integral Eqs. (4) and (5) couples the components $\alpha^>$ and $\alpha^<$ since the summation over the index β on the right-hand side runs over all components below and above threshold.

Having solved the coupled equations for a given local interaction at a set of definite energies E_n and boundary conditions, including possible bound states, the corresponding wave functions $v_n=\Psi^\lambda_i(E_n)$ can be used to construct the separable representation,

$$V_N=\sum_{n,m=1}^{N} V|u_n\rangle D_{nm}\langle u_m|V \ . \tag{6}$$

Here the form factors $V|u_m\rangle$ have the explicit form in coordinate space,

$$\langle r;\alpha|V^\lambda|u_n\rangle=\sum_\beta V^\lambda_{\alpha\beta}(r)\Psi_{\beta i}(r) \ . \tag{7}$$

For a given energy E_n one would choose various kets $|u_n\rangle$ corresponding to different boundary conditions. In practice it turns out that it is sufficient to consider for each energy only one scattering state with a specific boundary condition. It follows from Eq. (2) that at the given selected energy E_n, the EST method ensures that the reaction matrix K, calculated from the separable potential, reproduces both on- and half-off-the-energy shell the ones calculated from the original potential along the row and the column specified by the boundary condition. One must then carefully choose a set of energies and boundary conditions in such a way that the original K matrix is well reproduced by the separable potential in a wide energy interval. This can be a difficult task above threshold where the number of open channels components can be quite large. However, one is usually interested only in energies below the $N\Delta$ threshold where only nucleon-nucleon components are presented, and these are never larger than two for each channel.

III. SEPARABLE REPRESENTATION OF THE ARGONNE v_{28} POTENTIAL

The v_{28} model uses the basic 14 operators in the NN channels, which have been proved to give satisfactory fits to the data. In order to introduce delta degrees of freedom, other 14 operators were added to include all possible $\pi N\Delta$ and $\pi\Delta\Delta$ couplings plus a central repulsion in the $N\Delta$ and $\Delta\Delta$ channels. Some restrictions have been imposed to the transition operators to simplify the structure of the potential, and make it specially suitable for nuclear structure calculations. The $\pi N\Delta$ coupling constant was fixed to the phenomenological value in the framework of the Chew-Low theory. The $\pi\Delta\Delta$ vertex was taken from the quark model. This fixes the long-range part of the $N\Delta$ and $\Delta\Delta$ interactions. The short and intermediate range parts were obtained by fitting the NN scattering data up to $E_{\rm lab}=400$ MeV, and deuteron properties.

In Table I the channels for which we construct a separable representation are reported together with corresponding components. They are the dominant channels in nuclear matter calculations. To obtain the scattering states, we solved the set of coupled integral Eqs. (4) and (5) by discretizing the relative coordinate r, in a grid of 128 points in the interval $0\le r\le 6.5$ fm^{-1}. This interval was divided into 16 subintervals of 8 Gaussian points each. For the calculation of the deuteron wave function, we solved directly the coupled differential Eqs. (3), by the Runge-Kutta method, according to the procedure described in Ref. 2. Once the scattering states are calculated, the exact reaction matrix K can be computed from

$$K^\lambda_{ij}(k_i,k_j)=-\frac{2\mu_i}{\hbar^2}\frac{1}{k_i}\sum_\alpha \int_0^{+\infty} dr'\phi^\lambda_{l_i}(k_i,r')V^\lambda_{i\alpha}(r')\Psi^\lambda_{\alpha j}(r') \ , \tag{8}$$

where i,j indicate the components above threshold. Analogously, the corresponding K matrix computed from the separable expansion for a generic energy, can be written

$$K^{\lambda N}_{ij}(k_i,k_j)=\sum_{n,m=1}^{N}\langle\phi^\lambda_{l_i}|V^\lambda|u_n\rangle\Lambda^{\lambda N}_{nm}(k_j)\langle u_m|V^\lambda|\phi^\lambda_{l_j}\rangle \ ,$$
$$(\Lambda^{\lambda N^{-1}})_{nm}=\langle u_n|V^\lambda|u_m\rangle+\langle u_n|V^\lambda G(k_j)V^\lambda|u_m\rangle \ , \tag{9}$$

where G is the free Green's function, diagonal in the channel components, with the boundary conditions defined according to the preceding section.

TABLE II. Selected energies and boundary conditions for the separable potential in the isospin triplet channels. The number indicating the boundary conditions follows the order in which the partial waves are reported in Table I.

Channel	Selected energies in MeV and boundary conditions									
$^1S_0$	3	30	83	212	332	478	651	750	904	1075
	1	1	1	1	1	1	2	2	2	2
$^1D_2, ^3P_0$	30	83	212	332	478	651	850	1075		
	1	1	1	1	1	2	2	2		
$^3P_1$	30	83	212	332	478	651	850	1075		
	1	1	1	1	1	3	3	3		
$^3P_2 - ^3F_2$	30	83	163	212	332	402	478	651	850	
	1	2	1	2	1	2	1	4	4	

A. Spin singlet isospin triplet channels

These channels contain both $N\Delta$ and $\Delta\Delta$ components. However, in Ref. 2 it was shown that by a small readjustment of the parameters one can get an equally good fit of the experimental data, by neglecting the $\Delta\Delta$ components. We have followed this parametrization and the chosen energies and boundary conditions for the EST procedure in the $^1S_0$ and $^1D_2$ channels are reported in Table II.

It has to be noted that the energy range considered in the expansion is larger than the $N\Delta$ threshold. The reason for this large energy domain is that the separable representation is devised specially for nuclear matter calculations where one needs to average the potential U_Δ felt by the Δ inside nuclear matter. The latter requires the calculation of the $N\Delta - N\Delta$ scattering matrix above threshold.[12] This can be seen from the explicit form of the average potential, namely

$$U_\Delta(k) = \frac{1}{16} \sum_{\lambda\alpha} \sum_{k' \le k_F} (2J_\lambda + 1)(2T_\lambda + 1) \times \langle kk';\lambda\alpha | G_B^{N\Delta}(\epsilon) | kk';\lambda\alpha \rangle , \quad (10)$$

where $G_B^{N\Delta}$ is the Brueckner reaction matrix in the $N\Delta$ sector, k' the momentum of a single nucleon confined by the Fermi momentum k_F, and k the momentum of the Δ. Notice that only the diagonal part of the reaction matrix appears. The latter has to be evaluated on-the-energy shell $\epsilon = \epsilon_k^N + \epsilon_{k'}^\Delta$.

If only the region below threshold has to be considered, then one can reduce considerably the rank of the separable potentials without any appreciable change in the phase shifts and mixing parameters, by restricting the set of EST states to the ones below the threshold. Above

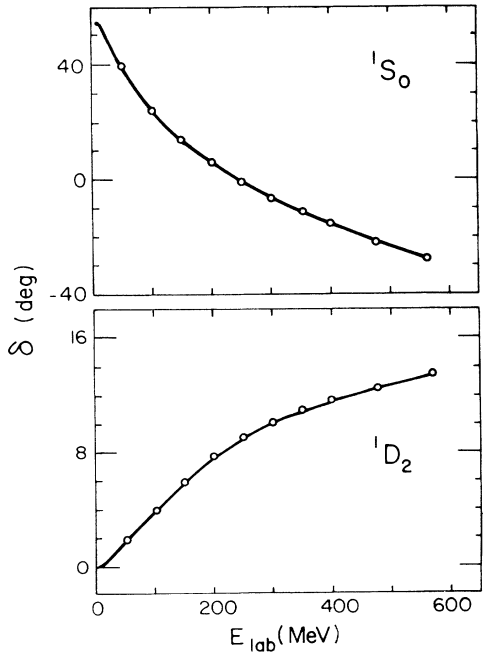

FIG. 4. The $^1S_0$ and $^1D_2$ phase shifts calculated with the separable potential (full line), compared with the ones obtained from the original local v_{28} potential (full dots) as a function of the total kinetic energy attached to the NN channel in the laboratory system.

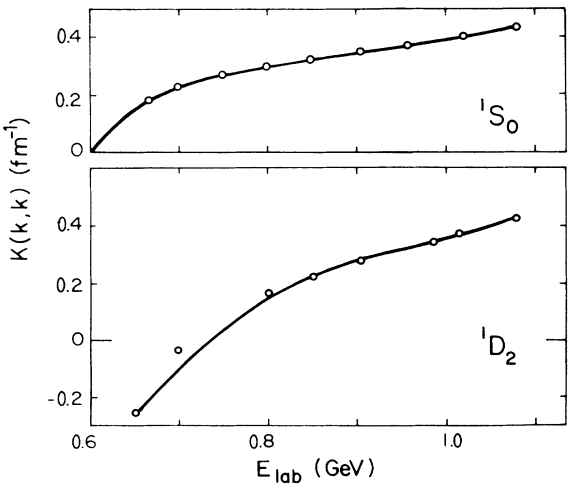

FIG. 5. The on-shell scattering K matrix above $N\Delta$ threshold for the $^1S_0$ and $^1D_2$ channels calculated with the separable potential (full line), compared with the ones obtained from the original potential (full dots). Here we display the $N\Delta$ components $^5D_0$ and $^5S_2$, respectively. The latter is the dominant one for the $^1D_2$.

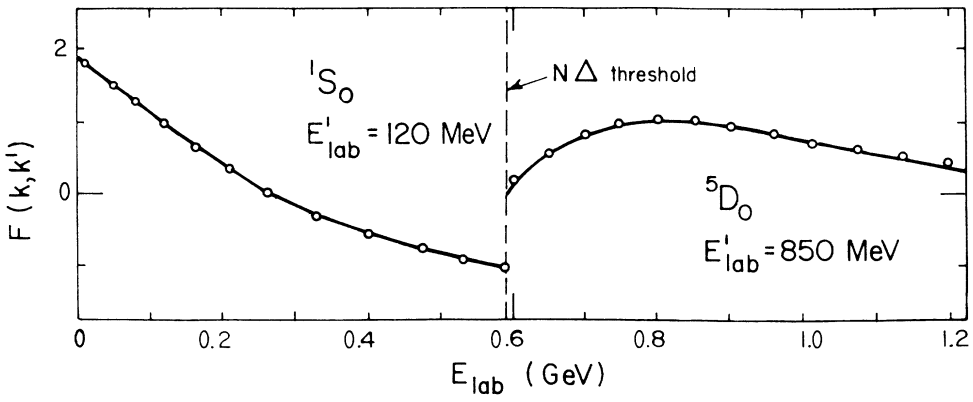

FIG. 6. Kowalski-Noyes function in the NN and $N\Delta$ sectors of the $^1S_0$ channel as a function of the total kinetic energy in the NN channel at a fixed energy E'_{lab}. The full dots and full line correspond to the exact and separable calculation, respectively.

threshold, the number of linearly independent solutions increases. There are two for the $^1S_0$ channel and five for the $^1D_2$ one. It is therefore important to select properly the boundary conditions without increasing too much the rank of expansion. Fortunately it turns out that among the components of the $N\Delta$ sector only one for each channel gives the dominant contribution to the Δ potential. They are the $^5D_0$ component for the $^1S_0$ channel, and the $^5S_2$ component for the $^1D_2$ channel. This dominance of a single ΔN component, holds for the $T=1, S=1$, channels too. The inclusion only of these dominant components in the calculation of the Δ potential should give the correct result within 10%. We should emphasize that, in solving the coupled channel equations for the scattering states and scattering matrix, all the possible components of each channel were included.

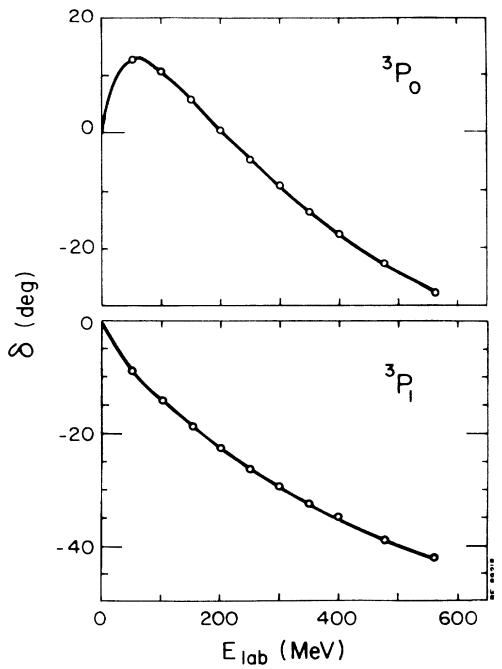

FIG. 7. As in Fig. 4 for the $^3P_0$ and $^3P_1$ channels.

In Fig. 4 we compare the NN phase shifts for these two channels, calculated with the separable potential and the original one. This excellent reproduction of the original on-shell reaction matrix can be obtained using a rank 5 potential for both the $^1S_0$ and the $^1D_2$ channel. The energy range where this separable expansion is performed is quite sufficient for conventional nuclear matter calculations, even in the framework of the continuous choice of the single-particle potential. We extended the expansion also to energies above threshold going to rank 10 and 9, respectively, for the previous channels. The corresponding K matrices are compared with the exact ones in Fig. 5 for the above-mentioned dominant components.

Finally, the half-off-the-energy shell behavior of the K matrix was checked by comparing the Kowalski-Noyes function

$$F(k,k')=K(k,k')/K(k,k)$$

of the original and separable potentials, at energies different from the selected ones. As an example, we present in Fig. 6 the Kowalski-Noyes function for the $^1S_0$

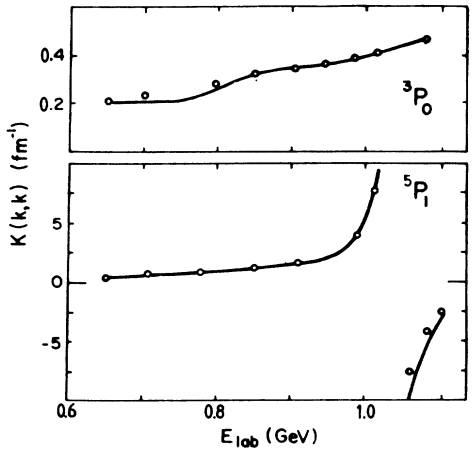

FIG. 8. As in Fig. 5 for the $^3P_0$ and $^3P_1$ channels. Here the dominant components in each sector are the $^3P_0$ and $^5P_1$.

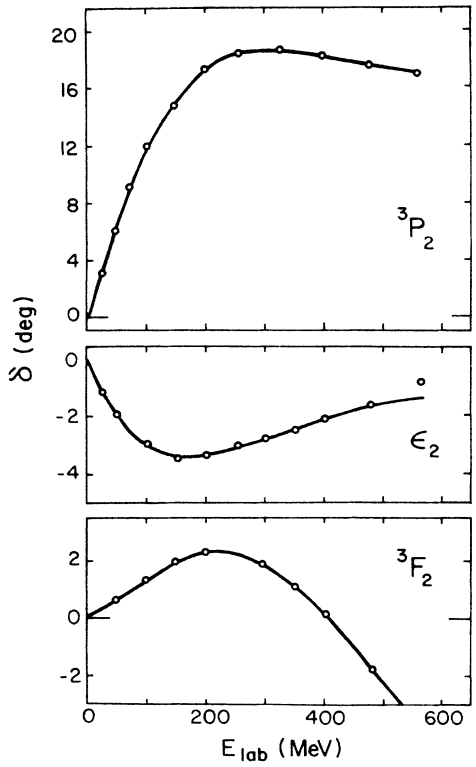

FIG. 9. As in Fig. 4 for the $^3P_2-{}^3F_2$ channel.

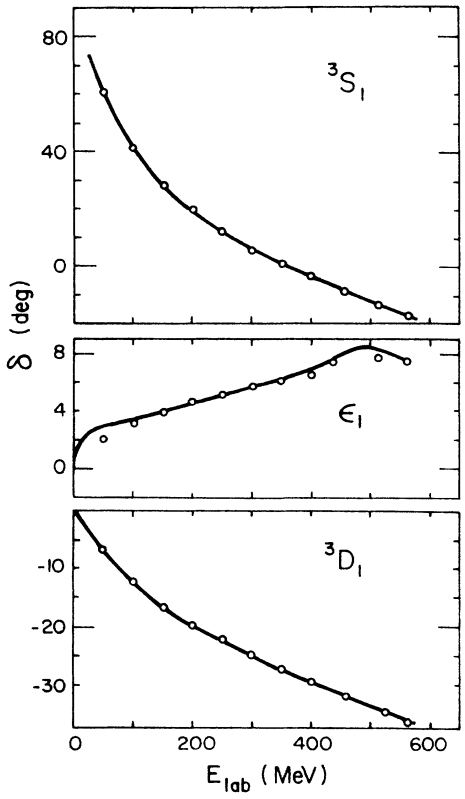

FIG. 11. As in Fig. 4 for the $^3S_1-{}^3D_1$ channel.

channel in the diagonal NN component, the $^1S_0$, up to the $N\Delta$ threshold, and the $N\Delta$ component, the $^5D_0$, above the $N\Delta$ threshold. As it can be seen, the off-the-energy shell behavior is very well reproduced. The same accuracy was also observed for the other channels including the ones analyzed in subsections B and C. Consequently only the on-the-energy shell behavior will be discussed there.

B. Spin triplet isospin triplet channels

In these channels one has to include both the $N\Delta$ and $\Delta\Delta$ components. For the $^3P_0$ and $^3P_1$ channels we followed the same method of the preceding subsection selecting for the dominant $N\Delta$ components above threshold the $^3P_0$ and $^5P_1$, respectively (see Table I). In Fig. 7 the phase shifts obtained with the separable potential are compared with the exact ones. This expansion requires rank 5 for both channels. The exact and approximate re-

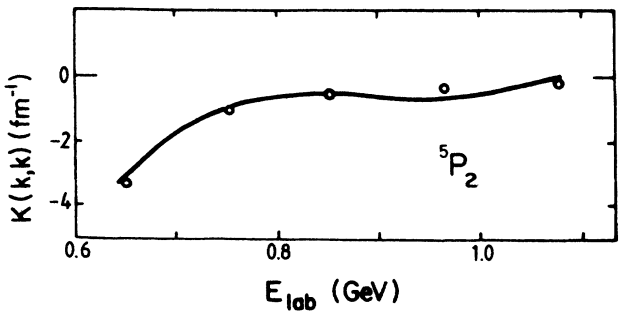

FIG. 10. As in Fig. 5 for the $^3P_2-{}^3F_2$ channel. Here the dominant component is the $^5P_2$.

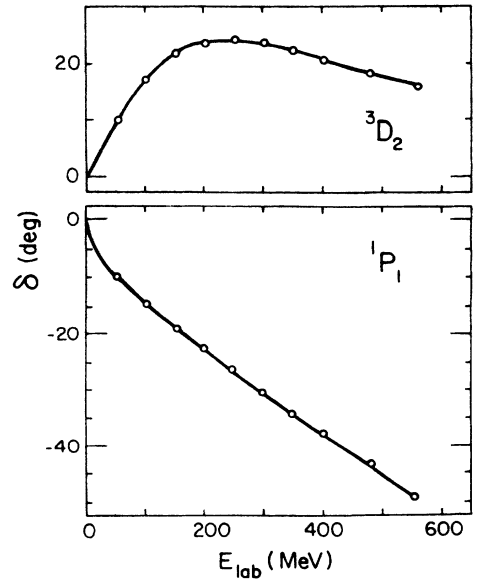

FIG. 12. As in Fig. 4 for the $^3D_2$ and $^1P_1$ channel.

TABLE III. The same as in Table II for the isospin single channels.

Channel	Selected energies in MeV and boundary conditions								
$^3S_1-{}^3D_1$	−2.26	30	53	83	120	212	332	402	561
		2	1	2	1	2	1	2	1
$^3D_2,{}^1P_1$	30	83	212	332	487	651			
	1	1	1	1	1	1			

action matrices above threshold, are compared in Fig. 8. This extension of the expansion requires an increase of the rank to 8. The corresponding EST parameters are displayed in Table II. In the $^3P_1$ channel we found that the $^5P_1$ $N\Delta$ component produces a diagonal K matrix with a singularity around a total energy of 1 GeV. In general the K matrix can have singular behavior, for example if the phase shift passes decreasing through $\pi/2$, which does not imply the existence of a resonance. We think then that this singular behavior has no physical meaning.

The $^3P_2-{}^3F_2$ channel has 11 components. The phase shifts and mixing parameters obtained with a rank 7 expansion for this channel are reported in Fig. 9, in comparison with the exact ones. Finally in Fig. 10 the K matrix of the separable potential above $N\Delta$ threshold for the dominant component $^5P_2$, is compared with the exact one. For this extension rank 9 was required.

C. Isospin singlet channels

For these channels besides the NN part, only the $\Delta\Delta$ components are present. Consequently they will not contribute to the Δ averaged potential in nuclear matter. Having in mind such calculations, the expansion was then restricted essentially up to the $N\Delta$ threshold for the $T=1$ channels. The ranks of the expansion for the $^3S_1-{}^3D_1$, $^3D_2$ and $^1P_1$ channels are 9, 6, and 6, respectively. Since the $^3S_1-{}^3D_1$ channel is one of the most contributing ones to the nuclear matter binding, we have used a high rank in the expansion in order to get a very high degree of accuracy for both phases and mixing parameters of this channel, in the relevant energy range. They are reported in Fig. 11. In Fig. 12 we report the phases for the $^3D_2$ and $^1P_1$ channels. The corresponding EST parameters are included in Table III.

IV. CONCLUSIONS

We have presented a separable representation of the Argonne v_{28} potential which includes Δ degrees of freedom through the coupling of the NN channel to both the $N\Delta$ and $\Delta\Delta$ ones. The separable representation is specially devised for nuclear matter calculations in particular for Bethe-Brueckner calculations where one needs to compute the mean nuclear potential felt by the Δ particle inside nuclear matter. These calculations are in progress and will be presented by the authors elsewhere. Preliminary results can be found in Ref. 13.

The separable potential is determined by the strength parameters D_{nm} and form factors $V|u_n\rangle$ of Eqs. (6) and (7). The latter were calculated numerically in a set of discrete points between zero and 6.5 fm as described in Sec. II. No fitting of these form factors to analytic forms was attempted. We feel that, with modern computing facilities they can be handled directly in numerical form.

In view of the large number of components for each channel, the rank of the potential has been chosen large enough to get the high degree of accuracy needed in nuclear matter calculations for a wide energy range. In this way, we were able to construct a separable potential which incorporates the Δ degrees of freedom suitable for practical applications and compatible with all experimental data of the two-nucleon system.

ACKNOWLEDGMENTS

We would like to thank Professor L. Streit for the kind hospitality at the University of Bielefeld, where this work was accomplished. This work was partially supported by Volkswagen Forschung, Partnership Program.

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