Theory of the isotensor term in the pion-nucleus optical potential

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Phenomenological analysis of double charge exchange scattering of pions shows that there is a sizable isotensor term in the optical potential at resonant energies. In an earlier paper we proposed that this term could be expected to reflect coupling of the elastic and analog channels to particular nonelastic channels with specific isospin structure. In this paper we present detailed models for this channel coupling, based on pion absorption and correlated double scattering. It is shown that any of the models could contribute to the isotensor potential with the appropriate phase. New predictions of isospin dependence of the potential are given.

I. INTRODUCTION

Elastic scattering of pions on nuclear targets, together with single charge exchange (SCX) and double charge exchange (DCX) scattering to analog states may be treated by use of an optical potential U, to be used in a wave equation (e.g., Klein-Gordon)

$$(\nabla^2+k^2)\psi=U\psi \ . \tag{1.1}$$

In the isospin space of the pion (operator ϕ) and the target analog states (operator T), this potential may be written in the form

$$U=U_0+U_1(\phi\cdot T)+U_2(\phi\cdot T)^2 \ , \tag{1.2}$$

based on isospin invariance alone. The term $U_2(\phi\cdot T)^2$ is the quadratic or "isotensor" term.

This term is the subject of this paper. The main interest in the U_2 term is that it involves many-body effects, that is, two or more correlated nucleons in the nuclear target. In the multiple-scattering theory of the optical potential[1] for pions, U_0 and U_1 are both largely determined by one-nucleon properties of the target, e.g., the nucleon density $\rho(r)$, and the neutron excess density $\Delta\rho(r)$, respectively, while U_2 depends on (at least) two-nucleon properties.[2]

For the present work we are interested in pion energies near resonance, in particular for $T_\pi\simeq 164$ MeV, for which there has been a systematic study by Greene *et al.*[3] of the optical potential, using elastic, SCX, and DCX data. In the phenomenological analysis of Greene *et al.*, U_2 is taken to have the p-wave form

$$U_2=\lambda(T)\nabla\cdot\frac{\nabla\rho^2(r)}{\rho_0}\nabla \ , \tag{1.3}$$

quadratic in the neutron-excess density, with ρ_0 the standard nucleon density ($\equiv 0.16$ fm^{-3}), and a strength parameter $\lambda(T)$. Following Johnson and Siciliano,[4] $\lambda(T)$ is assumed to be independent of target A, but has the following T dependence:

$$\lambda_{JS}(T)=\lambda_2^{(2)}/T(2T-1)+\lambda_4^{(2)}/T^2 \ , \tag{1.4}$$

in terms of two parameters: $\lambda_2^{(2)}$ fit to data, and $\lambda_4^{(2)}$ calculated. The data used are heavily weighted by $T=1$ nuclear targets; therefore, we shall interpret their fit parameter as determining only the $T=1$ combination of (1.4)

$$\lambda_G\equiv\lambda_{JS}(1)=\lambda_2^{(2)}+\lambda_4^{(2)} \ . \tag{1.5}$$

In addition, we shall also consider an s-wave isotensor potential of the form

$$U_2=-k^2\bar{\lambda}(T)\frac{\Delta\rho^2(r)}{\rho_0} \ , \tag{1.6}$$

where $\bar{\lambda}_G\equiv\bar{\lambda}(1)$, in analogy with (1.5).

The assumed dependence of U_2 on $\Delta\rho^2$ in (1.3) or (1.6) follows from multiple-scattering theory: this would be the lowest-order term (in $\Delta\rho$) of the form $(\phi\cdot T)^2$. However, the requirement that U_2 be proportional to $\Delta\rho^2(r)$, and not, say to $\Delta\rho(r)\Delta\rho(r')$ (nonlocal) is a zero-range assumption. In Johnson and Siciliano, this simplification comes about as a result of the strong optical absorption, which restricts the scattering to the nuclear surface. The zero-range form also obtains as a limiting approximation for any mechanism contributing to U_2 for which short distances are favored on dynamical grounds. In the present work we consider pion absorption, and scattering from correlated pairs, for which this is usually assumed, and consider only the zero-range limit, so that we can compare our results with the analysis of Ref. 2.

The main result of Greene *et al.*, for our purposes, is the determination of λ_G, which they find to have a large, positive imaginary component, and a smaller, less well determined real component, as we discuss in Sec. VIII. Although the precise value of λ_G depends on other assumptions in the parametrization, the qualitative result seems to be required to produce the experimental forward DCX cross sections (to which λ_G was fitted), and the interference minimum seen at $\sim 20°$ in the angular distribution for several targets (see Ref. 2, Fig. 4). It is these experimental features which give the best evidence for the existence of the isotensor potential U_2. In an earlier paper[5] we gave a qualitative argument for the phase of U_2 (or λ_G).

There have been several previous attempts to understand the dynamics behind the magnitude and phase of λ_G. Johnson and Siciliano[4] gave an estimate based on correlated multiple scattering, which gave an effect of the right magnitude but wrong sign. (We discuss this result further in Sec. IX.) Oset *et al.*[6] estimated the contribution of meson exchange currents, finding them to be out of phase with the multiple-scattering results, which is equivalent to a prediction λ_G be real. Similarly, attempts[7,8] to explain λ_G by a Δ model in which a Δ produced in π^+ scattering charge exchanges in the target before emitting a final π^-, also predict a λ_G essentially real.

Our picture of the underlying mechanism of U_2 follows from the observation that $\mathrm{Im}\lambda_G > 0$ can be related, through unitarity, to loss of flux to other open channels with appropriate isospin structure. The unitary argument was given in Ref. 5, where we also derived an inequality relating DCX amplitudes to nonelastic cross sections. In the present paper we give three specific models for such nonelastic reactions, spelling out the consequences for DCX.

The paper is organized as follows: In Sec. II we characterize the isospin structure of any model based on two-body correlations, in the zero-range limit. In Sec. III we derive the density forms (1.3) and (1.6), under restrictive assumptions about the target structure. This allows us to make a connection with the work of Refs. 3 and 4. We also find different isospin dependence of U_2 than given in these papers. The isospin algebra is reduced by projection methods in Sec. IV. The three models are developed in Secs. V, VI, and VII. Relations to the phenomenological results of Greene *et al.*[3] are given in Sec. VIII, and further consequences of the models in Sec. IX. Conclusions follow.

II. ZERO-RANGE MODELS

In this section we discuss the general features of two-nucleon operators which represent inelastic processes coupled to the elastic-analog channels.

We may represent a general local two-nucleon operator in the form

$$M(\mathbf{r},\mathbf{r}')=\sum_{i\neq j}^{A} Q_{ij}\delta(\mathbf{r}-\mathbf{x}_i)\delta(\mathbf{r}'-\mathbf{x}_j)\ , \tag{2.1}$$

where Q_{ij} is a spin-isospin operator, and $\mathbf{x}_i,\mathbf{x}_j$ are nucleon coordinates.

We shall assume in this work that the processes of interest take place at short distances, and therefore represent them by zero-range operators,

$$M(\mathbf{r})=\sum_{i\neq j}^{A} Q_{ij}\delta(\mathbf{r}-\mathbf{x}_i)\delta(\mathbf{r}-\mathbf{x}_j)\ . \tag{2.2}$$

We shall also include p-wave (nonlocal) operators obtained from (2.2) in the usual (Kisslinger) derivative form:

$$M'(\mathbf{r})=\nabla\cdot M(\mathbf{r})\nabla\ . \tag{2.3}$$

Since we are interested in operators which can contribute to DCX, we restrict the isospin to that for $T=1$ pairs only. Since $M(r)$ is zero range in (2.2) and (2.3), the spin is automatically restricted to $S=0$ pairs, and Q_{ij} may be written without spin dependence, in the form

$$Q_{ij}=A_0P_{ij}+A_1\phi\cdot(\tau_i+\tau_j)+A_2B_{ij} \tag{2.4}$$

in terms of the pion isospin operator ϕ and the nucleon isospin operator $\frac{1}{2}\tau_i$, where P_{ij} is the $T=1$ projection operator

$$P_{ij}=\tfrac{1}{4}(3+\tau_i\cdot\tau_j) \tag{2.5}$$

and B_{ij} is a scalar product of second-rank tensor operators in ϕ and T space, given by

$$B_{ij}=(\phi\cdot\tau_i)(\phi\cdot\tau_j)-\tfrac{2}{3}\tau_i\cdot\tau_j\ . \tag{2.6}$$

Equation (2.4) gives the isospin tensor decomposition of $M(r)$ into (iso)scalar, vector, and tensor components.

Consider the expectation value of $M(r)$ in the target ground state, for a neutron excess $n, T=n/2, T_3=-T$. We assume that the isospin is carried entirely by the excess neutrons; the core has zero isospin. We are not concerned about the contribution of the core particles, since they do not contribute to the isotensor term. Therefore, we consider the expectation value for the neutron excess only:

$$\langle -T|M(r)|-T\rangle=\langle T,-T|\sum_{i\neq j}^{n} Q_{ij}\delta(\mathbf{r}-\mathbf{x}_i)\delta(\mathbf{r}-\mathbf{x}_j)|T,-T\rangle\ . \tag{2.7}$$

The isospin structure is easily evaluated, since all pairs have the same isospin structure, yielding

$$\begin{aligned}\langle -T|Q_{ij}|-T\rangle&=[n(n-1)]^{-1}\sum_{k\neq l}\langle -T|Q_{kl}|-T\rangle\\&=[n(n-1)]^{-1}\langle -T|[A_0n(n-1)+A_1(n-1)4(\phi\cdot T)+4A_2\tilde{B}]|-T\rangle\ ,\end{aligned} \tag{2.8}$$

where

$$\tilde{B}=(\phi\cdot T)^2+\tfrac{1}{2}(\phi\cdot T)-\tfrac{2}{3}T^2 \tag{2.9}$$

is the isotensor operator for the target. By simple Wigner-Eckart arguments, (2.8) must hold for off-diagonal matrix elements (in ϕ_3,T_3) as well. Then (2.7) may be expressed in the isospin matrix form

$$[M(r)]_n=\rho_n(r,r)\{4A_0T(T-\tfrac{1}{2})+8A_1(T-\tfrac{1}{2})(\phi\cdot T)+4A_2\tilde{B}\}\ , \tag{2.10}$$

where we use $n=2T$, and $[\ldots]_n$ is a matrix (operator) in ϕ_3, T_3. We have introduced the two-neutron correlation function

$$\rho_n(\mathbf{r},\mathbf{r}')=[n(n-1)]^{-1}\langle T,-T|\sum_{i\neq j}^{n}\delta(\mathbf{r}-\mathbf{x}_i)\delta(\mathbf{r}'-\mathbf{x}_j)|T,-T\rangle \tag{2.11}$$

for the neutron excess, which is normalized to unity:

$$\int d\mathbf{r}\,d\mathbf{r}'\rho_n(\mathbf{r},\mathbf{r}')=1 .$$

The contributions to $[M(r)]_n$ from core particles may also be calculated in terms of correlation functions, but will not include an isotensor contribution, $\tilde{B}$. (Note: for simplicity we have excluded the case of valence protons as well as neutrons outside a $T=0$ core.)

III. CONNECTION WITH THE DENSITY FORMS

The correlation function in (2.10), and its p-wave analog in (2.3) can be related to the assumed density dependence of Johnson and Siciliano,[4] and of Greene *et al.*[3] [see Eqs. (1.3) and (1.6)] for the special case that the excess neutrons are in a single valence shell, with the same radial wave function $R_{nl}(r)$. We now denote the neutron-excess density by $\Delta\rho_n(r)$, with the normalization

$$\int d\mathbf{r}\,\Delta\rho_n(r)=n . \tag{3.1}$$

Consider only $J=0$ (even n) target ground states, for which $\Delta\rho_n(r)$ is a scalar function of $\mathbf{r}$, in fact,

$$\Delta\rho_n(r)=(4\pi)^{-1}nR_{nl}^2(r) . \tag{3.2}$$

It is easily shown that the correlation function (2.11) at $\mathbf{r}=\mathbf{r}'$ is proportional to the square of (3.2):

$$\rho_n(r,r)\propto\Delta\rho_n^2(r) , \tag{3.3}$$

which is the only possible scalar function of the wave functions. The coefficients may be calculated directly [see the Appendix] for a filled neutron shell ($n=n_F$):

$$\rho_F(r,r)=\Delta\rho_F^2(r)/2n_F(n_F-1) , \tag{3.4}$$

and for $n=2$:

$$\rho_2(r,r)=(n_F-1)\rho_F(r,r)=n_F\Delta\rho_2^2(r)/8 . \tag{3.5}$$

For any even n, if the ground state has good seniority zero ($v=0$), it can be shown that the general relation becomes $n(n-1)\rho_n(r,r)\propto n$. This can be seen from the linear n dependence of a δ function potential in $v=0$ states (see the Appendix). Then,

$$\rho_n(r,r)=\frac{(n_F-1)}{(n-1)}\rho_F(r,r)=\frac{n_F\Delta\rho_n^2(r)}{2n^2(n-1)} . \tag{3.6}$$

The result is that for a single valence shell of neutrons, the zero-range theory of Sec. II does give the $\Delta\rho_n^2$ density dependence of Siciliano and Johnson, and Greene *et al.* However, the isospin dependence predicted is not as given in those papers, as we now see.

Combining (2.10) and (3.6), keeping only the isotensor term of $[M(r)]_n$

$$[M_2(r)]_n=\frac{2A_2n_F}{n-1}\frac{\Delta\rho_n^2(r)}{n^2}\tilde{B} . \tag{3.7}$$

The coefficient of $\Delta\rho_n^2(r)(\phi\cdot T)^2$ may be written (with $n=2T$)

$$A_2n_F/2T^2(2T-1) . \tag{3.8}$$

The result is that the T dependence of the isostensor term in a zero-range model, with the excess neutrons fully paired (seniority zero) in a single shell, is given by

$$\lambda(T)=\frac{g}{T^2(2T-1)} \tag{3.9}$$

with g independent of T [and similarly for $\bar{\lambda}(T)$]. This may be compared to the different T dependence obtained by Johnson and Siciliano, shown in (1.4), which comes about as follows. These authors assume that the excess neutrons can be treated as if they were in a closed shell that is subject only to Pauli correlations, but not pairing. This is equivalent to using (3.4) for each n:

$$\rho_n'(r,r)=\Delta\rho_n^2(r)/2n(n-1) , \tag{3.10}$$

which gives the T dependence (with $n=2T$)

$$\lambda'(T)=\lambda_2^{(2)}/T(2T-1) . \tag{3.11}$$

This accounts for the first term in (1.4), which can be associated with $\rho_n(r,r)$. The second term in (1.4) depends on T^{-2}, in contrast to (3.8) and (3.9); it is associated with a correction term in double scattering, as will be explained in Sec. VII.

IV. PROJECTION OF TWO-BODY ISOSPIN CHANNELS

The isospin structure of the two-body operators of Sec. II is given by the operator Q_{ij} of (2.4). Now we concentrate on a specific nucleon pair ($ij=12$) with $T=1$ and rewrite Q_{12} using (2.8) and (2.9)

$$\begin{aligned}Q_{12}&=A_0+2A_1(\phi\cdot T)+2A_2\tilde{B}\\&=(A_0-\tfrac{8}{3}A_2)+(2A_1+A_2)(\phi\cdot T)+2A_2(\phi\cdot T)^2 .\end{aligned} \tag{4.1}$$

It is useful to reexpress the operator in terms of the total isospin ($\theta=\phi+T$) of the $\pi+2N$ channel, by means of projection operators P_θ with $\theta=0,1,2$:

$$Q_{12}=\sum_\theta R_\theta P_\theta , \tag{4.2}$$

where

$$P_0=\tfrac{1}{3}[(\phi\cdot T)^2-1]\ ,$$
$$P_1=\tfrac{1}{2}[2-(\phi\cdot T)-(\phi\cdot T)^2]\ , \tag{4.3}$$
$$P_2=\tfrac{1}{6}[2+3(\phi\cdot T)+(\phi\cdot T)^2]\ .$$

Equating coefficients in (4.1) and (4.2), we find

$$A_0=\tfrac{1}{9}R_0+\tfrac{1}{3}R_1+\tfrac{5}{9}R_2\ ,$$
$$A_1=-\tfrac{1}{12}R_0-\tfrac{1}{8}R_1+\tfrac{5}{24}R_2\ , \tag{4.4}$$
$$A_2=\tfrac{1}{6}R_0-\tfrac{1}{4}R_1+\tfrac{1}{12}R_2\ .$$

This allows us to characterize the isospin structure of $M(r)$ of Sec. II by the contributions of specific θ channels.

V. DIRECT ABSORPTION MODEL

In this model we assume that pion absorption on pairs of valence neutrons takes place at zero range, and contributes a term to the optical potential of the form

$$U_a(r)=[M(r)]_n \tag{5.1}$$

with $[M(r)]_n$ given by (2.10). Using (3.7) and (3.8), the "isotensor" optical potential becomes

$$U_2(r)=\frac{A_2 n_F}{2T^2(2T-1)}\Delta\rho_n^2(r)\ . \tag{5.2}$$

The coefficient A_2 can be related to the parameter $\bar{\lambda}_G$ of (1.5) for a $T=1$ target, by

$$\frac{A_2 n_f}{2}=-\frac{k^2\bar{\lambda}_G}{\rho_0}\ . \tag{5.3}$$

The phenomenological fits to DCX of Greene *et al.* give $\mathrm{Im}\lambda_G>0$, which in turn requires $\mathrm{Im}\,A_2<0$, if we assume $\bar{\lambda}_G\simeq\lambda_G$. (See Sec. VIII for discussion.) This has consequences for which isospin channels contribute to U_a, which can be seen as follows.

Consider absorption by a $T=1$ nucleon pair, with the final nucleon pair isospin T_f:

$$\pi+(NN)_{T=1}\rightarrow(NN)_{T_f}\ . \tag{5.4}$$

Now $T_f=\theta$, the total isospin in this channel (see Sec. IV), and $T_f=0,1$ only for (NN).

Now write $U_a(r)$ for the $n=2$ case, in the form [see Eqs. (2.10), (2.11), and (4.1)].

$$U_a(r)=2\rho_2(r,r)Q_{12}\ . \tag{5.5}$$

Using the θ-channel decomposition of Q_{12} in (4.2), we have $R_2\equiv0$ (no $\theta=2$ channel). In addition, $\mathrm{Im}U_a(r)<0$ for an absorptive potential, which must hold for each θ channel separately. Since $\rho_2(r,r)>0$, it follows that $\mathrm{Im}R_0<0$ and $\mathrm{Im}R_1<0$.

From (4.4) we have, from the requirement $\mathrm{Im}\,A_2<0$, with $R_2=0$,

$$\tfrac{1}{6}\,\mathrm{Im}R_0-\tfrac{1}{4}\,\mathrm{Im}R_1<0\ , \tag{5.6a}$$

or

$$2\,\mathrm{Im}R_0<3\,\mathrm{Im}R_1<0\ , \tag{5.6b}$$

which constrains the relative strength of the $\theta=0,1$ channels.

As an example, consider absorption in a pure $\theta=0$ mode (and which therefore has no ΔN component). Then $R_1\equiv0$, and, from (4.3),

$$Q_{12}=R_0P_0=\frac{R_0}{3}[(\phi\cdot T)^2-1]\ . \tag{5.7a}$$

From (4.4) we find the coefficients

$$9A_0=-12A_1=6A_2=R_0\ . \tag{5.7b}$$

VI. DELTA ABSORPTION MODEL

First, we characterize the optical potential in a Δ model with no nuclear medium effects (and no recoil) as in (2.3), in isospin matrix form

$$U(r)=-\nabla\cdot[G_0(r)]\nabla\ , \tag{6.1}$$

with

$$[G_0(r)]=[V^\dagger(\mathbf{r})V(\mathbf{r})]/D_0\ . \tag{6.2}$$

Here $V(r)$ is the $\pi N\rightarrow\Delta$ transition operator at point $\mathbf{r}$, for a p-wave pion; the expectation value is in the target state, where [ ] now includes the core particles, as well as the neutron excess. The (free) resonant denominator is given by

$$D_0=\omega-\omega_R+i\Gamma/2\ , \tag{6.3}$$

where ω is the pion c.m. energy, ω_R ($=268$ MeV) is the resonance energy, and Γ is the full width (actually a function of ω). The one-body operator $V^\dagger V$ may be represented in the form [in analogy with (2.1)]

$$V^\dagger(\mathbf{r})V(\mathbf{r})=\gamma\sum_{i=1}^{A}Q_{3/2}(i)\delta(\mathbf{r}-\mathbf{x}_i)\ , \tag{6.4a}$$

with the $T(\pi N)=\frac{3}{2}$ projector:

$$Q_{3/2}(i)=\tfrac{1}{3}[2+\phi\cdot\tau(i)]\ . \tag{6.4b}$$

(We have omitted the nucleon spin-term here.) The target expectation (in isospin matrix form) can be expressed in terms of $\rho(r)$ and $\Delta\rho_n(r)$ (note: $\rho=\rho_0+\Delta\rho$):

$$[V^\dagger(\mathbf{r})V(\mathbf{r})]=\tfrac{2}{3}\gamma\left\{\rho(r)+\frac{1}{n}\Delta\rho_n(r)(\phi\cdot T)\right\}\ , \tag{6.5a}$$

with an amplitude factor

$$\gamma=4\pi\Gamma/\kappa^3(1+\omega_L/M)\ ,$$

where κ is the pion momentum in the πN c.m. and ω_L is the *lab* pion energy. This, with (6.1)–(6.3), gives an optical potential of the conventional form with isoscalar and isovector terms. As in Sec. II, we have assumed the isospin to be carried entirely by the neutron excess. The term of (6.5a) due to the excess can be written

$$[V^\dagger(\mathbf{r})V(\mathbf{r})]_n=\frac{2\gamma}{3n}\Delta\rho_n(r)\{n+(\phi\cdot T)\}\ . \tag{6.5b}$$

Now we introduce higher-order effects due to the nuclear target by introducing a Δ-nucleus interaction, modifying (6.1)–(6.3) as follows:

$$\tilde{U}(r)=-\nabla\cdot[G(r)]\nabla\ , \tag{6.6}$$

$$[G(r)]=[V^{\dagger}(\mathbf{r})D^{-1}(r)V(\mathbf{r})]\ , \tag{6.7}$$

with a modified denominator

$$D(r)=D_0-W(r)\ . \tag{6.8}$$

$W(r)$ is the Δ interaction with the residual target at the point $\mathbf{r}$, which we take to be the location of a second nucleon (the first having transformed into the Δ). Again, we make a zero-range assumption by associating both nucleons with point $\mathbf{r}$. We work to first order in $W(r)$, expanding (6.1)–(6.3):

$$\tilde{U}(r)=U(r)+\Delta U(r)\ , \tag{6.9a}$$

$$\Delta U(r)=-\nabla\cdot[\Delta G(r)]\nabla\ , \tag{6.9b}$$

$$[\Delta G(r)]=[V^{\dagger}(\mathbf{r})W(r)V(\mathbf{r})]/D_0^2\ . \tag{6.9c}$$

The target expectation value in the numerator of (6.9c) is of a two-nucleon operator $V^{\dagger}(\mathbf{r})W(r)V(\mathbf{r})$, which we may treat by the methods of Secs. II and IV.

Again, we concentrate on the contribution of the neutron excess only. First, take $n=2$. We rewrite (6.5b) in terms of the projectors P_i $(i=0,1,2)$ of (4.3):

$$[V^{\dagger}(\mathbf{r})V(\mathbf{r})]_2=\gamma\Delta\rho_2(r)\{\tfrac{1}{3}P_1+P_2\}\ . \tag{6.10}$$

The right-hand side of (6.10) gives the relative contribution of the $\theta=1,2$ channels $(\pi+2N)$ in the Δ model. [Note that $\theta=0$ is forbidden, since $\theta(\Delta N)=1,2$ only.]

Turning to the numerator of (6.9c) (for $n=2$), we write

$$[V^{\dagger}(\mathbf{r})W(r)V(\mathbf{r})]_2=\gamma\rho_2(r,r)\{\tfrac{1}{3}W_1P_1+W_2P_2\}\ . \tag{6.11}$$

The coefficients W_1, W_2 give the relative strength of $W(r)$ in the two θ channels. Using (2.10) and (4.4), one may obtain the explicit form of $\Delta U(r)$.

To be specific, consider a model of pion absorption through the transition $\Delta N\rightarrow NN$. In this case, only the $\theta=1$ channel contributes. Then (6.11) becomes

$$[V^{\dagger}(\mathbf{r})W(r)V(\mathbf{r})]_2=\frac{\gamma}{3}W_1\rho_2(r,r)P_1\ . \tag{6.12}$$

The form of the optical potential is given by (6.9a) and (6.9b), with

$$[\Delta G(r)]_n=\frac{\gamma W_1}{3D_0^2}\rho_n(r,r)\{\tfrac{4}{3}T(T-\tfrac{1}{2}) -(T-\tfrac{1}{2})(\phi\cdot T)-\tilde{B}\}\ , \tag{6.13}$$

using (2.10) and (4.4). We find that the isospin structure of $\Delta U(r)$ is completely determined by assuming the Δ model, with $\Delta N\rightarrow NN$, and the form is now specified (with the zero-range assumption) up to one (complex) parameter, W_1. Since $W(r)$ is assumed to be an absorptive interaction, we require $\mathrm{Im}W_1<0$.

The form of the isotensor potential U_2 can be obtained from (6.13), using (2.9) and (3.6),

$$U_2(r)=\frac{\gamma W_1 n_F}{6n^2(n-1)D_0^2}\nabla\cdot\Delta\rho_n^2(r)\nabla\ . \tag{6.14}$$

If we compare this form with that given in (1.3), for $n=2$, we have

$$\frac{\lambda(1)}{\rho_0}=\frac{\gamma W_1 n_F}{24D_0^2}\ . \tag{6.15}$$

On resonance, $D_0^2=(i\Gamma/2)^2<0$. Thus, from (6.15), with $\mathrm{Im}W_1<0$ (above), the model predicts $\mathrm{Im}\lambda(1)>0$, consistent with the empirical findings of Greene *et al.* [See Eq. (8.3)].

Another way of interpreting the result (6.14) is to consider the potential $W(r)$ to be a linear function of the nuclear density in the form

$$W(r)=v_0\rho(r)+\frac{v_1}{n}\Delta\rho_n(r)(\phi\cdot T) \tag{6.16}$$

with isoscalar and isovector strength parameters v_0, v_1. (Note: we refer to the pion, not the Δ isospin here.) Then, expanding $\tilde{U}(r)$ as in (6.9), and keeping only the coefficient of $(\phi\cdot T)^2$, we obtain

$$U_2=\frac{-2\gamma v_1}{3n^2D_0^2}\nabla\cdot\Delta\rho_n^2(r)\nabla\ , \tag{6.17}$$

which, by comparison with (6.14), gives

$$v_1=-\frac{W_1 n_F}{4(n-1)}\ . \tag{6.18}$$

Note that the effective strength v_1 varies as $(n-1)^{-1}$, because of the correlations, as in (3.6).

VII. CORRELATED DOUBLE SCATTERING

In the fixed-scatterer (or closure) approximation, double scattering contributes a second-order term to the optical potential of the form (nonlocal)

$$U^{(2)}(\mathbf{r},\mathbf{r}') =(4\pi)^2\left[\sum_{i\neq j}f_if_j\delta(\mathbf{r}-\mathbf{x}_i)(1-P_0)\delta(\mathbf{r}'-\mathbf{x}_j)\right]G(\mathbf{r},\mathbf{r}')\ , \tag{7.1}$$

where f_i is the πN scattering amplitude, $G(\mathbf{r},\mathbf{r}')$ the pion propagator in the target, the projector $(1-P_0)$ operates on nuclear states, and removes the target ground state (and analog states) from the intermediate state spectrum, and [ ] denotes an isospin matrix in the analog space, as in (2.11). (See, e.g., Ref. 1, Sec. 4.3b). With further assumptions, this potential term becomes local (below). The πN amplitudes have the standard isospin form

$$f_i=f_0+f_1\phi\cdot\tau_i \tag{7.2}$$

with τ_i the nucleon isospin operator.

We can use the notation of Sec. II to reexpress (7.1)

$$U^{(2)}(\mathbf{r},\mathbf{r}')=(4\pi)^2[N(\mathbf{r},\mathbf{r}')]_nG(\mathbf{r},\mathbf{r}')\ , \tag{7.3}$$

where

$$N(\mathbf{r},\mathbf{r}')=\sum_{i\neq j} Q_{ij}\delta(\mathbf{r}-\mathbf{x}_i)(1-P_0)\delta(\mathbf{r}'-\mathbf{x}_j) \tag{7.4}$$

and

$$Q_{ij}=f_0^2+\tfrac{2}{3}f_1^2\tau_i\cdot\tau_j+f_0f_1\cdot\phi(\tau_i+\tau_j)+f_1^2B_{ij}\ . \tag{7.5}$$

[See Eqs. (2.1), (2.4), and (2.6).] Then, restricting consideration to the excess-neutron space (as before), we find, as in (2.8)–(2.10) (with $n=2T$)

$$[N(\mathbf{r},\mathbf{r}')]_n=\{\rho_n(\mathbf{r},\mathbf{r}')-\Delta\rho_n(r)\Delta\rho_n(r')/(2T)^2\}\times\{(f_0^2+\tfrac{2}{3}f_1^2)T(T-\tfrac{1}{2})+8f_0f_1(T-\tfrac{1}{2})\phi\cdot T+4f_1^2\tilde{B}\}\ . \tag{7.6}$$

The two terms in the curly brackets come from the two terms in $(1-P_0)$: the first term is the two-body correlation, as in (2.10) and (2.11), while the second is simply quadratic in the neutron-excess density. The isospin form (in square brackets) is identical with that of (2.10) for Q_{ij} of (7.5).

We shall assume that the range of nonlocality of the potential (7.1) or (7.3) is small compared to the nuclear size. This is only partly due to the correlation structure of (7.6), when we include shell model (pairing) correlations, which are not necessarily of short range. But since we are considering pions of resonant energies, the propagator $G(\mathbf{r},\mathbf{r}')$ should include damping due to optical absorption (this is a higher-order effect) with a mean free path of the order of 1 fm. Therefore, we replace (7.3) by the local approximation

$$U^{(2)}(r)=\int d\mathbf{r}'U^{(2)}(\mathbf{r},\mathbf{r}')=(4\pi)^2\int d\mathbf{r}'[N(\mathbf{r},\mathbf{r}')]_nG(\mathbf{r},\mathbf{r}')\ , \tag{7.7}$$

where the integral is understood to be limited to the domain of 1–2 fm about $\mathbf{r}$. The integral can be performed in the high-energy approximation, using an eikonal form for G (see Ref. 1, pp. 162–164), obtaining

$$I=\int d\mathbf{r}'\{\rho_n(\mathbf{r},\mathbf{r}')-\Delta\rho(\mathbf{r})\Delta\rho(\mathbf{r}')/(2T)^2\}G(\mathbf{r},\mathbf{r}')\simeq\frac{-i}{2k}\frac{\Delta\rho^2(r)}{(2T)^2}R_c\ , \tag{7.8}$$

where R_c (a real quantity with dimension of length) is a measure of the effective range of the integral (7.7). It is assumed that $\Delta\rho(r)$ varies slowly over that range.

A more illuminating representation of (7.8) is obtained by separating it into two terms

$$I\simeq\frac{-i}{2k}\left\{\alpha\rho_n(r,r)-\frac{\beta\Delta\rho_n^2(r)}{(2T)^2}\right\}\ , \tag{7.9}$$

where $\rho_n(r,r)$ represents the shell-model correlation (at $\mathbf{r}'=\mathbf{r}$). The coefficients α,β are positive quantities (of dimension of length) which include the optical damping. Any additional short-range correlation effect appears in α. The form (7.9) makes explicit the T dependence of (7.7), as we discuss later.

Combining terms, we write (7.7)

$$U^{(2)}(r)=(4\pi)^2I\{(f_0^2+\tfrac{2}{3}f_1^2)T(T-\tfrac{1}{2})+8f_0f_1(T-\tfrac{1}{2})\phi\cdot T+4f_1^2\tilde{B}\} \tag{7.10}$$

from which we extract the isotensor term, in two forms;

$$U_2(r)=\frac{-8\pi^2i}{k}f_1^2\frac{\Delta\rho_n^2(r)}{T^2}R_c\ , \tag{7.11a}$$

or

$$U_2(r)=\frac{-8\pi^2}{k}f_1^2\{4\alpha\rho_n(r,r)-\beta\Delta\rho_n^2(r)/T^2\}\ . \tag{7.11b}$$

Finally, using the form (3.6) for $\rho_n(r,r)$, we relate the two forms in (7.11) for the case of a single shell, by

$$R_c=\frac{\alpha n_F}{2(2T-1)}-\beta\ . \tag{7.12}$$

Since we have evaluated U_2 in a strictly local form (using the eikonal approximation), we can relate (7.11a) to the parameter $\bar{\lambda}(T)$ of (1.5), by

$$\frac{8\pi^2i}{kT^2}f_1^2R_c=\frac{k^2\bar{\lambda}(T)}{\rho_0}\ . \tag{7.13}$$

At resonance, f_1 is pure imaginary, so that (7.12) gives $\mathrm{Im}\bar{\lambda}(T)>0$ (as required by Greene *et al.*) only if $R_c<0$.

The optical potential of (7.11) is approximately equivalent to that obtained by Johnson and Siciliano,[4] who follow a somewhat different treatment of the integral I without the eikonal and forward-scattering approximations. They do, however, obtain a zero-range result similar to (7.11), with complex parameters equivalent to α,β. As discussed earlier, their treatment of the shell-model correlation function leads to (3.10) instead of our result (3.6), from which the factor $\{\ \}$ in (7.11b) takes the form

$$\left\{\frac{\alpha}{T(2T-1)}-\frac{\beta}{T^2}\right\}\Delta\rho_n^2(r)\ , \tag{7.14a}$$

or

$$R_c'=\frac{\alpha T}{(2T-1)}-\beta\ . \tag{7.14b}$$

This indeed gives the T dependence quoted in (1.4), where $\lambda_2^{(2)}\propto\alpha$ and $\lambda_4^{(2)}\propto-\beta$.

VIII. RELATION OF MODEL RESULTS TO OPTICAL ANALYSIS

We now connect the models of the previous sections to the isotensor potential determined by Greene *et al.* As we discussed in the Introduction, the optical-model analysis of Greene *et al.* fits free parameters to a large data set for 164-MeV pions. We are most interested in the coefficient λ_G of (1.5), which fixes the strength of the isotensor potential U_2 of (1.2) for $T=1$ targets. As mentioned earlier, $\lambda_2^{(2)}$ is fit to data. Using two different sets of assumptions, these authors calculate two different values of $\lambda_4^{(2)}$, and therefore obtain two different fits to $\lambda_2^{(2)}$, or to λ_G (all in fm^3):

$$\lambda_2^{(2)}=1.66+10.8i \text{ fm}^3 ,$$
$$\lambda_4^{(2)}=2.89-1.13i \text{ fm}^3 , \quad (8.1)$$
$$\lambda_G=4.55+9.7i \text{ fm}^3 ,$$

and

$$\lambda_2^{(2)}=-5.5+8.3i \text{ fm}^3 ,$$
$$\lambda_4^{(2)}=5.3+2.6i \text{ fm}^3 , \quad (8.2)$$
$$\lambda_G=-0.2+10.9i \text{ fm}^3 .$$

It is clear that $\mathrm{Re}\lambda_G$ is not well determined, and in any case, smaller than $\mathrm{Im}\lambda_G$, which is consistently fit by these two procedures, to an average value of

$$\mathrm{Im}\lambda_G=10.3 \text{ fm}^3 . \quad (8.3)$$

The actual values of λ_G are, of course, sensitive to all the other fitting assumptions: densities, isoscalar potential strengths, etc. However, the sign and order of magnitude of (8.3) are more or less fixed by the interference in the DCX angular distribution.

Each of the three models discussed in Secs. V–VII is characterized by one free complex parameter, which can be chosen to give a positive value of $\mathrm{Im}\lambda_G$. For the local models we shall simply assume that $\bar{\lambda}_G=\lambda_G$, which is equivalent to assuming that the local potential (1.6) gives the same contribution to DCX as the nonlocal one of (1.3). This is a good first approximation for forward scattering, for which the parameter fitting was done. A better treatment would require a refitting of $\bar{\lambda}_G$ using (1.6). Only one component of the free parameter for each model is determined by the value of $\mathrm{Im}\lambda_G$ or $(\mathrm{Im}\bar{\lambda}_G)$ given by (8.3). As seen above, the value of $\mathrm{Re}\lambda_G$ is not well determined; for what follows we shall take $\mathrm{Re}\lambda_G\equiv 0$. Then we can fix the value of the undetermined parameter for each model in turn, assuming it gives the total contribution to λ_G.

For the direct s-wave absorption model, we found in (5.3):

$$\frac{-A_2 n_F}{2k^2}=\frac{\bar{\lambda}_G}{\rho_0} , \quad (8.4)$$

where A_2 is a free parameter.

For the delta absorption model, (6.15) gives

$$\frac{\gamma W_1 n_F}{24 D_0^2}=\frac{\lambda_G}{\rho_0} , \quad (8.5)$$

with W_1 the free parameter (with $\mathrm{Im}W_1<0$). Lastly, for correlated double scattering, using (7.13) with $T=1$,

$$\frac{8\pi^2 i}{k^3} f_1^2 R_c=\frac{\bar{\lambda}_G}{\rho_0} , \quad (8.6)$$

with R_c the free parameter.

To match coefficients to (8.3), we use $\rho_0=0.16 \text{ fm}^{-3}$ (Greene), and $k=1.37 \text{ fm}^{-1}$ (laboratory momentum), to get for (8.4)

$$\mathrm{Im}(A_2 n_F)=-241 \text{ fm}^4 , \quad (8.7)$$

where n_F is the number of neutrons in the (assumed) filled valence shell. [Note the constraint of (5.6).] This value fixes the size of the absorption cross section for the s-wave absorption model, as discussed in the following section.

For the delta absorption model, we evaluate (8.5) at 164 MeV, using $\omega_R=268$ MeV, $\Gamma=105$ MeV, and $D_0=(-0.321+i)\Gamma/2$, which yields

$$\mathrm{Im}(W_1 n_F)=-23.3 \text{ fm}^2 . \quad (8.8)$$

This sets the scale of the absorption cross section for the delta model, as discussed in Sec. IX, Eq. (9.5) and following. We can also use (8.8) to fix the strength (6.18) of the imaginary part of the isovector Δ-nucleus potential (6.16), (6.18), taking $n=2$:

$$\mathrm{Im}v_1=5.83 \text{ fm}^2 . \quad (8.9)$$

Combining (6.3), (6.8), and (8.9), we can write (6.16) as a density-dependent correction to the delta width Γ of the form

$$\Delta\Gamma=\mathrm{Im}(v_1\rho_0)(\Delta\rho_n/\rho_0)$$
$$=0.933 \text{ fm}^{-1} (\Delta\rho_n(r)/\rho_0) . \quad (8.10)$$

For $\Delta\rho/\rho_0=0.1$ as in Ref. 4, this gives $\Delta\Gamma=18.4$ MeV.

Last, for the correlated double scattering model, we use the resonant formula for the forward (laboratory) isovector scattering amplitude

$$kf_1=-\frac{2}{3}\frac{\Gamma}{D_0}\frac{k}{\kappa}=0.502+1.562i \quad (8.11)$$

to evaluate (8.6). Then, with (8.3), we find

$$R_c=(-1.188-0.852i) \text{ fm} . \quad (8.12)$$

Note that R_c is complex; it would be real exactly at resonance (for $\mathrm{Re}\lambda_G\equiv 0$). The order of magnitude of R_c (1 fm) is not unreasonable for a nuclear correlation. We cannot choose among the three models without further information given in the following section.

IX. FURTHER CONSEQUENCES OF THE MODEL RESULTS

We now discuss some specific features of the three models which make possible further tests of the dynamics of the isotensor potential. First, consider the two absorption models of Secs. V and VI which have the following aspects in common: (1) They imply a connection between the isotensor potential U_2 and the cross section $\sigma_a(n)$ for absorption of a pion on the excess neutrons. The direct and delta models have different connections, as we shall show. (2) Both models predict a T dependence of U_2 as given in (3.9).

For the direct absorption model, the absorption cross section $\sigma_a(n)$ is given by

$$\sigma_a(n)=-k^{-1}\int \mathrm{Im}\langle U_a(\mathbf{r})\rangle|\psi^{(+)}(\mathbf{r})|^2 d^3r , \quad (9.1)$$

where $\langle U_a\rangle$ is the isospin expectation of the potential given by (5.1), and $\psi^{(+)}$ is the elastic optical wave. [See

Ref. 5: Note that in the present paper, U is defined by (1.1), i.e., $U=2\omega V$, where V is the potential of Ref. 5.] We wish to compare this to $\sigma_c^{(2)}$, a closely related quantity introduced in Ref. 5, which enters into the inequality (18) of that paper. In our present notation, dropping the superscript (2) for clarity,

$$\sigma_c=-k^{-1}\int \mathrm{Im}U_2(r)\langle(\phi\cdot T)^2\rangle|\psi^{(+)}(\mathbf{r})|^2d^3r\ . \tag{9.2}$$

This quantity defines the contribution to the reaction cross section due to the isotensor potential alone. To compare the two, consider the ratio, which may be obtained from (2.9) and (2.10):

$$\frac{\sigma_a(n)}{\sigma_c}=\frac{\mathrm{Im}[4A_0T(T-\frac{1}{2})+8A_1(T-\frac{1}{2})\langle\phi\cdot T\rangle+4A_2\langle\tilde{B}\rangle]}{4\,\mathrm{Im}A_2\langle(\phi\cdot T)^2\rangle}\ . \tag{9.3}$$

For example, for a $T=1$ target with absorption only in the $\theta=0$ mode, given by (5.7), for a pion of charge c,

$$\frac{\sigma_a(2)}{\sigma_c}=\frac{[-1+\langle c|(\phi\cdot T)^2c\rangle]}{\langle c|(\phi\cdot T)^2c\rangle}\ . \tag{9.4}$$

For $c=1$ (π^+), $\langle c|(\phi\cdot T)^2|c\rangle=2$, so that

$$\sigma_1=2\sigma_a(2) \tag{9.5}$$

in this case. (See Ref. 5, footnote 13.)

For the delta absorption model, the connection of σ_c and σ_a is less direct than (9.3) or (9.4), because of interference effects in $\Delta U(r)$ of (6.9). In this case, the absorption cross section can be shown to be

$$\sigma_a(n)=k^{-1}\int \mathrm{Im}\langle V^\dagger(\mathbf{r})W(r)V(\mathbf{r})]\rangle\left|\frac{\nabla\psi(\mathbf{r})}{D_0}\right|d^3r\ . \tag{9.6}$$

In contrast, the change in reaction cross section due to ΔU is given by

$$\Delta\sigma\equiv k^{-1}\int \mathrm{Im}\langle\Delta G(r)\rangle|\nabla\psi(\mathbf{r})|^2d^3r\ , \tag{9.7}$$

where $\Delta G(r)$ is given in (6.13). The cross section σ_c is defined to be only the U_2 contribution to $\Delta\sigma$, i.e.,

$$\sigma_c=k^{-1}\int \mathrm{Im}U_2(r)\langle(\phi\cdot T)^2\rangle|\nabla\psi(\mathbf{r})|^2d^3r\ . \tag{9.8}$$

In all three expressions, we have integrated by parts to obtain the $|\nabla\psi|^2$ forms.

The ratio of (9.6) to (9.7) is easily seen to be (for W_1 pure imaginary)

$$\frac{\sigma_a(n)}{\Delta\sigma}=[|D_0|^2\mathrm{Re}(D_0^{-2})]^{-1}=\left[1-\frac{\Gamma^2}{2|D_0|^2}\right]^{-1}, \tag{9.9}$$

which goes from a value of 1 far off resonance, to a value of -1 at resonance (where $\Delta\sigma$ is negative). The ratio of (9.7) to (9.8) is given by the isospin ratio, as in (9.3), but using (6.13) for this model:

$$\frac{\Delta\sigma}{\sigma_c}=\frac{[\frac{4}{3}T(T-\frac{1}{2})-(T-\frac{1}{2})\langle(\phi\cdot T)\rangle-\langle\tilde{B}\rangle]}{-\langle(\phi\cdot T)^2\rangle}\ . \tag{9.10}$$

For a $T=1$ target, this ratio becomes

$$\frac{\Delta\sigma}{\sigma_c}=\frac{2-\langle c|\phi\cdot T|c\rangle-\langle c|(\phi\cdot T)^2|c\rangle}{-\langle c|(\phi\cdot T)^2|c\rangle}\ . \tag{9.11}$$

For π^+ $(c=1)$, $\langle(\phi\cdot T)\rangle=-1$, $\langle(\phi\cdot T)^2\rangle=2$, so that $2\Delta\sigma=-\sigma_1^{(2)}$. Finally, the relation of σ_a to σ_c for $T=1$ becomes

$$\sigma_1=-2\sigma_a(2)\left[1-\frac{\Gamma^2}{2|D_0|^2}\right]\ . \tag{9.12}$$

The result for other T is obtained immediately from (9.8) and (9.9). This result for the delta model $(\theta=1)$ may be compared to that of (9.5) for the direct absorption model with $\theta=0$. Note that σ_a is always positive (as a true cross section for absorption), while σ_c is not.

We turn now to the model of correlated double scattering (Sec. VII). We noted following (7.13) that at resonance, to obtain $\mathrm{Im}\lambda_G>0$ requires $R_c<0$ [or $\mathrm{Re}R_c<0$, near resonance, as in (8.12)]. This means that the β term of (7.12) must exceed the α term, which would be the case if the repulsive anticorrelation is sufficiently strong at short distance. It is not clear that this should be the case, since the repulsion competes with the attractive correlation due to pairing, as in (3.6). In fact, the model calculation of double scattering by Johnson and Siciliano shows the problem: they find (Table I, Ref. 4, first paper, 180 MeV)

$$\begin{aligned}\lambda_2&=-3.64-8.01i\ ,\\ \lambda_4&=3.45+5.52i\ ,\end{aligned} \tag{9.13}$$

which results in $\mathrm{Im}\lambda_G<0$, even with a repulsive correlation, and *no* pairing effect [since they use (7.14), not (7.12)]. This suggests that the correlated double scattering is not likely, by itself, to explain the origin of isotensor potential.

In principle, the contribution of correlated double scattering to U_2 should also be measurable as a cross section (σ_c of Ref. 5). In this case, the nonelastic reaction would be the double knockout, $(\pi,\pi'2N)$. Since we are assuming that short-range effects dominate (zero-range approximation), one might expect the relevant kinematic region to be $\mathbf{p}_1+\mathbf{p}_2\cong\mathbf{q}$, where $\mathbf{p}_1,\mathbf{p}_2$ are the recoil nucleon momenta, and $\mathbf{q}=\mathbf{k}-\mathbf{k}'$ the momentum transfer of the pion.

A further point is the T dependence: if the β term (λ_4) dominates, the dependence will be αT^{-2}, rather than $T^{-2}(2T-1)^{-1}$, as in (3.9), which is predicted for the absorption models. The consequences are discussed in the following section.

X. CONCLUSIONS

We have proposed three models for channel coupling as a dynamical mechanism for the isotensor term U_2 of the pion optical potential. Each of the models can give the value of the phenomenological (complex) parameter λ_G, by the adjustment of one parameter, as we have shown in Sec. VIII. For the two absorption models, the parameters will necessarily give $\mathrm{Im}\lambda_G > 0$, as required; for the correlated double scattering model, this condition imposes a restriction on the correlation length, $\mathrm{Re}R_c < 0$. However, it is not immediately clear that the magnitudes of these parameters are physically reasonable, that is, how much the proposed processes actually do contribute to DCX through U_2.

The method discussed in Ref. 5 leads to a somewhat model independent test for the case of absorption, by use of the inequality (18),

$$\sigma_c \geq \frac{4\pi}{k}\mathrm{Im}\Delta f_{cc} \tag{10.1}$$

in which the quantity defined in (9.2) is compared to the DCX amplitude (up to a factor) contributed by U_2 (to lowest order). In Ref. 5, we found for the case of ^{18}O, that the right-hand side of (10.1) is 4.8 mb for the calculations of Greene *et al.* An estimate of absorption gives $\sigma_a \simeq 1.5-1.8$ mb. In Sec. IX we have derived the relation of σ_1 to σ_a for the two absorption models. The direct absorption model (s wave) gives $\sigma_1 = 2\sigma_a \simeq 3.0-3.6$ mb, from (9.5), which is a substantial part of the right-hand side of the inequality (10.1). For the Δ model, the ratio is given by (9.12), which at 164 MeV gives $\sigma_1 = 1.626\sigma_a \simeq 2.4-2.9$ mb. [See Eq. (8.8).] So, at least for the case of ^{18}O, it is not unreasonable to conclude that a substantial fraction of ($\sim\frac{1}{2}$) of the isotensor contribution to DCX may be due to absorption, based on either model. We expect that, e.g., correlated double scattering supplies the rest of the inequality.

However, on other physical grounds, we expect the Δ absorption to be suppressed relative to the s-wave absorption, at least at short range. The reason is that the ΔN s wave does not contribute to absorption, because of selection rules. Therefore there should be little effect in the zero-range NN limit; the contribution would be zero, except to Δ recoil.

We have already argued that correlated double scattering give a reasonable order of magnitude effect, as seen in (8.12). However, here it is least clear that the zero-range assumption is justified. This effect seems to require a delicate balance between the attractive effect of pairing correlations (in α) and the repulsive effect of short-range correlations (in β) in (7.9) and (7.11b). A better treatment of this mechanism is probably required.

That brings us to the limitations of the present work. We have taken the zero-range assumption from the beginning, and restricted the nuclear structure to a single j^n-shell model of the neutron excess. These approximations did allow us to compare our results directly with those of Greene *et al.*, as shown in Sec. III. They also lead to the simple T dependence predicted for absorption models, in (3.9), and for correlated double scattering, in (7.11). For finite range, as might obtain in more realistic treatment of these mechanisms, the changes are not so great for nuclei for which the j^n and good seniority assumptions are valid. For these cases, the nonzero range introduces a finite "monopole" contribution to the correlation, which vanishes in the zero-range limit. The correction will have the "monopole" T dependence of (3.11). (The monopole and zero-range terms correspond to the A,B terms of Ref. 9.) The j^n model for nuclear structure is probably not adequate for all aspects of the DCX problem, and should be improved by including configuration mixing. One would expect this to change the spatial dependence of U_2, both through changes of the form [e.g., in (3.7)], and in the densities themselves. The T dependence, however, is less likely to change, as long as the effective seniority scheme does not break down. The extension to include valence protons can also be made, although the forms are more complex.

The question of T dependence is under further investigation, to see that the effects are predicted to be under different assumptions given in this paper. The calcium isotopes seem obvious candidates for study, but angular distributions at resonant energies are not measured. A prediction of the zero-range models is that the interference minimum should change with T for different isotopes.

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APPENDIX: CORRELATION FUNCTIONS FOR SINGLE j SHELL OF NEUTRONS

Write the single-particle wave function for the orbit $\{nljm\}$ in the coupled form

$$\phi_m(\mathbf{r})\rangle = \sum_{M\mu}(l\tfrac{1}{2}M\mu|jm)\psi_M(\mathbf{r})|\mu\rangle \tag{A1}$$

with

$$\psi_M(\mathbf{r}) = R_{nl}(r)Y_{lM}(\hat{r})$$

and $|\mu\rangle$ a spinor ($\mu = \pm\frac{1}{2}$).

For a filled shell, $n = n_F = (2j+1)$, the neutron density is easily obtained

$$\begin{aligned}\Delta\rho_F(r) &= \sum_m \langle \phi_m^\dagger(\mathbf{r})\phi_m(\mathbf{r})\rangle \\ &= \sum_{\mu,m}\sum_M (l\tfrac{1}{2}M\mu|jm)^2|\psi_M(\mathbf{r})|^2 \\ &= \frac{(2j+1)}{(2l+1)}\sum_M |\psi_M(\mathbf{r})|^2 \,.\end{aligned} \tag{A2}$$

The correlation function is directly calculated from (2.11), for $\mathbf{r} = \mathbf{r}'$, for a simple antisymmetric state:

$$n_F(n_F-1)\rho_F(r,r)=\sum_{m\neq m'}\langle\phi_m^\dagger(\mathbf{r})\phi_{m'}^\dagger(\mathbf{r})\{\phi_m(\mathbf{r})\phi_{m'}(\mathbf{r})\rangle-\phi_{m'}(\mathbf{r})\phi_m(\mathbf{r})\rangle\}$$
$$=\sum_{m,m'}\{\langle\phi_m^\dagger(\mathbf{r})\phi_m(\mathbf{r})\rangle\langle\phi_{m'}^\dagger(\mathbf{r})\phi_{m'}(\mathbf{r})\rangle-\langle\phi_m^\dagger(\mathbf{r})\phi_{m'}(\mathbf{r})\rangle\langle\phi_{m'}^\dagger(\mathbf{r})\phi_m(\mathbf{r})\rangle\}\ . \tag{A3}$$

This is simply the sum of two-body matrix elements in orbits (m,m') of the operator in (2.11), written in the conventional *direct* minus *exchange* form. The first term equals $\Delta\rho_F^2(r)$, from (A2). The second term may be evaluated as follows:

$$\sum_m\phi_m(r)\rangle\langle\phi_m^\dagger(r)=\sum_m\sum_{\substack{\mu'M'\\ \mu M}}(l\tfrac{1}{2}M\mu|jm)(l\tfrac{1}{2}M'\mu'|jm)\psi_M(\mathbf{r})\psi_{M'}^*(\mathbf{r})|\mu\rangle\langle\mu'|\ . \tag{A4}$$

It follows from the reflection relations:

$$(l\tfrac{1}{2}M\mu|jm)(l\tfrac{1}{2}M'\mu'|jm)=(l\tfrac{1}{2}-M-\mu|j-m)(l\tfrac{1}{2}-M'-\mu'|j-m)\ , \tag{A5a}$$

and

$$\psi_M(\mathbf{r})\psi_{M'}^*(\mathbf{r})=(-1)^{M-M'}\psi_{-M}^*(\mathbf{r})\psi_{-M'}(\mathbf{r})\ , \tag{A5b}$$

that the coefficient of $|\mu\rangle\langle\mu|$ in (A4) is

$$\langle\mu|\phi_m(\mathbf{r})\rangle\langle\phi_m^\dagger(\mathbf{r})|\mu\rangle=\sum_{mM}(l\tfrac{1}{2}M\mu|jm)^2|\psi_M(\mathbf{r})|^2=\sum_{m'M'}(l\tfrac{1}{2}M'-\mu|jm')^2|\psi_{M'}(\mathbf{r})|^2\ , \tag{A6a}$$

that is, independent μ. Therefore

$$\langle\mu|\phi_m(\mathbf{r})\rangle\langle\phi_m^\dagger(\mathbf{r})|\mu\rangle=\tfrac{1}{2}\sum_{\mu mM}(l\tfrac{1}{2}M\mu|jm)^2|\psi_M(\mathbf{r})|^2=\tfrac{1}{2}\Delta\rho_F(r)\ . \tag{A6b}$$

By a similar argument, using (A5), the coefficient of $|\mu\rangle\langle-\mu|$ in (A4) vanishes. Then (A3) becomes

$$\rho_F(r,r)=[n_F(n_F-1)]^{-1}\{\Delta\rho_F^2(r)-\tfrac{1}{2}\Delta\rho_F^2(r)\}=\Delta\rho_F^2(r)/2n_F(n_F-1)\ , \tag{A7}$$

which gives Eq. (3.4).

The correlation function (at $\mathbf{r}=\mathbf{r}'$) for the $n=2$ paired state is calculated directly from the $(J=0)$ coupled wave function

$$\psi(r,r')=\sum_m\phi_m(\mathbf{r})\phi_{-m}(\mathbf{r})\rangle(-1)^{j-m}/\sqrt{2j+1}\ ,$$
$$=\sum_{M\mu m}(l\tfrac{1}{2}M\mu|jm)(l\tfrac{1}{2}-M-\mu|j-m)(-1)^{j-m}\psi_M(\mathbf{r})\psi_{-M}(\mathbf{r})|\mu,-\mu\rangle/\sqrt{2j+1}\ . \tag{A8a}$$

(Note that only $S=0$ contributes for $\mathbf{r}=\mathbf{r}'$.) Then using the reflection properties of the Clebsch-Gordan coefficients and spherical harmonics for $-M\to M$, etc., it can be shown that

$$\psi(r,r)=\sum_\mu|\mu,-\mu\rangle\frac{(-1)^{l+1/2-\mu}}{\sqrt{2j+1}}\sum_{Mm}(l\tfrac{1}{2}M\mu|jm)^2|\psi_M(\mathbf{r})|^2$$
$$=\sum_\mu|\mu,-\mu\rangle(-1)^{l+1/2-\mu}\Delta\rho_F(r)/2\sqrt{2j+1}\ , \tag{A8b}$$

where we use (A6b). Then, using $n_F=2j+1$, we find

$$\rho_2(r,r)=\langle\psi^\dagger(r,r)\psi(r,r)\rangle$$
$$=\Delta\rho_F^2(r)/2n_F\ . \tag{A9}$$

The two forms given in (3.5) follow immediately from (A9), (A7), and (3.2).

The n dependence of (3.6) may be obtained by calculating the expectation value of a δ-function interaction in the n even, $v=0$ ground state [see (2.11)]:

$$\langle\delta\rangle_n\equiv\left\langle\tfrac{1}{2}\sum_{i\neq j}\delta(\mathbf{x}_i-\mathbf{x}_j)\right\rangle_n$$
$$=\frac{n(n-1)}{2}\int d\mathbf{r}\,\rho_n(r,r)\ . \tag{A10}$$

The result can be expressed as follows [see Ref. 10, Eqs. (20.34), (28.63), and (28.66)]:

$$\langle\delta\rangle_n=\frac{n}{2}V_0+\frac{n(n-2)}{2(2j-1)}(E_0-V_0)\ , \tag{A11}$$

where

$$E_0=\frac{2}{(2j+1)}\sum_J(2J+1)V_J \tag{A12}$$

and V_J are the two-body matrix elements in states of (j^2,J), which for the δ function give

$$V_J=A\begin{bmatrix}j & j & J\\ \frac{1}{2} & -\frac{1}{2} & 0\end{bmatrix}^2 \tag{A13}$$

with A independent of J (J even throughout). Summing (A12) gives immediately

$$E_0 = V_0 = A/(2j+1) . \tag{A14}$$

Evaluating (A10) using (A11) and (A14) yields

$$\int d\mathbf{r}\,\rho_n(r,r) = V_0/(n-1) . \tag{A15}$$

This, with (3.3) and (3.4) leads to (3.6).

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[1]See, e.g., J. M. Eisenberg and D. S. Koltun, *Theory of Meson Interactions with Nuclei* (Wiley, New York, 1980), Chaps. 4 and 5.

[2]The isotensor term for pions was first discussed by M. Ericson and T. E. O. Ericson, Ann. Phys. (N.Y.) **36**, 323 (1966).

[3]S. J. Greene, C. J. Harvey, P. A. Seidl, R. Gilman, E. R. Siciliano, and M. B. Johnson, Phys. Rev. C **30**, 2003 (1984).

[4]M. B. Johnson and E. R. Siciliano, Phys. Rev. C **27**, 730 (1983); **27**, 1647 (1983).

[5]D. S. Koltun and M. K. Singham, Phys. Rev. C **49**, 704 (1989).

[6]E. Oset, D. Strottman, M. J. Vicente-Vacas, and Ma Wei-Hsing, Nucl. Phys. **A408**, 461 (1983).

[7]M. B. Johnson, E. R. Siciliano, H. Toki, and A. Wirzba, Phys. Rev. Lett. **52**, 593 (1984).

[8]E. Oset, M. J. Vicente-Vacas, M. B. Johnson, D. Strottman, H. T. Fortune, and R. Gilman, Nucl. Phys. **A483**, 514 (1988).

[9]N. Auerbach, W. R. Gibbs, J. N. Ginocchio, and W. B. Kaufmann, Phys. Rev. C **38**, 1277 (1988).

[10]A. de-Shalit and I. Talmi, *Nuclear Shell Theory* (Academic, New York, 1963).