^{12}C-^{12}C potential by inversion

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The potentials for ^{12}C-^{12}C scattering at 360, 1016, 1449, and 2400 MeV are obtained by inversion of S functions of an absorption model of scattering. The parameters of those S functions were determined by fitting the elastic scattering differential cross sections. A semiclassical inverse scattering method at fixed energy was used to produce potentials uniquely related to the S functions in the so-called sensitive radial regions around the strong absorption radii, potentials which do not depend upon an assumed shape. Comparison of the inversion potentials with those obtained by optical model potential fits to data show close agreement in the sensitive regions. It is emphasized that inversion is an easily performed, more systematic, method of analysis of intermediate energy heavy-ion scattering data than the usual optical model approach having the additional advantage of model independence.

I. INTRODUCTION

The nucleus-nucleus potential in the proximity of the strong absorption radius is of central interest in data analyses of heavy-ion reactions.[1] Of particular interest is that potential in the intermediate energy range (10–100 MeV/nucleon), as with increasing kinetic energy it should reflect a decrease in the importance of nucleon correlation and antisymmetrization effects.[2] At the highest energies the essential elements needed to define that potential should be the nuclear density distributions and the appropriate two-nucleon t matrices.[3] Theoretical calculations show that such changes in the factors determining heavy ion interactions manifest themselves as a strong energy dependence of the potential, particularly in the region of the "sensitive" radius.[2–4]

But, for convenience if not necessity, most studies choose a parametrized form of the density distribution and, often, of the potential itself.[1,5–7] Consequently, the conventional microscopic and macroscopic (phenomenological) models of the nucleus-nucleus potential invariably have a bias in radial form. Also, in a number of cases, the phenomenological optical model potential calculations gave ambiguous results while, to facilitate calculations based upon the microscopic structure theories, numerous approximations must be made.

Our approach is quite different. We seek the radial forms of (local) nucleus-nucleus potentials by solution of the inverse scattering problem[8] at fixed energy. Such methods are designed to use S functions as the basic data and so are natural extensions to the older, and most widely applied, method of analyzing heavy-ion scattering data, i.e., the strong absorption model (SAM) approach that was pioneered by Frahn.[9] The SAM approach directly defines smooth S functions that have been determined by fitting data. The question then arises whether or not it is possible to invert SAM type S functions that have been so determined. If that is possible then the SAM fits relate directly to the corresponding optical potential but they are obtained without any preconceptions about their radial shape. Such is not the case with the conventional optical model (OM) approach. Nevertheless, as the OM approach is quite widely used, the SAM S functions and the potentials obtained by their inversion afford, at the very least, a much improved scheme to that of "notch testing" to define sensitive regions of the optical potential.

Our interest in this study of ^{12}C-^{12}C scattering data is founded upon the reasons given by Brandan, Fricke, and McVoy[10] in their study of potential ambiguities. First, the data are quite extensive and taken at energies for which semiclassical methods of data analyses will be valid. Second, the differential cross-section data have shapes characteristic of intermediate "transparency," i.e., the ^{12}C-^{12}C cross-section shapes are intermediary to those normally found with very heavy-ion (strong absorption) scattering (e.g., Kr+Bi) with those typical of weak absorption, light-ion (e.g., alpha) scattering. Finally, previous analyses[1,10] of ^{12}C-^{12}C data suggest that the S-matrix values (at discrete integer values of l for each energy) required to fit the cross sections follow a smooth trend with l, one which can be well reproduced using a simple functional form of the S function.

Fixed energy, cross-section data are limited by scattering angle to span a finite range of momentum transfer and so there must be ambiguities in any set of $S_l(k)$ values obtained by fitting those data. For ^{12}C-^{12}C scattering, the measured data taken with energies in excess of 300 MeV are sufficient that such ambiguities are primarily those associated with small values of l (less than the "rainbow" value l_R defined by Satchler[1]). But those angular momenta components contribute very little to calculated cross sections in these cases.[1] As a consequence, although the conventional optical model potential of

Woods-Saxon form is subject to the continuous Igo ambiguity between strengths and radii,[10] the ^{12}C-^{12}C data above 300 MeV are of a quality not only to minimize that problem but also to assure that a smooth variation of $S_l(k)$ with l suitable for use in (semiclassical) inversion can be found that fits the data.

Inverse scattering theory defines scattering potentials, and by Loeffel's theorem,[8,9] when the S function is known for all l, those scattering potentials are unique. Use of an inversion method necessitates interpolation upon the table of $S_l(k)$ that "best fit" the measured data, however that table may be generated. For the higher energy heavy-ion scattering (e.g., ^{12}C-^{12}C) in which we are interested, many partial waves are required in data analysis and the associated $S_l(k)$ values follow a smooth trend with l. Furthermore, the great successes of the strong absorption models (SAM) in fitting heavy-ion data in general[9] suggests that S functions should be smooth continuous functions of quite simple form. Thus a convenient and yet physically motivated form of the S function can be specified and its use in inversion will then give a unique scattering potential.

Various methods for the solution of the inverse scattering problem at fixed energy have been developed.[11-18] In principle, they allow the determination of local potentials starting with a set of S functions that have been obtained by fitting cross-section data. The resultant potentials are phase equivalent to the unknown underlying exact microscopic (and nonlocal) interactions. But while those methods have been applied quite successfully in many (nuclear) scattering situations, there have been relatively few applications to heavy-ion scattering. In most of those applications, semiclassical inversion was used. Fully quantal inversion to date remains mostly of theoretical interest. But fully quantal inversion was employed to reconstruct a potential that was obtained by an OM approach to fit the scattering of ^{16}O from ^{12}C at 168 MeV. With exactly known phase shifts, the starting potential was reproduced[12] to a high degree of accuracy and down to very small distances, by using the so-called mixed (rational-nonrational) inversion scheme.[14] This result was confirmed subsequently[13] by using the modified Newton-Sabatier inversion scheme. Note, however, that to achieve this reconstruction, "data" of infinite precision were assumed. Another application of quantal inversion consisted of the determination of a potential due to the addition of a single Regge pole to the background scattering function given by a Woods-Saxon potential fit to the elastic ^{16}O$+^{18}$Si scattering cross section. The Regge pole was required to explain the backward rise of the angular distribution.[15]

An interative-perturbation method[18] is another technique to give potentials from low energy heavy-ion scattering. It is not a true inversion scheme since it involves a continual adjustment of a starting potential form until a set of phase shift values are achieved. But an inverse relationship is used to map changes to phase shifts to changes in the potential. Perhaps this scheme more appropriately belongs to the class of model independent techniques referred to by Satchler[1] and for which he has noted that, while such procedures have been quite successful in analyses of scattering of projectiles such as alpha particles, their applications to the low energy scattering of heavier ions have given much more ambiguous results. Low energy heavy-ion scattering frequently has a "phase shift ambiguity." Such an ambiguity poses problems for fully quantal inversion techniques as well, of course.

But at high energies the S functions of the SAM type do fit heavy-ion scattering data very well. Of course, data must be sufficient to give little uncertainties in the exact values of the $S_l(k)$ "data." When that is so, such S functions are particularly suitable for a systematic study of the energy dependence of heavy-ion potentials when used in conjunction with a semiclassical inversion scheme.[11] With precise criteria for uniqueness, as discussed in Sec. II, this method enables us to fix the region where the potential is uniquely determined by the data. Specifically, the ^{12}C$+^{12}$C local optical potential can be defined for energies at and above 360 MeV and for radial distances extending well inside the strong absorption radius. The WKB inversion has been used before[16] to study the ^{12}C-^{12}C elastic scattering potential at 1016 MeV. In that case, a strong absorption S function of a McIntyre type was successfully inverted using the WKB inversion scheme to obtain a potential different from the usual Woods-Saxon potential but which gave as good a fit to cross-section data. Specifically this inverted potential did not have the extremely strong real attraction characteristic of the optical model analysis,[16] a dichotomy that was the starting point for our study.

With data available at 360, 1016, 1449, and 2400 MeV to define SAM and OM potential parameters, a wide range of phase shifts and scattering functions are available for study. The resultant potentials, be they derived from folding the two nucleon t matrices, from conventional optical potential fitting of the data, or from the WKB inversion prescription, have been used in the coupled channels code, ECIS.[19] With that computer program, using a collective rotational model of spectroscopy, the effects of coupling to the 2_1^+ (4.44 MeV) excited states can be assessed. No other states have been considered since our interest herein was but to assess general effects of channel coupling both upon the shape of the "best fit" OM potential and as a possible variation between OM and inverted potentials.

A brief review of the WKB inversion method is given next and the results of our calculations made with it for the ^{12}C-^{12}C potential are presented in Sec. III. The conclusions one may draw are given in Sec. IV.

II. INVERSE SCATTERING AT FIXED ENERGY IN THE WKB APPROXIMATION

As details of the inverse scattering, fixed energy, problem and of the use of the WKB approximation to facilitate evaluation of inversion potentials have been presented in the literature,[8,11-17] only the salient features will be given herein. The input data for inversion are the scattering phase shifts (equivalently the empirical scattering, S functions). In the WKB approximation, phase shifts relate to a quasipotential, $Q(\sigma)$, by[11]

$$\delta(\lambda)=-\frac{1}{2E}\int_{\lambda}^{\infty} Q(\sigma)\frac{\sigma}{(\sigma^2-\lambda^2)^{1/2}}d\sigma\ , \qquad (1)$$

where $\lambda=l+\frac{1}{2}$ and E is the center-of-mass energy, and that quasipotential is defined in terms of the classical deflection functions,

$$\theta(\lambda)=2\,d\delta(\lambda)/d\lambda \qquad (2)$$

by

$$Q(\sigma)=\frac{2E}{\pi}\int_{\sigma}^{\infty}\frac{\theta(\lambda)}{(\lambda^2-\sigma^2)^{1/2}}d\lambda$$
$$\equiv\frac{4E}{\pi}\frac{1}{\sigma}\frac{d}{d\sigma}\left[\int_{\sigma}^{\infty}\frac{\delta(\lambda)\lambda}{(\lambda^2-\sigma^2)^{1/2}}d\lambda\right]\ . \qquad (3)$$

Then, with wave number k, the inverse potential is related to $Q(\sigma)$ by[20]

$$V(kr)=E\{1-\exp[-Q(\sigma)/E]\} \qquad (4)$$

with

$$r=(\sigma/k)\exp[Q(\sigma)/2E]\ . \qquad (5)$$

The potential so specified is unique provided that there is a one to one correspondence between r and σ.

The key feature in this prescription is the integral of Eq. (3) and the rational representation of $\delta(\lambda)$ defined by Lipperheide and Fiedeldey,[14] namely

$$\delta(\lambda)=\frac{1}{2i}\ln(S(\lambda))=\frac{1}{2i}\ln\left[\prod_{n=1}^{N}\frac{\lambda^2-\beta_n^2}{\lambda^2-\alpha_n^2}\right]\ , \qquad (6)$$

which makes that integral analytic. Thus by using empirical S functions $(S(\lambda)\equiv S_l(k))$ and mapping them with the rational representation, evaluation of the quasipoten-

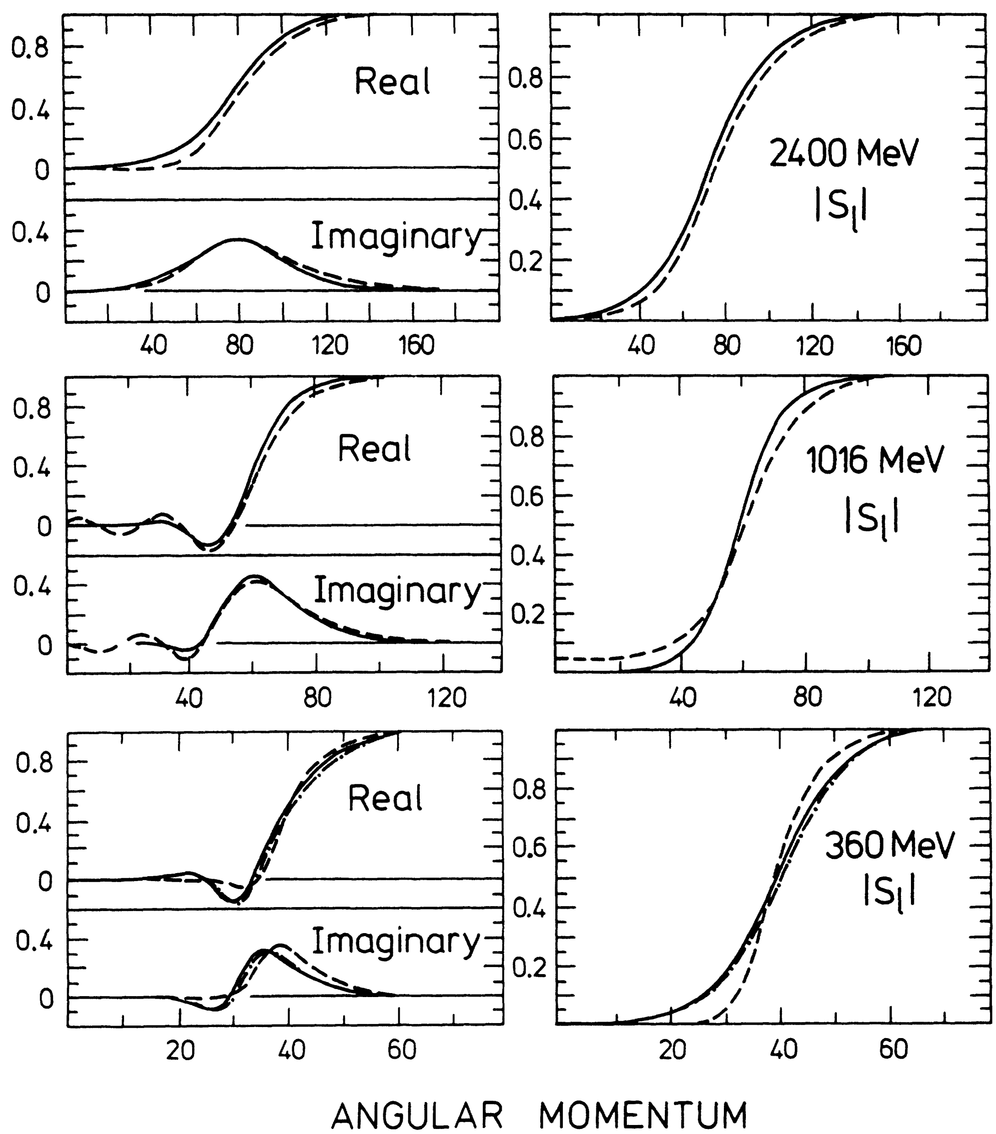

FIG. 1. The S functions (real, imaginary, and modulus) from the elastic scattering of ^{12}C from ^{12}C at 360, 1016, and 2400 MeV. The INV and OM calculation results are displayed by the continuous and dashed curves, respectively. The INV results virtually coincide with the McIntyre parametrizations at 1016 and 2400 MeV, so only at 360 MeV is the latter displayed by the dash-dotted curve.

tial (and thence the inverse scattering potential) is straightforward. But the experimental S functions in the presence of Coulomb forces are not readily represented in this way. Thus, with the identification

$$S_{\mathrm{exp}} = S_{\mathrm{nucl}} S_{\mathrm{Coul}} , \tag{7}$$

wherein S_{nucl} are the nuclear S functions and S_{Coul} are those of point Coulomb scattering, viz.,

$$S_{\mathrm{Coul}}(\lambda) = \Gamma(\lambda + 0.5 + i\eta)/\Gamma(\lambda + 0.5 - i\eta) \tag{8}$$

(η is the Coulomb parameter), we seek instead inversion of the modified S functions of rational form,

$$S_{\mathrm{mod}} = S_{\mathrm{exp}} / S_{\mathrm{back}} , \tag{9}$$

where S_{back} is a "background" scattering function of the form[11]

$$S_{\mathrm{back}}(\lambda) = \exp[i\eta \ln(\lambda^2 + \lambda_c^2)] \tag{10}$$

that involves λ_c, a cutoff parameter. The corresponding inversion potential to S_{back}, V_{back}, is a quasi-Coulomb potential which asymptotically (large r) behaves as the Coulomb potential but it is not singular at the origin. In this way we avoid the problems experienced by Kujawski[11] in using a point Coulomb background potential. But of particular importance, besides giving modified S functions that can be represented very well in rational form, S_{back} itself can be inverted classically to give V_{back} to high accuracy.

With the inverted potentials, V_{mod} and V_{back}, we construct

$$V_{\mathrm{exp}} = V_{\mathrm{mod}} + V_{\mathrm{back}} \equiv V_{\mathrm{nucl}} + V_{\mathrm{Coul}}^{\mathrm{FS}} , \tag{11}$$

wherein the Coulomb potential is taken to be that of a charged sphere of radius R_c, namely

$$V_{\mathrm{Coul}}^{\mathrm{FS}} = \begin{cases} 2\eta/r, & r > R_c , \\ \dfrac{\eta}{R_c}(3 - r^2/R_c^2), & r < R_c , \end{cases} \tag{12}$$

so that the nuclear potential is given by

$$V_{\mathrm{nucl}}(r) = V_{\mathrm{mod}} + V_{\mathrm{back}} - V_{\mathrm{Coul}}^{\mathrm{FS}} . \tag{13}$$

III. RESULTS OF CALCULATIONS

We have used the McIntyre parametrizations of S functions[21] for a strong absorption model of scattering as input to the WKB inversions and mapped them to rational representations ($S_{\mathrm{mod}}(\lambda)$) thereby extending them to the required complex values of λ. For the cases of 360, 1016, 1449, and 2400 MeV ^{12}C-^{12}C elastic scattering to be discussed herein, the specific parameter values have been determined by Mermaz *et al.*[22,23] and the potentials deduced from the inversion procedures have been used as external input to the code ECIS.[19] Those potentials as well as the S functions and differential cross sections (ratio to Rutherford scattering) so obtained are identified hereafter by the tag INV. The INV results will be compared with others designated by OM (optical model) and which are the results of using standard Woods-Saxon optical model potentials and a search procedure in ECIS to determine (OM) parameter values that yield a "best fit" to measured data. But deformation effects must be included in this case to get very good fits to cross-section data, as will be shown. Specifically, a quadrupole deformation ($\beta_2 = -0.6$) was considered in the ^{12}C ground state density, and modulations to the standard Woods-Saxon shapes to all orders in β_2 were taken into account.

The S functions for the elastic scattering of ^{12}C on ^{12}C as functions of angular momentum and at energies of 360, 1016, and 2400 MeV are shown in Fig. 1. Therein the real and imaginary parts of the McIntyre parametrizations are compared with INV and OM results in the left hand panel. The moduli of the corresponding S functions are compared in the right hand panel. The McIntyre parametrization values are displayed by the dot-

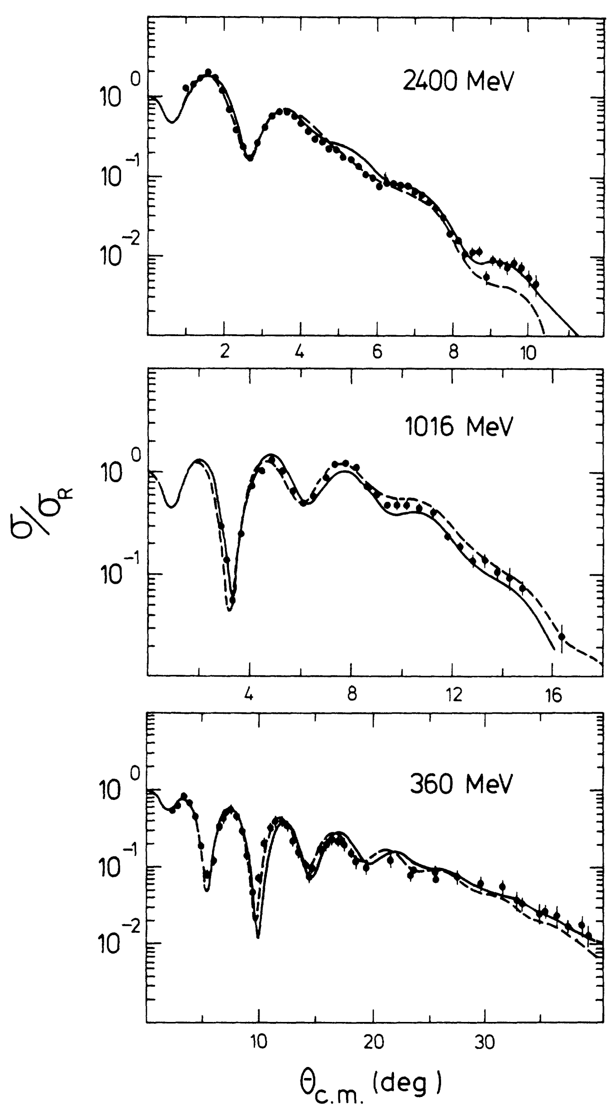

FIG. 2. The ^{12}C-^{12}C elastic scattering cross sections (ratio to Rutherford) calculated using the INV potentials (continuous lines) and OM potentials (dashed lines) and compared with the data at 360, 1016, and 2400 MeV.

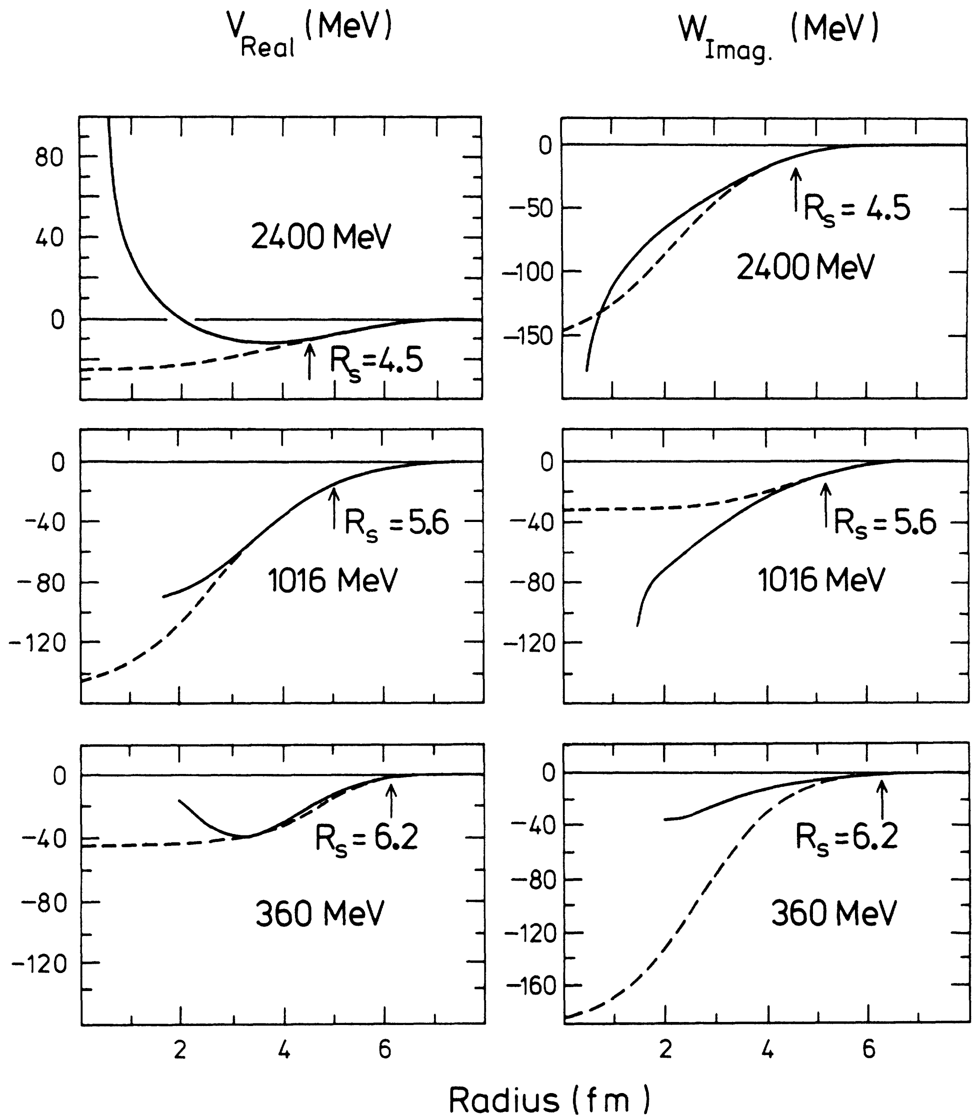

FIG. 3. The INV and OM potentials from which the results given in Fig. 2 were calculated. The real and imaginary parts of those potentials are given in the left and right hand panels, respectively, and the INV and OM potentials are shown by the continuous and dashed curves, respectively.

dashed curves, those of the OM calculations by the dashed curves, while the INV results are displayed as the continuous curves. The close agreement between the McIntyre parametrization values and the INV results at all energies is a direct reflection of the propriety of the WKB methods used in the inversion procedure as well as of the stability of the numerical methods used. Indeed at 1016 and 2400 MeV the INV results and the McIntye parametrizations are indistinguishable in this diagram.

At the higher energies, the OM results closely reproduce the strong absorption model values with relatively small deviations that are not very significant. At the lowest energy, however, the $S_l(k)$ given by the OM calculations has quite similar structure to the McIntyre parametrization but now with more noticable shifts in distribution. Those shifts nevertheless affect the fits to cross sections and lead to noticeable differences between the inverted and best fit OM potentials.

The INV and OM calculations give the cross sections represented by the continuous and dashed curves, respectively, in Fig. 2. Those results are compared therein with the available data.[5,6] Clearly the data are all fit quite well and there is little to choose between the OM and INV results at the energies of 1016 and 2400 MeV. At 360 MeV, the OM result is a better fit to the data and the (small) differences between the two calculated cross sections are commensurate with the differences between their respective $S_l(k)$. Thereby we anticipate the need to choose the functional form for $S_l(k)$ carefully prior to use in inversion. This is more clearly demonstrated with the results of calculations of the 1449 MeV ^{12}C on ^{12}C reaction and which are discussed later.

The optical model potential parameters are given in Table I (in the columns designated OM). Those designated CC relate to the results of coupled channels calculations also to be discussed later. Clearly the 1016 MeV

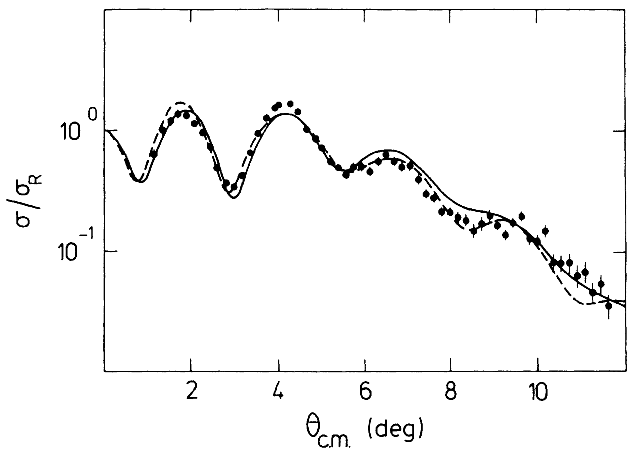

FIG. 4. The ^{12}C-^{12}C cross sections at 1449 MeV compared with data and with the INV and OM calculation results that are given by the continuous and dashed curves, respectively.

OM potential has quite a different character to those appropriate at 2400 and 360 MeV. The real part of the 1016 MeV OM potential is strong and attractive, and much stronger than its imaginary part. The reverse is the case for both of the 360 and 2400 MeV OM potentials as is clearly demonstrated in Fig. 3. Therein the OM potentials (obtained from searching to get the "best fits" to the elastic scattering cross sections) are displayed by the dashed curves. The continuous curves are the potentials obtained by inversion. Clearly the OM and INV potentials, except perhaps for the 360 MeV imaginary potential, are in very good agreement from at least 4 fm outwards. This region lies well inside the strong absorption radii (indicated by the arrows) in all cases and spans the sensitive radial regimes. The variation between the imaginary potentials for 360 MeV is the most pronounced of the set. Such differences, nevertheless, are still small in the sensitive region with those (small) differences being specifically the cause of variance between the OM and INV S functions and cross sections at this energy.

The inverted potentials do not extend to the origin although the method does best for the highest energy. This is as one would expect for any method based upon the WKB approximation. At 360 and 1016 MeV, and for radii well inside the strong absorption radii, the WKB inversion tends to become inaccurate. At even smaller radii the method may even break down altogether. But the potentials inside 3–4 fm are quite irrelevant. Even within the context of Woods-Saxon type optical potentials it is well known,[1] by means of the "notch test," that ^{12}C-^{12}C scattering does not allow a unique determination of the optical potential within a radius of about 4 fm. Indeed the region of maximum sensitivity so defined lies between 4 and 6 fm but moves outward with decreasing energy. Our approach, which determines optical potentials by inversion of the McIntyre S functions fitted to the data, represents a more stringent test of these conclusions, since it does not depend upon the assumed shape of the optical potential and only upon the shape of the S function. But the data allow a far less ambiguous determination of the parameters of the McIntyre S functions than those of a Woods-Saxon potential.[10] Consequently, our inverted potentials in the sensitive region, being uniquely associated with the McIntyre S functions, are more directly related to the data and therefore determine that sensitive region of the potential to a higher degree of reliability than do previous methods (notch test, etc.). From these results we conclude that given the currently available data base, it is not very meaningful at present to be concerned with detailed calculations of the optical potentials and wave functions for heavy ions at short distances,[24] and to compare these with Woods-Saxon optical potential fits.

A. The parametrized S functions

While inversion requires but a table of S functions, the SAM parametrizations are very convenient representations of data. They also give a smooth behavior for $S_l(k)$ with l, which is convenient for use in the WKB inversion. In fact the accuracy of the semiclassical inversion depends upon this smoothness. But it is imperative that the parameter values be chosen from an optimal fit to the scattering cross section. Such has not been the case with an analysis of the 1449 MeV data,[23] albeit that the reported parameter values do lead to a quite reasonable comparison with the data. In this case the OM potential, its associated S functions, and the calculated cross section all differ from those of the McIntyre parametrization (SAM) calculations. Those calculated cross sections are displayed in Fig. 4 wherein the INV and OM results are again represented by the continuous and dashed lines, respectively. The OM parameter values required to obtain the "best fit" results are given in Table I. The INV result is not a bad fit to the data but the quality of that fit is not the same as those obtained from the 360, 1016, and 2400

TABLE I. The optical model potential parameter values. The Coulomb potential is that from the overlap of two uniformly charged spheres, each of radius 1.3 $A^{1/3}$ (2.98 fm).

	360		1016		1449		2400	
	OM	CC	OM	CC	OM	CC	OM	CC
V_0	43.77	47.15	153.23	78.16	42.67	51.07	25.2	19.1
r_0	0.9442	1.027	0.6098	0.8456	0.9368	0.8847	0.9173	1.0314
a_0	0.6242	0.5776	0.9685	0.8357	0.9599	0.9815	0.9350	0.7171
W_0	190.71	79.66	31.21	52.47	33.56	45.15	157.8	56.8
r_d	0.5816	0.7172	0.9625	0.8865	0.9679	0.9055	0.4823	0.7272
a_d	0.815	0.8231	0.7557	0.6367	0.5414	0.6047	0.8820	0.7409

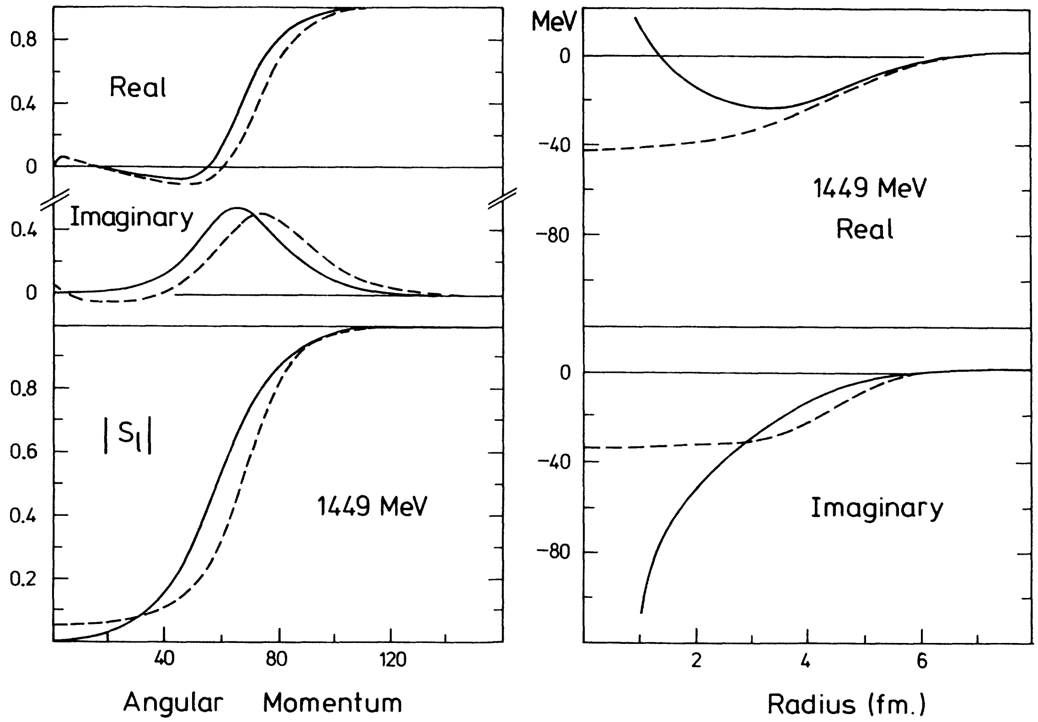

FIG. 5. The S functions (real, imaginary, and modulus) and the potentials (real and imaginary) from the calculations of 1449 MeV ^{12}C on ^{12}C scattering. INV and OM results are shown by the continuous and dashed curves, respectively.

MeV SAM parametrizations. The cause is quite evident when one compares the associated S functions and potentials as is done in Fig. 5. The S functions are displayed on the left in this figure and the potentials are presented on the right with the INV and OM results portrayed by the continuous and dashed curves, respectively. The S functions are similar but the SAM values (which also result from computation using the INV potential in ECIS, thereby justifying the inversion method) essentially have smaller grazing angular momenta (l_g and $l_{g'}$) in the McIntyre parametrization), the very small l values being of no consequence. With the potentials, the most significant difference between the INV and OM results occurs in the imaginary parts, with the variations for $r<4$ fm being of no importance. Clearly, therefore, to use an SAM parametrization of S functions with inversion to define the optical potential necessitates a careful study to give a good fit to data at the outset. In general, inversion requires high quality data and S functions deduced from very good fits to those data.

B. Channel coupling effects

At some energies, accurate measurements of inelastic scattering differential cross sections have been made, and of the excitation of the collective, 2_1^+ state at 4.44 MeV in particular. It is well known that this inelastic transition, when treated by standard collective models, requires a deformation parameter value of -0.6, hence our previous choice for deformation corrections to the conventional Woods-Saxon forms in OM calculations. But the specific coupled equations, let alone the need to simultaneously fit both elastic and inelastic scattering cross sections, have not been considered in any of our foregoing discussion. We limit consideration to just the 2_1^+ excitation and note that others[3,5] also couple the 3_1^- state in their studies. But the 2_1^+ transition is the essential additional reaction channel and for our purpose it suffices.

The results of our coupled channels calculations to simultaneously fit the differential cross sections from the elastic and 2_1^+ inelastic scattering of ^{12}C on ^{12}C are given in Fig. 6. Those coupled channels results are portrayed at each of the four energies by the continuous curves and are compared with the data and with the best fit OM results (dashed lines) for the elastic, ratio to Rutherford, cross section. The 2_1^+ results are given in mb/sr and although no error bars have been displayed, the empirical values[5,6] were used in the coupled channels search calculations. The resultant ^{12}C-^{12}C potential parameters from those searches are listed in Table I.

At the three highest energies, the fits to the cross-section data are very good with little loss in the quality of fit of the ratio to Rutherford data from that obtained using the "best fit" OM parameters. But the fits to the 360 MeV data are not as good. From the third minimum (at 14° in σ/σ_R) onward, the coupled channel analysis is poor. The 2_1^+ data are also not very well fit at this energy.

The potentials are given in Fig. 7 with the OM, coupled channels (CC), and INV potentials for each designated energy being represented by the continuous, dashed,

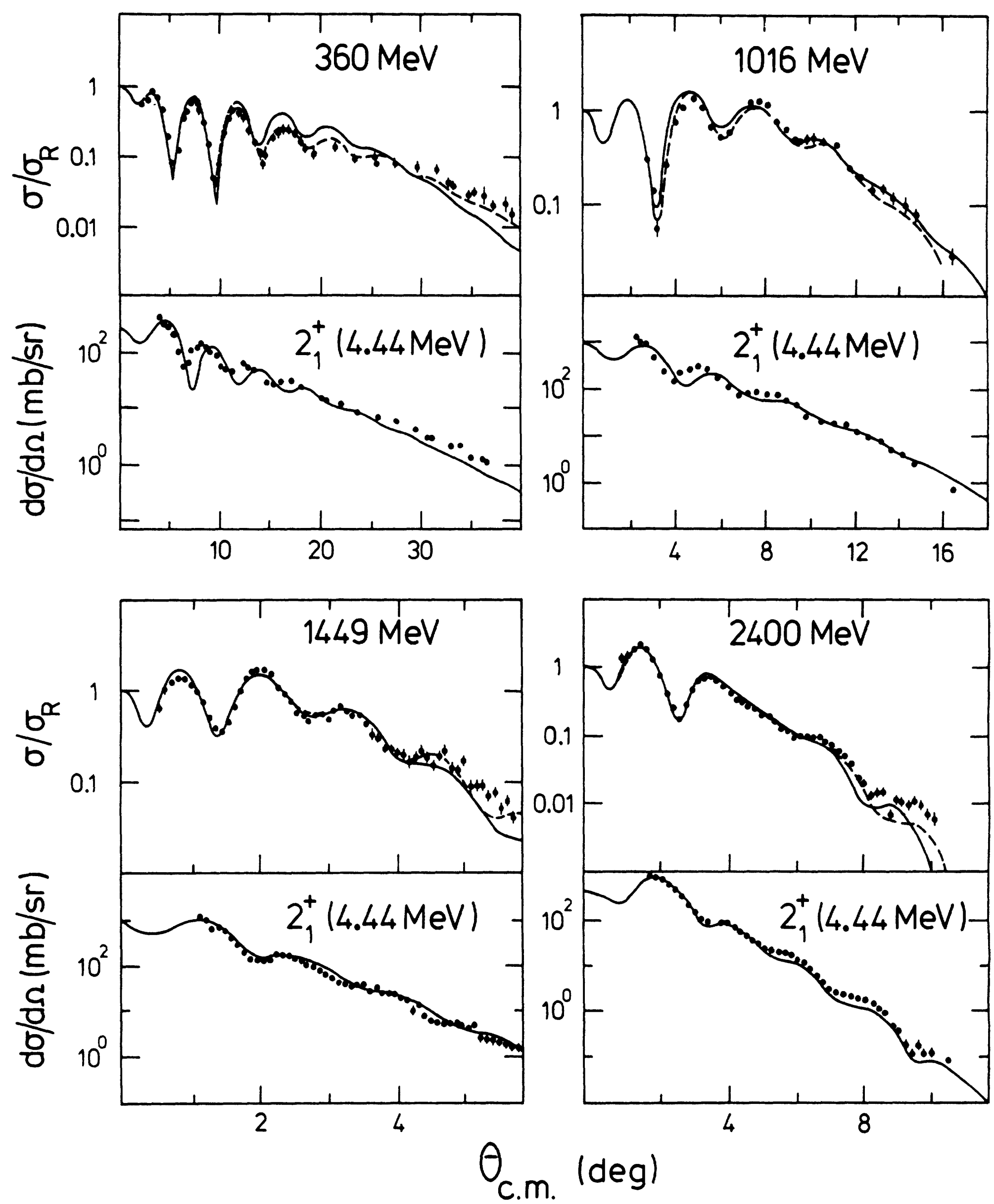

FIG. 6. The results of coupled channels calculations for ^{12}C-^{12}C scattering at all four energies and with elastic and 2_1^+ (4.44 MeV) channels coupled. The dashed curves give the best fit OM potential cross sections for comparison.

and dash-dotted curves, respectively. Clearly, except at 360 MeV, the OM and CC potentials within the sensitive 4–6 fm region are very similar but that similarity is affected by considerable variation of the parameters as given in Table I. It is customary to consider potentials as deep or shallow in accordance with their behavior at small radii; thus, at 2400 MeV, the CC potentials (real and imaginary parts) are shallow in comparison to the OM ones, the imaginary part particularly so. For 1449 MeV the reverse is the case while at 1016 MeV, the real (imaginary) part of the CC potential is stronger (weaker) than the OM counterpart. In view of this variation of the influence of coupled equations upon "best fit" optical model potentials, past concern about deep versus shallow potentials for heavy-ion interactions seems of no great significance. The essential thing instead is that with good fits to data the potentials are very similar, irrespective of the method of calculation, in the sensitive region. That point is confirmed first by the 1449 MeV results for which the inversion was made using unsatisfactory SAM S functions. At that energy the CC and OM potentials from which very good fits to data result are still very similar in the sensitive region. The 360 MeV data results give the second confirmation of the sensitive region criterion. At this energy the INV and OM calculated cross sections were both quite good fits to the data but the CC result was much poorer. The potentials, the real parts especially, reflect that diversity.

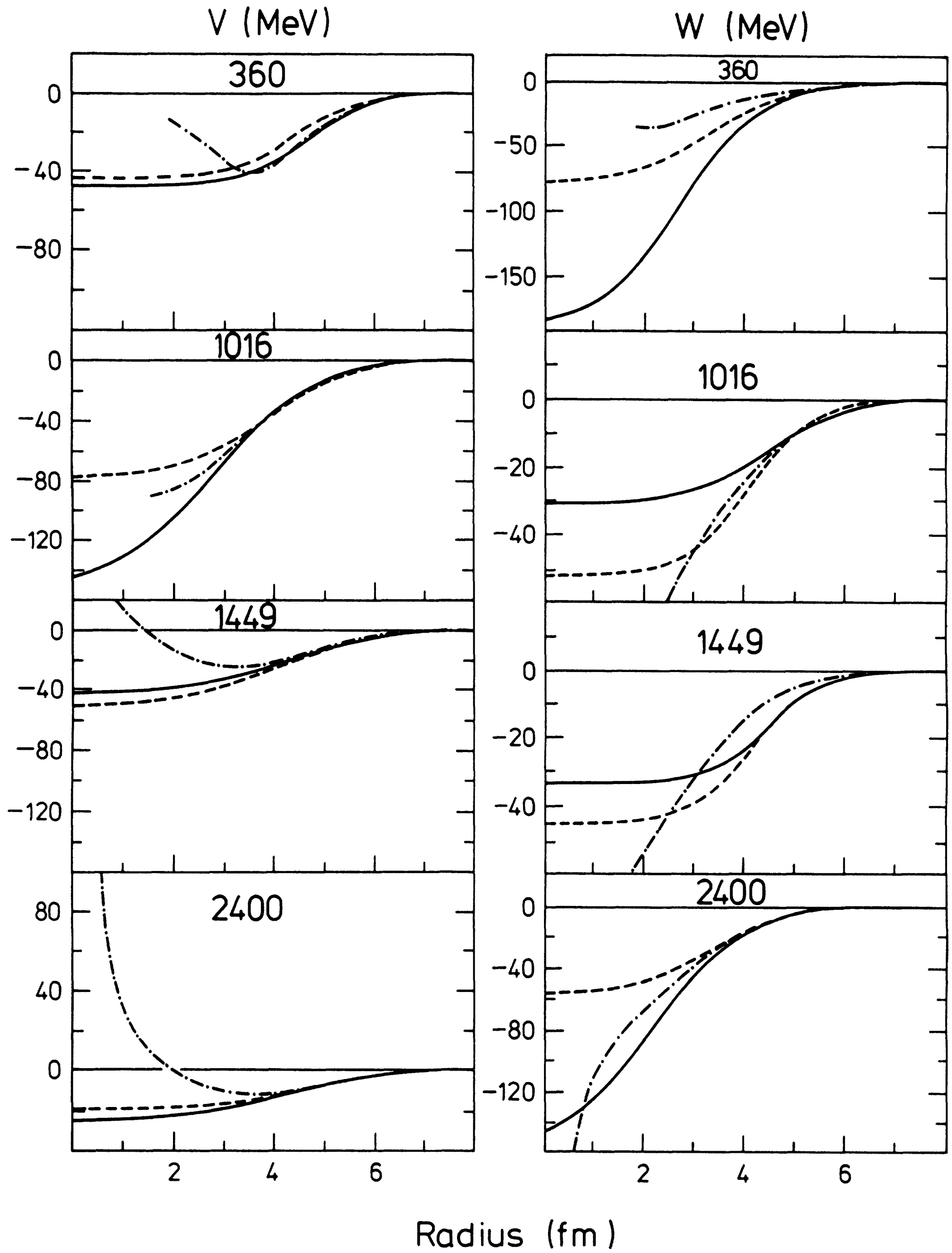

FIG. 7. A comparison of the elastic channel radial potentials given by inversion (INV: dash-dotted curves), by best fit optical model calculations (OM: continuous curves), and by the coupled channels calculations (CC: dashed curves).

C. The potential values in the sensitive region

It is interesting to compare the actual values of the different potentials at radii in the sensitive region. This we have done by tabulation. In Table II the potential strengths from each of the OM, CC, and INV calculations and at the pertinent strong absorption radii are given. With the exceptions of the 360 MeV (CC in particular) and 1449 MeV INV calculations, both of which gave less than satisfactory fits to the data, the potentials

TABLE II. Potential strengths (negative) at the strong absorption radii, R_s.

E (MeV)		360	1016	1449	2400
R_s (fm)		6.2	5.6	5.2	4.5
	OM	2.04	7.93	11.91	10.60
V (MeV)	CC	3.28	8.77	12.09	11.03
	INV	2.28	7.65	9.74	9.93
	OM	2.44	5.33	6.54	10.92
W (MeV)	CC	2.24	4.29	6.73	9.71
	INV	3.30	4.43	3.83	10.29

in any set agree to within 1 MeV and usually to within 10%. The trend is that the potential values also increase with decreasing strong absorption radius. In Table III, the potential strengths at each energy but at a fixed radius of 5 fm are presented. Again the 360 MeV and 1449 MeV INV values reflect the unsatisfactory fits to data obtained using the relevant model interactions. Otherwise the agreement within each set of potential values is very good.

TABLE III. Potential strengths (negative) at a radius of 5 fm.

E (MeV)		360	1016	1449	2400
	OM	10.95	14.13	13.78	7.52
V (MeV)	CC	17.63	16.09	14.07	7.73
	INV	12.50	14.28	11.68	7.18
	OM	10.19	9.78	8.70	6.39
W (MeV)	CC	8.81	9.75	8.85	5.40
	INV	7.04	9.50	4.85	5.71

IV. CONCLUSIONS

Smooth, strong absorption like S functions, such as those given by the McIntyre parametrization, have been found to be eminently suitable in the use of semiclassical methods to specify scattering potentials by inversion. For ^{12}C-^{12}C scattering above 360 MeV, and with McIntyre parameters fixed by fits to cross-section data, unique potentials are obtained accurately to radii well inside of the known sensitive radial regions. Through those sensitive radial regions the potentials obtained by inversion agreed well with the best optical model interactions we could specify. As there are no biases in radial form, and as the important $S_l(k)$ values (insofar as fits to the cross-section data are concerned) are given by the starting S functions, inversion is a means of specifying the nucleus-nucleus interaction alternate, and possibly preferable, to that of the conventional optical model approach. At the very least, such inversion provides a connection between fits to data made at the S-matrix level and those made via direct potential scattering theory. The whole inversion procedure is greatly simplified when conditions favor the semiclassical approximation to the full quantal inversion scheme, so much in fact that the prescription could be incorporated routinely into a program fitting S functions of the strong absorption kind to data and thence defining the corresponding potentials.

Comparison of the potentials obtained by inversion with the best fit optical model potentials then gives a most reliable determination of the sensitive radial region since the short range behavior of the inverted potential is often quite different from that of a conventional Woods-Saxon shape. Using this comparison procedure we have confirmed that, with the ^{12}C-^{12}C system, not only does the sensitive region extend from 4 to 6 fm at the higher energies and from 5 to 7 fm at 360 MeV, but also that the sensitive region tends to smaller radii with increasing energy.[5,6] Likewise we have confirmed that the absorptive potential at 5 fm decreases with energy[5,6] in contrast to predictions of microscopic models of scattering.[3,4] Inside the sensitive region the inverted potentials tend to become repulsive like those found in Muller *et al.*[3]

We note, however, that one need not perform or have the results of any optical model calculation to define the sensitive region. One merely need modify the McIntyre S functions within the measured accuracy of the data and overlay the corresponding potentials obtained by their inversion.

Inclusion of coupled channels does not vitiate any of our conclusions. For the ^{12}C-^{12}C system, at least, the only significant role of coupling the 2_1^+ state in calculations seems to be the variation of the elastic channel potential from that of a standard Woods-Saxon form.

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