Single pion production in nucleon-nucleon collisions

Francesca Sammarruca
Department of Physics, University of Idaho, Moscow, Idaho 83843

Tetsuro Mizutani*
Paul Scherrer Institut, CH-5232 Villingen, Switzerland
(Received 2 November 1989)

In this work, we explore the $NN \rightarrow \pi NN$ reaction within a relativistic model for the NN and πNN coupled systems consistent with two- and three-body unitarity. After describing the theoretical input, we concentrate on some exclusive $pp \rightarrow pn\pi^+$ observables. These are compared with the available data and with predictions from other models. In particular, we examine systematically the dependence of some spin observables on various components of the dynamical input. We identify and isolate some problems related to the present approach and we point out possible directions for future research.

I. INTRODUCTION

Our present work is devoted to the problem of single-pion production in two-nucleon collisions at energies where the delta resonance dominates the process (i.e., incident laboratory energy between 600 and 800 MeV).

Pion production from two-nucleon collisions is not really a new topic. Naturally, the interest in this subject started with the advent of the first pion producing accelerators. We can locate one of the earliest measurements of $pp \rightarrow pn\pi^+$ in the mid 1950's,[1] followed by several experiments carried out in the early 1960's.[2–4] Theoretical interpretations were given in terms of either the peripheral model with absorption[5] (above 1 GeV), or the so-called "one-pion-exchange model"[6] (at lower energies). The latter overlaps, to some extent, with the Mandelstam model[7] which, in particular, emphasises the importance of the intermediate $\Delta(1232)$ excitation. The early experiments were mostly conducted in emulsions or bubble chambers, which usually meant poor statistics; in particular, the spin observables were hard to measure. Theoretical models reached a moderate degree of success in explaining the data, but were quite phenomenological and thus did not provide a deep insight into the detailed mechanism of pion production. By the end of the 1960's, the interest in this subject had gone down.

A renewed interest developed in the early 1970's and has been continuing since then. The driving force behind this revival originated essentially from experimental progress: (i) high quality and/or intensity nucleon beams became available from the three major meson factories as well as the KEK and SATURNE facilities (the latter with beam energies up to 2 GeV); (ii) improved polarization techniques for beams and target made the measurement of crucial spin observables possible, and (iii) there was considerable progress in counter experiments detecting at least two particles in coincidence (the energy of one of the particles being measured), with high statistics over a wide range of solid angles. This allows a complete kinematical determination of the three-particle final state.

In spite of this considerable experimental progress, the present database is by no means comparable to the one available on the NN elastic scattering and $NN \leftrightarrow \pi d$ reactions. This is in part due to the much larger phase space to be covered in three-particle final-state reactions. Inclusive reactions, of the type $NN \rightarrow N(X)$ and $NN \rightarrow \pi(X)$, have been measured,[8–15] as well as the exclusive processes $pp \rightarrow pn\pi^+$ (Refs. 16–21) and $pp \rightarrow pp\pi^0$.[22,23] In each of those experiments, a limited part of the phase space was investigated, and some measurement of spin observables was performed. In all cases, the energy range covered the region where the Δ excitation is the prevailing mechanism, and some of the experiments were aimed at extracting information on the $N\Delta$ interaction as well.[10,20,21]

On the theoretical side, there has been considerable progress in the treatment of the coupled NN-πNN system during the last decade.[24–28] By solving a set of coupled integral equations [which we shall refer to as the "πNN equation" (PNNE)], we are now able to obtain *simultaneously* the amplitudes for the process $NN \rightarrow NN$, $NN \leftrightarrow \pi d$, $NN \rightarrow \pi NN$, $\pi d \rightarrow \pi d$, and $\pi d \rightarrow \pi NN$. The equations respect two- and three-body unitarity, and so are appropriate for energies up to about 1 GeV (in terms of the laboratory kinetic energy in the NN channel), where the inelastic process is essentially limited to single-pion production.

The coupled-channel nature of the equations implies a very strong constraint on the phenomenological parameters of the model, since the theory must provide a reasonable description of all the coupled channels simultaneously, rather than just one specific process. This is an enormous advantage over the earlier models mentioned above for the purpose of a detailed understanding of the pion-production mechanism.

When the PNNE was applied to processes with two-body final state,[29–31] in general a good qualitative and, in some cases, a good quantitative description of the data was achieved. Now it is only natural to explore within the same framework the remaining processes, namely those with a three-particle final state, $NN \rightarrow \pi NN$, $\pi d \rightarrow \pi NN$. Notice that these channels can be obtained directly from "breakup" of the half-off-shell isobar amplitudes produced by solving the PNNE on the real axis.

Already motivated by the ongoing experimental and theoretical progress, the interest in the $NN \rightarrow \pi NN$ reaction has been further boosted by the possible existence of dibaryon resonances in the Δ region.[32] The common assumption was that the alleged dibaryons would have a large inelastic decay width into the πNN three-body channel as compared with the two-body channels (e.g., πd); therefore, the most suitable process for their hunting was seen in the pion production from two-nucleon collisions.

The first PNNE calculation was performed by Dubach *et al.*;[33] the model, already applied to NN elastic scattering at medium energies,[29] was based on a set of three-body coupled equations for the NN and the $N\Delta$ channels driven by a nonstatic one-pion exchange. In contrast to the earlier nonunitary approaches (like the one by Mandelstam and its variants), the model was able to reproduce the general tendency of the data qualititively, but was not satisfactory in quantitative terms. The same group then extended the calculation to other observables,[34–36] with various modifications and improvements (e.g., taking into account short-range effects on the lowest partial waves, or including the nucleon-nucleon final state interaction).

Later, Matsuyama and Lee[27] as well as Ueda[31] and Garcilazo[28] performed similar calculations based upon their own versions of the PNNE. Essentially all those unitary models are able to describe qualitatively the global features of the data, but none of them is quantitatively satisfactory. Also, not all the existing data were compared with predictions by the models: the comparison has been centered around the multiple differential cross section and the proton beam asymmetry for the reaction $pp \rightarrow pn\pi^+$, with coplanar final state kinematics and the neutron as the undetected particle.[16]

The general quantitative disagreement with the data may quite well be the reflection of the insufficient NN spin triplet inelastic strength shown in almost all PNNE results for the $NN \rightarrow NN$ and $NN \leftrightarrow \pi d$ processes.[37] Also, the treatment of the $\pi N - P_{11}$ amplitude, which is responsible for the mechanism of pion absorption and emission from off-shell intermediate nucleons, is rather crude in most of those models. Thus an improvement in this sector may result in a better agreement with the data.

Mainly those two aspects have motivated us to look into the pion-production channel; our work is built upon the model developed by the Lyon Group and applied so far to the two-body reactions involving the πNN system.[30]

Among the features of our model is the inclusion of πN interactions other than the (dominant) P_{33} and the P_{11}, which we describe realistically.

Short-range contributions are also taken into account and they include exchange of heavy bosons in the NN channel as well as the $N\Delta$ channel. Interestingly, the model of Kloet and Silbar,[29] by emphasizing the long-range part of the NN interaction, does obtain a good degree of success in describing the bulk of the pion-production data, which seems to indicate the peripheral nature of the process. This is also a rather fundamental point, which deserves further attention.

This work is subdivided as follows: in Sec. II we review the model used to calculate $NN \rightarrow N\Delta$ half-off-shell amplitudes; in Sec. III A, we explain how the pion-production amplitude is constructed; in Sec. III B, we present and discuss some results for exclusive $pp \rightarrow pn\pi^+$. Our present conclusions as well as future plans are discussed in Sec. IV.

II. THE MODEL

A. The scattering equations

The equations for the coupled $NN \rightarrow \pi NN$ system derived by Avishai and Mizutani,[24] Rinat and Thomas,[26] Afnan and Blankleider,[25] are formally similar to the Faddeev-Lovelace equations. They are by now well known and therefore we will not discuss them in detail here. We limit ourselves to recall that the two-body subsystems are assumed to interact via a separable nonlocal potential and to be dominated by bound states or resonances. Following the Aaron, Amado, and Young (AAY)[38] procedure, the Blanckenbecler-Sugar (BbS) reduction allows integration over the relative energy, leaving a set of three-dimensional coupled equations. Their final form is[39]

$$T_{\alpha\beta}(p',p,s)=B_{\alpha\beta}(p',p,s)+\frac{1}{(2\pi)^3}\sum_{\gamma}\int\frac{d^3p''}{2E_{p''}}B_{\alpha\gamma}(p',p'',s)D_{\gamma}^{-1}(\sigma_{p''})T_{\gamma\beta}(p'',p,s)\ , \tag{1}$$

where $E=(p^2+m^2)^{1/2}$. T's are the three-body amplitudes and B's the driving terms describing single-particle exchange between two bound or resonating pairs; α,β, and γ denote the states with a particle and an interacting pair, and D^{-1} is the two-body propagator in the three-body Hilbert space in the presence of an off-shell spectator particle. The function D is obtained directly from πN scattering theory, assuming the two particles interact in the c.m. frame via a separable potential

$$v(p',p)=\lambda g(p')g(p)\ , \tag{2}$$

where λ is the strength and g the form factor. For this system, the two-body BbS equation reads

$$t(p',p,\sigma)=v(p',p)+\frac{1}{(2\pi)^3}\int\frac{d^3p''}{2E_1(p'')E_2(p'')}v(p',p'')\frac{E_1(p'')+E_2(p'')}{\sigma-[E_1(p'')+E_2(p'')]^2}t(p'',p,\sigma)\ , \tag{3}$$

where σ is the c.m. total energy squared. The solution of Eq. (3) is of the form

$$t(p',p,\sigma)=g(p')D^{-1}(\sigma)g(p) , \tag{4}$$

where

$$D(\sigma)=\lambda^{-1}-\frac{1}{(2\pi)^3}\int\frac{d^3k}{2E_1(k)E_2(k)}g^2(k)\frac{E_1(k)+E_2(k)}{\sigma-[E_1(k)+E_2(k)]^2} . \tag{5}$$

The two-body interactions, Eq. (4), in the presence of a third particle (in the three-body center of mass) must be defined in a Lorentz invariant way, since we are in a relativistic framework. This is accomplished by expressing the vertex functions $g(p),g(p')$ in terms of relativistic relative momenta (instead of the Galilean invariant relative momenta), as proposed by Aaron.[39]

Finally, angular momentum reduction will cast Eq. (1) in a convenient one-dimensional form, suitable for practical solutions.

B. The two-body systems

All the S, and P πN waves are taken into account, and parametrized as rank-one separable potentials, Eq. (2). Following previous work by Lamot and collaborators,[30] the form factor chosen for the S_{11}, S_{31}, P_{13}, and P_{31} is of the form

$$g_l(p)=p^l\left[\frac{A}{p^2+a^2}+\frac{B}{p^2+b^2}\right] . \tag{6}$$

The parametrization[40] adopted for each wave are chosen so to fit the phase shifts and the scattering lengths and volumes.

A form factor as in Eq. (6), with $B=0$, is also used to reproduce the P_{33} on-shell behavior. In this case, the strength λ in Eq. (2) is taken to be energy-dependent to reproduce the position of the resonance. We recall that the P_{33} scattering matrix, on- and off-shell, is written as

$$t(p',p,\sigma)=g(p')D^{-1}(\sigma)g(p) , \tag{7}$$

where

$$D^{-1}(\sigma)=\frac{1}{\sigma-M_\Delta^2-I(\sigma)} \tag{8}$$

(σ is the c.m. total energy squared). In the last equation, $I(\sigma)$ is the integral appearing in Eq. (5); M_Δ, the bare Δ mass, is taken to be 1320 MeV.[41] The parameters of the form factor which is chosen for the on-shell case as

$$g(p)=\frac{\alpha p}{p^2+\beta^2} \tag{9}$$

are then constrained by the $\pi N-P_{33}$ scattering data. A satisfactory fit to the P_{33} phase shifts is obtained with $\alpha=218.58\ \text{fm}^{-1}$, $\beta^2=2.17\ \text{fm}^{-2}$ (corresponding to about 300 MeV for β, a very "soft" form factor).

The P_{11} partial wave is parametrized as proposed by Mizutani *et al.*.[42] Here, we just recall that one writes the total amplitude as a sum of two contributions, the "pole" and the "non-pole" part, and the energy-dependent vertex function for the pole part is obtained by dressing the πNN vertex and the nucleon propagator to take into account the virtual pion emission and absorption.[42]

C. Off-shell nature of P_{33} and P_{11}

As we are studying the channel which contains most of the NN inelasticity, we have been particularly concerned with the major source of inelasticity in this energy region, namely the Δ resonance. Within the πNN three-body framework, this amounts to choosing a model for the "off-shell" $\pi N-P_{33}$ vertex which actually describes the mechanism of pion production through isobar dissociation. Here, we have tried to investigate the "off-shell freedom" resulting from the fact that the πN scattering data cannot constrain the off-shell behavior.

In fact, when the form factor Eq. (9) is used for "off-shell $\pi N-P_{33}$ scattering, the applied cutoff turns out to be too strong (i.e., the associated range too long) which naturally results in a substantial underestimation of pion production.

As a remedy to this problem, various prescriptions are available from theories of off-shell continuation of the πN scattering matrix. One way, already used in $NN\to\pi d$ calculations,[39] consists in defining a new form factor

$$F(p)=v(k)\frac{h(p)}{h(k)} , \tag{10}$$

where k is the center of mass momentum for on-shell π-N scattering, defined by

$$\sigma=[(m^2+k^2)^{1/2}+(\mu^2+k^2)^{1/2}]^2 \tag{11}$$

and $h(p)$ a new form factor to be chosen. Typically, we take a monopole form for $h(p)$

$$h(p)=\frac{k^2+\lambda_{33}^2}{p^2+\lambda_{33}^2} \tag{12}$$

and a value of 800 MeV for the cutoff mass λ_{33}. The choice of this parameter is however rather delicate, as shown later in this section.

In a similar fashion, we perform off-shell modifications on the P_{11} (pole and nonpole), with a monopole form for $h(p)$ and a value of 1.2 GeV for the cutoff mass.[30]

When working within the intermediate energy range, the NN partial waves bearing most of the available inelasticity are ${}^1D_2$ and ${}^3F_3$, since they couple to very central $N\Delta$ states through the transitions $NN({}^1D_2)\to N\Delta({}^5S_2)$ and $NN({}^3F_3)\to N\Delta({}^5P_3)$, respectively. Concerning the inelasticities in these waves, a common tendency of the

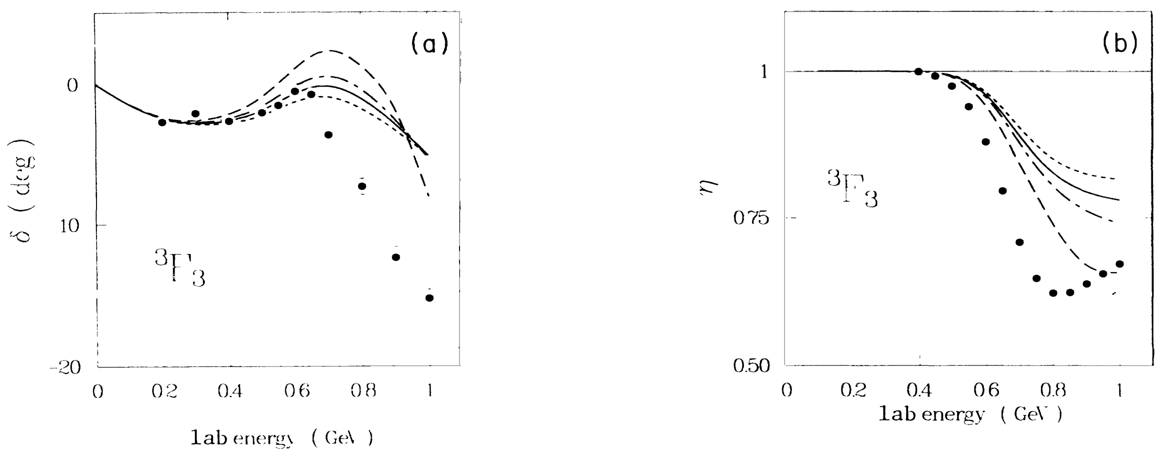

FIG. 1. Variation of the $^3F_3$ phase parameters with the cutoff mass of the off-shell πN-P_{33} vertex: $\lambda_{33}=0.7$ GeV (dotted line), $\lambda_{33}=0.8$ GeV (solid line), $\lambda_{33}=0.9$ GeV (dashed-dotted line), and $\lambda_{33}=1.2$ GeV (dashed line). Data from Arndt *et al.* (Ref. 43).

existing models can be observed; for $^1D_2$, a rapid decrease (or, at least, saturation) of the inelasticity is seen above 700 MeV (laboratory energy) in disagreement with the phase shifts analysis.[43] As far as $^3F_3$ is concerned, the inelasticity is generally insufficient everywhere.

Some comments are due here concerning the model by Kloet and Silbar (KS),[44] since it does not share this tendency. Actually, the inelasticity parameter is highly overestimated for $^1D_2$ and only slightly so for $^3F_3$ (below 1 GeV) while the phase shifts are unrealistically large. Notice that a reasonable inelastic strength in the triplet channel together with an overly strong singlet may imply that the relative singlet/triplet strength remains problematic. We recall that the KS model involves rank-one separable potentials for both P_{11} and P_{33} (no other πN interaction is allowed). Moreover, the same parametrization is adopted on- and off-shell, while we perform off-shell modifications as explained above, Eqs. (10–12). Therefore, we were concerned with understanding which mechanism can produce such a large inelasticity within a similar but somewhat simpler model. When comparing our results with those from the KS model, one must keep in mind the following: our model includes only one of the two possible time orderings in the Feynman pion propagator, but this can be compensated by the off-shell modifications with harder cutoffs. In trying to reproduce the KS results, we then noticed the following: the overestimation of the inelasticity, especially in $^1D_2$, was to a large extent related to the choice of the πN-P_{11} vertex–in particular, the lack of attraction typically produced by a rank-one (pole part only) potential. Therefore, it seems that both P_{33} *and* P_{11} do control the physics of pion production; moreover, a substantial and realistic improvement of the inelastic parameters in the critical partial waves cannot be simply achieved within a rank-one potential model for the P_{33}. Increased strength in the P_{33} can in principle be obtained by controlling the range of the form factor, but a corresponding over-attraction in the real phases is hard to avoid. This is seen in Fig. 1, where we show the sensitivity of the $^3F_3$ phase parameters to the cutoff mass λ_{33} of the off-shell form factor, Eq. (12). Even though this tendency may be somewhat less pronounced in some models,[27] none of them is immune from the typical underestimation of the triplet inelastic strength; thus the problem, which most likely originates in the complicated off-shell structure of the meson-baryon vertex, remains open.

The effect and relevance of the off-shell freedom on pion production are discussed in Sec. III.

D. Short-range contributions

We now discuss how short-range forces are included in our model. The three-body equation generates isobar amplitudes on iteration of the one-pion-exchange driving terms, which represent the long range part of the nuclear force. Any other contribution (potential) is independently evaluated and added to the driving terms.

The short-range contribution to the NN interaction comes primarily from the ω, ρ, and σ bosons (the η, δ, and ϕ mesons play only a small role). The one-boson-exchange potential (OBEP) is defined as a sum of one-particle-exchange amplitudes of certain bosons with given mass and coupling. Thus,

$$V_{\mathrm{OBEP}} = \sum_{\alpha=\eta,\rho,\omega,\delta,\sigma} V_\alpha^{\mathrm{OBE}} \tag{13}$$

which is then added to the one-pion-exchange amplitude.

A form factor of the type

$$\left[\frac{\Lambda_\alpha^2 - m_\alpha^2}{\Lambda_\alpha^2 + (\mathbf{q}' - \mathbf{q})^2} \right]^{n_\alpha} \tag{14}$$

is applied to each meson-nucleon vertex, with the cutoff mass Λ and n_α as free parameters (see Table I). Our parameters are in good agreement with those used by the Bonn group.[46]

Besides the exchange of heavy mesons between two nucleons, one-boson-exchange involving an isobar can also occur. (We will limit ourselves to the case of a P_{33} isobar, given its dominant role).

TABLE I. Meson parameters used for one-boson exchange.

	m_α (MeV)	$g^2/4\pi$	Λ_α (MeV)	n_α
σ	570	7	1300	1
ω	782.8	22	1650	$\frac{3}{2}$
ρ	769	0.7(6.1)[a]	1400	$\frac{3}{2}$

[a]The tensor/vector ratio of the ρ coupling constants, f/g, is given in parenthesis. For the ω this ratio is assumed to be zero.

The selection rules substantially reduce the number of bosons which can be exchanged in those diagrams: having the nucleon isospin $\frac{1}{2}$ and the Δ isospin $\frac{3}{2}$, only an iso-vector boson ($T=1$) can be present at the $\alpha N\Delta$ vertex, namely the pion and the ρ (besides the scalar δ, which we shall not consider).

To evaluate the contributions $V^{\alpha}_{N\Delta}$ and $V^{\alpha}_{\Delta\Delta}$, the procedure we adopt is to simply include the ρ along with the pion in the three-body calculation. One writes the ρNN and the $\rho N\Delta$ (potential theoretic) vertices exactly as the corresponding vertices with the pion, and sums over the two possible values of the spin of the ρN pair ($\frac{1}{2}$ and $\frac{3}{2}$). The thus constructed potentials are then added to the corresponding pion-exchange terms, to give the full $N\Delta$ and $\Delta\Delta$ transition potentials

$$V_{N\Delta}=V^{\pi}_{N\Delta}+V^{\rho}_{N\Delta}\ , \tag{15}$$

$$V_{\Delta\Delta}=V^{\pi}_{\Delta\Delta}+V^{\rho}_{\Delta\Delta}\ . \tag{16}$$

For the $\rho N\Delta$ coupling constant, the quark model predicts[45]

$$f^2_{\rho N\Delta}=\frac{f^2_{\pi n\Delta}}{f^2_{\pi NN}}g^2_{\rho NN}\left[\frac{m_\rho}{2M}\right]^2\left[1+\frac{f_{\rho NN}}{g_{\rho NN}}\right]^2, \tag{17}$$

with

$$f^2_{\pi N\Delta}=\frac{72}{25}f^2_{\pi NN} \tag{18}$$

and

$$f^2_{\pi NN}=\left[\frac{m_\pi}{2M}\right]^2 g^2_{\pi NN} \tag{19}$$

($g_{\rho NN}$ and $f_{\rho NN}/g_{\rho NN}$ are given in Table I.) At the $\rho N\Delta$ vertex, a cutoff as in Eq. (14) is applied, with $n_\alpha=2$ (a dipole form factor is required to suppress the high momentum component at the vertex in the case of a vector meson) and $\Lambda_\alpha=1000$ MeV.

The parametrization adopted (Table I) was chosen so to achieve a good fit of ${}^1D_2$ (Fig. 2); actually, a satisfactory fit of this wave was first obtained with the inclusion of heavy-meson exchange in the NN channel only, and then, after including the $\rho N\Delta$ contribution, the parameters

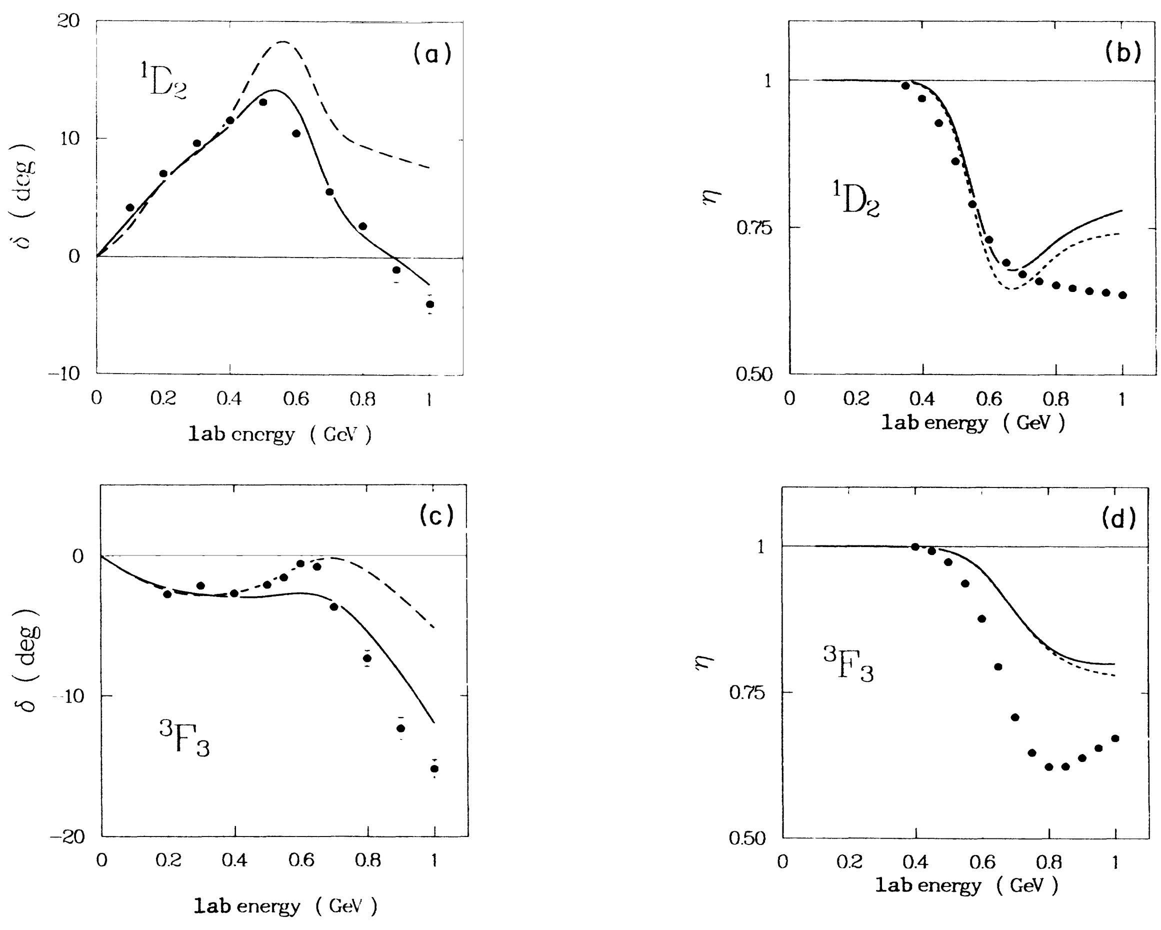

FIG. 2. Phase shifts and inelasticity parameters for ${}^1D_2$ and ${}^3F_3$. Solid line with, and dashed line without short-range contributions. Data from Arndt *et al.* (Ref. 43).

were readjusted so to regain the previous fit of ${}^1D_2$. In conventional low-energy NN models,[47] a large effect is observed by applying this "refit" procedure to the S waves at low energy. Presumably, a more careful study of the ρ parametrization is needed: for instance, choosing to fit ${}^1D_2$ to begin with may already prejudice the analysis against the lowest partial waves. In other words, the net effect of a heavy meson, since it is introduced up to a certain extent phenomenologically, does depend on one's reference point in defining the effect itself (we do not perform a global χ^2 fit).

Notice that for most πNN theories, it is especially difficult to reproduce quantitatively phase shift analysis results for the P waves, typically showing insufficient repulsion above 500 MeV in ${}^3P_1$ and ${}^3P_0$. The procedure adopted by Lee[27] of using a subtracted Paris potential reduces this tendency, but a lack of repulsion at medium energy can still be observed. The behavior of the P waves in the inelastic region is characterized by the fact that the coupled $N\Delta$ channels are also in a relative P wave, so that the NN and $N\Delta$ intermediate states should be equally important. In the πNN models, the two-baryon intermediate state is represented by a spectator nucleon and an interacting pair [the isobar—see Eqs. (4) and (5)]. The resulting structure of the NN propagator in the πNN framework, and its different energy dependence (as compared with conventional two-body NN models), does in fact contribute to the typical attractive tendency of the predicted NN P waves. In particular, the phase introduced by the self-energy diagram seems to disturb these waves, which are in fact better reproduced (as far as the real part is concerned) by a simple OBEP model.[46] On the other hand, an accurate description of these waves is essential to correctly predict elastic NN observables. Ideally, one would like to describe equally well elastic and inelastic processes, but this turns out to be a very difficult task and has not yet been achieved. Some realistic NN models[47] which rather carefully describe the low energy NN data, do not do much better above the pion-production threshold[46] than potential theoretic approaches.[27,29] This could signify (as we tend to believe) that the problems go beyond the choice between a field theoretic or a potential theoretic, separable ansatz.

Presently, it may be safer to suspect that the models for higher energy have not yet been worked out as carefully as for the low-energy regime, rather than a general limitation of all meson models.

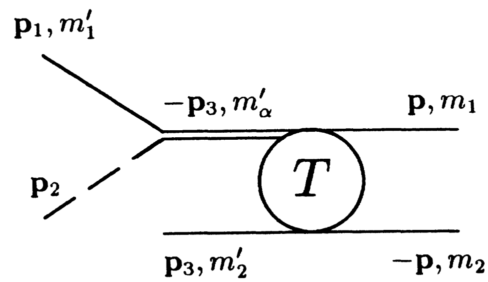

FIG. 3. Schematic representation of the 2→3 body amplitude. The underlying time axis is horizontal, pointing left into the future.

III. THE PION-PRODUCTION CHANNEL

A. The pion-production amplitude

The $NN\rightarrow\pi NN$ reaction has been studied rather intensively in the past few years, but still not exhaustively. Because of its much richer final-state phase space, the $2\rightarrow 3$ body unbound process is technically more difficult to handle, theoretically as well as experimentally. As we discussed in the Introduction, a growing interest is developing around this process (for instance, the database is rapidly expanding),[48,21] as it knowledge is indispensable for a detailed understanding of the NN inelasticities.

In this section, we describe how the production mechanism is incorporated in our model; we will concentrate on exclusive processes (kinematically complete experiments).

For the reaction we are studying, the initial channel is always a two-nucleon state on-shell; therefore, we solve the integral equation, Eq (1), for half-off-shell isobar amplitudes (the final state being off-shell) describing the process $NN\rightarrow N\alpha$ (α is the isobar label).[49] The subsequent isobar dissociation is then calculated from these amplitudes, as schematically described in Fig. 3. In the figure, $\mathbf{p}_1$ and $\mathbf{p}_3$ are the momenta of the final nucleons, and $\mathbf{p}_2$ is the pion momentum (the proton or the neutron can be at the breakup vertex, depending on the charge of the isobar).

The LSJ projections of the isobar amplitudes are then summed over to provide the (two-body) spin amplitude. Choosing the quantization axis along the incident direction, which we take to be the z axis, we obtain

$$\langle m'_\alpha, m'_2, \mathbf{p}_3 | T | m_1, m_2, \mathbf{p}\rangle = \sum_{L,S,L',S',J} \langle \tfrac{1}{2},\tfrac{1}{2},m_1,m_2|S,\gamma\rangle \langle L,S,0,\gamma|J,\gamma\rangle \langle \tfrac{1}{2},s_\alpha,m'_2,m'_\alpha|S',\gamma'\rangle \langle L',S',\mu',\gamma'|J,\gamma\rangle \times Y_{L'}^{\mu'}(\hat{\mathbf{p}}_3)\left[\frac{2L+1}{4\pi}\right]^{1/2} \langle L',S',J,p_3|T|L,S,J,p\rangle \; . \tag{20}$$

After summation over the spin projections, m_α, the product of the plane-wave amplitude, the isobar propagator, and the breakup vertex provides the $2\rightarrow 3$ body spin amplitude

$$\begin{aligned} T_{23} &= \langle m'_1, m'_2, \mathbf{p}_1, \mathbf{p}_2 | T | \mathbf{p}, m_1, m_2\rangle \\ &= \sum_{m_\alpha} \langle \mathbf{p}_1, m'_1 | \alpha, -\mathbf{p}_3, m'_\alpha\rangle R(\sigma_{p3}) T_{N\alpha} \end{aligned} \tag{21}$$

with $T_{N\alpha}$ as in Eq. (20), and σ the invariant mass of the isobar.

The full (ninefold) differential cross section equals, up to phase space and flux factors, the square of T_{23}:

$$d\sigma = \frac{(2\pi)^4 |T_{23}|^2}{4F} \delta^4\left[P - \sum_{i=1}^{3} p_i\right] \prod_{i=1}^{3} \frac{d^3 p_i}{(2\pi)^3 2E_i} \; , \tag{22}$$

where F is the invariant flux (in the laboratory system, $F = p_{\mathrm{lab}} m$). Actually, only five variables are needed to fully specify the final state, given the energy-momentum conservation relations expressed by the δ function.

As is usually done in the case of two charged particles, we choose to specify the proton and pion directions and the proton final momentum; the remaining variables will be formally integrated over, giving the fivefold differential cross section.

Asymmetries can be expressed in terms of differences of polarized cross sections.

B. Results and discussion

In what follows, the two specified angles (θ_p, θ_π) refer to the outgoing proton and pion, respectively, in the laboratory system, and are located in the scattering plane.

First, we concentrate on understanding what are the major contributions to the differential cross section. Naturally, we expect the inefficiency of the predicted inelasticities in the major singlet and triplet to reflect itself in the cross section. This is fully confirmed by Fig. 4, where the cross section is compared with the data[16] for two different kinematical situations, and where the solid curve is obtained with our standard model as described in Sec. II, with off-shell modifications for the P_{11} and the P_{33} (cutoff of 1.2 GeV for the pole and nonpole parts of the P_{11} and 0.8 GeV for the P_{33}).

A check of consistency can be performed by looking at the same quantities in other models[27,36] in relation to the predicted inelasticities. As mentioned above, the KS model[36] substantially overpredicts $^1D_2$ (about 70° for the inelasticity parameter at 800 MeV, against 25° from the phase shift analysis); the triplet $^3F_3$ is also above the experimental value, but not as much (and it is slightly underestimated above 1 GeV). On the other hand, their differential cross section is not drastically overestimated; for certain forward kinematical configurations, as the one considered in Fig. 4, it is actually only slightly overpredicted.[36] This raises the question on the relative importance of the singlet and the triplet at this energy, or, in other words, if and where the triplet starts to dominate the cross section. We will come back to this point later in this section.

The smaller rise of the data at low momentum, due to final state interaction (FSI), is not predicted by our model, as we do not perform any FSI correction. While this effect is essential to describe all the features of the cross section, the off-shell πN physics, which is of interest to us, is reflected in the Δ-dominated region (right peak).

To better establish the correspondence between the major inelasticities and the size of the predicted cross section, we have performed some variations of the model. For the dashed curve shown in Fig. 4, we have used a rank-one P_{11} (pole part only) with an energy-independent monopole form factor (cutoff of 1.2 GeV), as in the Kloet-Silbar work.[29] We have kept our model for the P_{33} as well as the off-shell modifications but have "artificially" increased the inelasticity with a cutoff of 1.2 GeV for the P_{33} (a value of 0.8 GeV is used for the solid curve). We recall how this choice creates attractive side

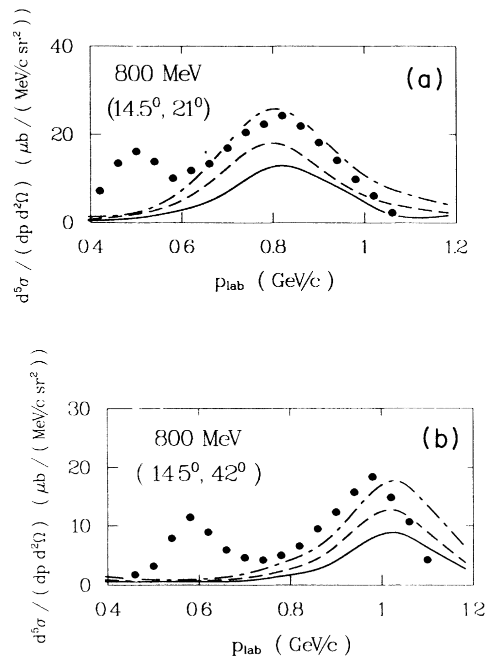

FIG. 4. Predicted $pp \to pn\pi^+$ cross section obtained with our model (solid line) and variations thereof (dashed and dashed-dotted line) as explained in the text. Data from Hancock *et al.* (Ref. 16).

effects (see Fig. 1). The inelasticities in $^1D_2$ and $^3F_3$ corresponding to the dashed curve in Fig. 4, in terms of η, are 0.40 and 0.73 respectively (the values from the phase shift analysis[43] are 0.65 and 0.66).

The dashed-dotted curve in Fig. 4 is obtained as above but with an off-shell cutoff of 1.5 GeV for the P_{11} (such a large cutoff mass, used in a rank-one P_{11} potential, considerably reduces the attraction on the NN P waves, since the two-pion exchange diagram becomes more repulsive).

We notice that the overestimation of the inelasticity in $^1D_2$ does not imply a similar overestimation of the cross section. We must also mention that, in achieving larger inelasticities, a strong attraction has been produced: $\delta(^1D_2)$ is about 20° at 800 MeV. This is a behavior rather similar to that found by Kloet and Silbar,[29] as discussed in Sec. II C.

Moreover, we notice that the model for the P_{11} seems to strongly affect the inelasticity in $^1D_2$ (in favor of a rank-one potential, namely a less realistic description of the P_{11}). To investigate the sensitivity of the results to the P_{11} description, a study of pion production in proton-neutron collisions would certainly be more suitable; there, production in isospin zero channel, (although expected to be small), comes into the picture, and therefore production through a P_{11} wave becomes more explicit. It is then possible that a more consistent treatment of the P_{11} would produce substantially different results than those by Kloet and Silbar. This has not yet been investigated by our group.

Next, we show how the clear underprediction of the solid curve in Fig. 4 is seen in other kinematical configurations (which correspond to smaller cross sections) i.e., (Fig. 5). As less production occurs, the model does better at reproducing the experimental production, as seems reasonable: the cross section is basically "background" in the sense that the isobar is not at the resonance. For these cases, the modified models tend to overpredict the cross section.

We stress that a simple scaling of the cutoff mass cannot be expected to produce a systematic improvement: indeed, if the problem is in the lack of strength of a particular partial wave or spin state, the relative strength (of various partial waves with respect to each other) would not be consistently altered by this manipulation.

Remaining on the question of the relative importance of ${}^1D_2$ and ${}^3F_3$, notice that the effect on the differential cross section from omitting the major triplet wave is strikingly larger than the one obtained from omitting ${}^1D_2$ (Figs. 6), even though the predicted inelasticity for the latter is numerically larger, at 800 MeV. So, it appears that, at this energy and for typical exclusive kinematics, production from a $N\Delta$ channel in a relative P wave is the preferred mechanism (we performed the same test at some lower energies and found that, below 700 MeV, ${}^1D_2$ still dominates the cross section).

If the behavior of the cross section can be easily understood, the same cannot be said for the beam asymmetry A_{N0} which depends sensitively on relative phases between different amplitudes. It is therefore of crucial importance that the phase information of each amplitude is treated correctly.

From Fig. 7, we observe a reasonable agreement with the data at low proton momentum; the model, however, fails to follow the rise of the data in the high momentum part, where the theoretical curve shows no or little structure. For certain kinematics, a general qualitative tendency to follow the data is observed (Fig. 8). We point

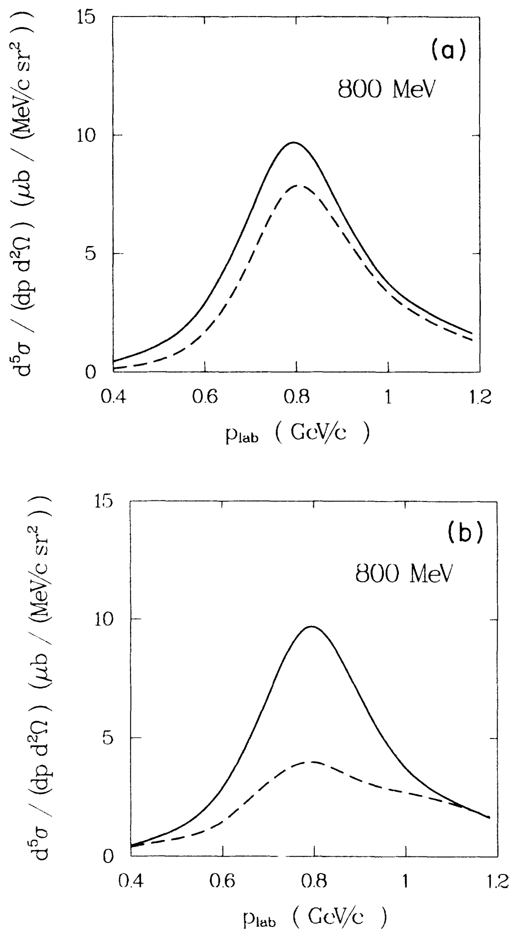

FIG. 6. Effect on the differential cross section at (14.5°, 21°) of omitting (a) ${}^1D_2$ (dashed line), or (b) ${}^3F_3$ (dashed line).

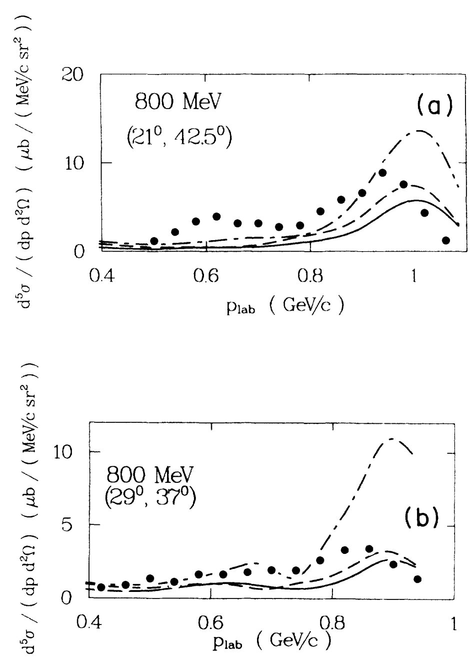

FIG. 5. Same as Fig. 4 for different kinematical conditions. Data from Hancock *et al.* (Ref. 16).

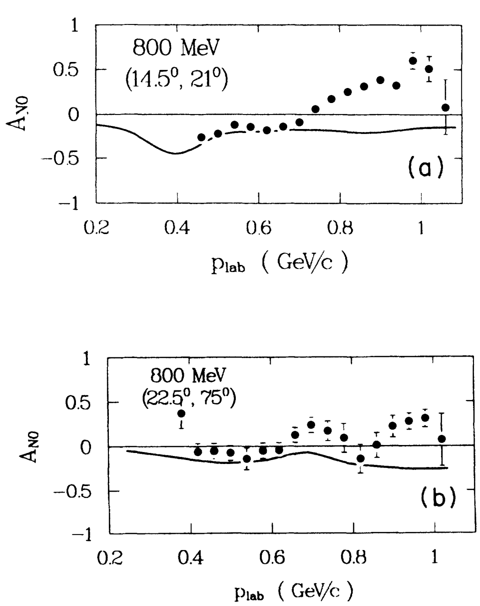

FIG. 7. Beam asymmetry as predicted by our standard model for two different kinematical conditions. Data from Hancock *et al.* (Ref. 16).

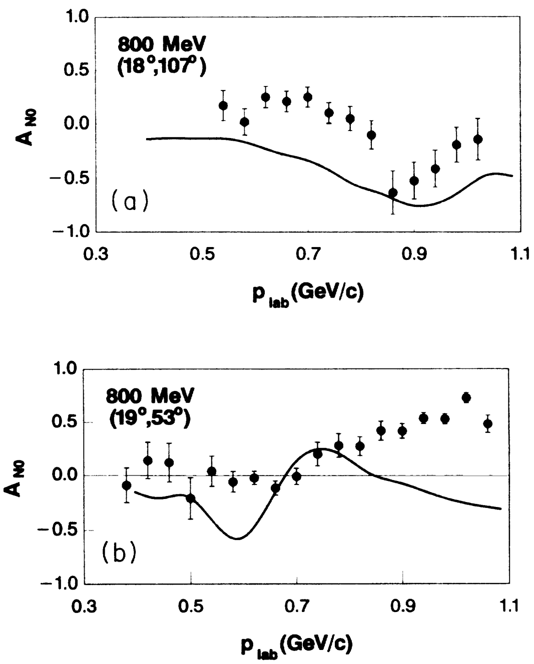

FIG. 8. Same as Fig. 7, at different kinematical configurations. Data from Hancock *et al.* (Ref. 16).

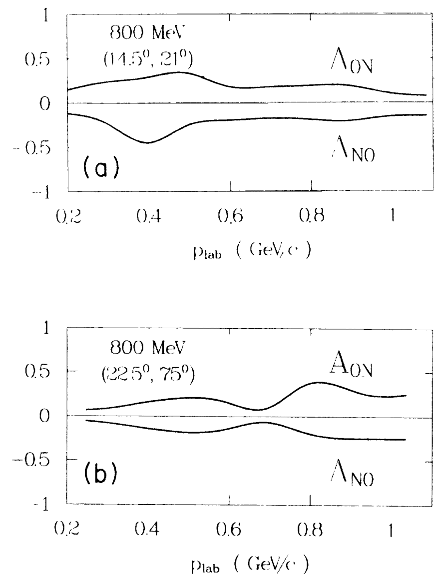

FIG. 9. Comparison of the beam and target asymmetries for two different kinematical conditions.

out that no relevant change in the structure of the spin observables was obtained by the same modifications as performed in Figs. 4–5.

To better understand the structure of the polarization, we have compared A_{N0} with A_{0N}, the target asymmetry (Fig. 9): clearly, $A_{N0} \approx -A_{0N}$: in fact, it can be shown that if these two spin observables are only due to the interference of two partial waves of opposite parity the equality is exact.[36] The approximate equality of our predicted A_{N0} and $-A_{0N}$ suggests that we are looking at quantities controlled (within the present model) by the interference of two major amplitudes both of which lack strength; naturally, one immediately thinks of the major singlet and triplet waves, ${}^1D_2 \to {}^5S_2$ and ${}^3F_3 \to {}^5P_3$. A similar correlation between the lack of inelasticity in the triplet channel and the quality of the predicted longitudinal asymmetry, $\Delta\sigma_L$, can be observed.[36,44]

The structure of the asymmetry in the absence of the major singlet or triplet wave (Fig. 10) may provide some insight on how different spin states contribute to this observable. Since a small but positive asymmetry (in the high momentum region) is seen in the absence of ${}^1D_2$, it is possible that the flat A_{N0} of our result indicate the lack of triplet strength as compared to the singlet. We recall that other studies of pion production (within the Deck model)[50,51] have attributed the failure to reproduce some spin observables to an underestimation of the triplet waves.

Next, we show how short-range contributions, which are, of course, expected to affect mostly the central waves, seem to play a minor role (Fig. 11). The cross section is slightly reduced, consistent with the fact that the inelasticities are reduced by the inclusion of short-range forces, and no significant change in the structure of the asymmetry is observed. Recalling our discussion of Sec. II B, the above statement could be model dependent (namely, depending on how short-range contributions are introduced and how mild/strong the effect is); therefore, we have performed a more drastic test, by totally omitting the lowest partial waves (Fig. 12); while the cross section increases, indicating perhaps that the central waves interfere destructively with higher J amplitudes, still a rather small effect is observed on A_{N0}.

Finally, we show our predictions for some spin-spin

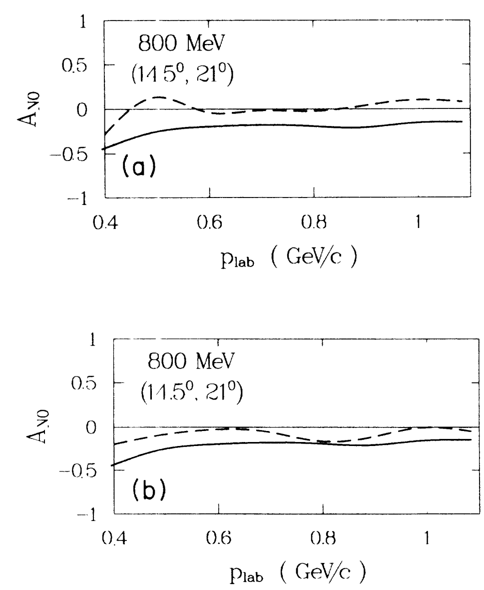

FIG. 10. Effect of the beam asymmetry at (14.5°,21°) of omitting (a) ${}^1D_2$ (dashed line) or (b) ${}^3F_3$ (dashed line).

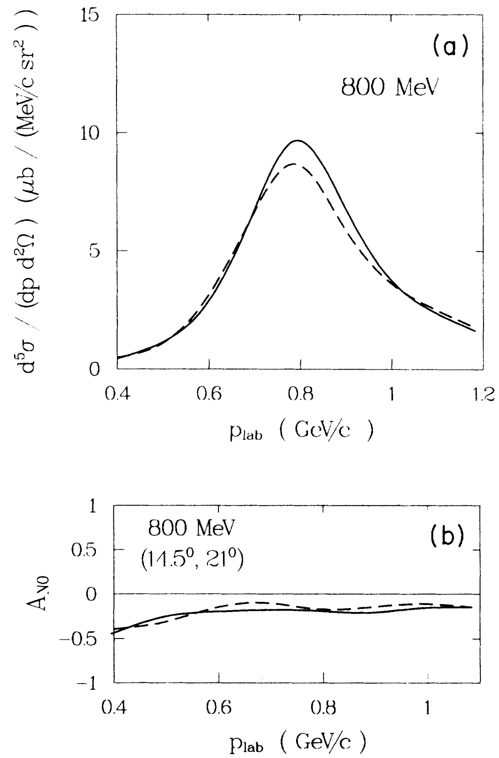

FIG. 11. Cross section and polarization at (14.5°,21°) with short-range contributions (dashed line) and without (solid line).

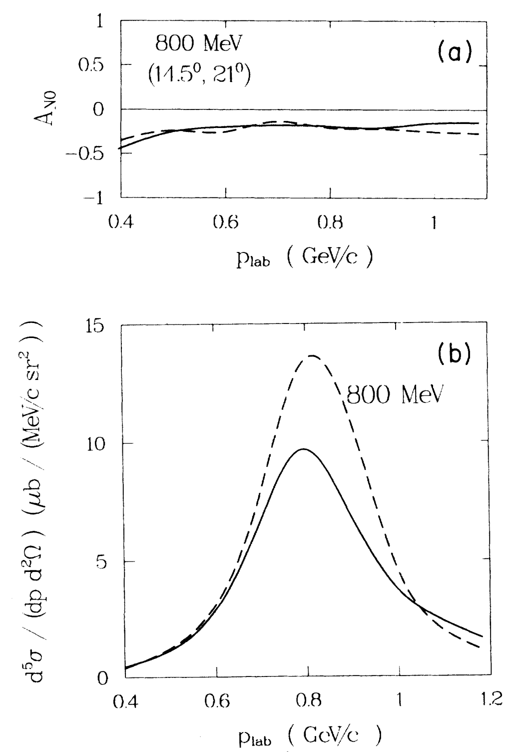

FIG. 12. Effect of the beam asymmetry and the cross section at (14.5°,21°) of omitting the lowest partial waves ($J=0,1$) (dashed line).

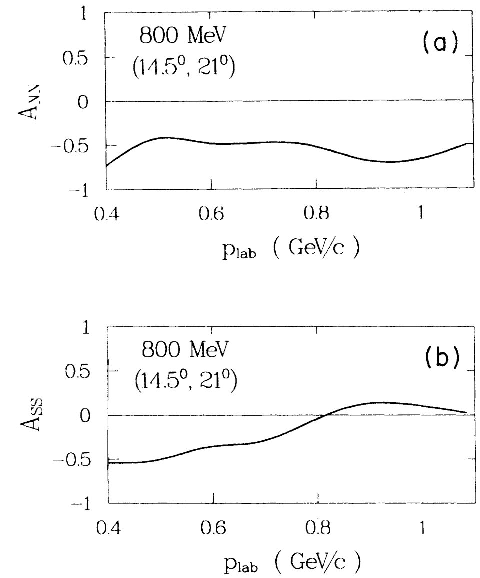

FIG. 13. Predictions for some spin-spin correlations.

correlations (both beam and target polarized)—see Figs. 13 and 14). Unlike for A_{N0}, some qualitative similarity can be observed with the corresponding predictions by Dubach *et al.*,[36] (for instance, the large and negative A_{NN}), which may suggest less model dependence of these observables.

Depending on moduli squared of amplitudes, with opposite sign for singlet and triplet initial states, the A_{LL} parameter is quite revealing. Notice the large values of A_{LL} (Fig. 14) in the absence of either $^1D_2$ or $^3F_3$. The expression for A_{LL} contains the difference of two cross sections with initial spins parallel and antiparallel respectively; the fact that in the absence of $^3F_3$ ($^1D_2$) A_{LL} is close to $-1(+1)$ is then an indication that $\sigma(L,L) \approx \sigma(^3F_3)$ and $\sigma(L,-L) \approx \sigma(^1D_2)$.

In general, not enough can be said on where the models go wrong by looking at the Hancock's representation of the data,[16] since it is not clear how to physically interpret the structure of certain observables (for instance, the rise of A_{N0} at high momentum).

In the above, we have nevertheless tried to analyze critically the major ingredients of the model; in doing so, we were able to identify some typical features, as well as

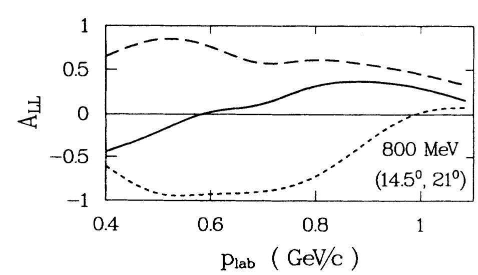

FIG. 14. Effect on A_{LL} at (14.5°,21°) of omitting $^1D_2$ (dashed line) or $^3F_3$ (dotted line).

deficiencies. Before attempting further predictions, we believe we can draw some definite conclusions from the present work, which are suggestive of a possible way to proceed.

IV. CONCLUSIONS

In this work, we have been concerned with a relativistic unitary model for the NN and the πNN system. Our model is conventional in the sense that it involves only known mesons and baryons together with effective meson-baryon interactions. In particular, we have examined exclusive π^+ production from the NN system with the goal of further understanding and constraining our theoretical input and achieving some insight to what controls this mechanism.

From our analysis it seems that the pion-production process is dominated by the $^1D_2$ and $^3F_3$ partial waves. In particular, sufficient inelasticity in those two waves guarantees a reasonable prediction of the differential cross section (even when better inelasticity implies less realistic real phases). However, the same does not apply to A_{N0}, which seems to be controlled by a delicate interplay of spin states, rather than being dominated by a particular partial wave.

A definite conclusion we may draw from our results is that short-range effects do not seem to play a very crucial role in describing pion production. Yet, even though consistent with previous findings,[27,36] the above could be a model dependent statement; in other words, a more careful parametrization of the short-range forces (so as to achieve more realistic S and P wave) may result in a somewhat different conclusion.

It appears that a major aspect of the model needs further study, namely, the way the inelasticities are taken into account (i.e., the model for the Δ resonance). Therefore, at the present time, one facet of our research is devoted to whether some of the problems can be solved through an improved model for P_{33} (keeping in mind a realistic description of the P_{11}). In this respect, a possible solution has been put forth in the use of a higher rank potential representation for the P_{33}; namely, the introduction of a nonresonant background interaction in addition to the commonly used Δ-pole part. In fact, our preliminary results in this direction show a consistent and significant improvement of pion-production observables. While this would probably not settle every open question, it may be an indication that our present difficulties in describing inelastic processes are due to limitations of the models rather than to a general failure of meson-baryon theory to explain the NN data above pion-production threshold.

The new data on exclusive π^+ production[21] will be of great help in constraining the models. They should provide more concrete information on the partial wave content of the pion-production process, since they bear a direct relation to the $NN \rightarrow N\Delta$ transition, which dominates the physics.

Also, the data on $np \rightarrow \pi^+(X)$ of Kleinschmidt *et al.*[15] are of interest: an attempt to describe these data in a perturbative approach[52] has shown serious problems, indicating some missing physics in the np channel. This will be a next step in our pursuit.

We emphasize that no model is, so far, in better than qualitative agreement with experiment for the $NN \rightarrow \pi NN$ reaction. At the present time, it appears reasonable to attribute the problems to a large latitude left in the meson-baryon models. Other limitations in today's models, besides the description of the πN input, are more "intrinsic." For instance, the ways we implement relativistic kinematics and the form of the meson propagator do introduce a certain arbitrariness and/or model dependence.

This work, by extending the model to the breakup channel, has basically set up the tools for a parallel or unified study of the two- and three-body NN reactions. This type of study will present a stronger constraint on the theoretical input and we hope it will bring the model to its full degree of maturity.

ACKNOWLEDGMENTS

One of us (F.S.) expresses deep appreciation to Richard Silbar for enlightening discussions and valuable advice. This work was supported in part by the U.S. Department of Energy under Contract No. DE-FG05-84ER40413, the National Science Foundation under Contract No. PHY-8906944, and the National Science Foundation San Diego Supercomputer Center.

*Permanent address: Department of Physics, Virginia Polytechnic Institute, Blacksburg, Virginia 24061.

[1]V. P. Dzhelepov *et al.*, Dokl. Acad. Nauk SSSR **104**, 380 (1955).

[2]B. Baldoni, S. Focardi, H. Hromadnik, L. Monari, S. Saporetti, S. Fuminò, F. Mezzaneres, E. Bertolini, and G. Gialanella, Nuovo Cimento **26**, 1376 (1962).

[3]V. M. Guzhavin, G. K. Kliger, V. Z. Kolganov, A. V. Lebedev, K. S. Marish, Yu. D. Prokoshin, V. T. Smolyankin, A. P. Sokolov, L. M. Soroko, and Ts'ui Wa-Ch'nang, Zh. Ekp. Teor. Fiz. **46**, 1245 (1964) [Sov. Phys.—JETP **19**, 847 (1964)].

[4]D. V. Bugg, A. J. Oxley, J. A. Zoll, J. G. Rushbrooke, V. E. Barnes, J. B. Kinson, W. P. Dodd, G. A. Doran, and L. Riddiford, Phys. Rev. B **133**, 1017 (1964).

[5]R. J. Eden, *High Energy Collision of Elementary Particle* (Cambridge University Press, Cambridge, 1967).

[6]E. Ferrari and F. Selleri, Nuovo Cimento **21**, 1028 (1961).

[7]S. Mandelstam, Proc. R. Soc. London, Ser. A **244**, 491 (1958).

[8]J. A. McGill, C. Glashausser, K. Jones, S. K. Nanda, M. Barlett, R. Fergerson, J. A. Marshall, E. C. Milner, and G. W. Hoffmann, Phys. Lett. **134B**, 157 (1984).

[9]B. E. Bonner, C. L. Hollas, C. R. Newsom, P. J. Riley, G. Glass, Mahavir Jain, and B. J. VerWest, Phys. Rev. D **27**, 497 (1983).

[10]G. Bizard, F. Bonthonneau, J. L. Laville, F. Lefebvres, J. C. Malherbe, and R. Regimbart, Nucl. Phys. **B108**, 189 (1976).
[11]G. Glass *et al.*, Phys. Lett. **127B**, 27 (1983).
[12]D. R. F. Cochran, P. N. Dean, P. A. M. Gram, E. A. Knapp, E. R. Martin, D. E. Nagle, R. B. Perkins, W. J. Shlaer, H. A. Thiessen, and E. D. Theriot, Phys. Rev. D **6**, 3085 (1972).
[13]F. H. Cverna, D. R. Berrington, M. W. McNaughton, H. B. Willard, N. S. P. King, and D. R. Giebink, Phys. Rev. C **23**, 1698 (1981).
[14]Hudomalj-Gabitzsch *et al.*, Phys. Lett. **60B**, 215 (1976).
[15]M. Kleinschmidt, Th. Fischer, G. Hanmel, W. Hürster, K. Kern, L. Lehmann, E. Rössle, and H. Schmitt, Z. Phys. **A298**, 253 (1980).
[16]A. D. Hancock *et al.*, Phys. Rev. C **27**, 2742 (1983).
[17]T. S. Bhatia *et al.*, Phys. Rev. C **28**, 2071 (1983).
[18]C. L. Hollas *et al.*, Phys. Rev. Lett. **55**, 29 (1985).
[19]C. E. Waltham *et al.*, Nucl. Phys. **A433**, 649 (1985).
[20]A. B. Wicklund *et al.*, Phys. Rev. D **35**, 2670 (1987).
[21]R. L. Shypit *et al.*, Phys. Rev. Lett. **60**, 901 (1988).
[22]P. J. Riley *et al.*, Phys. Lett. B **197**, 23 (1987).
[23]P. C. Gugelot, S. Kullander, G. Landau, F. Lemeilleur, and J. Yonnet, Nucl. Phys. **B37**, 93 (1972).
[24]Y. Avishai and T. Mizutani, Phys. Rev. C **27**, 312 (1983).
[25]B. Blankleider and L. R. Afnan, Phys. Rev. C **23**, 1384; **24**, 1572 (1981).
[26]A. S. Rinat and A. W. Thomas, Nucl. Phys. **A282**, 365 (1977).
[27]A. Matsuyama and T.-S. H. Lee, Phys. Rev. C **34**, 1900 (1986).
[28]H. Garcilazo, Few-Body Systems, Suppl. **1**, 456 (1986).
[29]W. M. Kloet and R. R. Silbar, Nucl. Phys. **A338**, 281 (1980).
[30]G. H. Lamot, J. L. Perrot, C. Fayard, and T. Mizutani, Phys. Rev. C **35**, 239 (1987).
[31]T. Ueda, Prog. Theor. Phys. **76**, 729 (1986).
[32]I. P. Auer, E. Colton, D. Hill, K. Nield, B. Sandler, H. Spinka, Y. Watanabe, A. Yokosawa, and A. Beretvas, Phys. Lett. **67B**, 113 (1977); **70B**, 475 (1977).
[33]J. Dubach, W. M. Kloet, A. Cass, and R. R. Silbar, Phys. Lett. **106B**, 29 (1981).
[34]J. Dubach, W. M. Kloet, and R. R. Silbar, J. Phys. G **8**, 475 (1982).
[35]J. Dubach, W. M. Kloet, R. R. Silbar, J. A. Tjon, and E. van Fassen, Phys. Rev. C **34**, 944 (1986).
[36]J. Dubach, W. M. Kloet, and R. R. Silbar, Nucl. Phys. **A466**, 573 (1987).
[37]T. Mizutani, in *Proceedings of the International Conference on the Theory of Few-Body and Quark-Hadronic Systems, Dubna, 1987,* edited by V. K. Lukyanov (JINR, Dubna, 1987), p. 173.
[38]R. Aaron, R. D. Amado, and J. E. Young, Phys. Rev. **174**, 2022 (1968).
[39]R. Aaron, in *Modern Three-Hadron Physics,* edited by A. W. Thomas (Springer, Berlin, 1977), Chap. 5.
[40]K. Schwarz, H. F. K. Zingl, and L. Mathelitsch, Phys. Lett. **83B**, 297 (1979).
[41]R. M. Woloshyn, E. J. Moniz, and R. Aaron, Phys. Rev. C **12**, 909 (1975).
[42]T. Mizutani, C. Fayard, G. H. Lanout, and S. Nahabetian, Phys. Rev. C **24**, 2633 (1981).
[43]R. A. Arndt, L. D. Roper, R. A. Bryan, R. B. Clark, B. J. VerWest, and P. Signell, Phys. Rev. D **28**, 97 (1983).
[44]W. M. Kloet and R. R. Silbar, Nucl. Phys. **A364**, 346 (1981).
[45]G. E. Brown and W. Weise, Phys. Rep. C **22**, 281 (1975).
[46]R. Machleidt, Adv. Nucl. Phys. **19**, 189 (1989).
[47]R. Machleidt, K. Holinde, and Ch. Elster, Phys. Rep. **149**, 1 (1987).
[48]D. V. Bugg, in *Proceedings of the Rensselaer Conference on Excited Baryons, Troy, NY, 1988* (World Scientific, Teaneck, NJ, 1989).
[49]F. Sammarruca, Ph.D. thesis, Virginia Polytechnic Institute, 1988 (unpublished).
[50]W. Jauch, A. König, and P. Kroll, Phys. Lett. **143B**, 509 (1984).
[51]A. König and P. Kroll, Nucl. Phys. **A356**, 345 (1981).
[52]F. Wellers, Proceedings of the International Symposium on Mesons and Light Nuclei, Bechyne, 1988 [Czech, J. Phys. B **39**, 72 (1989)].