Negative-parity states in the spectrum of ^{6}Li from elastic scattering and radiative capture of ^{3}He by ^{3}H

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The results of a new resonance analysis of the elastic scattering of ^{3}He by ^{3}H in odd-parity states and the radiative capture of ^{3}He by ^{3}H in the first excited state of ^{6}Li are presented. We give evidence for a very broad level of ^{6}Li at $E_{\rm Li}=26.59\pm0.065$ MeV with $J^\pi=2^-$, $S,T=1,1$ not previously reported. The shape of the angular distributions and the broad maximum observed in the radiative capture cross section at high energies are explained in terms of the interference of three very wide $^{33}F_J$ overlapping resonances. Numerical values of the resonance parameters are given.

I. INTRODUCTION

The isobaric diagram of nuclei with $A=6$ shows that all excited states of ^{6}He and ^{6}Li, and all states of ^{6}Be, are unbound with respect to nucleon and to cluster decay.[1] In ^{6}Li, the study of the ^{4}He$+d$ system has led to good information about the $T=0$ states.[1,2] The relatively small widths of the 0^+ and 2^+, $T=1$, states below 6 MeV have made it possible to study them in several reactions and in inelastic scattering. However, the other states of ^{6}Li are very poorly known, except that it is clear that there are no other sharp or fairly sharp $T=1$ states of ^{6}Li below=17 MeV.[3] In this work we are particularly interested in the $T=1$, negative-parity, unbound states of ^{6}Li that are strongly coupled to the ^{3}He$+^3$H channel. Evidence for these odd-parity states comes from the elastic scattering and radiative capture of ^{3}He by ^{3}H in odd orbital angular momentum states.

The elastic scattering of polarized ^{3}He by ^{3}H was measured by Vlastou *et al.*[4] over the energy range from 20 to 33 MeV, ^{3}He bombarding energy. These authors represented the cross section and polarization of the scattered and recoil particles from 5 MeV up to 33 MeV in terms of 18 real phase shifts, which they called solution I. They made a parametrization of the real resonant phase shifts in terms of the single level R-matrix formalism. They found resonances in the $^{33}P_J$ waves ($L=1$) at $E_{\rm Li}=21.0$ and 21.5 MeV, with $J^\pi=2^-,T=1$ and $J^\pi=0^-,T=1$, respectively, and in the $^{33}F_J$ waves ($L=3$) at $E_{\rm Li}=25.7$ and 26.7 MeV, with $J^\pi=4^-,T=1$ and $J^\pi=3^-,T=1$, respectively, but no resonance is reported with $J^\pi=2^-,T=1$ for $L=3$. A second, more realistic, representation of the same elastic scattering data, called solution II, was made in terms of 18 complex phase shifts to account for the possible influence of the open reaction channels. Vlastou *et al.*[4] made no resonance analysis of the complex phase shifts of their solution II.

The cross section for the radiative capture of ^{3}He by ^{3}H to form the first excited state of ^{6}Li ($J^\pi=3^+,E_{\rm Li}=2.185$ MeV) was measured by Blatt *et al.*,[5] in the energy range from $E_{\rm He}=1$ MeV up to 5 MeV, and by Ventura *et al.*[6] in the energy range from $E_{\rm He}=6$ MeV up to 26 MeV. The excitation function shows broad maxima centered at 5.0 and 20.6 MeV, ^{3}He bombarding energy, indicating the presence of resonances in ^{6}Li at excitation energies around 18.3 MeV and 26 MeV, respectively. The broad structure around 20.6 MeV has a shape suggestive of two very wide overlapping resonances. Ventura *et al.*[6] calculated the capture cross section to the first excited state of ^{6}Li assuming that the reaction mechanism is a direct radiative capture in a single channel configuration. Since in L-S coupling, the first excited state of ^{6}Li is described as a ^{13}D state,[7] there are only four partial waves $^{33}P_2$, $^{33}F_2$, $^{33}F_3$, and $^{33}F_4$, which can contribute to an electric dipole transition to this state. The calculation was made with a computer code developed by D. R. Thompson and Y. C. Tang.[8] The potential used by these authors to represent the two-body nucleon-nucleon interaction was purely central. Therefore, in the calculation by Ventura *et al.*,[6] the effective interaction between the ^{3}He and ^{3}H clusters was also purely central and no splitting of the states of good orbital angular momentum in states of different total angular momentum could result. Ventura *et al.*[6] found good agreement between the measured and computed values of the excitation function and the photon angular distribution at $E_{\rm He}$ energies below 10 MeV. In the region of the peak at $E_{\rm He}=20.6$ MeV they found a large disagreement between measured and computed values which they interpreted as evidence of the breakdown of the ^{3}He-^{3}H cluster configuration into a different intermediate configuration of ^{6}Li, with the same spin and parity, which then decays by photon emission to the first excited state of ^{6}Li.

More recently, P. Kramer and D. Schenzle[9] computed the energy spectrum of ^{6}Li taking into account the noncentral terms in the nucleon-nucleon interaction. They found the three $^{33}F_J$ states of negative parity split in one $^{33}F_4$ state at $E_{\rm Li}=24.9$ MeV, and the $^{33}F_2$ and $^{33}F_3$ states, degenerate at 27.8 MeV. It was also found that these states are almost pure [3,3] partitions with very little admixture of [4,11]. References to other theoretical

studies may be found in the monographs by P. Kramer[9] and R. A. Eramzhyan *et al.*[10]

Since the partition [3,3] couples strongly to the odd orbital angular momentum states in the ^{3}He+^{3}H channel, the results of Kramer and Schenzle[9] strongly suggest that the wide bump observed in the excitation function of the radiative capture cross section to the first excited state of ^{6}Li around $E_{\rm He}=27$ MeV is due to a direct capture process in which the ^{3}He is first captured by ^{3}H to form an unbound excited state of ^{6}Li with an almost pure ^{3}He-^{3}H cluster structure, which then may decay by gamma-ray emission to the first excited state of ^{6}Li. The double humped structure apparent in the high energy bump of the excitation function, and the change in sign of the curvature of the photon angular distribution, would then be due to interference between two very wide $^{33}F_J$ overlapping resonances of the same parity. In order to test this conjecture we proceed as follows: To make apparent the resonant structure in the expression for the direct capture cross section, in Sec. II we reformulate the theory of the direct radiative capture expanding the wave function of the relative motion in the entrance channel in terms of Gamow states of the relative motion of the two clusters. In Sec. III, we extract from the complex phase shifts, which Vlastou *et al.*[4] used to represent the elastic scattering of ^{3}He by ^{3}H, the relevant elements of the scattering matrix. A single level resonance plus background formula is fitted to the derived elements of the scattering matrix. The values of the resonance energies, elastic widths, and total widths obtained as described in Sec. III are inserted in the expression for the resonant direct radiative capture cross section found in Sec. II. In Sec. IV we report the results of a fit of the expressions found in this way to the radiative capture data of Ventura *et al.*[6] in order to obtain numerical values of the radiative widths. Section V is devoted to a discussion of the results. The paper ends with some conclusions stated in Sec. VI.

II. RESONANCES IN THE DIRECT RADIATIVE CAPTURE

The radiative capture of ^{3}He by ^{3}H to form the first excited state of ^{6}Li will be described as a direct process in which the nucleus-nucleus interaction in the entrance channel produces resonances, that is, it gives rise to the formation of short lived unbound quasimolecular excited states of ^{6}Li with a dominant ^{3}He-^{3}H structure. Once formed, these states may decay by gamma-ray emission to the final bound state of ^{6}Li. In the formation of the quasimolecular states, the single particle degrees of freedom participate only insofar as the properties of the nucleus-nucleus interaction are determined by the internal states of motion of the colliding nuclei in their ground states. In the gamma-ray transitions, the internal degrees of freedom may play a more active role, especially when the dominant cluster structure of the final state is different from that of the unbound resonant states.

In this picture, the energy dependence of the cross section is dominated by contributions from the entrance channel rather than from the dynamics of the nuclear interior. Therefore, any resonances present in the elastic scattering cross section due to the nucleus-nucleus interaction in the entrance channel will also be present in the radiative cross section.

A. Decomposition in partial transition amplitudes

The differential cross section for direct capture from the continuum to a bound state of the ^{3}He-^{3}H system with emission of electric dipole radiation, computed in perturbation theory to first order in the electromagnetic field, is given by

$$d\sigma/d\Omega=\frac{K_\gamma}{hv(2S+1)}\sum_{M_i,M_f}\sum_{P}|M_{fi}^{(P)}|^2\ , \tag{1}$$

where K_γ and P are the wave number and circular polarization of the gamma radiation, respectively, v is the relative speed of the ^{3}He and ^{3}H nuclei, S is the channel spin, and i and f refer to the initial state in the continuum and the final bound state, respectively.

The matrix element of the interaction Hamiltonian for electric dipole radiation is given by[11]

$$M_{fi}^{(P)}=-iD_{M,P}^{(1)*}(\phi_\gamma,\theta_\gamma,0)\langle fM_f|M_{E1}^{(M)}|iM_i\rangle\ , \tag{2}$$

where M is the magnetic quantum number of the radiation, $D_{M,P}^{(1)}(\phi_\gamma,\theta_\gamma,0)$ are the elements of the rotation matrices as functions of the angles describing the direction of the gamma-ray emission with respect to the beam axis, and $\langle fM_f|M_{E1}^{(M)}|iM_i\rangle$ is the nuclear matrix element of the electric dipole operator.

Substitution of (2) in (1) gives

$$d\sigma/d\Omega=\frac{2\pi K_\gamma m}{(2S+1)\hbar^2k^2}\sum_{M_iM_f}\left[|\langle fM_f|M_{E1}^{(M)}|iM_i\rangle|^2\sum_{P=\pm1}|D_{M_f-M_i,P}^{(1)*}(\phi_\gamma,\theta_\gamma,0)|^2\right] \tag{3}$$

where k is the wave number of the relative motion of the colliding nuclei and m is the reduced mass of the system.

The initial state continuum wave function for ^{3}He incident along the z axis will be labeled by the channel spin $S=1$, and its z component M_i. It may be written as an expansion in vector spherical harmonics

$$\Psi_i^{M_i}=\sum_{J=1}^{\infty}\sum_{l'=|J-1|}^{J+1}\sum_{l=|J-1|}^{J+1}\sum_{M_s}\left[[4\pi(2l+1)]^{1/2}i^l\exp[i(\sigma_l-\sigma_0)]\right.$$
$$\left.\times C(l1J;OM_sM_i)\mathcal{A}\left[\Phi_{\rm He}\Phi_H\frac{R_{l'l}^{(J)}(kr)}{kr}\mathcal{Y}_{Jl'1}^{M_s}(\theta,\phi)\right]Z(R_{\rm c.m.})\right]\ . \tag{4}$$

In Eq. (4), σ_l is the Coulomb phase shift, r, θ, and ϕ are the coordinates of the relative motion, $R^{(J)}_{l'l}(kr)$ is the radial wave function, $\mathcal{Y}^{M_s}_{Jl'1}$ is a spin-isospin angle function, appropriate for $T=1$, with orbital angular momentum l' and spin 1 coupled to total angular momentum J. Since the nucleus-nucleus potential acting between ^{3}He and ^{3}H is in general noncentral and mixes waves of different orbital angular momentum, $R^{(J)}_{l'l}(kr)$ has two orbital angular momentum indices, l', which indicates the component of the wave function belonging to a specific orbital angular momentum, and l, which refers to the angular momentum of the incident beam, i.e., to the boundary condition at $t\rightarrow-\infty$. The functions $\Phi_{\rm He}$ and $\Phi_{\rm H}$ in Eq. (4) represent the internal spatial structures of the ^{3}He and ^{3}H clusters, respectively. The function $Z(R_{\rm c.m.})$ describes the motion of the total center of mass.

In the case under consideration, the final state is the first excited state of ^{6}Li, at $E_x=2.185$ MeV, with $J^\pi=3^+, S=1$. In L-S coupling, this state belongs to a triplet of $^{13}D_3$ states, all with $T=0$.[7] Aleksandrov *et al.*[12] have found that this state is strongly excited in the reaction ^{3}He(^{7}Li,α)^{6}Li, showing that it has an appreciable ^{3}He-^{3}H structure. These same authors have shown that it also has a considerable ^{4}He-^{2}H clustering component.[2,9] It is generally recognized that this state is well described as predominantly a [4,2] partition.[2,9,10] In the cluster model of nuclear structure and nuclear reactions, the channels ^{3}He+^{3}H and ^{4}He+^{2}H correspond to the weights $w=(3,3)$ and $w=(4,2)$, respectively. Now, in terms of partitions we get $[3]\times[3]\rightarrow[4,2]+[3,3]$ and $[4]\times[2]\rightarrow[4,2]$. Hence, both weights or cluster structures may be used to build the partition $f=[4,2]$, but only the first one induces the partition $f=[33]$.[9] For bound state calculations this implies that both weights would contribute to the states with $f=[4,2]$. Therefore, following Kramer *et al.*,[9] the final state wave function will be written as

$$\Psi^{M_f}_{J_f}=C^{(3,3)}_{J_f}\mathcal{A}\{[(\Phi_{^3\mathrm{He},s=1/2}\otimes\Phi_{^3\mathrm{H},s=1/2})_1\otimes\mathcal{Y}_{L_f}(\hat{r})]^{M_f}_{J_f}\frac{w^{(3,3)}_{J_fL_f}(r)}{r}Z_{CM}(\mathbf{R})\}$$
$$+C^{(4,3)}_{J_f}\mathcal{A}\{[(\Phi_{^4\mathrm{He},s=0}\otimes\Phi_{^2\mathrm{H},s=1})_1\otimes\mathcal{Y}_{L_f}(\hat{r})]^{M_f}_{J_f}\frac{w^{(4,2)}_{J_fL_f}(r)}{r}Z_{CM}(\mathbf{R})\}\ , \tag{5}$$

where $C^{(\alpha,\beta)}_{J_f}$ is a normalization factor and $w^{(\alpha,\beta)}_{J_fL_f}(r)$ is the radial function of the relative motion. The spin functions of the two clusters are first coupled to $S=1$ and then coupled to the orbital angular momentum L_f of the relative motion to give the total angular momentum J_f of the final state. In the case of the ^{3}He-^{3}H cluster structure, its explicit form is

$$[(\Phi_{^3\mathrm{He},s=1}\otimes\Phi_{^3\mathrm{H},s=1})^{M'_s}_1\otimes\mathcal{Y}^{\mu}_{L_f}(\hat{r})]^{M_f}_{J_f}=\Phi_{^3\mathrm{He}}(\mathbf{r}_i)\Phi_{^3\mathrm{H}}(\mathbf{r}_j)\sum_{\mu,M_s}C(L_f1J_f;\mu M'_sM_f)\mathcal{Y}^{\mu}_{L_f}(\hat{r})\xi^{M'_s}_1\ ,$$

with $C(L_f1J_f;\mu M'_sM_f)$ being a Clebsh-Gordan coefficient in the notation of Rose[13] and $\xi^{M'_s}_1$ being a spin-isospin function having M'_s for the z component of the spin angular momentum.

Substitution of (4) and (5) for the initial and final state wave functions in the matrix element of the electric dipole operator gives

$$\langle fM_f|M^{(M)}_{E1}|iM_i\rangle=(-1)^{J_f-M_f}\sum_{J=1}^{\infty}\sum_{l=|J-1|}^{J+1}(2l+1)^{1/2}C(l1J;OM_iM_i)\begin{bmatrix}J_f & 1 & J\\ -M_f & M_f-M_i & M_i\end{bmatrix}Q_{Jl}(E)\ , \tag{6}$$

where

$$Q_{Jl}(E)=i^l\exp[i(\sigma_l-\sigma_0)]\sum_{l'=|J-1|}^{J+1}(J_fL_f||M_{E1}||Jl'l)\ . \tag{7}$$

The angular dependence of the cross section is fully contained in the function

$$\sum_{P=\pm1}|D^{(1)*}_{M_f-M_i;P}(\phi_\gamma,\theta_\gamma,0)|^2=\tfrac{2}{3}[1+(\tfrac{3}{2}\delta_{|M_f-M_i|,1}-1)P_2(\cos\theta_\gamma)]\ . \tag{8}$$

Upon substitution of (8) and (6) in (3), the summation over the spin projections in the initial state and the average over the spin projections in the final state may be performed; the result may be written as

$$d\sigma/d\Omega=\frac{1}{4\pi}\sigma_T(E)+a_2(E)P_2(\cos\theta_\gamma)\ , \tag{9}$$

where

$$\sigma_T(E)=\frac{\pi\lambda^2}{(2S+1)}\sum_{J=|J_f-1|}^{J_f+1}\sum_{l=|J-1|}^{J+1}|Q_{Jl}|^2 \tag{10}$$

and

$$a_2(E)=\frac{3\lambda^2}{8(2S+1)}\sum_{J=|J_f-1|}^{J_f+1}\sum_{J'=|J_f-1|}^{J_f+1}\sum_{l=|J-1}^{J+1}\sum_{l'=|J-1|}^{J+1}(\{\tfrac{1}{3}\delta_{ll'}\delta_{JJ'}-[(2l+1)(2l'+1)]^{1/2}Z(ll';JJ')\}Q_{Jl}^{*}Q_{J'l'})\ ; \tag{11}$$

the coefficient $Z(ll';JJ')$ is given by

$$Z(ll';JJ')=\sum_{M_i}\left[C(l1J;OM_iM_i)C(l'1J';OM_iM_i)\begin{bmatrix}J_f & 1 & J\\ -M_i & 0 & M_i\end{bmatrix}\begin{bmatrix}J_f & 1 & J\\ -M_i & 0 & M_i\end{bmatrix}\right]. \tag{12}$$

The energy dependence of the cross section is determined by the energy dependence of the partial transition amplitudes $Q_{Jl}(E)$, which is expressed in Eq. (7) in terms of the reduced matrix elements of the electric dipole operator. These reduced matrix elements are labeled with the partial wave orbital angular momentum l', and the angular momentum l of the incident beam at $t\rightarrow-\infty$.

In Sec. IV, we will compare the energy dependence of the radiative capture cross section with the energy dependence of the elastic scattering data, but the data on elastic collisions of particles with spin are usually parameterized in the formalism of eigenphases and mixing parameters. It is therefore convenient to give expressions for the partial transition amplitudes for radiative capture also in this formalism. In order to do this, recall that the radial function $R_{l'l}^{(J)}(kr)$, for very large values of r, behaves as the sum of an incoming Coulomb wave of angular momentum l, plus outgoing Coulomb waves of angular momentum l'. The coefficients of the outgoing waves are the elements of the collision matrix $S_{l'l}^{(J)}$ in the elastic channel. Since, in general, $S_{l'l}^{(J)}$ is nondiagonal, for very large values of r, $R_{l'l}^{(J)}(k,r)$ is a linear combination of waves with different phase shifts

$$R_{l'l}^{(J)}(k,r)=\sum_{\Lambda=|J-1|}^{J+1}U_{Jl'\Lambda}(k,r)B_{\Lambda l} \tag{13}$$

and

$$U_{Jl'\Lambda}(k,r)\rightarrow\frac{i}{2}B_{l'\Lambda}[\exp(-i\sigma_{l'})W_{l'}^{(-)}(kr)-\exp(i2\delta_{J\Lambda})\exp(+i\sigma_{l'})W_{l'}^{(+)}(kr)]\ . \tag{14}$$

The wave functions $U_{l'\Lambda}^{(J)}(k,r)$ are sometimes called eigenwaves and their phase shifts, eigenphaseshifts. The coefficients $B_{l'\Lambda}^{(J)}$ and $B_{\Lambda l'}^{(J)}$ are elements of the orthogonal matrix that diagonalize the collision matrix $S_{l'l}^{(J)}$. Explicit expressions for $B_{l'\Lambda}^{(J)}$ and $B_{\Lambda l'}^{(J)}$, in terms of the mixing parameter ϵ_{Jl} for scattering of ^{3}He by ^{3}H, are given at the end of Sec. II D.

Substitution of (13) in (4) and (7) gives

$$Q_{Jl}(E)=\sum_{\substack{\Lambda=|J-1|\\ \Lambda\neq J}}^{J+1}\hat{Q}_{J\Lambda}(E)B_{\Lambda l}\ . \tag{15}$$

$\hat{Q}_{J\Lambda}(E)$ is the transition amplitude for radiative capture in the representation of eigenphases. Substitution of (15) in (10) and (11) gives the total and differential cross sections for radiative capture in the formalism of eigenphases and mixing parameters.

B. The partial transition amplitude in the cluster model

Let us consider now the energy dependence of the partial radiative transition amplitude $Q_{Jl}(E)$. Keeping only the spin independent terms, the electric dipole operator is[11]

$$M_{E1}=\sum_{i=1}^{6}\frac{e}{2}[1+\tau_3(i)]\left[\frac{d}{d\rho_i}\rho_i j_1(K_\gamma\rho_i)\right]Y_{1M}(\hat{\rho}_i); \tag{16}$$

the nucleon spatial coordinate ρ_i is measured relative to the coordinate of the total center of mass, i.e.,

$$\boldsymbol{\rho}_i=\mathbf{r}_i-\mathbf{R}_{\text{c.m.}}\ . \tag{17}$$

e is the electron charge, $\tau_3(i)$ is the third component of the isospin of the ith nucleon, and K_γ is the wave number of the photon and $j_1(K_\gamma\rho)$ is the spherical Bessel function of order one.

Then, according to (4), (5), and (7), the energy dependence of the partial transition amplitude $Q_{Jl}(E)$ comes from two sources, the dependence of the electric dipole operator $M_{E1}^{(M)}$ on the wave number K_γ of the photon, and the dependence of the radial wave function $R_{l'l}^{(J)}(kr)$ on the wave number k of the relative motion of the two clusters in the entrance channel. It will be shown that, in this picture, a resonance in the partial transition amplitude can only be due to a resonance in the radial wave function, since the electric dipole operator $M_{E1}^{(M)}$ is an entire function of k in the finite part of the complex k plane.

The dependence on K_γ of the transition amplitude Q_{Jl} can be made explicit integrating over the single nucleon degrees of freedom. For the evaluation of the integrals we make use of the identity

$$\frac{d}{d\rho_i}[\rho_i j_1(K_\gamma\rho_i)]Y_{1M}^{*}(\hat{\rho}_i)=\frac{1}{2\pi}\lim_{\Lambda\rightarrow1}\left[\frac{d}{d\Lambda}\Lambda\int Y_{1M}^{*}(\hat{K}_\gamma)\exp(i\Lambda\mathbf{K}_\gamma\cdot\boldsymbol{\rho}_i)d\Omega_K\right]. \tag{18}$$

Hence, the partial transition amplitude may be written as

$$Q_{Jl}(E)=i^l\exp[i(\sigma_l-\sigma_0)]\sum_{l'=|J-1|}^{J+1}\left[\frac{1}{2\pi}\lim_{\Lambda\to 1}\left[\frac{d}{d\Lambda}\Lambda\int d\Omega_k\, Y^*_{1M}(\hat{K})\right.\right.$$
$$\left.\left.\times\left\langle\Psi_{J_fL_f}\left|\left|\sum_{i=1}^{6}\frac{e}{2}[1+\tau_3(i)]\exp[i\Lambda\mathbf{K}_\gamma\cdot(\mathbf{r}_i-\mathbf{R}_{\text{c.m.}})]\right|\right|\Psi_{Jl'l}\right\rangle\right]\right]. \tag{19}$$

The reduced matrix element in the right-hand side of Eq. (19) may be written as the sum of two terms, each one having a bra state of well-defined clustering structure, ^{3}He-^{3}H or ^{4}He-^{2}H, coming from the two components of well-defined clustering structure in the wave function $\Psi_{J_fL_f}$ of the final state. Since the functional form of the energy dependence of the matrix elements is determined by the properties of the radial wave function in the entrance channel the analysis of the two cases proceeds exactly along the same lines. Therefore, for the purpose of this section, which is to make explicit the resonant behavior of the partial transition amplitude $Q_{Jl}(E)$, it will be sufficient to consider in detail only the case which is simpler to evaluate, namely, when the clustering structure is the same in the initial and final states.

In this case, the matrix element which appears in the right hand side of this equation is of the same type as the matrix element considered by Kanada, Liu, and Tang in the calculation of the charge form factor of ^{7}Li,[14] and in the computation of the radiative capture of ^{3}He by ^{4}He in the long-wavelength limit with resonating group method wave functions.[15] These authors give quite general formulas which can be applied to any two-cluster $A+B$ system, with A and B being s shell clusters. Thus, we made the computation of $Q_{Jl}(E)$ following essentially the same procedure as described in these references but, since we are interested in the explicit form of the energy dependence of the matrix element, we avoided the long-wavelength limit approximation.

It is thus assumed that the functions Φ_{He} and Φ_{H}, which represent the internal spatial structures of the ^{3}He by ^{3}H clusters, have the normalized forms

$$\Phi_{^3\text{He}}=\left[\frac{\alpha^2}{3\pi^2}\right]^{3/4}\exp\left[-\frac{1}{2}\alpha\sum_{i=1}^{3}(\mathbf{r}_i-\mathbf{R}_{\text{He}})^2\right] \tag{20a}$$

and

$$\Phi_{^3\text{H}}=\left[\frac{\alpha^2}{3\pi^2}\right]^{3/4}\exp\left[-\frac{1}{2}\alpha\sum_{i=4}^{6}(\mathbf{r}_i-\mathbf{R}_{\text{H}})^2\right], \tag{20b}$$

where $\mathbf{R}_{\text{He}}$ and $\mathbf{R}_{\text{H}}$ are, respectively, the center of mass coordinates of the two clusters, and α is the width parameter. The function $Z(\mathbf{R}_{\text{c.m.}})$ describes the motion of the total center of mass. It is conveniently chosen as

$$Z(\mathbf{R}_{\text{c.m.}})=\left[\frac{6\alpha}{\pi}\right]^{3/4}\exp(-3\alpha R^2_{\text{c.m.}}). \tag{21}$$

Substitution of (20a), (20b), and (21) in (4), (5), and (19), and a straightforward calculation, give

$$Q_{Jl}(E)=(-1)^{M_f+m_l}(-1)^l i^l\exp[i(\sigma_l-\sigma_0)][(2J+1)(2J_f+1)]^{1/2}$$
$$\times\sum_{l'=|J-1|}^{J+1}(2l'+1)^{1/2}\begin{Bmatrix}L_f & 1 & J_f\\ l' & 1 & J\\ 1 & 0 & 1\end{Bmatrix}C(l'1L_f;000)144\sqrt{5}|C_f|\frac{e}{\hbar}$$
$$\times\left[\frac{m}{k}K_\gamma\right]^{1/2}\left\{I_0(Jl'l;k)+\left[\frac{3}{2L_f+1}\right]^{1/2}[2I_1(Jl'l;k)+I_2(Jl'l;k)]\right\}. \tag{22}$$

The radial integrals $I_x(Jl'l;k)$ may be written as

$$I_x(Jl'l;k)=\int_0^\infty\int_0^\infty w_{J_fL_f}(r)M_x^{(E1)}(r,r')R_{l'l}^{(J)}(kr')dr\,dr', \tag{23}$$

with $x=0,1,2$.

The integrand in the direct term is

$$M_0^{(E1)}(r,r')=[\Theta_{E1}(\tfrac{1}{2}K_\gamma r)-2\tilde{\mu}_0K_\gamma^2 j_1(K_\gamma r)]\delta(r-r'). \tag{24}$$

Θ_{E1} is the radial part of the electric dipole operator

$$\Theta_{E1}(\tfrac{1}{2}K_\gamma r)=3\left[j_1(\tfrac{1}{2}K_\gamma r)+r\frac{dj_1(\tfrac{1}{2}K_\gamma r)}{dr}\right], \tag{25}$$

where r is the radial coordinate of the relative motion in the entrance channel, and $j_1(\frac{1}{2}K_\gamma r)$ is the spherical Bessel function of order one.

The Galilei-invariant exchange internal matrix element $M_x^{(E1)}(r,r')$ in (23) is given by

$$M_x^{(E1)}(r,r')=O_{E1}^{(x)}(\lambda_{bx}K_\gamma r,\lambda_{bx}K_\gamma r')k^{(x)}_{L_f,l}(r,r'), \tag{26}$$

where

$$O_{E1}^{(x)}=\left\{\left[\tilde{\rho}\frac{dj_1(\tilde{\rho})}{d\tilde{\rho}}j_0(\tilde{\rho}')+j_1(\tilde{\rho})\tilde{\rho}'\frac{dj_0(\tilde{\rho}')}{d\tilde{\rho}'}+j_0(\tilde{\rho})\tilde{\rho}'\frac{dj_1(\tilde{\rho}')}{d\tilde{\rho}'}+\tilde{\rho}\frac{dj_0(\hat{\rho})}{d\tilde{\rho}}j_1(\tilde{\rho}')\right]+(1-2\tilde{\mu}_x K_\gamma^2)\times[j_0(\tilde{\rho})j_1(\tilde{\rho}')+j_1(\tilde{\rho})j_0(\tilde{\rho}')]\right\}\exp(-\tilde{\mu}_x K_\gamma^2) \tag{27}$$

with

$$\tilde{\rho}=\lambda_{bx}K_\gamma r\ ,$$

$$\tilde{\rho}'=\lambda_{bx}K_\gamma r'\ ,$$

and

$$k_{L_f',l}^{(x)}(r,r')=k_{L_f}^{(x)}(r,r')+k_l^{(x)}(r,r') \tag{28}$$

with

$$k_{L_f}^{(x)}(r,r')=(-1)^{L_f}(-1)^x 4\pi\left[\frac{27\alpha}{16\pi}\right]^{3/2}rr'\times\left[\frac{\pi}{2C_x rr'}\right]^{1/2}I_{L_f+1/2}(C_x rr')\times\exp\left[-\frac{15\alpha}{16}(r^2+r'^2)\right]\ . \tag{29}$$

The constants that appear in the above equations are

$$\lambda_{bx}=\frac{3}{4(3-x)},\quad x=1,2\ , \tag{30}$$

$$\tilde{\mu}_x=\frac{1}{24\alpha}\frac{(12-5x)}{(3-x)},\quad x=0,1,2\ , \tag{31}$$

and

$$C_x=-\frac{9\alpha}{4}\frac{(3-2x)}{x(3-x)},\quad x=1,2\ ; \tag{32}$$

$I_{L_f+1/2}(C_x rr')$ is a modified spherical Bessel function of the first kind.[16]

The long-wavelength limit of the Galilei-invariant internal matrix element of the electric dipole operator is readily obtained from (24), (25), and (27). The result is

$$\lim_{K_\gamma\to 0}\frac{1}{K_\gamma}M_0(\tfrac{1}{2}K_\gamma r)\delta(r-r')=r\delta(r-r') \tag{33}$$

and

$$\lim_{K_\gamma\to 0}\frac{1}{K_\gamma}M_x(r,r')=\frac{2\lambda_{bx}}{3}[(r)^1(r')^0+(r)^0(r')^1]\times k_{L_f,l}^{(x)}(r,r'),\quad x=1,2\ . \tag{34}$$

Then, it may be verified that in the long-wavelength limit, our expression for the partial transition amplitude $Q_{Jl}(E)$, Eq. (22), reduces to the expression obtained from the general formulas given by Liu, Kanada, and Tang,[15] Eqs. (26)–(31) of that reference in the special case of the ^{3}He-^{3}H system with $\alpha_A=\alpha_B=\alpha$.

C. Analytical structure of the transition amplitude in the complex k plane

In order to make apparent the resonant behavior of the transition amplitude, the radial wave function of the relative motion of the two clusters in the entrance channel will be expressed in terms of the Green's function of the integrodifferential radial equation of the relative motion in that channel. The resulting expression for the transition amplitude will be expanded as a summation over singularities of the Green's function plus an integral over a continuum of scattering wave functions of complex wave number.

The radial wave function may be written in terms of the Green's function as

$$R_{l'l}^{(J)}(kr)=F_{l'}(kr)\delta_{l'l}+\sum_{l''=|J-1|}^{J+1}\int_0^\infty\int_0^\infty G_{Jl'l''}^{(+)}(k;r,s)\times V_{l''l}^{(J)}(s,t)F_l(kt)ds\,dt\ , \tag{35}$$

where $F_l(kr)$ is the regular Coulomb function, $G_{Jl'l''}^{(+)}(k;r,s)$ is the Green's function of the equation of the relative motion of the ^{3}He-^{3}H system, and $V_{l''l}^{(J)}(s,t)$ is the nucleus-nucleus potential in the entrance channel, which, in general, is noncentral and nonlocal. The regular Coulomb function is proportional to the regular Whittaker function $M_{i\eta,l+1/2}(2ikr)$; η is the Sommerfeld parameter.

Substitution of (35) in the radial integral (23) gives

$$I_x(Jl'l;k)=[I_x^{(0)}(jl'l;k)+I_x^{(1)}(jl'l;k)]\frac{C_l(k)}{(2i)^{l+1}}\ , \tag{36}$$

where

$$I_x^{(0)}(Jl'l;k)=\int_0^\infty\int_0^\infty w_{J_f L_f}(r)M_x^{(E1)}(k;r,r')M_{i\eta,l+1/2}(2ikr')dr\,dr' \tag{37}$$

and

$$I_x^{(1)}(Jl'l;k)=\sum_{l''=|J-1|}^{J+1}\int_0^\infty\int_0^\infty\int_0^\infty\int_0^\infty w_{J_f L_f}(r)M_x^{(E1)}(k;r,r')G_{Jl'l''}^{(+)}(k;r',s)V_{l''l}^{(J)}(s,t)M_{i\eta,l+1/2}(2ikt)dr\,dr'\,ds\,dt\ , \tag{38}$$

and the Gamow factor $C_l(k)$ is given by

$$C_l(k)=\frac{1}{(2l+1)!!}\prod_{s=1}^{l}\left[1+\left[\frac{\eta}{s}\right]^2\right]^{1/2}\left[\frac{2\pi\eta}{\exp(2\pi\eta)-1}\right]^{1/2}, \tag{39a}$$

$$\eta=\frac{Z_{\mathrm{He}}Z_{\mathrm{H}}me^2}{\hbar^2k}. \tag{39b}$$

The analytical properties of $I_x^{(0)}(Jl'l;k)$ and $I_x^{(1)}(Jl'l;k)$ as functions of complex k may be derived from the properties of the functions in the integrands.

Let us first recall that the radial functions $R_{l'l}^{(J)}(kr)$ of the resonating group method are equivalent to a radial Schrödinger equation, the equivalence being realized by means of an integral transform independent of k.[17] Hence, the analytical properties of the Green's function of the integrodifferential equation of the resonating group method, as functions of k, are essentially the same as those of the Green's function of the corresponding radial Schrödinger equation. In the case of a nonlocal nucleus-nucleus potential, the analytical properties of the complete Green's function of the radial Schrödinger equation, as functions of the complex variable k, are somewhat more general than in the well known case of a local potential. A careful discussion of these properties when the nonlocal potential $V_l^{(J)}(s,t)$ is central and decreases faster than exp(-as) and/or exp(-at), when s and/or t go to infinity, and is well behaved at the origin of coordinates in both variables, may be found in the work of von der Heydt.[18] The method for generalizing these results from the case of central to the case of noncentral potentials is given in Newton.[19] An expansion of matrix elements of operators containing the complete Green's function in terms of a biorthonormal set of bound and resonant states plus an integral over a continuum of scattering functions of complex wave number in the case of nonlocal potentials in momentum space representation has been given by E. Hernández and A. Mondragón,[20] and in configuration space representation by A. Mondragón, E. Hernández, and J. M. Velázquez-Arcos.[21]

Some properties of the functions in the integrand of (37) and (38), relevant to our discussion, are the following:

(i) The complete Green's function $G_{Jl'l}^{(+)}(k;r,s)$ as function of the complex variable k, for fixed r and s, is meromorphic in the half-plane $\mathrm{Im}k>-a$, excluding the negative real axis where it has a cut extending from 0 to $-\infty$; a is the constant that characterizes the exponentially decreasing asymptotic behavior of $V_{l'l}^{(J)}$. In the half-plane $\mathrm{Im}k\geq 0$, the poles of $G_{Jl'l}^{(+)}(k;r,s)$ may lie only on the positive imaginary axis or on the real axis.[21] In the strip $0>\mathrm{Im}k>-a$, $G_{Jl'l}^{(+)}(k;r,s)$ has a finite number of poles located symmetrically with respect to the real axis. In case there is a pole at $k=0$, it may be simple or double.[18]

(ii) The residue of $G_{Jl'l}^{(+)}(k;r,s)$ on a simple pole is the product of two energy eigenfunctions properly normalized, which belong to the energy eigenvalue $E_n=-\hbar^2k_n^2/2m$. When the pole is located in the strip $0>\mathrm{Im}k>-a$, with $\mathrm{Re}k_n>0$, $\mathrm{Im}k_n<0$, and $\mathrm{Re}k_n>|\mathrm{Im}k_n|$, the energy eigenvalue is complex with $\mathrm{Im}E_n<0$, and the eigenfunctions are the resonant states or Gamow functions.[18]

(iii) For fixed values of r, the regular Whittaker function $M_{i\eta,l+1/2}(2ikr)$ is an entire function of k in the finite part of the complex k plane, excluding the negative real axis where it has a cut extending from 0 to $-\infty$.[22]

(iv) According to (24), (25), (26) and (27), the Galilei-invariant internal matrix elements of the electric dipole operator, $M_x^{(E1)}(r,r')$, are entire functions of the photon wave number K_γ. When the motion of the nuclei in the entrance and exit channels is nonrelativistic, the wave number of the photon is given by

$$K_\gamma(k)=\frac{Q_1}{\hbar c}\left[1-\frac{Q_1}{Mc^2}\right]+Ak^2, \tag{40}$$

where

$$A=\frac{1}{2}\frac{\hbar c}{mc^2}\left[1-\frac{Q_1}{Mc^2}\left[1-\frac{1}{2}\frac{Q_1}{Mc^2}\right]\right]. \tag{41}$$

In these expressions, M is the total mass of the ^{3}He+^{3}H system, m is the reduced mass, c is the velocity of light, and Q_1 is the difference of the Q value of the ^{3}H(^{3}He,γ)^{6}Li reaction and the excitation energy E_1 of the first excited state of ^{6}Li.

Hence, the internal matrix elements $M_x^{(E1)}(r,r')$ are entire functions of k^2 in the finite part of the complex k plane.

It follows that $I_x^{(0)}(Jl'l;k)$ is an entire function of k in the complex k plane, and $I_x^{(1)}(Jl'l;k)$ is a meromorphic function of k in the half-plane $\mathrm{Im}k>-a$, excluding the negative real axis where it has a cut extending from 0 to $-\infty$ (a is the real constant which characterizes the exponentially decreasing behavior of the potential).

In order to exhibit explicitly the singularities of $I_x^{(1)}(Jl'l;k)$ it is convenient to have the Green's function in Eq. (38) written in its spectral representation. To do this, we make use of the completeness of bound and scattering states and insert between $G_{Jl''l}^{(+)}(k;r,s)$ and $V_{l''l}^{(J)}(s,t)$ a $\delta(s-s')$ written as

$$\delta(s-s')\delta_{l'l''}=\sum_m v_{mJl'}(s)v_{mJl''}(s')+\frac{1}{2\pi}\sum_{\Lambda=|J-1|}^{J+1}\int_0^\infty U_{Jl'\Lambda}(k's)\times U_{Jl''\Lambda}^*(k's')dk', \tag{42}$$

where $v_{mJl}(s)$ is a bound state eigenfunction corresponding to the negative energy E_{mJ}, and $U_{Jl\Lambda}(k's)$ is a scattering state function of wave number k'. In this way we obtain

$$I_x^{(1)}(Jl'l;k)=\sum_{\substack{m\\ \text{bound states}}}\left[\int_0^\infty\int_0^\infty w_{J_fL_f}(r)M_x^{(E1)}(r,r')v_{mJl'}(r')dr\,dr'\right]$$
$$\times\frac{1}{E+|E_{mJ}|}\left[\sum_{l''=|J-1|}^{J+1}\int_0^\infty\int_0^\infty v_{mJl''}(r)V_{l''l}^{(J)}(r,r')M_{i\eta,l+1/2}(2ikr')dr\,dr'\right]$$
$$+\frac{1}{2\pi}\sum_{\Lambda=|J-1|}^{J+1}\int_0^\infty dk'\left\{\left[\int_0^\infty\int_0^\infty w_{J_fL_f}(r)M_x^{(E1)}(r,r')U_{Jl'\Lambda}(k'r')dr\,dr'\right]\frac{1}{(k+i\epsilon)^2-k'^2}\right.$$
$$\left.\times\left[\sum_{l''=|J-1|}^{J+1}\int_0^\infty\int_0^\infty U_{Jl''\Lambda}^*(k's)V_{l''l}^{(J)}(s,s')M_{i\eta,l+1/2}(2ik's')ds\,ds'\right]\right\}. \quad (43)$$

Now, from the spectral representation of the resolvent for outgoing and incoming particles and the Plemelj formulas,[23] it follows that

$$\sum_\Lambda U_{Jl'\Lambda}(k's)U_{Jl\Lambda}^*(k's')$$
$$=i\frac{k'}{2\pi}[G_{Jl'l}^{(+)}(k';s,s')-G_{Jl'l}^{(-)}(k';s,s')]\ ; \quad (44)$$

as it stands, this expression is valid only when k' is real and positive. A generalization of (44), valid for complex values of k', may be obtained from the relations,[18]

$$G_{Jl'l}^{(+)}(z;s,s')-G_{Jl'l}^{(-)}(z;s',s)$$
$$=(-1)^l i\frac{2\pi}{z'}\sum_\Lambda U_{Jl'\Lambda}(z,s)U_{Jl\Lambda}(-z,s') \quad (45)$$

and

$$U_{Jl'\Lambda}(-z,s')=(-1)^{l+1}U_{Jl'\Lambda}^*(z^*,s')\ ; \quad (46)$$

substitution of (46) in (45) gives

$$\sum_\Lambda U_{Jl'\Lambda}(z,s)U_{Jl\Lambda}^*(z^*,s')$$
$$=(-1)^{l+1}i\frac{z}{2\pi}[G_{Jl'l}^{(+)}(z;s,s')-G_{Jl'l}^{(-)}(z;s,s')]\ , \quad (47)$$

which is valid when z is in the strip $a>\mathrm{Im}z>-a$, and reduces to (44) when $z=k'$ on the real axis. This result suggests the substitution of the adjoint of $U_{Jl\Lambda}(k',s')$ defined as

$$\tilde{U}_{Jl\Lambda}(k',s')=(-1)^{l+1}U_{Jl\Lambda}(-k',s') \quad (48)$$

for $U_{Jl\Lambda}^*(k',s)$ in (43) and (44).

Thus, the integral over k' in the second term on the right hand side of (43) may be written as

$$\int_0^\infty\left[\int_0^\infty\int_0^\infty\int_0^\infty w_{J_fL_f}(r)M_x^{(E1)}(r,r')U_{Jl'\Lambda}(kr')\tilde{U}_{Jl''\Lambda}(k',s)\right.$$
$$\left.\times V_{l''l}^{(J)}(s,s')M_{i\eta,l+1/2}(2iks')dr\,dr'ds\,ds'\right]\frac{1}{(k+i\epsilon)^2-k'^2}dk'$$
$$=\int_0^\infty\left[\int_0^\infty\int_0^\infty\int_0^\infty w_{J_fL_f}(r)M_x^{(E1)}(r,r')[G_{Jl'l''}^{(+)}(k',r',s)-G_{Jl'l''}^{(-)}(k';r',s)]\right.$$
$$\left.\times V_{l''l}^{(J)}(s,s')M_{i\eta,l+1/2}(2iks')dr\,dr'ds\,ds'\right]\frac{1}{(k+i\epsilon)^2-k'^2}dk'\ . \quad (49)$$

The resolvent for incoming particles may be analytically continued to the lower half of the k' plane without crossing the real axis and it has no singularities in that part of the plane. The analytical continuation of the resolvent for outgoing particles may have poles in the lower half of the k plane. Near a complex pole at k_ν, the behavior of $G_{Jl'l''}^{(+)}(k';r',s)$ can be expressed as

$$G_{Jl'l''}^{(+)}(k';r',s)=\frac{u_{\nu Jl'}^{(\Lambda)}(r')u_{\nu Jl''}^{(\Lambda)}(r)}{k'^2-k_{\nu J\Lambda}^2}+\chi_\nu Jl'l''(k';r',s)\ , \quad (50)$$

where $u_{\nu Jl'}^{(\Lambda)}(r)u_{\nu Jl}^{(\Lambda)}(r')$ is the product of two normalized Gamow functions corresponding to the complex energy $E_{\nu J\Lambda}$, and $\chi_{\nu Jl'l''}(k;r's)$ is regular at $k_{\nu J\Lambda}$; Λ is the label of the element of the diagonal scattering matrix which has the pole.

Now, the integration contour in Eq. (50) is deformed into the lower half-plane as shown in Fig. 1. When the deformed contour C crosses over resonant poles, but avoids other singularities of the analytically continued Green's functions, the theorem of the residue yields

$$\int_0^\infty \left\{ \int_0^\infty [w_{J_f L_f}(r) M_x^{(E1)}(r,r') U_{Jl'\Lambda}(k'r') \tilde{U}_{Jl''\Lambda}(k',s) V_{l''l}^{(J)}(s,s') M_{i\eta,l+1/2}(2iks') dr\, dr' ds\, ds'] \frac{1}{(k^2-k'^2)} \right\} dk'$$

$$= \sum_{\substack{\nu \\ \text{(resonant states)}}} \left[\int_0^\infty \int_0^\infty w_{J_f L_f}(r) M_x^{(E1)}(r,r') u_{\nu Jl'}^{(\Lambda)}(k'r') dr\, dr' \right]_{k'=k_\nu}$$

$$\times \frac{1}{k^2-k_\nu^2} \left[\int_0^\infty \int_0^\infty u_{\nu Jl''}^{(\Lambda)}(k's) V_{l''l}^{(J)}(s,s') M_{i\eta,l+1/2}(2iks') ds\, ds' \right]_{k'=k_\nu}$$

$$+ \int_C \left\{ \left[\int_0^\infty \int_0^\infty w_{J_f L_f}(r) M_x^{(E1)}(r,r') U_{Jl'\Lambda}(k'r') dr\, dr' \right]_{k'=z} \right.$$

$$\left. \times \frac{1}{k^2-z^2} \left[\int_0^\infty \int_0^\infty \tilde{U}_{Jl''\Lambda}(k',s) V_{l''l}^{(J)}(s,s') M_{i\eta,l+1/2}(2iks') ds\, ds' \right]_{k'=z} \right\} dz \ . \tag{51}$$

The integrals on the right-hand side of this equation are defined with k' in the upper half of the k' plane. After the integration over the radial coordinates is performed, the resulting functions of k' are analytically continued to k_ν or z in the lower half of the k' plane. Defined in this way, it is not obvious that the coefficients in the expansion may be interpreted as transition amplitudes from a state in the continuum to resonant or quasibound states.

Let us consider first the analytical continuation of the integrals of the potential. The scattering wave function may be written as the sum of an incoming plus an outgoing wave. The Gamow function is a purely outgoing wave. When $k'=k_\nu$, the amplitude of the outgoing waves grows with r as $\exp(-\mathrm{Im}k_\nu r)$. But, since $|\mathrm{Im}k_\nu| < a$, the exponentially decreasing behavior of the potential dominates at large distances. Hence, the same conditions on the nonlocal potential that guarantee the existence of bound states and scattering solutions of the radial equation are sufficient to ensure the existence of the integrals of products of the potential and scattering or Gamow functions with k_ν in the argument. Since $U_{Jl\Lambda}(k'r)$ and $u_{\nu Jl}^{(\Lambda)}(k'r)$ are analytical functions of k', continuous across the real positive axis,[21] the analytical continuation of the integrals may be performed under the integration sign.

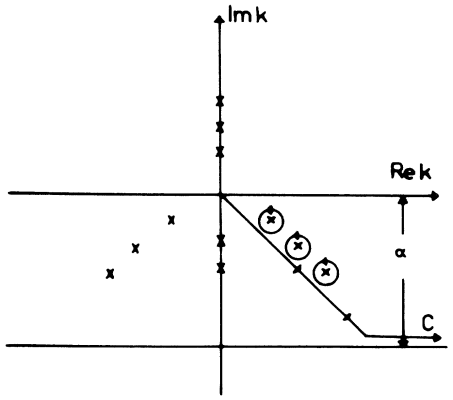

FIG. 1. The integration contour C, in the wave number plane k, that appears in Eq. (51) and (53), starts from the origin as a straight line with slope -1, it goes down to $\mathrm{Im}=\alpha+\epsilon$, then it continues as a straight line parallel to the real axis.

Let us consider now the analytic continuation of the integrals of the Galilei-invariant matrix elements of the electric dipole operator $M_x^{(E1)}(k;r,r')$. In the case of $x=1,2$, the energy independent Gaussian factor in $M_x^{(E1)}(k;r,r')$ dominates over the exponentials for large values of r or r', making the integrals finite even when the wave number k' in the argument of the scattering and Gamow functions is equal to k_ν with $\mathrm{Im}k_\nu<0$. In this case, as in the previous one, the limit $k'\to k_\nu$ may be taken under the integration sign. In the case of the direct integrals, $x=0$, both $U_{Jl\Lambda}(k_\nu r)$ and the Gamow function $u_{\nu Jl}^{(\Lambda)}(k_\nu r)$ oscillate asymptotically between exponentially growing envelopes. But, since for large values of r, $w_{J_f L_f}(r)$ is an exponentially decreasing function of r, independent of k', we may have two possibilities: First, when k_ν is such that, for large values of r, the absolute value of the integrand is an exponentially decreasing function of r, the integrals defined with k_ν in the argument of the scattering state function and the Gamow function are well defined and the limit $k'\to k_\nu$, indicated in (51), may be taken under the integration sign. Second, when $k_{\nu J\Lambda}$ is such that, for large values of r, the absolute value of the integrand grows exponentially with r, the integrand oscillates asymptotically between exponentially growing envelopes and the integrals with $k_{\nu J\Lambda}$ in the argument of the scattering function or the Gamow function are not well defined. In this case the integrals are defined with the help of a Gaussian regulator. Zel'dovich[24] has shown that the analytic continuation to $k_{\nu J\Lambda}$ in the lower half of the k' plane of the integral defined with k' in the upper half of the k' plane is equal to the limit of the integral defined with the Gaussian regulator when the parameter of the regulator goes to zero.

Substitution of (51) in (43), (38), and (36) gives

$$I_x(J,l'l;k)=\sum_{\substack{\nu \\ \text{(resonant states)}}}(w_{J_fL_f}|M_x^{(E1)}|u_{\nu Jl'}^{(\Lambda)})_{k'=k_\nu}\frac{1}{k^2-k_\nu^2}\sum_{l''=|J-1|}^{J+1}(u_{\nu Jl''}^{(\Lambda)}|V_{l''l}^{(J)}|M_{i\eta,l+1/2})(2i)^{-l-1}C_l(k)+I_x^{(B)}(Jl'l;k) \tag{52}$$

with

$$I_x^{(B)}(Jl'l;k)=\frac{C_l(k)}{(2i)^{l+1}}I_x^{(0)}(Jl'l;k)+\sum_{\substack{m \\ \text{(bound states)}}}(w_{J_fL_f}|M_x^{(E1)}|v_{mJl'})\frac{1}{k^2+|k_m|^2}\sum_{l''=|J-1|}^{J+1}(v_{mJl''}|V_{l''l}^{(J)}|M_{i\eta,l+1/2})$$
$$+\frac{2}{\pi}\sum_{\Lambda}\int_C dz\left[(w_{J_fL_f}|M_x^{(E1)}|U_{Jl'\Lambda})_{k'=z}\frac{1}{k^2-z^2}\sum_{l''=|J-1|}^{J+1}(U_{Jl''\Lambda}|V_{l''l}^{(J)}|M_{i\eta,l+1/2})_{k'=z}\right], \tag{53}$$

where, for $x=0$,

$$(w_{J_fL_f}|M_0^{(E1)}|u_{\nu Jl'}^{(\Lambda)})_{k'=k_\nu}=\lim_{\mu\to 0}\int_0^\infty \exp(-\mu r^2)[w_{J_fL_f}(r)M_0^{(E1)}(k;r)u_{\nu Jl'}^{(\Lambda)}(k_\nu r)dr] \tag{54}$$

and for $x=1,2$,

$$(w_{J_fL_f}|M_x^{(E1)}|u_{\nu Jl'}^{(\Lambda)})_{k'=k_\nu}=\int_0^\infty w_{J_fL_f}(r)M_x^{(E1)}(k;r,r')u_{\nu Jl'}^{(\Lambda)}(k_\nu,r')dr\,dr' , \tag{55}$$

and similar expressions for the corresponding terms in (53) where a scattering function of complex argument z occurs in place of the Gamow function. The integrals of the potential are

$$(u_{\nu Jl''}^{(\Lambda)}|V_{l''l}^{(J)}|M_{i\eta,l+1/2})_{k'=k_\nu}=\int_0^\infty\int_0^\infty u_{\nu Jl''}^{(\Lambda)}(k_\nu r)V_{l''l}^{(J)}(r,r')M_{i\eta,l+1/2}(2ikr')dr' \tag{56}$$

and

$$(U_{Jl''\Lambda}|V_{l''l}^{(J)}|M_{i\eta,l+1/2})_{k'=z}=\int_0^\infty\int_0^\infty \tilde{U}_{Jl''\Lambda}(z,r)V_{l''l}^{(J)}(r,r')M_{i\eta,l+1/2}(2ikr')dr\,dr' . \tag{57}$$

As explained above, the notation $\tilde{U}_{Jl''\Lambda}(z,r)$ is used in the right-hand side of (57) since the adjoint of the scattering state function $U_{Jl''\Lambda}(z,r)$ is not equal to its complex conjugate when the wave number z is complex.

Since, in this work, the Gamow states are defined as eigenfunctions of the integrodifferential equation of the resonating group method, they are normalized according to the rule

$$(u_{J\nu}^{\Lambda}|u_{J\nu}^{\Lambda})=\sum_l\lim_{\mu\to 0}\int_0^\infty\exp(-\mu r^2)u_{\nu Jl}^{(\Lambda)}(k_\nu r)u_{\nu Jl}^{(\Lambda)}(k_\nu r)dr$$
$$+\sum_{x=1}^{2}\sum_{l=|J-1|}^{J+1}\int_0^\infty\int_0^\infty u_{\nu Jl}^{(\Lambda)}(k_\nu,r)N_{xl}(r,r')u_{\nu Jl}^{(\Lambda)}(r')dr\,dr'=1 , \tag{58}$$

where $\delta(r-r')-\sum_{x=1}^{2}N_{xl}(r,r')$ is the radial part of the norm kernel.

D. Resonances

The results expressed in Eq. (52) and (53) may be cast in more familiar terms when we identify the elastic and radiative partial widths. The elastic partial width is proportional to the square of the absolute value of the transition amplitude from a state in the continuum to the resonant state. Hence,

$$\exp(i\phi_{\nu J\Lambda})\Gamma_{el,\nu J(\Lambda)l}^{1/2}(K)=\left[\frac{4m}{\hbar^2 k}\right]^{1/2}\sum_{\substack{l''=|J-1| \\ l''\neq J}}^{J+1}[u_{\nu Jl''}^{(\Lambda)}(k_\nu)|V_{l''l}^{(J)}|M_{i\eta,l+1/2}(2ik)](2i)^{-l-1}C_l(k) . \tag{59}$$

Since the Coulomb function is real, the phase $\phi_{\nu J\Lambda}$ comes from the Gamow function and it is independent of k.

The partial width of the resonant state for decay into a bound state of lower energy by emitting γ radiation is proportional to the square of the reduced transition amplitude.[25] Hence, we may write

$$\exp(i\chi_{vJ\Lambda})\Gamma^{1/2}_{\gamma,vJ\Lambda}(k)=(-1)^{M_f+M_i}(-1)^{l}[(2J_f+1)]^{1/2}$$
$$\times\sum_{l'=|J-1|}^{J+1}[(2l'+1)]^{1/2}\begin{Bmatrix} L_f & 1 & J_f \\ l' & 1 & J \\ 1 & 0 & 1 \end{Bmatrix}C(l'1L_f;000)144\sqrt{5}|C_f|$$
$$\times\frac{e}{\hbar}\left[\frac{mK_\gamma}{k}\right]^{1/2}\left[(w_{J_fL_f}|M_0^{(E1)}|u^{(\Lambda)}_{vJl'})+\left[\frac{3}{(2L_f+1)}\right]^{1/2}(2(w_{J_fL_f}|M_1^{(E1)}|u^{(\Lambda)}_{vJl'})\right.$$
$$\left.+(w_{J_fL_f}|M_2^{(E1)}|u^{(\Lambda)}_{vJl'}))\right]. \tag{60}$$

The expansion of the partial transition amplitude for direct radiative capture as a sum of resonances plus a background term is obtained substituting (59) and (60) in (52) and (22). The result is

$$Q_{Jl}(E)=(2J+1)^{1/2}i^{l+1}\exp[i(\sigma_l-\sigma_0)]\exp[i(\phi_{vJ\Lambda}+\chi_{vJ\Lambda})]\frac{\Gamma^{1/2}_{\gamma,vJ\Lambda}(k)\Gamma^{1/2}_{el,vJ(\Lambda)l}(k)}{(E-E_{vJ\Lambda})+i\frac{1}{2}\Gamma_{vJ\Lambda}}+Q_{Jl}^{(B)}(E)\ . \tag{61}$$

The nonresonant terms in (50) and (53) have been collected in the background amplitude $Q_{Jl}^{(B)}(E)$.

The expression (60) gives only the contribution of the ^{3}He-^{3}H clustering component of $\Psi_{J_fL_f}$ to the radiative width. In a realistic microscopic computation one must add to the right hand side of (60) the contribution of the ^{4}He-^{2}H clustering component of $\Psi_{J_fL_f}$. However, expression (61) is valid independently of the cluster composition of the final state wave function since its derivation depends only on the analytical properties of the wave function in the entrance channel.

In (61), both partial widths are functions of k. At low energies, in the long-wavelength regime, the gamma width depends on k only very weakly, and it may be approximated by its value at the resonance energy.

The elastic partial width $\Gamma_{vJ(\Lambda)l}(k)$, at low energies, is a rapidly varying function of k due to the strong dependence on k of the Gamow factor $C_l(k)$ and the weaker variation with k of the Whittaker function. When a wide resonance is close to threshold, it is essential to take into account this energy variation to have a good representation of the data. In the case of a potential decreasing exponentially or faster with the radial distance, when k is small the main contribution of the Whittaker function to the integral in (59) comes from small values of the argument. For small values of the argument, the Whittaker function behaves as[26]

$$M_{i\eta},l+1/2(i2kr)=(2ikr)^{l+1}\left[1+\frac{\eta}{l+1}(kr)\ldots\right]. \tag{62}$$

This expression suggests the following approximation:

$$\Gamma^{1/2}_{el,vJ(\Lambda)l}(k)\simeq\Gamma^{1/2}_{el,vJ(\Lambda)l}(E_{vJ\Lambda})N_l(k_v,k)\ , \tag{63}$$

where $N_l(k_v,k)$ is the Coulomb penetration factor[27]

$$N_l(k_v,k)=\left[\frac{k}{k_v}\right]^{l+1}\frac{C_l(k)}{C_l(k_v)}\ ; \tag{64}$$

k_v is the wave number corresponding to the resonance energy $E_{vJ\Lambda}$.

In this approximation, the partial transition amplitude may be written as

$$Q_{Jl}(E)=(2J+1)^{1/2}i^{l}\exp[i(\sigma_l-\sigma_0)]\exp[i(\phi_{vJ\Lambda}+\chi_{vJ\Lambda})]\frac{\Gamma^{1/2}_{\gamma,vJ\Lambda}(E_{vJ\Lambda})\Gamma^{1/2}_{el,vJ(\Lambda)l}E_{vJ\Lambda})}{(E-E_{vJ\Lambda})+i\frac{1}{2}\Gamma_{vJ\Lambda}}N_l(k_v,k)+Q_{Jl}^{(B)}(E)\ . \tag{65}$$

Multiplying both sides of Eq. (65) by the orthogonal matrix $B_{l\Lambda}$, and summing over l, we obtain the transition amplitude in the representation of eigenphases

$$\hat{Q}_{J\Lambda'}=\sum_{l=|J-1|}^{J+1}\sum_{\Lambda''=|J-1|}^{J+1}\left\{(2J+1)^{1/2}B_{\Lambda''l}i^{l}\exp[i(\sigma_l-\sigma_0)]B_{l\Lambda}\exp[i(\phi_{vJ\Lambda}+\chi_{vJ\Lambda})]\frac{\Gamma^{1/2}_{\gamma,vJ\Lambda}\Gamma^{1/2}_{el,vJ\Lambda}}{(E-E_{vJ\Lambda})+i\frac{1}{2}\Gamma_{vJ\Lambda}}\mathcal{F}_{\Lambda\Lambda'}\right\}+\hat{Q}^{(B)}_{J\Lambda'}\ . \tag{66}$$

The elastic partial width, in the representation of eigenphases, is given by

$$\Gamma^{1/2}_{el,vJ\Lambda}\delta_{\Lambda\Lambda'}=\sum_{l=|J-1|}^{J+1}\Gamma^{1/2}_{el,vJ(\Lambda)l}B_{l\Lambda'}\ , \tag{67}$$

and the Coulomb penetration factor is now

$$F_{\Lambda\Lambda'}=\sum_{l=|J-1|}^{J+1}B_{\Lambda l}N_l(k_v,k)B_{l\Lambda'}\ . \tag{68}$$

For a spin $\frac{1}{2}$ nucleus colliding with a spin $\frac{1}{2}$ nucleus, the orthogonal matrix $B_{l\Lambda}$ is written in terms of the mixing parameter ϵ_J:

$$B_{J-1,J-1}=B_{J+1,J+1}=\cos\epsilon_J \ , \tag{69a}$$

$$-B_{J-1,J+1}=B_{J+1,J-1}=-\sin\epsilon_J \ . \tag{69b}$$

Therefore, the Coulomb penetration factor in the representation of eigenphases is given by

$$F_{\Lambda\Lambda=J\mp1}(k_{\Lambda=J\mp1};k)=\cos^2\epsilon_J N_{J\mp1}+\sin^2\epsilon_J N_{J\pm1} \ . \tag{70}$$

The upper sign applies when the diagonal scattering matrix has a pole in the element in the first row and first column ($\Lambda=J-1$); the lower sign corresponds to a pole in the third row and third column ($\Lambda=J+1$). The element in the second row and second column has opposite parity and does not get mixed with the others. Hence, when $\Lambda=J$,

$$F_{\Lambda=J}(k_J;k)=N_J(k_J,k) \ . \tag{71}$$

III. RESONANCES IN THE ELASTIC SCATTERING OF ^{3}He BY ^{3}H

A. The experimental data

The elastic scattering of polarized ^{3}He by ^{3}H was measured by Vlastou *et al.*[4] in the energy range from 20 to 33 MeV. These authors also made a detailed phase shift analysis of the elastic scattering of ^{3}He by ^{3}H over the energy range from 5 to 33 MeV. The experimental data consisted of the differential cross sections, analyzing power and polarization of the scattered particles, and polarization of the recoil particles measured by them[4] and other authors.[28–31] The computation of the cross section and polarization of the scattered and recoil particles was made in the Blatt and Biedenharn[32] general formalism for the elastic scattering between particles of spin $\frac{1}{2}$, based in the L-S coupling scheme, as developed by Tombrello *et al.*[33] The scattering matrix, $S^{(J)}_{l,l'}$, for each value of the total angular momentum J, parity $\pi=(-1)^{J\mp1}$, spin $S=1$, and isospin $T=1$, was written in the eigenphase representation and parametrized in terms of three parameters, the two eigenphases δ_{Λ_J} with $\Lambda=J\mp1$, and the mixing parameter ϵ_J,

$$S^{(J)}_{J-1,J-1}=\cos^2\epsilon_J\exp(i2\delta_{(J-1)_J})+\sin^2\epsilon_J\exp(i2\delta_{(J+1)_J}) \ , \tag{72}$$

$$S^{(J)}_{J+1,J+1}=\sin^2\epsilon_J\exp(i2\delta_{(J-1)_J})+\cos^2\epsilon_J\exp(i2\delta_{(J+1)_J}) \ , \tag{73}$$

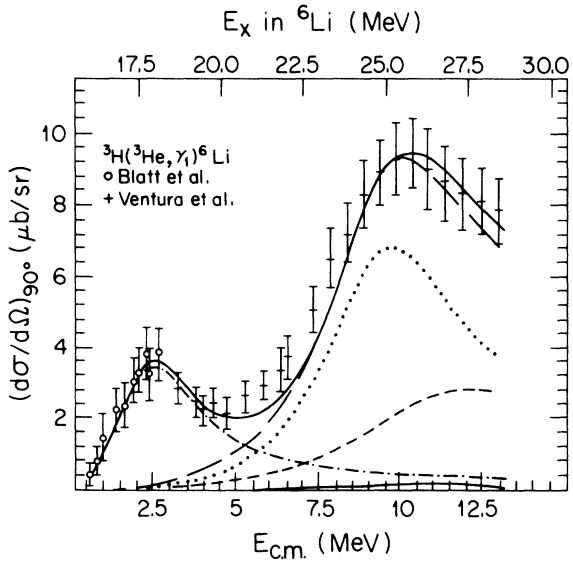

FIG. 2. The 90° differential cross section for the reaction ^{3}H(^{3}He,γ_1)^{6}Li* as a function of center of mass energy (Ref. 6). The solid line represents the best fit of interfering resonances formula to the data. The dot-dashed line represents the contribution of the $^{33}P_2$ wave, the short dashed line shows the $^{33}F_2$ wave, the dotted line represents the $^{33}F_4$ wave, the solid line at the bottom represents the $^{33}F_3$ wave, and the long dashed line represents the sum of the $^{33}F_J$ waves ($J=2$, 3, and 4).

$$S^{(J)}_{J-1,J+1}=S^{(J)}_{J+1,J-1}=\sin\epsilon_J\cos\epsilon_J[\exp(i2\delta_{(J-1)_J})-\exp(i2\delta_{(J+1)_J})] \ . \tag{74}$$

When $\Lambda=J$, $\pi=(-1)^J$, and $S=1$, there is no mixing of states of different orbital angular momentum, and the corresponding element of the collision matrix was parameterized simply as

$$S^{(J)}_{J,J}=\exp(i2\delta_{(J)_J}) \ . \tag{75}$$

In a first analysis of the data, which Vlastou *et al.*[4] call solution I, these authors took into account up to 18 real phase shifts in the calculation of the cross section and polarization of the scattered and recoil particles, with l from 1 up to 4. Although the quality of the fit is good, a second, more realistic analysis of the same elastic scatter-

TABLE I. Level parameters extracted from the analysis of the elastic scattering and radiative capture of ^{3}He by ^{3}H in the first excited state of ^{6}Li.

State	$E_{vJ\Lambda}$ (MeV)	$\Gamma_{vJ\Lambda}$ (MeV)	$\Gamma_{el,vJ\Lambda}$ (MeV)	$\Gamma_{\gamma,vJ\Lambda}$ (eV)
$^{33}P_2$	2.190±0.025	3.012±0.007	2.983±0.001	80±12.5
$^{33}F_2$	10.795±0.065	8.684±0.125	8.422±0.110	1164^{+140}_{-324}
$^{33}F_3$	8.984±0.054	6.754±0.110	6.252±0.090	7^{+80}_{-7}
$^{33}F_4$	9.095±0.055	5.316±0.112	4.233±0.070	807^{+84}_{-238}

$\Delta\chi_{12-32}=-55°$ $\Delta\chi_{12-34}=87°$ $\Delta\chi_{32-34}=-142°$

ing data, called solution II, was made, in terms of 18 complex phase shifts, to account for the possible influence of the open reaction channels. Vlastou *et al.*[14] parametrized the resonant real phase shifts of their solution I in terms of a single level R-matrix formalism. In the case of the real $^{33}F_J$ phase shifts, they were able to extract meaningful resonance parameters for the $^{33}F_3$ and the $^{33}F_4$ phase shifts, but not for the $^{33}F_2$ phase shift. They did not analyze the complex phase shifts of their solution II.

B. New analysis of the data: Poles of the S matrix

In view of the possible influence of the open inelastic channels of the $^{33}F_J$ phase shifts in the energy region of the broad structure apparent in the excitation function for radiative capture of ^{3}He by ^{3}H, Fig. 2, we decided that a parametrization of the complex phase shifts, solution II of Vlastou *et al.*,[4] should be made in terms of a formula more realistic than the single level R-matrix expression. With this purpose in mind, we made a χ^2 fit of the single level $S_\Lambda^{(J)}$ matrix formula[34]

$$S_\Lambda^{(J)}=\exp[i2\phi_{HS}(a)]\left[\exp(i2\beta_{\nu J\Lambda})|B_{J\Lambda}|-i\exp(i2\phi_{\nu J\Lambda})\frac{\Gamma_{el,\nu J\Lambda}}{(E-E_{\nu J\Lambda})+i\frac{1}{2}\Gamma_{\nu J\Lambda}}F_\Lambda^2(k)\right] \tag{76}$$

to the resonant elements of the scattering matrix in the representation of eigenphases obtained from the $^{33}P_2$, $^{33}F_3$, and $^{33}F_4$ complex phase shifts of solution II of Vlastou *et al.*[4] In the notation of Vlastou *et al.*,[4]

$$S_\Lambda^{(J)}=\eta_{J\Lambda}\exp(i2\,\mathrm{Re}\delta_{\Lambda_J}) \tag{77a}$$

and

$$\eta_{J\Lambda}=\exp(-2\,\mathrm{Im}\delta_{\Lambda_J})\ . \tag{77b}$$

Since these authors found it unnecessary to take into account phase shifts for values of the orbital angular momentum larger than four, in their analysis the eigenwave $^{33}F_4$ has no partner and only the $^{33}P_2$ and $^{33}F_2$ eigenwaves are mixed with mixing parameter ϵ_2.

In expression (76), $F_\Lambda(k)$ is the Coulomb penetration factor defined in (70); $\phi_{HS}(a)$ is the phase shift due to a charged hard sphere of radius a and charge $Z_{\mathrm{He}}Z_{\mathrm{H}}e$,

$$\phi_{HS}(a)=\tan^{-1}\frac{F_l(ka)}{G_l(ka)}\ . \tag{78}$$

$F_l(ka)$ and $G_l(ka)$ are the regular and irregular Coulomb functions, respectively. The magnitude of the background term is represented by a quadratic function of the energy

$$|B_{J\Lambda}(E)|=B_{J\Lambda}^{(0)}+m_{B_{J\Lambda}}E+r_{B_{J\Lambda}}E^2\ . \tag{79}$$

The phase of the background term $\beta_{J\Lambda}$ and the phase of the elastic width $\phi_{J\Lambda}$ are represented as linear functions of the energy with the same value of the slope,

$$\beta_{J\Lambda}(E)=\beta_{0J\Lambda}+m_\beta E\ , \tag{80}$$

$$\phi_{J\Lambda}(E)=\phi_{0J\Lambda}+m_\beta E\ ; \tag{81}$$

the resonance energy, $E_{\nu J\Lambda}$, the elastic width $\Gamma_{el,\nu J\Lambda}$, and the total width $\Gamma_{\nu J\Lambda}$ are not functions of the energy. Therefore, in expression (76) there are ten parameters that may be varied.

The analysis of the complex phase shifts over the energy range from 5 to 33 MeV was performed using the computer program FASFIT with an automatic search routine to minimize the quadratic deviations of the theoretical predictions computed with (76) from the resonant elements of the scattering matrix in the eigenphase representation obtained from the solution II of Vlastou *et al.*[4] The resonance formula in the right hand side of (76) was fitted to the elements of the scattering matrix obtained from the data analysis of Vlastou *et al.* by varying the parameters $B_{0J\Lambda}$, $\beta_{0J\Lambda}$, $\phi_{0J\Lambda}$, $\Gamma_{el,\nu J\Lambda}$, $\Gamma_{\nu J\Lambda}$, $E_{el,\nu J\Lambda}$, $m_{B_{J\Lambda}}$, $m_{\beta_{J\Lambda}}$, and $r_{B_{J\Lambda}}$ for different values of a, until the best set of parameters was found. The fit to the $^{33}P_2$ complex phase shift is good in the resonance energy region, $5\le E_{\mathrm{He}}\le 15$ MeV. The quality of the fit to all $^{33}F_2$, $^{33}F_3$, and $^{33}F_4$ complex phase shift data is very good over the whole range of energy, from 5 to 33 MeV, with a value of $\chi^2/N\le 0.4$ in all cases. We made an estimation of the sensitivity of the fit to variations in the resonance parameters: energies, total and partial widths. We let one parameter vary, keeping all the others fixed at their optimum values, in such a way that a variation within the allowed uncertainty should not produce any significant change in the fit to the data, defining this change to be less than 10% in χ^2. The results of the fit are shown in Fig. 3. The resonance parameters with their uncertainties are given in Table I.

IV. RESONANCES IN THE RADIATIVE CAPTURE OF ^{3}He BY ^{3}H

A. The experimental data

The cross section for the radiative capture of ^{3}He by ^{3}H to form the first excited state of ^{6}Li($J^\pi=3^+$, $E_{\mathrm{Li}}=2.186$ MeV) was measured by Blatt *et al.*,[5] in the energy range from $E_{\mathrm{He}}=1$ up to 5 MeV, and by Ventura *et al.*[6] in the energy range from $E_{\mathrm{He}}=6$ up to 26 MeV. The differential cross section, taken at 90°, as a function of the energy shows the presence of broad maxima centered at 4.0 and 21.8 MeV ^{3}He bombarding energy, indicating the presence of resonances in ^{6}Li at excitation energies in the region of 18 MeV and 26 MeV, respectively. The broad structure centered around $E_{\mathrm{He}}\simeq 20.6$ MeV has a shape suggestive of two very wide overlapping resonances, data points in Fig. 2. As a func-

tion of the angle, the cross section has the shape typical of electric dipole transitions, but it changes from concave, at low energies, to convex at energies in the region of the high energy bump. Ventura *et al.*[6] fitted the photon angular distributions taken at $E_{He}=8.0$, 13.1, and 21.7 MeV to a formula with terms up to and including $P_2(\cos\theta)$. No significant $P_3(\cos\theta)$ term was found.

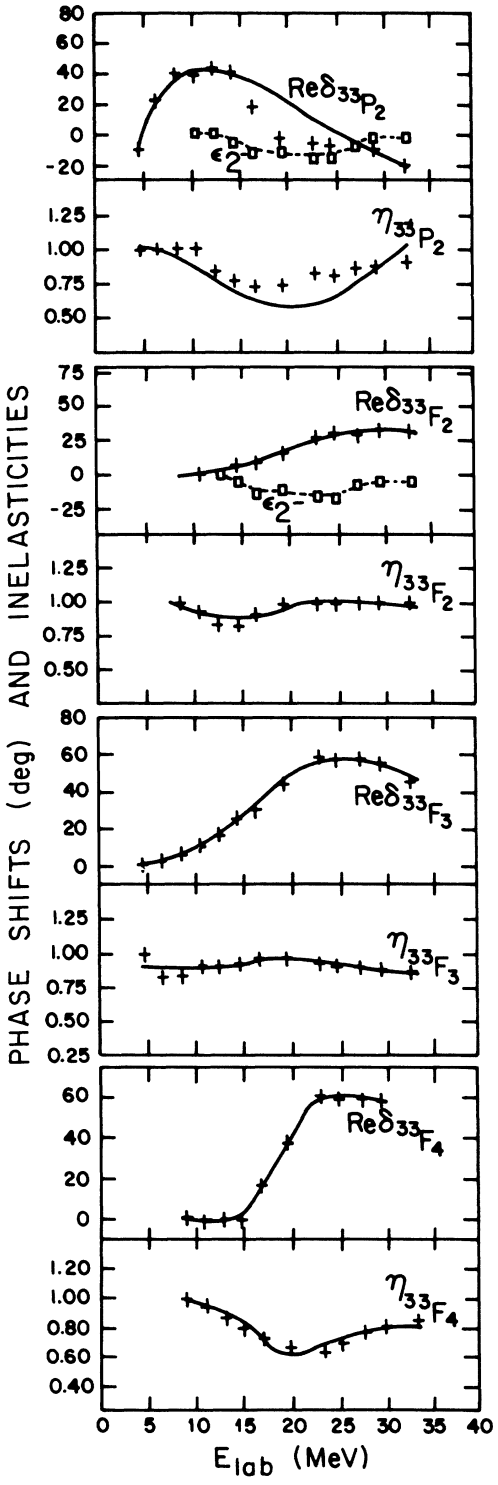

FIG. 3. The eignphase shifts and inelasticities of the Λ_J partial waves as functions of the bombarding energy. The dots show the experimental data (Ref. 4). The solid line represents the best fit of the single level plus background, Eq. (76), to the data.

These same authors[6] computed the capture cross section assuming a direct capture mechanism in a single channel configuration in the resonating group method formalism with a central nucleus-nucleus interaction.[8] Since in L-S coupling, the first excited state of ^{6}Li is described as a ^{13}D state, there are only four partial waves, $^{33}P_2$, $^{33}F_2$, $^{33}F_3$, and $^{33}F_4$, which can contribute to an electric dipole transition to this state. They found good agreement between the measured and computed values of the excitation function and the photon angular distribution at bombarding energies below 10 MeV. In the energy region of the maximum at $E_{He}\simeq 26$ MeV, they found large discrepancies between the measured and computed values of the excitation function, which they interpreted as evidence of the breakdown of the ^{3}He-^{3}H cluster configuration into a different intermediate configuration of the same spin and parity which then decays by photon emission to the first exited state of ^{6}Li. They also found a strong discrepancy between the measured and computed photon angular distributions; they found that their computed distribution is concave at all energies while the measured angular distribution changes from concave to convex when going from low to high energy. The effective interaction between ^{3}He and ^{3}H used by Ventura *et al.*[6] in their calculation was purely central and no splitting of states of good orbital angular momentum into states of different total angular momentum could result.

B. Resonance analysis of the cross section for radiative capture

More recently, Schenzle and Kramer[9] computed the energy spectrum of ^{6}Li taking into account the noncentral terms in the nucleon-nucleon interaction. They found the three $^{33}F_J$ states of negative parity split in one $^{33}F_4$ state at $E_{Li}=24.9$ MeV and the $^{33}F_2$, $^{33}F_3$ states degenerate at $E_{Li}=27.8$ MeV. It was also found that these states are almost pure $\{3,3\}$ partition with very little admixture of $\{4,11\}$.

Since the partition $\{3,3\}$ couples strongly to the odd orbital angular momentum states in the ^{3}He+^{3}H channel, the results of Kramer and Schenzle[9] strongly suggest that the wide bump observed in the excitation function of the radiative capture cross section to the first excited state of ^{6}Li around $E_{^3\mathrm{He}}=26.1$ MeV is due to a direct capture process in which the nucleus-nucleus interaction in the entrance channel gives rise to the formation of short lived unbound excited states of ^{6}Li with an almost pure ^{3}He-^{3}H cluster configuration. The double humped structure apparent in the high energy bump in the excitation function and the change in sign of the curvature of the angular distribution with energy are due to the presence of three wide $^{33}F_J$ overlapping resonances, two of which, $^{33}F_2$, $^{33}F_4$, are coherent and give rise to interfering contributions that may change the sign of $a_2(E)$.

In order to test this idea we made a resonance analysis of the experimental data of Blatt *et al.*[5] and Ventura *et al.*[6]

(a) The theoretical formula that was compared with the experimental data was obtained as follows:

(i) We wrote the partial amplitude $\hat{Q}_{J\Lambda}(E)$ as an expansion in resonances, as in Eq. (66).

(ii) We neglected the background term, since the energy denominators in the background integrals are much larger than the energy denominators in the resonant terms.

(iii) We kept only one resonant term in each partial amplitude, since the resonance analysis of the elastic scattering data shows that each of the four partial amplitudes contributing to the radiative capture has one resonance in the energy region of interest.

(iv) The resulting expression

$$\hat{Q}_{J\Lambda'}(E)=\sum_{l=|J-1|}^{J+1}\sum_{\Lambda''=|J-1|}^{J+1}(2J+1)^{1/2}B_{\Lambda''l}i^{l+1}\exp[i(\sigma_l-\sigma_0)]B_{l\Lambda'}\exp[i(\phi_{J\Lambda}+\chi_{J\Lambda})]\frac{\Gamma^{1/2}_{\gamma,J\Lambda}\Gamma^{1/2}_{el,J\Lambda}}{(E-E_{J\Lambda})+i\frac{1}{2}\Gamma_{J\Lambda}}F_{\Lambda\Lambda''}(k) \quad (82)$$

was substituted in the formulas (15), (9), (10), and (11) for the scattering amplitude.

(b) The numerical fit of the theoretical expression to the experimental data was made in the following way:

(i) Numerical values of the total yield σ_T, and the coefficient $a_2(E)$, were obtained from a least-squares fit of the expression

$$d\sigma/d\Omega=\frac{1}{4\pi}\sigma_T+a_2(E)P_2(\cos\theta) \quad (83)$$

to the angular distributions of γ_1 photons at energies $E_{He}=4.0$, 13.1, and 21.7 MeV taken from Ventura's Ph.D. thesis.[35] The values of the total cross section obtained in this way are shown as data points in Fig. 4.

(ii) We inserted the numerical values of the elastic channel parameter $E_{J\Lambda}$, $\Gamma_{J\Lambda}$, $\Gamma^{1/2}_{el,J\Lambda}$, $\phi_{J\Lambda}(E)$, and ϵ_2, obtained previously, in (82) and (10), and made a fit of the sum of resonant terms to the numerical values of σ_T and obtained numerical values of the radiative widths $\Gamma_{\gamma J\Lambda}$.

(iii) A fit of the resonant expressions (82), (15), and (11) to the numerical values of $a_2(E)$ gave us values of the differences of the radiative phases $\chi_{J\Lambda}-\chi_{J'\Lambda'}$.

(iv) Starting with this first set of resonant parameters we made a χ^2 fit of the resonant formulas (9), (10), and (11) to the excitation function taken at 90° lab, and optimized the fits of the resonant formulas to σ_T and $a_2(E)$.

(v) An estimation of the error in the determination of the values of the radiative widths was made as follows: The sensitivity of the fit to variations in the values of the radiative widths was estimated as in the previous case of the numerical analysis of the elastic data. To the resulting values of the uncertainty we added the estimated contribution to $\Delta\Gamma_\gamma$ from the experimental error in σ_T and $a_2(E)$ obtained from the least-squares fit (b-i). The radiative widths with their uncertainties are given in Table I. The results of the fit are shown in Figs. 2, 4, and 5.

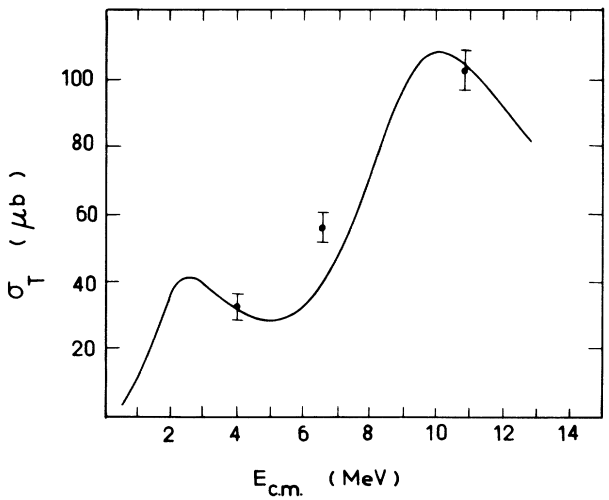

FIG. 4. Total yield of the reaction ^{3}H(^{3}He,γ_1)^{6}Li*. The solid line represents the best fit of the interfering resonances formula to the data.

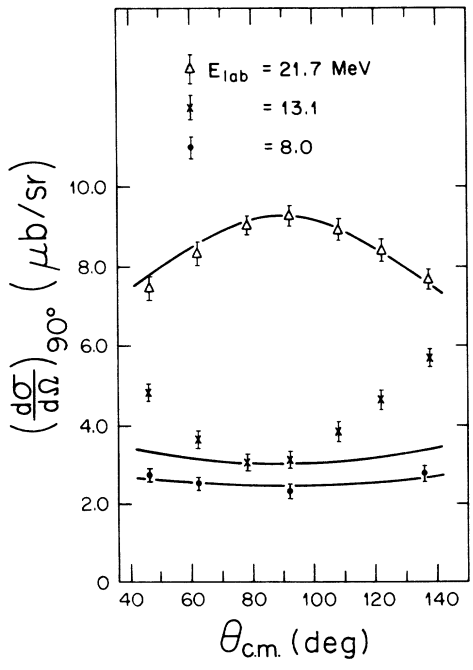

FIG. 5. The photon angular distribution for the reaction ^{3}H(^{3}He,γ_1)^{6}Li* (Ref. 6). The solid line represents the best fit of the interfering resonances formula to the data.

V. DISCUSSION OF THE RESULTS

In order to extract meaningful parameters for the resonances apparent in the radiative capture cross section of ^{3}He by ^{3}H to form the first excited state of ^{6}Li, we obtained the relevant scattering matrix elements $S_\Lambda^{(J)}$ from the complex eigenphase parametrization of the elastic scattering of ^{3}He by ^{3}H data of Vlastou *et al.*[4] A single level formula was fitted to the derived scattering matrix elements. In this formula the unitarity of the scattering matrix and the threshold behavior are exactly taken into account. We found one resonance in each of the four odd orbital angular momentum eigenwaves $^{33}P_2$, $^{33}F_2$, $^{33}F_3$, and $^{33}F_4$, that contribute to the radiative capture of ^{3}He by ^{3}H in the first excited state of ^{6}Li.

The resonance in the $^{33}P_2$ wave corresponds to a negative-parity state of ^{6}Li with $S=1$, $T=1$, and $J^\pi=2^-$ located at excitation energy of ^{6}Li equal to 17.985±0.025 MeV. In the elastic scattering data only the high energy side of this resonance is apparent, since the data were taken from $E_{\rm He}\simeq 5$ MeV ($E_{\rm Li}\simeq 20.8$ MeV) up to $E_{\rm He}\simeq 26$ MeV. However, in the radiative capture excitation function, Fig. 2, the resonance in the $^{33}P_2$ wave is clearly visible from $E_{\rm He}\simeq 1$ MeV upwards. It appears isolated with almost no background in the low energy side. Below $E_{\rm He}\simeq 6$ MeV the contribution of the resonances in the $^{33}F_J$ waves is negligible. Therefore, combining the data from elastic scattering and radiative capture of ^{3}He by ^{3}H, it is possible to determine the resonance parameters without ambiguity. To this resonance Vlastou *et al.*[4] had assigned an excitation energy value of $E_{\rm Li}=21.0$ MeV, extracted from a fit of the real phase shift of their solution I to a single level R-matrix formula. This value of the excitation energy is not compatible with the radiative capture data, since it would place the low energy peak of the radiative capture excitation function at an energy more than 3 MeV higher than the measured value. A negative-parity state of ^{6}Li, at $E_{\rm Li}=17.4$ MeV, and its isobaric analog in ^{6}He, at $E_{\rm He}=13.6$ MeV, were observed by Aleksandrov *et al.*[12] in the energy spectrum of α particles from the reaction ^{3}He(^{7}Li,α)^{6}Li and ^{3}H(^{7}Li,α)^{6}He, respectively. These authors computed the differential cross section in the framework of the optical model by means of the code LOLA. In their computation they assumed that these states could be described as a {3,3} partition with a microscopic configuration $(1s)^3(1p)^3$, which is compatible with a cluster relative motion wave function of $3s$ or $2p$ type. They could find agreement with the measured angular distribution only for the $2p$ type cluster relative motion wave function. On this basis, they assigned the quantum numbers $J^\pi=1^-, T=1$ to these two levels. Agreement with the angular distribution does not rule out a cluster relative motion in a triplet state, in which case the quantum numbers would be $J^\pi=2^-, T=1, S=1$. Therefore, it seems likely that the excited level of ^{6}Li with $J^\pi=2^-, S=1, T=1$ that we located at $E_{\rm Li}=17.95\pm0.025$ MeV is the same level as the one observed by Aleksandrov *et al.*[12] at $E_{\rm Li}=17.4$ MeV.

The resonance in the $^{33}F_4$ eigenwave corresponds to a negative-parity state of ^{6}Li with $S=1$, $T=1$, and $J^\pi=4^-$, and $E_{\rm Li}=24.890\pm0.055$ MeV, which is 0.7 MeV lower than the value found by Vlastou *et al.*[4] and quoted by Ajzenberg-Selove.[1] In our analysis, the level with $J^\pi=3^-, S=1, T=1$ was found at $E_{\rm Li}=24.780\pm0.054$ MeV, that is, at an energy almost 2 MeV lower than the value found by Vlastou *et al.*,[4] and quoted by Ajzenberg-Selove.[1]

We were able to identify a very broad resonance, $\Gamma_{CM}=8.68$ MeV, in the $^{33}F_2$, wave, corresponding to a negative-parity level of ^{6}Li with quantum numbers $J^\pi=2^-, S=1, T=1$ and $E_{\rm Li}=26.590\pm0.065$ MeV, which had not been reported in previous studies and which does not appear in Ajzenberg-Selove's review of "Energy levels of light nuclei."[1,3]

There are two main reasons that explain the difference between the results of our analysis of the elastic scattering phase shifts and the results obtained by Vlastou *et al.*[4]

(a) We made an analysis of the complex phase shifts, solution II of Vlastou *et al.*,[4] which is a more realistic representation of the elastic scattering data than the real phase shifts, solution I of Vlastou *et al.*,[4] which they analyzed.

(b) The single level resonance formula used in this work,[34] Eq. (76), takes into account the unitarity and the energy behavior of the scattering matrix near threshold in a more realistic fashion than the single level R-matrix formula used by Vlastou *et al.*[4] This difference makes the formula used in this work more appropriate to represent very wide resonances near threshold. This is particularly apparent in the case of the resonance in the $^{33}F_2$ eigenwave which could not be parametrized in terms of the R-matrix expression, but for which we obtained the excellent fit shown in Fig. 3.

Once the value of the elastic resonance parameters is known, the determination of the radiative widths and radiative phases is straightforward. The excitation function of the cross section is a sum of positive and negative terms. Hence, it is not possible to determine precise values of the radiative widths from the measured values of $(d\sigma/d\Omega)_{90^\circ}$ vs $E_{\rm He}$ even when the number of data points is large. An unambiguous determination of the value of the radiative widths can be made from the measured values of the total cross section, since this is a sum of positive terms. Unfortunately only three measured photon angular distributions were available for the numerical analysis, from which only three different values of σ_T could be extracted. The large estimated error in the results of our analysis is due to the small number of independent values of σ_T. However, the radiative phases are fixed with better accuracy since only one value of the three phase differences produces the change in sign of $a_2(E)$ at the right energy.

New and more precise measurements of the photon angular distributions at a large number of different values of the incident particle energy are required for an accurate determination of the values of the radiative widths.

VI. CONCLUSIONS

The reformulation of the theory of the direct radiative capture in terms of singularities of the scattering function

in the elastic channel and resonant or Gamow states seems appropriate for the description of quasimolecular resonances in the direct radiative capture of light nuclei by light nuclei. The connection with the cluster model of nuclear structure and nuclear reactions is explicitly exhibited. The elastic and radiative widths of a resonance are proportional to the transition amplitudes from a state in the continuum to a resonant state and from the resonant state to the final state, respectively. We give explicit expressions for these transition amplitudes in the formalism of the resonating group method in the particular case of the radiative capture of ^{3}He by ^{3}H in a final state with the same cluster structures. However, the resonance formulas derived in this formalism and used in the numerical analysis reported in this work are valid independently of the cluster configuration of the wave function of the final state, since its derivation depends only on some very general analytical properties of the wave function in the entrance channel. Using this form of the theory we have made a meaningful comparison of the experimental data on radiative capture of ^{3}He by ^{3}H to form the first excited state of ^{6}Li with the data extracted from the measured values of the elastic cross section and polarization of ^{3}He by ^{3}H.

The new analysis of the elastic complex phase shifts of Vlastou *et al.*,[4] presented in this work, gives evidence supporting the existence of a very broad level in the spectrum of ^{3}Li with $J^\pi=2^-, S=1, T=1$ at $E_{\rm Li}=26.59\pm0.065$ MeV, not reported in previous studies. We also give values of the energies and total and elastic widths of resonances in the $^{33}P_2$, $^{33}F_2$, $^{33}F_3$, and $^{33}F_4$ waves.

The resonance analysis of the radiative capture of ^{3}He by ^{3}H to form the first excited state of ^{6}Li is compatible with an almost pure ^{3}He-^{3}H cluster structure of the negative-parity unbound states of ^{6}Li, with $S=1$ and $T=1$, observed as resonances in the elastic scattering in the range of energies from 5 MeV $\leq E_{\rm He} \leq$ 33 MeV. We found no evidence of the existence of the negative-parity state of ^{6}Li at $E_{\rm Li}=25\pm1$ MeV, $\Gamma_{Cm}\simeq4$ MeV, $J^\pi=4^-, S=1, T=1$ with a cluster structure other than ^{3}He+^{3}H proposed by Ventura and quoted in Ajzenberg-Selove.[1] The low energy maximum in the excitation function at $E_{\rm He}\simeq2$ MeV is due to a resonance in the $^{33}P_2$ wave which corresponds to a state in the spectrum of ^{6}Li at $E_{\rm Li}=17.89\pm0.025$ MeV with $J^\pi=2^-$, $S=1$, and $T=1$.

The broad maximum observed in the excitation function around $E_{\rm He}\simeq21.8$ MeV is due to two very wide overlapping resonances in the $^{33}F_4$ and $^{33}F_2$ waves, corresponding to states of ^{6}Li at $E_{\rm Li}=24.890\pm0.055$ MeV with $J^\pi=4^-$ and $E_{\rm Li}=26.59\pm0.065$ MeV with $J^\pi=2^-$, respectively; both levels have $S=1$ and $T=1$. The contribution of the resonance in the $^{33}F_3$ wave to the radiative capture cross section is very small.

The identification of the state of $E_{\rm Li}=26.590\pm0.065$ MeV with $S=1$ and $T=1$ in the elastic eigenwaves $^{33}F_2$ allows us to offer an explanation of the change in the sign of the curvature of the photon angular distribution, which is concave at low energies and convex at high energies. This change is produced by the interference of the coherent contributions of $^{33}P_2$, $^{33}F_2$, and $^{33}F_4$ resonances.

A better determination of the numerical values of the radiative widths of the resonances that contribute to the radiative capture will be possible when new and more precise measurements of the photon angular distribution are made at different values of the energy.

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