Contribution of the two-body transverse current to the non-energy-weighted sum rule for electron scattering

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The contribution of the two-body exchange currents to the nonweighted sum rule of ^{3}He is calculated for momentum transfer between 300 and 700 MeV/c. It is found that the inclusion of the mesonic pair current modifies the sum by 0.4% to 11% depending on the value of the momentum transfer q and the specifics of the strong interaction form factor.

Inelastic electron-scattering sum rules have been used for a long time to study the electromagnetic interaction in light, medium, and heavy nuclei. The derivation of the total sum rule (longitudinal plus transverse) in terms of both the nuclear charge and current operators had already been done[1-3] and used to study nucleon-nucleon correlations[1,2] and the role of the charge exchange forces,[1] among other things. Since the charge and current operators can be complicated because of the mesonic degrees of freedom, separate measurements of the longitudinal and transverse responses are of great importance. Recently such measurements for the ^{3}He and ^{3}H nuclei had been performed[4,5] and the resulting data have been used to test different theoretical models. One advantage of working with the trinucleon system is the availability of accurate wave functions for the ground states.

The longitudinal response of ^{3}He and ^{3}H has been previously studied with the use of plane-wave impulse approximation (PWIA).[6,7] The discrepancy between the PWIA calculations and experimental results has been attributed to the missing effects of the final-state interaction (FSI).[8-10] Including such effects in deriving the wave functions of the continuum states lead to a better agreement with the experimental data especially in the case of ^{3}He. In addition, the longitudinal sum for ^{3}He has been calculated in Ref. 8 and found to have a reasonable agreement with the experimental sum. In all the previous calculations, only the single-nucleon charge operator was used because it is thought that meson-exchange currents do not play a major role in improving the discrepancy between theory and experiment in comparison to other factors such as the treatment of the three-body breakup contributions.[9,10]

The transverse response of ^{3}He and ^{3}H has also been calculated with the use of PWIA in Refs. 6 and 7 and with the inclusion of FSI in Refs. 9 and 10. In this case, the PWIA method produces better agreement with the experimental results than it does in the case of the longitudinal response. The introduction of FSI in Refs. 9 and 10 does not lead to a conclusive improvement over the PWIA because the results appear to be dependent on the method used in deriving the continuum states. Since only the single-nucleon current operator was used in calculating the transverse response, the existing discrepancy between theory and experiment might be attributed to the effect of meson-exchange currents,[10] among other factors.

In this work, we investigate the effect of including the meson-exchange currents in addition to the single-nucleon current in calculating the transverse sum rule for the ^{3}He nucleus. The sum is estimated for the momentum transfer q in the range 300 MeV/c–700 MeV/c and with the use of the wave function of Ref. 11. The sum rule for the transverse current is written as

$$S_T(q)=\sum_n |\langle n|\mathbf{J}_\perp(\mathbf{q})|0\rangle|^2=\langle 0|\mathbf{J}_\perp^+(\mathbf{q})\cdot\mathbf{J}_\perp(\mathbf{q})|0\rangle \ , \quad (1)$$

where $\mathbf{J}(\mathbf{q})$ is the vector current and is given by

$$\mathbf{J}(\mathbf{q})=\mathbf{J}^{(1)}(\mathbf{q})+\mathbf{J}^{(2)}(\mathbf{q}) \ , \quad (2)$$

where $\mathbf{J}^{(1)}(\mathbf{q})$ is the single-nucleon current operator and we only consider the spin (magnetic) part because the contribution of the convection current to the sum is very small (less than 1% of the spin current) in the range of q considered in this work. $\mathbf{J}^{(1)}(\mathbf{q})$ is given by

$$\mathbf{J}^{(1)}(\mathbf{q})=\sum_i \frac{i}{2m}[0.44+2.35\tau_z(i)](\boldsymbol{\sigma}_i\times\mathbf{q})e^{i\mathbf{q}\cdot\mathbf{r}_i} \ , \quad (3)$$

where 0.44 and 2.35 are the isoscalar and isovector magnetic moments of the nucleon in μ_N units. $\mathbf{j}^{(2)}(\mathbf{q})$ is the two-body exchange current; here we only consider the pair current that has the following expression:

$$\mathbf{J}^{(2)}(\mathbf{q})=\frac{-1}{4\pi}\frac{g^2}{4m^2}\sum_{i,j}\left\{(\boldsymbol{\tau}_i\times\boldsymbol{\tau}_j)_z(\boldsymbol{\sigma}_j\cdot\hat{r})\boldsymbol{\sigma}_i e^{i\mathbf{q}\cdot\mathbf{r}_i}\left[(1+x_\pi)e^{-x_\pi}-\left[1-\frac{\Lambda^2-\mu^2}{2\Lambda}\frac{\partial}{\partial\Lambda}\right](1+x_\Lambda)e^{-x_\Lambda}\right]\Big/ r^2+i\leftrightarrow j\right\} \ , \quad (4)$$

TABLE I. Numerical values of $X(q)$.

(q MeV/c)	$X(q)$ ($\Lambda=4m_\pi$)	$X(q)$ ($\Lambda=6m_\pi$)	$X(q)$ ($\Lambda=8m_\pi$)
300	0.028	0.068	0.11
400	0.015	0.037	0.06
500	0.009	0.022	0.037
600	0.006	0.015	0.025
700	0.004	0.01	0.018

where $\mathbf{r}=\mathbf{r}_i-\mathbf{r}_j$, g is the pseudoscalar πN coupling ($g^2/4\pi=15$), while $x_\pi=m_\pi r$ with m_π being the pion mass. $x_\Lambda=\Lambda r$ with Λ representing the cutoff mass in the πNN form factor $f_{\pi NN}^{(\mathbf{k}^2)}$, which is given by

$$f_{\pi NN}(\mathbf{k}^2)=\frac{\Lambda^2-m_\pi^2}{\Lambda^2+\mathbf{k}^2}\ , \tag{5}$$

where $\mathbf{k}$ is the momentum of the exchanged pion. It is customary to subtract the contribution due to elastic scattering from the right-hand side of Eq. (1). We find this contribution to be very small in comparison with the one coming from inelastic scattering for $q\geq 300$ MeV/c and we neglect it from now on. Equation (1) can then be written as

$$S_T(q)=S_1(q)+S_2(q)+S_{12}(q)\ , \tag{6}$$

where

$$S_1(q)=\langle 0|\mathbf{J}_\perp^{(1)^+}(\mathbf{q})\cdot\mathbf{J}_\perp^{(1)}(\mathbf{q})|0\rangle\ , \tag{7a}$$

$$S_2(q)=\langle 0|\mathbf{J}_\perp^{(2)^+}(\mathbf{q})\cdot\mathbf{J}_\perp^{(2)}(\mathbf{q})|0\rangle\ , \tag{7b}$$

$$S_{12}(q)=\langle 0|\mathbf{J}_\perp^{(1)^+}(\mathbf{q})\cdot\mathbf{J}_\perp^{(2)}(\mathbf{q})+\mathbf{J}_\perp^{(2)^+}(\mathbf{q})\cdot\mathbf{J}_\perp^{(1)}(\mathbf{q})|0\rangle\ . \tag{7c}$$

The symbol $S_1(q)$ gives the sum rule in the absence of meson-exchange currents. It contains a constant term multiplied by the square of the momentum transfer and a momentum-transfer dependent term. The latter goes to zero as the momentum transfer increases. The contribution of the meson-exchange current to the sum rule comes mainly from $S_2(q)$, since $S_{12}(q)$ is found to range between 2% and 8% of $S_2(q)$ depending on the value of q and on the specifics of the hadronic form factor. $S_2(q)$ itself contains both constant and momentum-transfer dependent terms which, again, decrease at higher momentum transfer. In Table I, we list the numerical values of $X(q)$ defined as

$$X(q)=\frac{S_2(q)+S_{12}(q)}{S_1(q)}\ ,$$

as a function of q for three different values of the cutoff mass Λ. As expected a higher value of Λ will allow for a larger contribution due to the meson-exchange currents.

In conclusion, we studied the effect of meson-exchange currents on the nonenergy-weighted sum rule for the ^{3}He nucleus. This effect depends on the value of the momentum-transfer q as a well as the size effects introduced by the strong interaction form factor. Before drawing any final conclusion, we believe it is necessary to calculate the response function for the same nucleus with the use of both one-body and two-body currents. Of course, repeating the same problem for ^{3}H, especially with the existing experimental data will help clear some of the questions regarding final-state interaction and the contribution of the mesonic degrees of freedom. We should mention that a study of the non-nucleonic degrees of freedom in connection with the sum rule of ^{3}He and ^{3}H has been presented in other publications.[12,13] We find it difficult, however, to compare between our work and that of Refs. 12 and 13 because of basic differences in the treatment of the problem. At the same time, the work of Ref. 12 concludes that the contribution of the meson-exchange currents to the sum rule is rather small, which is in agreement with our results.

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