Correlation observables in $(p,p'\gamma)$ reactions

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We investigate the importance of gamma-ray correlations in nucleon-nucleus scattering. Coincidence observables can give information about the scattering amplitude that is not accessible in noncoincident (p,p') experiments and can provide sensitive tests of different theoretical models. We have performed relativistic and nonrelativistic calculations with special emphasis on observables sensitive to composite spin-convection current amplitudes. We present calculations of coincidence cross sections as well as longitudinal and sideways analyzing powers using the $0^+\rightarrow 1^+$ excitation as an example. We have compared our results with coincidence cross section data for the first 1^+, $T=1$ state in ^{12}C. The experimental results seem to indicate a clear preference for the relativistic predictions.

I. INTRODUCTION

Nucleon-nucleus scattering at intermediate energies has always been regarded as an important testing ground for our understanding of nuclear structure and/or medium modifications to the elementary nucleon-nucleon (NN) interaction. The fact that at these energies the interaction between the incident nucleon and a nucleon bound in the nucleus closely resembles their interaction in free space makes intermediate energy physics potentially useful in our understanding of nuclear structure effects. Alternatively, by using *a priori* knowledge of nuclear structure these reactions might reveal interesting medium modifications to the elementary NN interaction. The use of the free NN interaction (impulse approximation) together with nuclear charge densities determined from electron-scattering experiments lead to a successful nonrelativistic, parameter-free description of elastic-scattering cross section. With the advent of better experimental facilities, spin observable data have become available providing a real challenge to theories of nucleon-nucleus scattering. At this date parameter-free calculations of elastic analyzing power and spin rotation parameters within a nonrelativistic impulse approximation formalism cannot account for the experimental data. This fact has stimulated researchers to try to understand the failure of the impulse approximation[1] and at the same time to look for alternative theoretical descriptions of nucleon-nucleus scattering.[2,3] At present, the most successful theory of elastic nucleon-nucleus scattering is the relativistic impulse approximation.[4] Good parameter-free descriptions of elastic-scattering cross section and spin observables have been obtained by solving the Dirac equation in the presence of an optical potential determined from NN data and either empirically or self-consistently determined nuclear charge densities.[5]

Since measurement of elastic-scattering observables can at most determine three independent quantities it is natural to investigate inelastic excitations. For these reactions, it is possible to determine up to eight independent quantities if one uses a polarized proton beam and detects the polarization of the scattered proton. Unfortunately, a complete measurement of all eight spin transfer coefficients it is not an easy task. Six of the eight spin transfer coefficients can only be determined after performing double-scattering experiments. Furthermore, knowledge of the inelastic-scattering amplitude would not be complete even after measuring all eight spin observables. For a parity-conserving $0\rightarrow J$ excitation there are $8J+3$ independent quantities to determine. It is only for $J=0$ excitations that the spin transfer coefficients are enough to determine the scattering amplitude and such excitations are seldom easy to measure.

Experiments in which the magnetic substate of the target becomes restricted, through measurement of p-γ correlations, can uncover the full richness contained in inelastic transitions.[6,7] By measuring coincidence observables one can address some interesting issues in a way that is sometimes simpler and in some cases even inaccessible in noncoincident, or "singles", measurements. Furthermore, theoretical calculations of coincidence observables do not require much more effort than the calculation of singles spin observables. In fact, by using an empirically determined quantity, namely, the branching ratio for the electromagnetic decay, all coincidence observables can be expressed in terms of the singles scattering amplitude. More importantly, the additional information available in coincidence measurements may provide stringent tests on different theoretical models of nucleon-nucleus scattering and may shed some light into the physics underlying these reactions.

We organize our paper in the following way. In Sec. II we present a brief discussion of the hadronic part of the amplitude. We discuss the structure of the inelastic-scattering amplitude and of the spin transfer coefficients. In Sec. III we briefly review the well-known structure of the electromagnetic decay amplitude. In Sec. IV we combine our earlier results to construct the $(p,p'\gamma)$ coincidence amplitude which we subsequently use in Sec. V to discuss properties of the coincidence observables. Finally, Sec. VI contains results of relativistic and nonrelativistic calculations together with our conclusions. We end each section by discussing the particular example of the $0^+ \rightarrow 1^+$ excitation. This reaction is of current experimental interest[8,9] and from a theoretical perspective simple enough to highlight the main features of the coincidence reaction.

II. THE (p,p') SCATTERING AMPLITUDE

The inelastic-scattering amplitude for the $0^+(p,p')J^\pi$ reaction can be written in the following way:

$$\hat{T}_J^p(\mathbf{p},\mathbf{p}') = \sum_M T_{JM}^p(\mathbf{p},\mathbf{p}')\hat{\Sigma}_{J^\pi M} , \tag{2.1}$$

where $\mathbf{p}(\mathbf{p}')$ is the initial (final) momentum of the proton, T_{JM}^p is the transition amplitude from the ground state to a nuclear state of spin-parity J^π and spin projection M, and

$$\hat{\Sigma}_{J^\pi M} \equiv |J^\pi M\rangle\langle 0^+| ,$$

is the polarization operator of the target. A convenient right-handed Cartesian coordinate system for this reaction is defined in terms of the initial and final projectile momenta,

$$\mathbf{n}=\mathbf{p}\times\mathbf{p}';\ \mathbf{K}=\mathbf{p}+\mathbf{p}';\mathbf{q}=\mathbf{n}\times\mathbf{K} , \tag{2.2}$$

where $\mathbf{n}$ is a vector perpendicular to the scattering plane, $\mathbf{K}$ is along the direction of the average momentum, and in the limit of a zero Q value for the reaction, $\mathbf{q}$ would be in the direction of the momentum transfer $\mathbf{p}'-\mathbf{p}$. If the final polarization of the nuclear state is undetected, the experimentally measurable quantities are customarily chosen to be the spin transfer coefficients defined by

$$\frac{d\sigma}{d\Omega_p} = \tfrac{1}{2}\mathrm{Tr}[\hat{T}_J^p\hat{T}_J^{p\dagger}] ,$$
$$D_{\alpha\beta} = \mathrm{Tr}[\sigma_\alpha\hat{T}_J^p\sigma_\beta\hat{T}_J^{p\dagger}]/\mathrm{Tr}[\hat{T}_J^p\hat{T}_J^{p\dagger}] , \tag{2.3}$$

where $\alpha, \beta = 0,\ n,\ K,\ q$, and $\sigma_0 \equiv 1$.

The form of the (p,p') transition amplitude is restricted by the requirements of invariance under rotations, and space inversion (parity). Since the polarization operator of the target, $\hat{\Sigma}_{J^\pi M}$, transforms as an irreducible tensor of rank J and parity π, T_{JM}^p must transform contragradiently to $\hat{\Sigma}_{J^\pi M}$ and have the same parity in order for the nuclear transition amplitude to transform as a scalar. A clear example is provided by the $0^+ \rightarrow 1^+$ transition. In this case the nuclear polarization operator,

$$\hat{\Sigma}^M = |1^+ M\rangle\langle 0^+|$$

transforms as a tensor of rank one and positive parity, i.e., an axial vector. Consequently, the most general rotational and parity invariant amplitude that one can write for the $0^+ \rightarrow 1^+$ transition is given by[10]

$$\hat{T}^p(\mathbf{p},\mathbf{p}') = A_{n0}(\hat{\Sigma}\cdot\hat{\mathbf{n}}) + A_{nn}(\hat{\Sigma}\cdot\hat{\mathbf{n}})(\sigma\cdot\hat{\mathbf{n}}) + A_{KK}(\hat{\Sigma}\cdot\hat{\mathbf{K}})(\sigma\cdot\hat{\mathbf{K}}) + A_{Kq}(\hat{\Sigma}\cdot\hat{\mathbf{K}})(\sigma\cdot\hat{\mathbf{q}}) + A_{qK}(\hat{\Sigma}\cdot\hat{\mathbf{q}})(\sigma\cdot\hat{\mathbf{K}}) + A_{qq}(\hat{\Sigma}\cdot\hat{\mathbf{q}})(\sigma\cdot\hat{\mathbf{q}}) , \tag{2.4}$$

or in a more compact way,

$$\hat{T}^p(\mathbf{p},\mathbf{p}') = \sum_{i\mu} A_{i\mu}\hat{\Sigma}_i\sigma_\mu;\ i=n,K,q;\ \mu=0,n,K,q , \tag{2.5}$$

where the individual amplitudes are scalar functions of the energy and momentum transfer. From the twelve possible amplitudes, $A_{i\mu}$, present in a $0^+(p,p')1$ reaction, only half of them contribute to the 1^+ excitation. Parity invariance splits these twelve amplitudes into two complementary sets of six with each set driving a different

TABLE I. Spin observables for the $0^+(p,p')1^+$ reaction.

$\frac{d\sigma}{d\Omega_p} = \lvert A_{n0}\rvert^2 + \lvert A_{nn}\rvert^2 + \lvert A_{KK}\rvert^2 + \lvert A_{Kq}\rvert^2 + \lvert A_{qK}\rvert^2 + \lvert A_{qq}\rvert^2$
$\frac{d\sigma}{d\Omega_p}D_{nn} = \lvert A_{n0}\rvert^2 + \lvert A_{nn}\rvert^2 - \lvert A_{KK}\rvert^2 - \lvert A_{Kq}\rvert^2 - \lvert A_{qK}\rvert^2 - \lvert A_{qq}\rvert^2$
$\frac{d\sigma}{d\Omega_p}D_{KK} = \lvert A_{n0}\rvert^2 - \lvert A_{nn}\rvert^2 + \lvert A_{KK}\rvert^2 - \lvert A_{Kq}\rvert^2 + \lvert A_{qK}\rvert^2 - \lvert A_{qq}\rvert^2$
$\frac{d\sigma}{d\Omega_p}D_{qq} = \lvert A_{n0}\rvert^2 - \lvert A_{nn}^2\rvert - \lvert A_{KK}\rvert^2 + \lvert A_{Kq}\rvert^2 - \lvert A_{qK}\rvert^2 + \lvert A_{qq}\rvert^2$
$\frac{d\sigma}{d\Omega_p}D_{0n} = 2[Re(A_{no}A_{nn}^*) + \mathrm{Im}(A_{KK}A_{Kq}^* + A_{qK}A_{qq}^*)]$
$\frac{d\sigma}{d\Omega_p}D_{n0} = 2[\mathrm{Re}(A_{n0}A_{nn}^*) + \mathrm{Im}(A_{Kq}A_{KK}^* + A_{qq}A_{qK}^*)]$
$\frac{d\sigma}{d\Omega_p}D_{Kq} = 2[\mathrm{Im}(A_{nn}A_{n0}^*) + \mathrm{Re}(A_{KK}A_{Kq}^* + A_{qK}A_{qq}^*)]$
$\frac{d\sigma}{d\Omega_p}D_{qK} = 2[\mathrm{Im}(A_{n0}A_{nn}^*) + \mathrm{Re}(A_{KK}A_{Kq}^* + A_{qK}A_{qq}^*)]$

parity state. The spin-transfer coefficients can now be written in a model-independent way in terms of these individual amplitudes,

$$\frac{d\sigma}{d\Omega_p}=\sum_{i\mu}|A_{i\mu}|^2 ,$$
$$D_{\alpha\beta}=\sum_{i\mu\nu}A_{i\mu}A^*_{i\nu}\tfrac{1}{2}\mathrm{Tr}[\sigma_\alpha\sigma_\mu\sigma_\beta\sigma_\nu]/\sum_{i\mu}|A_{i\mu}|^2 , \quad (2.6)$$

where we have used the identity

$$\mathrm{Tr}[(\hat{\Sigma}\cdot\hat{e}_i)(\hat{\Sigma}\cdot\hat{e}_j)^\dagger]=\mathrm{Tr}[(\hat{\Sigma}\cdot\hat{e}_i)^\dagger(\hat{\Sigma}\cdot\hat{e}_j)]=\hat{e}_i\cdot\hat{e}_j=\delta_{ij} . \quad (2.7)$$

Because of the form of the amplitude imposed by parity and rotational invariance, only *eight* of the spin transfer coefficients are nonzero: viz., $d\sigma/d\Omega_p$, $D_{0n}\equiv A_y$ (the analyzing power), $D_{n0}\equiv P$ (the polarization), D_{nn}, D_{KK}, D_{qq}, D_{Kq}, and D_{qK}. These eight quantities constitute *all* of the independent observables that can be measured without restricting the magnetic substate of the target. In Table I we show all spin observables for the $0^+(p,p')1^+$ reaction. Clearly, the information contained in these eight observables is not sufficient to determine the six complex amplitudes driving the $0^+\to1^+$ transition. In particular, Eq. (2.6) shows that the relative phase between individual amplitudes with $i\neq j$ (e.g., A_{KK} and A_{qK}), becomes impossible to determine whenever the magnetic substate of the target is unrestricted.

III. THE ELECTROMAGNETIC DECAY AMPLITUDE

The electromagnetic transition amplitude for the $J^\pi\to0^+$ gamma-ray deexcitation is given by

$$\hat{T}^\gamma_{J\lambda}(\mathbf{k})=\sum_M T^\gamma_{JM\lambda}(\mathbf{k})\hat{\Sigma}^\dagger_{J^\pi M} ,$$
$$T^\gamma_{JM\lambda}(\mathbf{k})=\langle 0^+|J_\lambda(\mathbf{k})|J^\pi M\rangle , \quad (3.1)$$

where $\mathbf{k}$ is the momentum of the gamma ray, $\lambda=\pm1$ its polarization, and $J_\lambda(\mathbf{k})$ a transverse component of the electromagnetic current operator,

$$J_\lambda(\mathbf{k})=\int d^3x e^{-i\mathbf{k}\cdot\mathbf{x}}\mathbf{J}(\mathbf{x})\cdot\hat{e}_\lambda(\hat{\mathbf{k}}) . \quad (3.2)$$

After an angular momentum decomposition, the electromagnetic current operator,

$$J_\lambda(\mathbf{k})=-\sum_{JM}(-i)^J\sqrt{2\pi(2J+1)}\mathcal{D}^J_{M\lambda}(\hat{\mathbf{k}})\times[\lambda T^{\mathrm{mag}}_{JM}(k)-T^{\mathrm{el}}_{JM}(k)] , \quad (3.3)$$

is written in terms of unnatural parity magnetic, $T^{\mathrm{mag}}_{JM}(k)$, and natural-parity electric, $T^{\mathrm{el}}_{JM}(k)$, multipole operators,[11]

$$T^{\mathrm{mag}}_{JM}(k)=\int d\mathbf{x}\, j_J(kx)\mathbf{Y}^M_{JJ}(\hat{\mathbf{x}})\cdot\mathbf{J}(\mathbf{x}) ,$$
$$T^{\mathrm{el}}_{JM}(k)=\frac{1}{k}\int d\mathbf{x}\nabla\times[j_J(kx)\mathbf{Y}^M_{JJ}(\hat{\mathbf{x}})]\cdot\mathbf{J}(\mathbf{x}) , \quad (3.4)$$

where $\mathcal{D}^J_{M\lambda}$ are Wigner $\mathcal{D}$ functions, $j_J(kx)$ is a spherical Bessel function of order J, and $\mathbf{Y}^M_{JJ}(\hat{\mathbf{x}})$ are vector spherical harmonics defined by,

$$\mathbf{Y}^M_{JJ}(\hat{\mathbf{x}})\equiv\sum_{m\mu}\langle J1m\mu|JM\rangle Y_{Jm}(\hat{\mathbf{x}})\hat{e}_\mu . \quad (3.5)$$

The preceding set of equations together with the Wigner-Eckart theorem enables one to write the electromagnetic amplitude in the following simple way:

$$T^\gamma_{Jm\lambda}(\mathbf{k})=T^\gamma_{J\lambda}(k)\mathcal{D}^{J*}_{M-\lambda}(\hat{\mathbf{k}}) , \quad (3.6)$$

where

$$T^\gamma_{J\lambda}(k)=i^J\sqrt{2}\pi\langle 0^+\|\lambda T^{\mathrm{mag}}_J(k)-T^{\mathrm{el}}_J(k)\|J^\pi\rangle . \quad (3.7)$$

The angular, as well as the magnetic substate, dependence of the gamma-ray amplitude is fully contained in the Wigner-$\mathcal{D}$ function. The nuclear structure information, on the other hand, is contained in the reduced matrix element.

As in the case of the (p,p') amplitude, the electromagnetic transition amplitude takes a very simple form for the $1^+\to0^+$ deexcitation. By using the identity,

$$\mathcal{D}^{(1)}_{M\lambda}(\hat{\mathbf{k}})=\hat{e}^*_M\cdot\hat{e}_\lambda(\hat{\mathbf{k}}) ,$$

the electromagnetic amplitude can be reduced to the following simple form:

$$\hat{T}^\gamma_\lambda(\mathbf{k})=T^\gamma_\lambda(k)[\sum_M(\hat{e}^*_M\hat{\Sigma}_M)\cdot\hat{e}_{-\lambda}(\hat{\mathbf{k}})]^\dagger=T^\gamma_\lambda(k)[\hat{\Sigma}\cdot\hat{e}_{-\lambda}(\hat{\mathbf{k}})]^\dagger . \quad (3.8)$$

which vanishes whenever the polarization of the nuclear state is orthogonal to the plane of polarization of the gamma ray; i.e., whenever the nuclear polarization is colinear with the gamma ray momentum $\mathbf{k}$.

IV. THE $(p,p'\gamma)$ REACTION

We now consider the $0^+(p,p')J^\pi$ transition followed by a $J^\pi\to0^+$ gamma-ray deexcitation. The transition amplitude for this two-step process is given by the product of the strong and electromagnetic transition amplitudes already discussed, i.e.,

$$\hat{T}^{p\gamma}_{J\lambda}(\mathbf{p},\mathbf{p}';\mathbf{k})=\hat{T}^\gamma_{J\lambda}(\mathbf{k})\hat{T}^p_J(\mathbf{p},\mathbf{p}')=T^\gamma_{J\lambda}(k)\sum_M T^p_{JM}(\mathbf{p},\mathbf{p}')\mathcal{D}^{J*}_{M-\lambda}(\hat{\mathbf{k}}) . \quad (4.1)$$

In analogy to the noncoincidence, or singles, observables defined in Eq. (2.3), we can define the coincidence spin-transfer coefficients by,

$$\frac{d^2\sigma}{d\Omega_p d\Omega_\gamma}=\tfrac{1}{2}\sum_\lambda\mathrm{Tr}[\hat{T}^{p\gamma}_{J\lambda}\hat{T}^{p\gamma\dagger}_{J\lambda}] ,$$
$$D_{\alpha\beta}(\hat{\mathbf{k}})=\sum_\lambda\mathrm{Tr}[\sigma_\alpha\hat{T}^{p\gamma}_{J\lambda}\sigma_\beta\hat{T}^{p\gamma\dagger}_{J\lambda}]/\sum_\lambda\mathrm{Tr}[\hat{T}^{p\gamma}_{J\lambda}\hat{T}^{p\gamma\dagger}_{J\lambda}] . \quad (4.2)$$

In particular, the coincidence cross section is, up to a normalization factor to be determined, given by

$$\frac{d^2\sigma}{d\Omega_p d\Omega_\gamma}=|T^\gamma_{J\lambda=1}|^2\sum_\lambda\sum_{MM'}\rho^J_{MM'}\mathcal{D}^{J*}_{M\lambda}(\hat{\mathbf{k}})\mathcal{D}^J_{M'\lambda}(\hat{\mathbf{k}}) , \quad (4.3)$$

where

$$\rho^J_{MM'}=\tfrac{1}{2}\mathrm{Tr}[T^p_{JM}T^{p\dagger}_{JM'}] , \qquad (4.4)$$

is the density matrix[12] describing the polarization of the excited nuclear state, and we have used the fact that $|T^\gamma_{J\lambda}|$ is independent of λ.

A simple way to evaluate the overall normalization of the cross section can be established by the following considerations. We assume that the gamma-ray lifetime is short enough so that, if a gamma decay occurs, it can unambiguously be identified with its associated inelastic-scattering event. This is an excellent assumption for many realistic experimental situations. In that case, if one detects gamma-ray coincidences throughout its entire solid angle one should recover the singles cross section *times the gamma-ray branching ratio R*. Therefore, integrating Eq. (4.3) over the entire gamma-ray solid angle, we obtain

$$\int \frac{d^2\sigma}{d\Omega_p d\Omega_\gamma}d\Omega_\gamma \equiv R\frac{d\sigma}{d\Omega_p}=\mathcal{N}\frac{8\pi}{2J+1}\sum_M \rho^J_{MM} =\mathcal{N}\frac{8\pi}{2J+1}\frac{d\sigma}{d\Omega_p} , \qquad (4.5)$$

where $\mathcal{N}$ is a normalization constant and we have used the equation,

$$\int d\hat{\mathbf{k}}\mathcal{D}^J_{M\lambda}(\hat{\mathbf{k}})\mathcal{D}^{J'*}_{M'\lambda}(\hat{\mathbf{k}})=\frac{4\pi}{2J+1}\delta_{JJ'}\delta_{MM'} . \qquad (4.6)$$

We then find the normalized coincidence cross section to be given by

$$\frac{d^2\sigma}{d\Omega_p d\Omega_\gamma}=\frac{2J+1}{8\pi}R\sum_\lambda\sum_{MM'}\rho^J_{MM'}\mathcal{D}^{J*}_{M\lambda}(\hat{\mathbf{k}})\mathcal{D}^J_{M'\lambda}(\hat{\mathbf{k}}) . \qquad (4.7)$$

We conclude this section by writing the coincidence amplitude and spin transfer coefficients for the $J^\pi=1^+$ excitation. Using Eqs. (2.5) and (3.8) for the strong and electromagnetic part of the amplitude respectively, we obtain

$$\begin{aligned}\hat{T}^{p\gamma}_\lambda(\hat{\mathbf{k}})&=\hat{T}^\gamma_\lambda(\mathbf{k})\hat{T}^p(\mathbf{p},\mathbf{p}')\\ &=T^\gamma_\lambda(k)[\hat{\boldsymbol{\Sigma}}\cdot\hat{\mathbf{e}}_{-\lambda}(\hat{\mathbf{k}})]^\dagger\sum_{i\mu}A_{i\mu}[\hat{\boldsymbol{\Sigma}}\cdot\hat{\mathbf{e}}_i]\sigma_\mu\\ &=T^\gamma_\lambda(k)\sum_{i\mu}A_{i\mu}[\hat{\mathbf{e}}_i\cdot\hat{\mathbf{e}}_{-\lambda}(\hat{\mathbf{k}})]\sigma_\mu .\end{aligned} \qquad (4.8)$$

Consequently, the coincidence spin observables for the $J^\pi=1^+$ excitation can be written entirely in terms of the individual amplitudes for the (p,p') reaction,

$$\begin{aligned}\frac{d^2\sigma}{d\Omega_p d\Omega_\gamma}&=\frac{3R}{8\pi}\sum_{ij\mu}A_{i\mu}A^*_{j\mu}t_{ij}(\hat{\mathbf{k}}) ,\\ D_{\alpha\beta}(\hat{\mathbf{k}})&=\sum_{ij\mu\nu}A_{i\mu}A^*_{j\nu}t_{ij}(\hat{\mathbf{k}})\\ &\quad\times\tfrac{1}{2}\mathrm{Tr}[\sigma_\alpha\sigma_\mu\sigma_\beta\sigma_\nu]\Big/\sum_{ij\mu}A_{i\mu}A^*_{j\mu}t_{ij}(\hat{\mathbf{k}}) ,\end{aligned} \qquad (4.9)$$

where we have used the identity,

$$\begin{aligned}t_{ij}(\hat{\mathbf{k}})&\equiv\sum_{\lambda=\pm1}[\hat{\mathbf{e}}_i\cdot\hat{\mathbf{e}}_\lambda(\hat{\mathbf{k}})][\hat{\mathbf{e}}_j\cdot\hat{\mathbf{e}}^*_\lambda(\hat{\mathbf{k}})]\\ &=[\delta_{ij}-(\hat{\mathbf{k}}\cdot\hat{\mathbf{e}}_i)(\hat{\mathbf{k}}\cdot\hat{\mathbf{e}}_j)] ,\end{aligned} \qquad (4.10)$$

The preceding formula closely resembles the expression for the singles spin observables given in Eq. (2.6). The singles spin transfer coefficients are obtained by simply replacing $t_{ij}(\hat{\mathbf{k}})$ by δ_{ij}. Even though this seems like a minor modification, it has some interesting consequences. For example, this result shows that while measuring relative phases of amplitudes with $i\neq j$ is impossible in a singles experiment, it becomes possible in a coincidence measurement provided the gamma-ray momentum has a nonvanishing component along the two basis vectors spanning the i-j plane. In addition, spin observables that should in principle be simple to measure, but vanish in a (p,p') reaction, e.g., longitudinal and sideways analyzing powers, are now nonzero and contain unique and possible interesting information. We therefore conclude that the added freedom contained in coincidence observables might have important consequences in our understanding of the nucleon-nucleus reaction.

V. PROPERTIES OF THE $(p,p'\gamma)$ OBSERVABLES

In this section we consider the relationship between specific coincidence and singles spin observables. We begin by considering out-of-plane ($\hat{\mathbf{k}}=\hat{\mathbf{n}}$) coincidence measurements for the specific case of $J^\pi=1^+$ excitations. For this case the only nonvanishing elements of the transverse photon polarization tensor $t_{ij}(\hat{\mathbf{k}})$ are

$$t_{KK}(\hat{\mathbf{n}})=t_{qq}(\hat{\mathbf{n}})=1 . \qquad (5.1)$$

As a consequence, terms in the (p,p') amplitude involving a target polarization along $\hat{\mathbf{k}}=\hat{\mathbf{n}}$, specifically the out-of-plane amplitudes A_{n0} and A_{nn}, do not contribute to the transition. This is a reflection of the fact that, for a gamma decay to a spin-zero final state, the helicity of the photon must have a nonzero component along the polarization of the excited nuclear state, [see Eq. (3.8)]. Since the photon is a spin-1 particle *and transverse*, only transverse target polarizations contribute to the reaction. Furthermore, from the structure of the scattering amplitude, Eq. (2.4), we observe that $\sigma\cdot\hat{\mathbf{K}}$ and $\sigma\cdot\hat{\mathbf{q}}$ are the only remaining operators in the projectile's spin space. Thus, out-of-plane measurements select only those processes involving a projectile spin flip with respect to the $\hat{\mathbf{n}}$ direction. With the help of equations (2.6) and (4.9) we can now derive useful relationships between the singles and out-of-plane coincidence observables,

$$\begin{aligned}\frac{d^2\sigma}{d\Omega_p d\Omega_\gamma}&=\frac{3R}{8\pi}\frac{d\sigma}{d\Omega_p}\left[\frac{1-D_{nn}}{2}\right] ,\\ D_{nn}(\hat{\mathbf{n}})&=-1 ,\\ P(\hat{\mathbf{n}})&=-A_y(\hat{\mathbf{n}})=\left[\frac{P-A_y}{1-D_{nn}}\right] ,\\ D_{Kq}(\hat{\mathbf{n}})&=D_{qK}(\hat{\mathbf{n}})=\left[\frac{D_{Kq}+D_{qK}}{1-D_{nn}}\right] ,\end{aligned} \qquad (5.2)$$

$$D_{KK}(\hat{\mathbf{n}})=-D_{qq}(\hat{\mathbf{n}})=\left[\frac{D_{KK}-D_{qq}}{1-D_{nn}}\right] .$$

These relations indicate that, for this example, out-of-plane gamma-ray coincidences *do not* provide information that is not already accessible, at least in principle, in singles measurements. In practice, however, coincidence measurements often provide a simpler way of obtaining this kind of information. In the work of Kovash *et al.*,[13] for example, the singles quantity $P\text{-}A_y$ has been obtained from $^{12}\mathrm{C}(p,p'\gamma)^{12}\mathrm{C}^*$ (15.11 MeV, $1^+T=1$) analyzing power measurements at $T_{\mathrm{lab}}=150$ MeV.

We now examine the properties of *in-plane* coincidence observables, by which we mean that the photon momentum lies in the (p,p') scattering plane, i.e., $\hat{\mathbf{k}}\cdot\hat{\mathbf{n}}=0$, $\hat{\mathbf{k}}\cdot\hat{\mathbf{K}}=\sin\theta_\gamma$, and $\hat{\mathbf{k}}\cdot\hat{\mathbf{q}}=\cos\theta_\gamma$. The in-plane coincidence cross section can now be derived straightforwardly from Eq. (4.2) and we obtain

$$\frac{d^2\sigma}{d\Omega_p d\Omega_\gamma}=\mathcal{A}(\theta_p)+\mathcal{B}(\theta_p)\cos2\theta_\gamma+\mathcal{C}(\theta_p)\sin2\theta_\gamma , \quad (5.3)$$

where

$$\frac{8\pi}{3R}\mathcal{A}=|A_{n0}|^2+|A_{nn}|^2 +\tfrac{1}{2}[|A_{KK}|^2+|A_{Kq}|^2+|A_{qK}|^2+|A_{qq}|^2] ,$$

$$\frac{8\pi}{3R}\mathcal{B}=\tfrac{1}{2}[|A_{KK}|^2+|A_{Kq}|^2-|A_{qK}|^2-|A_{qq}|^2] ,$$

$$\frac{8\pi}{3R}\mathcal{C}=-Re[A_{KK}A^*_{qK}+A_{Kq}A^*_{qq}] . \quad (5.4)$$

The in-plane coincidence cross section consists of an isotropic term $\mathcal{A}$, a term $\mathcal{B}\cos2\theta$ that is symmetric about the direction of $\hat{\mathbf{q}}$, and an antisymmetric term $\mathcal{C}\sin2\theta$. The isotropic piece can be expressed in terms of singles observables,

$$\mathcal{A}=\frac{3R}{8\pi}\left[\frac{3+D_{nn}}{4}\right]\frac{d\sigma}{d\Omega_p} , \quad (5.5)$$

and does not yield new information not accessible in singles measurements. In contrast the $\mathcal{B}$ and $\mathcal{C}$ terms *do* yield new information. In fact, it can be shown that, by measuring a complete set of observables in singles and at a single in-plane coincidence angle, it is possible to overdetermine the full $0^+\to1^+$ amplitude apart from an overall (unphysical) phase and *the relative phases between out-of-plane and in-plane amplitudes.* These latter phases, however, can be determined by measuring longitudinal or sideways analyzing power as we will show in the following.

In a direct only, relativistic plane-wave impulse approximation assuming zero Q value, the amplitudes A_{Kq} and A_{qK} are proportional to the composite spin-convection current amplitudes $\langle\sigma\times\mathbf{J}\rangle$ and $\langle\sigma\cdot\mathbf{J}\rangle$, respectively.[14,15] In standard nonrelativistic treatments these amplitudes arise exclusively from nonlocal processes like knock-on exchange.[16] Experimental information about these amplitudes would be of great interest in testing reaction models as well as nuclear structure. With this goal in mind, there have been efforts to measure the singles polarization-analyzing power difference $(P\text{-}A_y)$,[17–19] since it vanishes whenever A_{Kq} and A_{qK} are zero. Equation (5.4) shows that complementary information is accessible through the antisymmetric in-plane observable $\mathcal{C}$ which also vanishes when $A_{Kq}=A_{qK}=0$.

Longitudinal and sideways analyzing powers vanish in a singles experiment due to parity invariance. In a coincidence experiment these observables are no longer zero provided the photon polarization tensor, $t_{ij}(\hat{\mathbf{k}})$, has negative parity. This condition implies that longitudinal and sideways analyzing powers must be proportional to the pseudoscalar components of the photon polarization tensor, viz.,

$$t_{nK}=-(\hat{\mathbf{k}}\cdot\hat{\mathbf{n}})(\hat{\mathbf{k}}\cdot\hat{\mathbf{K}})$$

and

$$t_{nq}=-(\hat{\mathbf{k}}\cdot\hat{\mathbf{n}})(\hat{\mathbf{k}}\cdot\hat{\mathbf{q}}) .$$

Longitudinal, $D_{0L}(\hat{\mathbf{k}})$, and sideways, $D_{0S}(\hat{\mathbf{k}})$, analyzing powers,

$$\begin{bmatrix} D_{0L}(\hat{\mathbf{k}}) \\ D_{0S}(\hat{\mathbf{k}}) \end{bmatrix}=\begin{bmatrix} \cos\theta_{pK} & \sin\theta_{pK} \\ -\sin\theta_{pK} & \cos\theta_{pK} \end{bmatrix}\begin{bmatrix} D_{0K}(\hat{\mathbf{k}}) \\ D_{0q}(\hat{\mathbf{k}}) \end{bmatrix} , \quad (5.6)$$

are written in terms of the angle between the incoming beam direction and the average momentum, θ_{pK}, and $D_{0K}(\hat{\mathbf{k}})$ and $D_{0q}(\hat{\mathbf{k}})$ given respectively by

$$\frac{8\pi}{3R}\frac{d^2\sigma}{d\Omega_p d\Omega_\gamma}D_{0K}(\hat{\mathbf{k}}) =2[\mathrm{Re}(A_{n0}A^*_{KK})-\mathrm{Im}(A_{nn}A^*_{Kq})]t_{nK}(\hat{\mathbf{k}}) +2[\mathrm{Re}(A_{n0}A^*_{qK})-\mathrm{Im}(A_{nn}A^*_{qq})]t_{nq}(\hat{\mathbf{k}}) , \quad (5.7)$$

$$\frac{8\pi}{3R}\frac{d^2\sigma}{d\Omega_p d\Omega_\gamma}D_{0q}(\hat{\mathbf{k}}) =2[\mathrm{Re}(A_{n0}A^*_{Kq})+\mathrm{Im}(A_{nn}A^*_{KK})]t_{nK}(\hat{\mathbf{k}}) +2[\mathrm{Re}(A_{n0}A^*_{qq})+\mathrm{Im}(A_{nn}A^*_{qK})]t_{nq}(\hat{\mathbf{k}}) . \quad (5.8)$$

We first observe that measurement of longitudinal and sideways analyzing powers is enough to determine the relative phase between the out-of-plane amplitudes, (A_{n0}, A_{nn}), and the in-plane $(A_{KK}, A_{Kq}, A_{qK}, A_{qq})$, amplitudes. Furthermore, since these observables are proportional to t_{nK}, and t_{nq}, they vanish for out-of-plane as well as in-plane measurements. Therefore, it is only by performing part out-of-plane measurements that the $0^+\to1^+$ amplitude can be completely determined.

VI. RESULTS AND CONCLUSIONS

We now show relativistic impulse approximation and nonrelativistic impulse approximation calculations of coincidence observables for the 15.11 MeV, $1^+T=1$ state in ^{12}C at an incident proton energy of $T_{\mathrm{lab}}=400$ MeV. Cohen and Kurath[20] nuclear structure amplitudes were used in both cases and knock-on exchange processes were treated explicitly using the nonrelativistic code DW81 (Ref. 21) and its relativistic counterpart DREX.[22] The ele-

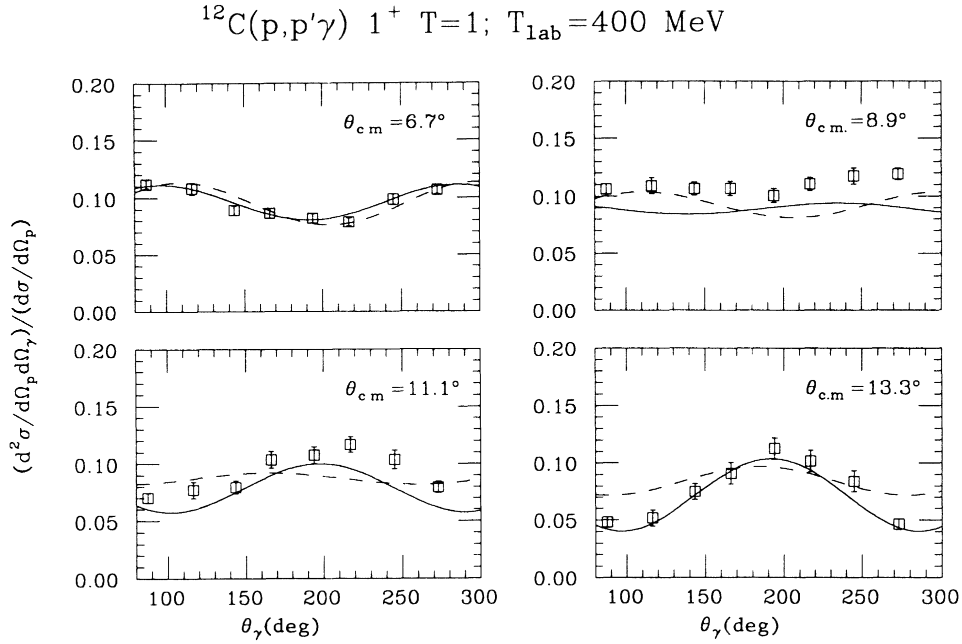

FIG. 1. Ratio of the in-plane coincidence to singles cros section as a function of the photon angle in the laboratory frame for the 15.11 MeV, $1^+T=1$ state in ^{12}C at an incident proton energy of $T_{\rm lab}=400$ MeV. The solid (dashed) lines are result of relativistic (nonrelativistic) calculations that are described in the text. Data is from Ref. 8.

mentary *NN t* matrix used in the claculations were based on nonrelativistic[23] and relativistic[24] parametrizations of the *NN* interaction with parameters constrained by Arndt phase-shift solutions.[25] In both cases distortions were calculated from the same *NN* interaction driving the transi-

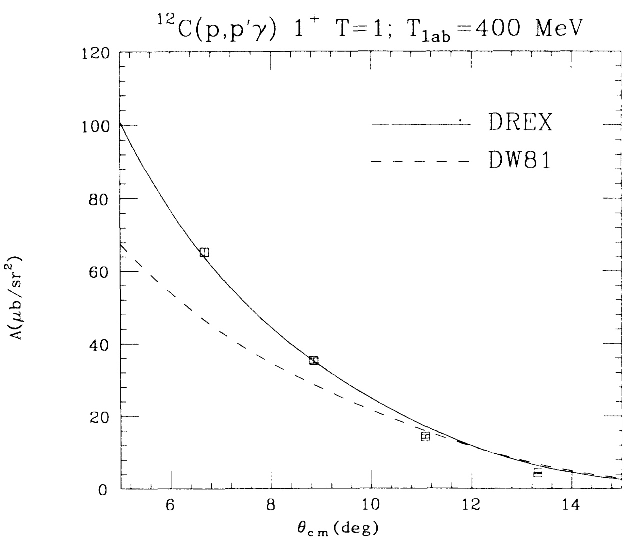

FIG. 2. Isotropic coefficient $\mathcal{A}$ for the in-plane coincidence cross section as a function of the center-of-mass scattering angle for the 15.11 MeV, $1^+T=1$ state in ^{12}C at an incident proton energy of $T_{\rm lab}=400$ MeV. The solid (dashed) lines are result of relativistic (nonrelativistic) calculations that are described in the text. Data is from Ref. 8.

tion. The individual Cartesian amplitudes in Eq. (2.4) were extracted from the helicity representation amplitudes calculated by the codes DW81 and DREX. In effect, then, the Cartesian amplitudes are simply a reexpression of the helicity amplitudes in a different coordinate system, namely the one defined in Eq. (2.2). However, the effects of distortions and nonzero Q values introduce ambiguities in the definition of the Cartesian coordinate system. These ambiguities contribute to differences between Cartesian and helicity observables. In this case it is the original helicity results that should be compared with experiment. The value, then, of the Cartesian formulation presented in this paper is the ease with which particular nuclear structure quantities can be identified with specific observables. For example, in connection with Eqs. (5.3) and (5.4), our Cartesian analysis shows that the antisymmetric, in-plane coincidence observable $\mathcal{C}$ should be maximally sensitive to composite currents. Such maximal sensitivity is also observed in the corresponding helicity observable. It is true in general that sensitivities suggested by the Cartesian expressions presented here survive in the helicity calculations even though the Cartesian and helicity observables are not identical.

In Fig. 1 we show the ratio of the in-plane coincidence to singles cross section as a function of the photon angle in the laboratory frame. (Note that the calculations shown here employ Cartesian amplitudes and as already discussed differ somewhat from the helicity results presented in Ref. 8.) From the cross section one can extract the isotropic, symmetric and antisymmetric terms of Eq. (5.3). The isotropic part of the cross section, shown in Fig. 2, is related to the singles D_{nn}, Eq. (5.5), and yields no new information. More interesting, howev-

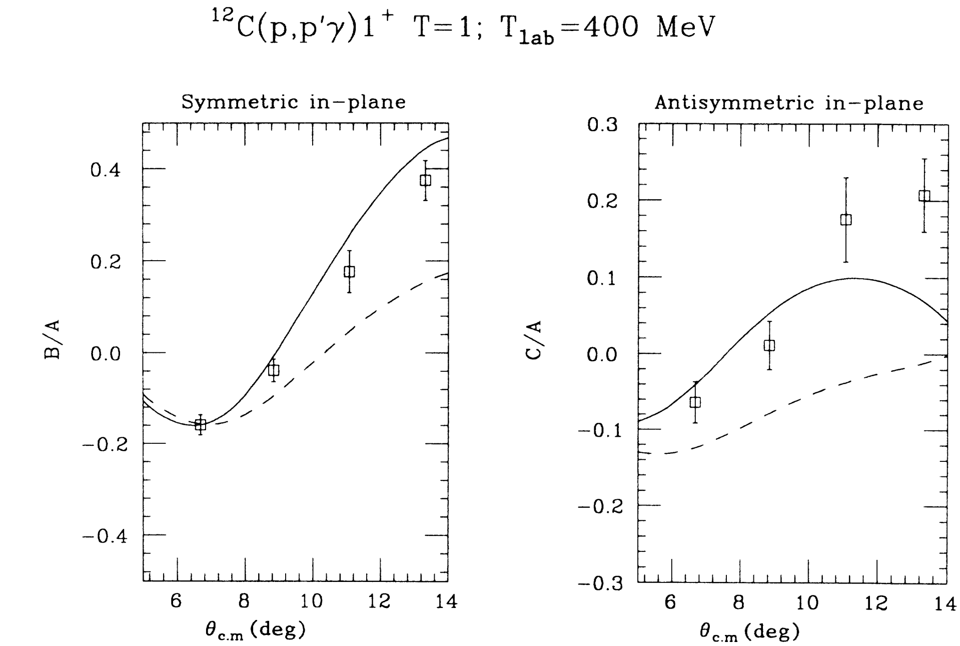

FIG. 3. Symmetric $\mathcal{B}/\mathcal{A}$ and antisymmetric $\mathcal{C}/\mathcal{A}$ coefficients for the in-plane coincidence cross section as a function of the center-of-mass scattering angle for the 15.11 MeV, $1^+T=1$ state in ^{12}C at an incident proton energy of $T_{\rm lab}=400$ MeV. The solid (dashed) lines are result of relativistic (nonrelativistic) calculations that are described in the text. Data is from Ref. 8.

er, are the deviations from isotropy seen in Fig. 3. In the nonrelativistic case (dashed lines) these deviations are seen to be small and do not reproduce the general trends of the data. In particular, the antisymmetric part of the coincidence cross-section data is seen to change sign as one goes from $\theta_{\rm c.m.}=6.7°$ to $\theta_{\rm c.m.}=13.3°$. This behavior is not reproduced by the nonrelativistic calculation and may indicate some deficiency in the calculation of the

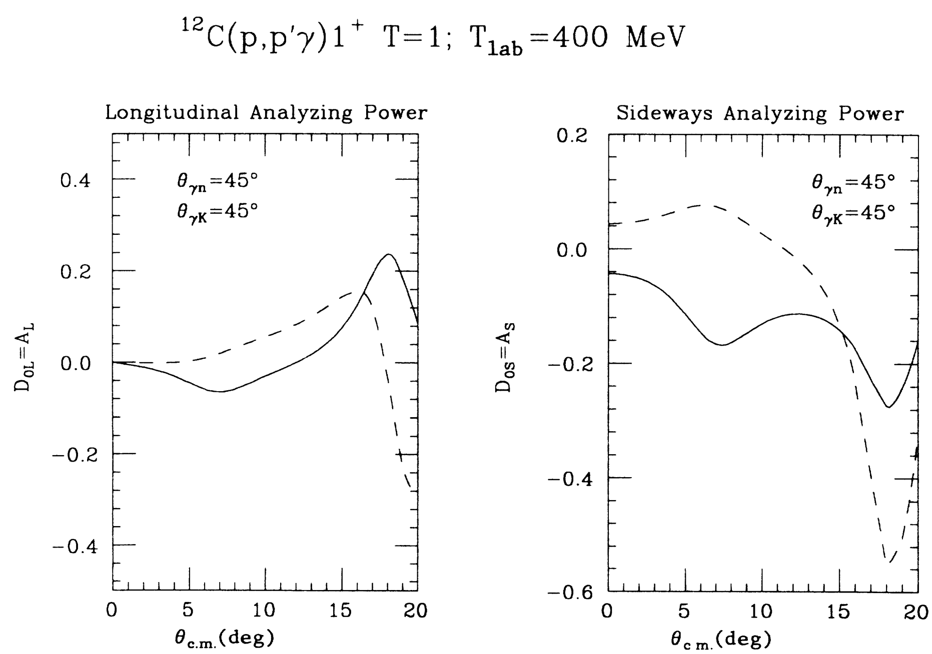

FIG. 4. Longitudinal and sideways analyzing powers as a function of center-of-mass scattering angle for the 15.11 MeV, $1^+T=1$ state in ^{12}C at an incident proton energy of $T_{\rm lab}=400$ MeV. The γ-ray angle is located in the n-K plane at an angle $\theta_{\gamma n}=\theta_{\gamma K}=45°$. The solid (dashed) lines are result of relativistic (nonrelativistic) calculations.

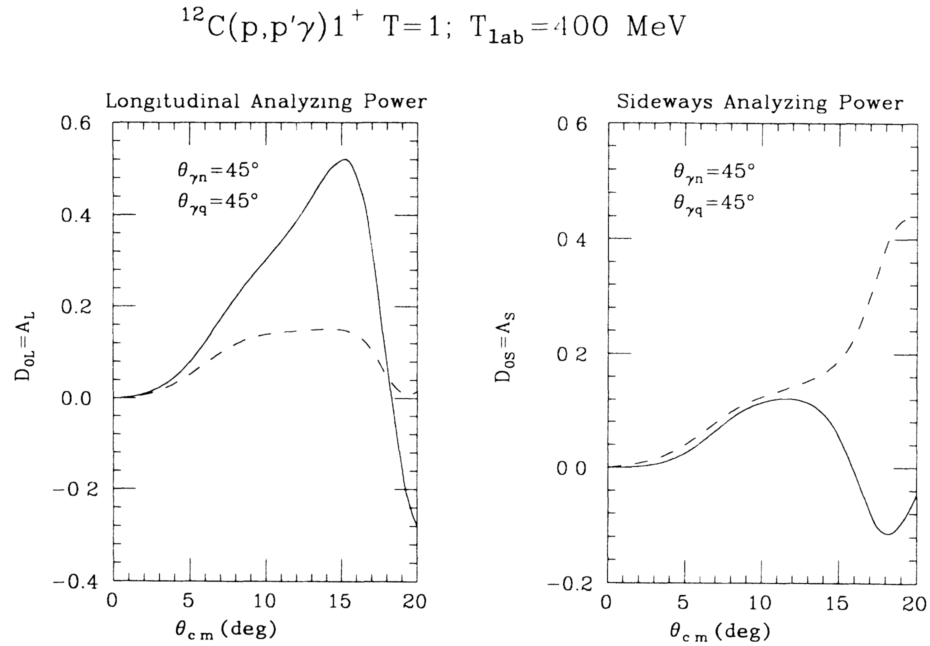

FIG. 5. Longitudinal and sideways analyzing powers as a function of center-of-mass scattering angle for the 15.11 MeV, $1^+ T=1$ state in ^{12}C at an incident proton energy of $T_{\rm lab}=400$ MeV. The γ-ray angle is located in the n-q plane at an angle $\theta_{\gamma n}=\theta_{\gamma q}=45°$. The solid (dashed) lines are result of relativistic (nonrelativistic) calculations.

composite spin-current amplitudes A_{Kq} and A_{qK}. These features are also apparent in the helicity calculations of Ref. 8. Based only on these limited set of experimental results we must conclude that the data shows a clear preference for the relativistic predictions. In Figs. 4 and 5 we show longitudinal and sideways analyzing power observables for a photon momentum in the $n-K$ plane ($\theta_{\gamma n}=\theta_{\gamma K}=45°$), and in the $n-q$ plane ($\theta_{\gamma n}=\theta_{\gamma q}=45°$), respectively. As mentioned before, part out-of-plane measurements are essential to determine the relative phase between the in-plane and out-of-plane amplitudes. From these figures one again observes large differences in the predictions of the two models. Experimental information about these as yet unmeasured analyzing powers would be of great interest.

In conclusion, we have examined the utility of coincidence $(p,p'\gamma)$ measurements. We have seen that some parts of the scattering amplitudes are difficult, and in some cases impossible, to determine in a conventional singles experiment. Coincidence measurements may then provide the only practical way to completely determine the scattering amplitude and therefore isolate those quantities that are particularly sensitive to differences between various theoretical models. We have shown, at least for the $0^+ \rightarrow 1^+$ excitation, how a simple modification in the formal structure of the singles spin observables yields the coincidence observables. This simple change has important consequences. For example, some unique properties of the scattering amplitude become accessible through the measurement of longitudinal and sideways analyzing powers. Many interesting issues can therefore be addressed by $(p,p'\gamma)$ experiments without having to detect the polarization of the scattered proton. Experimental information about these observables would provide unique information that is likely to shed some light on the physics underlying these reactions.

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[1]G. W. Hoffman *et al.*, Phys. Rev. Lett. **47**, 1436 (1981).
[2]L. G. Arnold, B. C. Clark, R. L. Mercer, and P. Schwandt, Phys. Rev. C **23**, 1949 (1981).
[3]B. C. Clark, R. L. Mercer, and P. Schwandt, Phys. Lett. **122B**, 211 (1983).
[4]J. A. McNeil, J. R. Shepard, and S. J. Wallace, Phys. Rev. Lett. **50**, 1439 (1983).
[5]J. R. Shepard, J. A. McNeil, and S. J. Wallace, Phys. Rev. Lett. **50**, 1443 (1983).
[6]N. Mobed, S. S. M. Wong, and X. Zhu, Nucl. Phys. **A456**, 644 (1986).
[7]J. Piekarewicz, Phys. Rev. C **37**, 719 (1988).
[8]K. H. Hicks *et al.*, Phys. Rev. Lett. **61**, 1174 (1989).
[9]H. O. Funsten (private communication).
[10]J. Piekarewicz, R. D. Amado, and D. A. Sparrow, Phys. Rev. C **32**, 949 (1985).

[11]D. M. Brink and G. R. Satchler, *Angular Momentum* (Clarendon, Oxford, 1968).
[12]G. R. Satchler, *Direct Nuclear Reactions* (Oxford University, New York, 1983).
[13]M. A. Kovash, *et al.*, submitted to Phys. Rev. C.
[14]D. A. Sparrow, J. Piekarewicz, E. Rost, J. R. Shepard, J. A. McNeil, T. A. Carey, and J. B. McClelland, Phys. Rev. Lett. **54**, 2207 (1985).
[15]J. R. Shepard, E. Rost, and J. A. McNeil, Phys. Rev. C **33**, 634 (1986).
[16]W. G. Love and J. R. Comfort, Phys. Rev. C **29**, 2135 (1984).
[17]T. A. Carey *et al.*, Phys. Rev. Lett. **49**, 266 (1982).
[18]J. B. McClelland *et al.*, Phys. Rev. Lett. **52**, 98 (1984).
[19]A. K. Opper *et al.*, Indiana University Cyclotron Facility Newsletter, 1989.
[20]S. Cohen and D. Kurath, Nucl. Phys. **73**, 1 (1965); T. H. S. Lee and D. Kurath, Phys. Rev. C **21**, 293 (1980).
[21]J. R. Comfort, G. L. Moake, C. C. Foster, P. Schwandt, and W. G. Love, Phys. Rev. C **26**, 1800 (1982).
[22]E. Rost and J. R. Shepard, computer code DREX (unpublished); E. Rost and J. R. Shepard, Phys. Rev. C **35**, 681 (1987).
[23]M. A. Franey and W. G. Love, Phys. Rev. C **31**, 488 (1985).
[24]C. J. Horowitz, Phys. Rev. C **31**, 1340 (1985).
[25]R. A. Arndt and L. S. Roper, code SAID (unpublished).