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Flow effects on transverse momentum spectra in ultrarelativistic nuclear collisions

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We discuss fits of the Landau-Milekhin model to the transverse momentum spectra measured in 200 GeV/c nucleus-nucleus collisions. It is observed that the data are fit by a range of anticorrelated values of the breakup temperature and the average transverse hydrodynamic velocity. These fits indicate that a better understanding of transverse flow in ultrarelativistic nuclear collisions is required to uniquely determine the breakup temperature of the system.

In this paper, we discuss transverse momentum spectra from collisions of ultrarelativistic heavy ions performed at CERN. The transverse momentum spectrum is of interest because one can deduce from it the breakup temperature T_k and possibly the critical temperature for the confinement phase transition. It is also speculated that the dependence of the average transverse momentum $\langle p_t \rangle$ on the number of charged particles per unit rapidity may prove to be a signature of the quark-gluon plasma.[1,2] If collective flow of the excited system does occur, one must consider the possibility that the transverse coordinates, sooner or later, also exhibit this flow.

The stage at which the system actually acquires these trasnverse degrees of freedom is important, because the "geometry" of the flow crucially affects the transverse spectra. There are two simple geometries which may be considered in this context. One of these geometries involves a homologous, spherically expanding fireball. A model considering such an expansion of a hadronic resonance gas in thermal and chemical equilibrium was investigated recently and good fits to the inclusive spectrum were obtained.[3] However, at ultrarelativistic energies the nuclei are compressed greatly along the beam axis in the center of mass system. Therefore the more likely geometry is that of an initial one-dimensional expansion along the beam axis followed by a three-dimensional stage including transverse expansion just prior to breakup. One such model was proposed initially by Landau[4] in the context of multiple production processes in hadron-hadron collisions. In the following discussion, we shall consider transverse flow within the framework of the Landau model.

The Landau hydrodynamic model proved very successful in predicting the rapidity and transverse momentum distributions of secondaries in hadron-hadron collisions for a wide range of energies. However, despite its phenomenological success, the model did not gain wide acceptance. This is so because some of the model's initial conditions are believed to be incompatible with recent ideas in high-energy physics.[5] Interestingly, many of these conceptual difficulties with the hydrodynamic model are resolved when the model is extended to discuss ultrarelativistic nucleus-nucleus collisions. For example, in experiments of ultrarelativistic nuclear collisions currently being performed at CERN (Ref. 6) and Brookhaven (Ref. 7), one observes (i) the production of a large number of secondaries and (ii) significant stopping of the projectile nuclei. Both (i) and (ii) are presumed by the model. Since we are primarily interested in the predictions of the hydrodynamic model we shall not discuss the conceptual implications of these observations any further but refer interested readers to Ref. 5 and 8.

The Landau hydrodynamical model was refined and generalized to nucleon-nucleus and nucleus-nucleus collisions by Milekhin.[9] We shall henceforth refer to the hydrodynamic model as the Landau-Milekhin model. This model has already been successfully applied to the study of transverse energy distributions from the CERN and Brookhaven experiments.[7] Furthermore, the model predicts a Gaussian shape to the rapidity distributions which is also observed in the CERN data.[10] We now apply the Landau-Milekhin model to the study of the transverse momentum spectra from these experiments.

It is generally believed that the transverse momentum spectrum at low p_t uniquely specifies the breakup temperature. However, this is only partially true since one has to account for the transverse flow or "Doppler widening" of the spectrum. In Landau's original formulation of the hydrodynamic model, the initial one-dimensional stage was fitted smoothly to the subsequent three-dimensional flow. Milekhin refined this procedure considerably by treating the subsequent evolution of the system by the method of characteristics. In his approach, the initial characteristic surface for the three-dimensional stage is parametrized in terms of the temperature and velocity at the end of the one-dimensional stage. This surface then propagates inward from the surface of the cylinder towards the center. The motion of the cylinder becomes wholly three-dimensional when the hypersurface reaches the center of the cylinder. The problem of three-dimensional flow with the Landau initial conditions was then solved numerically by Milekhin and he obtained an average hydrodynamic transverse velocity, $\langle v_\perp \rangle = \langle \tanh\xi \rangle$, where

$$\langle \sinh\xi \rangle = 0.53 \left[\frac{n+1}{2} \right]^{2/5} \left[\frac{E_{\rm lab}}{2m} \right]^{1/14} \times \exp\left[-\frac{(y_{\rm lab}-y_c)^2}{6L} \right] . \quad (1)$$

The quantity n is the ratio of the contracted target radius to the contracted projectile radius, $E_{\rm lab}$ is the projectile energy per nucleon, y is the rapidity, and $L=3\ln(T_i/T_k)$. T_i is the initial temperature at the start of the hydrodynamic expansion and $y_c=\ln\gamma(1+\beta)$, where γ is the Lorentz factor and β is the relative velocity between the laboratory and center of mass frames. The y_c are determined from fits to the rapidity data and are very close to the numbers predicted by the fireball kinetics.[11] The procedure adopted by Milekhin involved certain approximations[12] which have been criticized. A more accurate calculation making use of finite difference methods was performed by Andersson[13] in the context of hadron-hadron collisions but, to our knowledge, did not result in significantly more accurate formulas for the transverse hydrodynamic velocity. One may note that the coupling of the average transverse hydrodynamic velocity to the longitudinal degrees of freedom is very weak. If we then set the exponent in Eq. (1) to unity, for $E_{\rm lab}=200$ GeV/nucleon and $n=3.3$, we find $\langle v_\perp\rangle\sim 0.7$ in $c=1$ units.

The inclusive distribution, in the Landau-Milekhin model, of the thermalized secondaries at breakup is then given by the expression

$$F(y,p_t)=N_0F_1(y)F_2(y,p_t)\ , \tag{2}$$

where N_0 is the multiplicity of produced secondaries.

The function $F_1(y)$ represents the collective flow of the elements of the medium and the normalized $F_1(y)$ distribution can be expressed in the lab frame of the particles as

$$F_1(y)=\frac{1}{\sqrt{2\pi L}}\exp\left[-\frac{(y_{\rm lab}-y_c)^2}{2L}\right]. \tag{3}$$

Above L depends on the initial size of the system (or equivalently, on the initial temperature distribution). It can also be determined from fits to the experimental rapidity and pseudorapidity distributions. For instance, from the NA35 rapidity data for the S+S reaction, we obtain an excellent fit with $L=2$. Thus if one knows the breakup temperature T_k and the variance of the rapidity distribution, one should be able to estimate the initial temperature of the hydrodynamic expansion. For this particular reaction, since breakup temperatures vary between 70–120 MeV (this will be established later on), one can expect the initial temperature T_i to range from 135–235 MeV.

The function $F_2(y,p_t)$, on the other hand, represents the internal or thermal degrees of freedom. It is obtained by Doppler shifting the thermal distribution in the direction transverse to the direction of one-dimensional flow. One obtains,

$$F_2(y,p_t)=\frac{p_t}{T_k}\frac{m_t}{m}\frac{\displaystyle\sum_{n=1}^{\infty}(\mp 1)^{n-1}\left[\frac{m_t}{m}\langle\cosh\xi\rangle K_1(A)I_0(B)-\frac{p_t}{m}\langle\sinh\xi\rangle I_1(B)K_0(A)\right]}{\displaystyle\sum_{n=1}^{\infty}(\mp 1)^{n-1}K_2(nm/T_k)/n}\ . \tag{4}$$

Here, m is the mass of the secondary, m_t is the transverse mass, $A=n(m_t/T_k)\langle\cosh\xi\rangle$, $B=n(p_t/T_k)\langle\sinh\xi\rangle$, and K_i and I_i are the modified Bessel functions; (∓ 1) stands for Fermi (-1) and Bose $(+1)$ statistics.

The p_t distributions are generated by integrating the distribution function $F(y,p_t)$ over the range specified by the rapidity cuts y_1 and y_2 for central collisions. They take the form

$$\frac{dN}{dp_t^2}=\frac{1}{2m_tp_t}\int_{y_1}^{y_2}F_1(y)F_2(y,p_t)dy\ . \tag{5}$$

All the parameters on the right-hand side of the above equation, with the exception of the temperature T_k and the average transverse hydrodynamic velocity $\langle v_\perp\rangle=\langle\tanh\xi\rangle$ can be determined either from the fireball kinetics or from rapidity distributions. As a consequence of Milekhin's averaging over the radial coordinates in the three-dimensional stage, his expression for the average transverse velocity is rather approximate. Indeed, he estimates an error of $\sim 20\%$ (see Ref. 9) in Eq. (5). To account for possible variations in the magnitude of the average transverse velocity, we shall use

$$\langle\sinh\xi\rangle\rightarrow\langle\sinh\xi'\rangle=\lambda\langle\sinh\xi\rangle\ , \tag{6}$$

where λ is a multiplicative constant. We then fit (5) to the CERN p_t data varying values of T_k and λ with m as the pion mass. (In the CERN experiments the transverse momentum data refers to all negatively charged particles. The bulk of the produced secondaries are pions with contaminations from other negatively charged particles at the level of 10%.[14])

The results of our fits to the preliminary transverse momentum data of the NA35 (Ref. 10) group for the O+Au reaction at 200 GeV/nucleon and of the NA34 (Ref. 15) group for the O+W reaction at the same energy are shown in Figs. 1 and 2, respectively. We show here global fits of rms$\sim$30% up to $p_t\sim 1.75$ GeV for the NA35 data and up to $p_t\sim 2.4$ GeV for the NA34 data. In these fits, $T_k=80(110)$ MeV and $\lambda=0.9$ (0.7) for the NA35 (NA34) data. It is somewhat surprising that values of T_k and λ for the NA35 and NA34 data do not overlap although the beam and the energy per nucleon are the same for both cases. The quality of fit in Fig. 1 is decidedly better than the fit in Fig. 2 where one observes an experimental excess at both very low and high p_t. It may be argued that the rapidity cuts for the two experiments are different. However, in the model, the dependence of $F_2(y,p_t)$ on y is slight. One can therefore, to a good approximation, take $F_2(y,p_t)$ outside the integral in Eq. (5). The y integral then becomes the difference of two error functions which depend on the rapidity cuts y_1 and

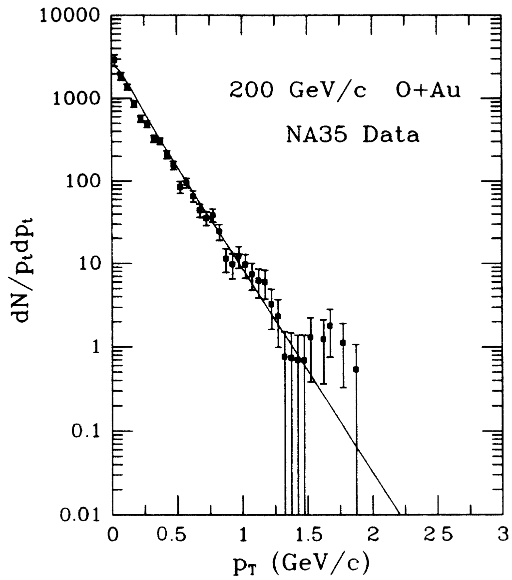

FIG. 1. Calculated pion transverse momentum spectrum, Eq. (5), compared with the experimental data (all negatives) for central events in the rapidity range $2<y_{\rm lab}<3$. The calculations are for a temperature $T_k=80$ MeV and $\lambda=0.9$.

y_2. This implies that the profile of the transverse momentum spectra in the Landau-Milekhin model is nearly independent of the rapidity cuts. If the preliminary data of the N35 collaboration at low p_t change[14] significantly we may have to look to sources outside the Landau-Milekhin model to explain the discrepancy at both low p_t and high p_t.

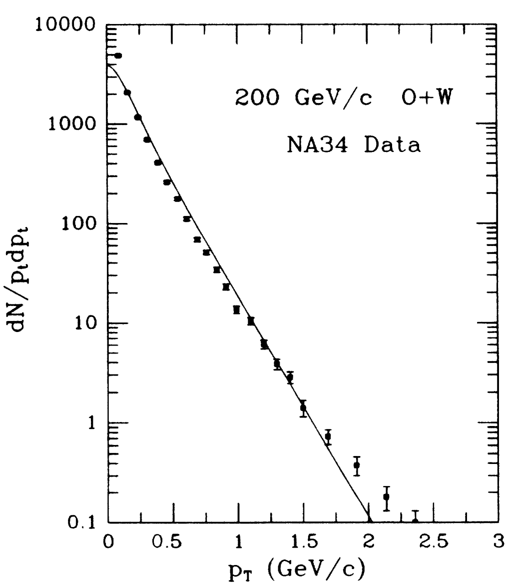

FIG. 2. Calculated pion transverse momentum spectrum, Eq. (5), compared with the experimental data (all negatives) for central events in the rapidity range $0.8<y_{\rm lab}<2$. The calculations are for a temperature $T_k=110$ MeV and $\lambda=0.7$.

TABLE I. Values of temperature T_k (in MeV) and the multiplicative constant λ [in Eq. (6)] that best fit (rms $\sim 30\%$) the transverse momentum data using Eq. (5).

T_k	λ	rms (%)
	NA35 data	
70	1.0	29.4
80	0.9	26.7
90	0.8	27.2
100	0.6	28.0
110	0.5	29.9
120	0.4	34.5
	NA34 data	
80	1.1	37.3
90	0.9	35.0
100	0.8	33.0
110	0.7	32.9
120	0.6	34.2

Even so, we find it very interesting that we obtain identical best fits for an entire range of T_k and λ from 70–120 MeV and 0.4–1.1, respectively. These are listed in Table I. Morever, we notice a striking anticorrelation between those values of T_k and λ which best fit the data. This result is intuitive because one expects that when the system acquires additional collective degrees of freedom in the transverse direction it does so at the expense of the thermal component of the expanding system.[16] When $\langle v_\perp \rangle \sim 0$ (i.e., when $\lambda \sim 0$) we do not obtain good fits to the data. This indicates that transverse flow effects cannot be ignored. Similarly, we find that temperatures below 70 MeV and above 140 MeV do not fit the data well.

In Fig. 3, we study the efects of varying λ for a fixed temperature $T_k=100$ MeV. The data shown in this figure are for the S+S reaction measured by the NA35 group. For $p_t \geq 0.25$ GeV/c, the spectral profile changes significantly as λ is varied from 0.4 to 1.4 (corresponding to a range of 0.4–0.8 in the transverse hydrodynamic velocity). Furthermore, it is seen that $\lambda \sim 0.8$ provides an excellent fit to the data. Interestingly enough, the spectrum below $p_t=0.25$ GeV is unaffected by the changes in λ. This is because the longitudinal velocities in this region are very close to the speed of light. This is less severe for heavier particles. In Fig. 4, we show the effect of the transverse hydrodynamic velocity on the kaon spectrum. The changes here are much more dramatic because the kaons at low p_t have longitudinal velocities considerably lower than those of the pions.

Though we obtain reasonable fits to the transverse momentum data for $p_t<1.75$ GeV/c, we believe better fits can be obtained by modifying the naive model to include a number of potentially significant and interesting effects. For example, following Milekhin, we take the velocity of sound to be that of a relativistic ideal gas. A fuller treatment would include the influence of final state interactions on both the collective and the internal distributions. We also assume that the transverse momentum

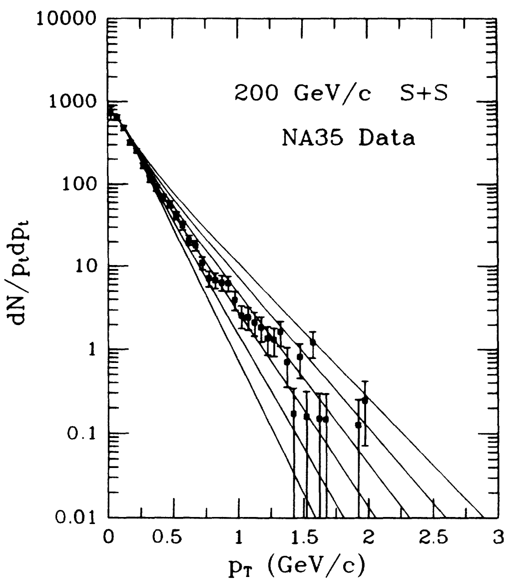

FIG. 3. Calculated pion transverse momentum spectra, Eq. (5), compared with the experimental data (all negatives) for central events in the rapidity range $2<y_{\rm lab}<3$. The calculations are for a fixed temperature $T_k=100$ MeV varying λ from 0.4 (lowermost curve) to 1.4 in steps of 0.2.

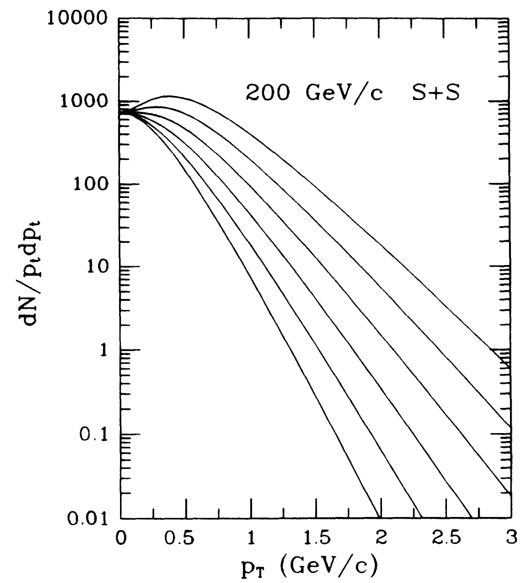

FIG. 4. Calculated kaon transverse momentum spectra, Eq. (5), for central impact parameters in the rapidity range $2<y_{\rm lab}<3$. The calculations are for a fixed temperature $T_k=100$ MeV varying λ from 0.4 (lowermost curve) to 1.4 in steps of 0.2

spectrum is composed entirely of negative pions. This is true for the bulk of the produced secondaries but, as stated earlier, the transverse momentum spectrum is contaminated up to $\sim 10\%$ by other secondaries—particularly negative kaons. We do not expect contributions at this level to alter the transverse momentum profile significantly. However, if one assumes that there is a more sizable contribution by the kaons, it is then not clear whether these more massive strange mesons have a chance to attain thermalization before breakup. Even if we include all of these corrections, it appears unlikely that we can fit the data for values of $p_t > 2.0$ GeV/c. This part of the spectrum would then depend greatly on nonequilibrium effects from the stage(s) prior to hydrodynamic expansion.

Before we conclude we digress briefly to discuss some of the similarities and differences of the Landau-Milekhin model to some of the other hydrodynamic models discussed in the literature. It has been noted[16] that the transverse momentum spectra obtained in hydrodynamic models with a boost invariant initial stage[17] are qualitatively similar to those in the Landau-Milekhin model. This is true primarily because, as noted earlier, the transverse hydrodynamic velocity is only weakly coupled to the longitudinal rapidity. Apart from this similarity, the initial conditions, the space-time evolution, and the rapidity distributions of these models are quite different. It is conceivable that the two models may predict the same distributions at higher energies. Earlier we briefly mentioned a hydrodynamic model with a spherical expansion.[3] One may note that the rapidity and transverse energy distributions in this model disagree with the results of recent experiments.[7,18]

In conclusion, we wish to draw attention to the fact that the Landau-Milekhin model provides good qualitative fits to the p_t spectrum. These fits demonstrate a striking anticorrelation in the values of the break-up temperature and the average transverse hydrodynamic velocity which fit the spectra. They also indicate that we need a better understanding of transverse flow in these systems in order to uniquely determine the breakup temperature. This is worthy of interest because other bulk features of the experiments such as the rapidity and transverse energy distributions are consistent with predictions of the model. Finally, it is highly desirable that means be devised which can isolate the average transverse hydrodynamic velocity from the breakup temperature. One possibility[19] is to compare predictions of a more complete model to transverse spectra of different particle species. It is clear that much work remains to be done in this direction.

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