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Vacuum polarization currents in finite nuclei

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Self-consistent relativistic Hartree calculations of odd-A nuclei, including vacuum polarization, are presented. The contribution to the baryon current from the deformed Dirac sea is calculated using a derivative expansion. This vacuum current does not affect magnetic moments; however, at intermediate momentum transfers it tends to cancel the current from the deformed occupied core, as expected from the random-phase approximation response in nuclear matter. Wave function mixing for the valence state, which arises in the self-consistent solution but not in random-phase approximations, significantly affects both Dirac and anomalous currents.

I. INTRODUCTION

Two types of relativistic mean-field approximations, which we shall refer to as the mean-field theory (MFT) and the relativistic Hartree approximation (RHA), have been applied to calculations of finite nuclei. The MFT is Walecka's original high-density approximation to nuclear matter; it includes self-consistent contributions to the energy and the source densities from only the occupied positive-energy nucleons.[1] The RHA is the full one-nucleon-loop approximation, which incorporates the effects of the mean fields on the states in the Dirac sea and so adds vacuum contributions to the energy and densities.[1] In this paper, we present the first fully self-consistent calculations of odd-A nuclei in RHA models, with an emphasis on the role of the three-vector vacuum polarization current. This work should be considered a sequel to Ref. 2, which treated MFT models of odd-A nuclei in detail.[3]

Relativistic MFT models provide a successful phenomenology for bulk properties (binding energies, rms radii, quadrupole moments, low-lying collective excitations) of nuclei, particularly when the model includes nonlinear scalar meson self-couplings.[4–8] Extensions of mean-field models to odd-A nuclei originally pointed to a failure of the phenomenology for isoscalar magnetic moments, namely, that unobserved enhancements seemed to be predicted,[9] but recent studies show that the problem is resolved by insisting on self-consistency. Specifically, the reduced nucleon effective mass M^* in the nuclear medium (due to the large scalar field) results in a valence nucleon convection current enhanced by M/M^* compared to a nonrelativistic single-particle current. However, the valence nucleon is a source of new meson fields, and self-consistency requires that the response of the core nucleons to these new fields be included in the current. The net result is a return to the isoscalar Schmidt moments.[10–14]

While the apparent failure of MFT models in predicting isoscalar magnetic moments is removed by restoring self-consistency, problems remain for the electromagnetic current at higher-momentum transfers. Unless further corrections generate large cancellations, enhancements relative to nonrelativistic currents are predicted in MFT models but are not reflected in the elastic magnetic scattering data.[2,15–18] This enhancement comes about partly because of the reduced effective mass of the valence nucleon and partly from the core response to it. The origin of the core response enhancement is the transverse vector particle-hole interaction (mediated by $\mathbf{V}$), which becomes increasingly attractive as M^* decreases, and even causes an instability of nuclear matter at high density and intermediate momentum transfer q.[15] In a finite odd-A nucleus, the valence nucleon is the source of a vector field that causes a particle-hole core polarization, which increases the convection current at finite q.

What about the current in RHA models? Vacuum effects dramatically change the nature of the core response at moderate momentum transfers.[15] In contrast to the MFT, RHA nuclear matter calculations show only small net changes in currents due to the medium response.[19] Since RHA M^* values are characteristically larger than those found in MFT models, RHA mean-field predictions for elastic magnetic scattering (using the RPA approach described below) are closer to the data. One could also argue that the RHA is a more consistent and complete mean-field treatment. In terms of linear response, the core contribution in the MFT can be identified as the usual particle-hole response minus the Pauli-blocked piece of the vacuum response. Keeping only part of the vacuum response seems unnatural at best; a theoretically complete treatment at the mean-field level would include the effects of the mean fields on the Dirac sea nucleons, as well as on the occupied positive-energy states.

Of course, while the RHA may be conceptually more complete than the MFT, the treatment of the vacuum might not approximate physical reality very well. After all, theoretical consistency does not always imply correct physics. Indeed, the inclusion of baryon loop corrections in hadronic field theories is controversial because of arguments that the composite nature of nucleons is not handled correctly.[20] For example, the authors of two recent papers argue that the physics of the loop corrections is wrong because it is not consistent with the $1/N_c$ expansion of QCD.[21] However, the RHA provides a consistent model of vacuum physics in a framework that has had considerable phenomenological success. We will not attempt to further justify the RHA, but will simply explore some of the consequences of including one-loop baryon corrections in self-consistent calculations of odd-A nuclei.

As was the case of MFT models, RHA models require self-consistency for a correct treatment of isoscalar magnetic moments. An added valence particle (or hole) is a source of new meson fields seen not only by the core nucleons, but also by the nucleons in the Dirac sea. The resulting polarization of the Dirac sea leads to new vacuum contributions to the scalar and baryon densities and to the baryon current. Once again, the core response contribution to the convection current is needed to return the magnetic moments to the Schmidt values.

In fact, this is a general result, going beyond the mean-field approximation. Lorentz covariance and the first law of thermodynamics imply that the current due to a valence nucleon in nuclear matter is k_F/μ, where k_F is

TABLE I. Summary of relativistic mean-field approximations used in calculations of currents in odd-A nuclei.

Approximation	Description
MFT	Mean field theory. Neglects contributions to the nucleon self-energy from the negative-energy Dirac sea. Source densities involve sums over positive-energy occupied states only. Used in Refs. 7, 8, and 34.
RHA/EP	Relativistic Hartree approximation/effective potential. Includes the effects of mean fields on the states in the Dirac sea in a local density approximation. Equivalent to keeping only the effective potential from the effective action. Used in Refs. 2 and 35.
RHA/DE	Relativistic Hartree approximation/derivative expansion. Includes the effects of mean fields on the states in the Dirac sea through leading order in the derivative expansion of the effective action (effective potential plus leading derivative correction). Used in Refs. 26 and 27.
RHA/RPA	Relativistic Hartree approximation/random-phase approximation. Includes the effects of mean fields on the states in the Dirac sea through a linear response approximation (RPA), with the polarization insertion evaluated in nuclear matter. Used in Refs. 16 and 25.

the Fermi momentum and μ is the chemical potential of the system,[22] and not k_F/M^* as expected naively. Thus, any approximation to the full relativistic field theory that preserves these physical principles should predict isoscalar magnetic moments close to the Schmidt values.[23] The MFT and RHA, *treated self-consistently,* are two such approximations and should predict roughly the same isoscalar moments.

There are two paths to self-consistency in an odd-A finite nucleus calculation. One approach is to start with the basis of the spherical core nucleus (which is easily calculated) and to include the core and vacuum response to the valence nucleon in a linear response approximation (RPA).[24,25] [One can include part or all of the RPA response using a local density approximation (LDA).] The other approach, used in Ref. 2 and in this paper, is to solve the Hartree equations for the (deformed) odd-A system directly. Note that the RPA is only an approximation to the fully self-consistent solution; the valence nucleon is treated as an *external* source of meson fields so that the core response does not act back on the valence wave function. In fact, we find that changes in the valence wave function in the fully self-consistent solution make significant contributions to the current.

The RPA and self-consistent calculations are very similar in physical content, although this is not always apparent in the formalism. It may be helpful to state the physics in different words to clarify the relationship between these approaches. If a nucleon is added to a nuclear medium such as nuclear matter or a closed shell nucleus, the current due to the added particle is "screened" by the medium, just as a test charge is screened in an electron gas. The added particle polarizes the medium (including the Dirac sea) and the current from this distortion (the "backflow current") is important.

In some RPA papers such as Ref. 25, this polarization is discussed as a modification of the vertex for the valence nucleon ("renormalization of the nucleon form factor"). To determine the vertex correction, the valence nucleon is essentially treated as an external test current, and the consistent linear response of the medium is calculated. (For Hartree, this means a ring sum.) The valence wave function is calculated in the presence of the *unpolarized* medium.

Alternatively, one can consider the core + valence system as a whole, treating all nucleons democratically, and solve this problem in a mean-field approximation (as a deformed nucleus). The difference from doing an RPA ring sum is that the valence wave function is allowed to change when the medium is distorted. This is a $1/A$ correction and therefore vanishes in nuclear matter. The equivalence of these two approaches in nuclear matter (without including vacuum polarization) is demonstrated in Ref. 14 and illustrated for finite nuclei in Ref. 2. The screening of the valence current from $N\bar{N}$ excitations in the RPA approach appears as the vacuum polarization current in the self-consistent approach.

A self-consistent RHA calculation in finite nuclei including the exact contribution from the vacuum (e.g., by summing over Dirac sea states) is not practical, so various approximations are adopted. In many calculations for finite spherical nuclei, the RHA vacuum corrections are included only in a local density approximation,[1] which we will label RHA/EP. (EP stands for effective potential.) Perry[26] and Wasson[27] have recently considered derivative corrections to the local density results, which they find to be non-negligible in finite nuclei. In this work, we extend these calculations to nonspherical nuclei, so as to include derivative corrections to the vacuum polarization current (RHA/DE). (Note: In an RHA/EP model, there is no vacuum polarization current.) Table I contains a summary of the relativistic mean-field approximations used in this work.

In Sec. II we review the formalism and solution method for self-consistent RHA calculations in odd-A nuclei. Since much of this material has appeared elsewhere, the discussion is abbreviated and the reader is referred to Ref. 2 for further details. In Sec. III we present calculations of isoscalar magnetic moments, convection currents, and elastic magnetic form factors for RHA models with and without vacuum polarization currents. Our conclusions are summarized in Sec. IV. Supplementary details on the derivative expansion are given in the Appendix.

II. FORMALISM

As in Ref. 2, we begin with the Walecka (σ-ω) model including scalar meson self-couplings. The Lagrangian density is[1]

$$\mathcal{L}=\bar{\psi}[\gamma_\mu(i\partial^\mu-g_v V^\mu)-(M-g_s\phi)]\psi+\tfrac{1}{2}(\partial_\mu\phi\partial^\mu\phi-m_s^2\phi^2)$$
$$-\tfrac{1}{4}(\partial_\mu V_\nu-\partial_\nu V_\mu)^2+\tfrac{1}{2}m_v^2 V_\mu V^\mu-U(\phi)+\delta\mathcal{L}\ , \quad (2.1)$$

where

$$U(\phi)=\frac{\kappa}{3!}\phi^3+\frac{\lambda}{4!}\phi^4$$

and $\delta\mathcal{L}$ is a counterterm Lagrangian.

We work in the relativistic Hartree approximation (RHA), with a zero-momentum renormalization prescription, as in Ref. 1. Part of the prescription is that the renormalized cubic and quartic couplings are chosen to be zero ($\kappa=\lambda=0$), and we do not consider them further in this work. We note, however, that nonzero couplings are essential to successful phenomenology in MFT models (at least for bulk properties). Extending the present formalism to accommodate nonlinear couplings is straightforward but will, in principle, also involve one-meson-loop contributions.[6,28] Here, the σ-ω model is extended to include ρ mesons and photons for a more realistic description of finite nuclei.

The mean-field equations for the neutral meson fields are

$$(\nabla^2-m_s^2)\phi(\mathbf{x})=-g_s\left[\sum_\alpha^{\text{occ}}\bar{U}_\alpha(\mathbf{x})U_\alpha(\mathbf{x})+\Delta\rho_s^{\text{vac}}(\mathbf{x})\right]\ , \quad (2.2)$$

$$(\nabla^2-m_v^2)V_0(\mathbf{x})=-g_v\left[\sum_\alpha^{\text{occ}}U_\alpha^\dagger(\mathbf{x})U_\alpha(\mathbf{x})+\Delta\rho_B^{\text{vac}}(\mathbf{x})\right]\ , \quad (2.3)$$

and

$$(\nabla^2 - m_v^2)\mathbf{V}(\mathbf{x}) = -g_v \left[\sum_\alpha^{\text{occ}} U_\alpha^\dagger(\mathbf{x})\boldsymbol{\alpha} U_\alpha(\mathbf{x}) + \Delta\mathbf{J}_B^{\text{vac}}(\mathbf{x}) \right] , \tag{2.4}$$

where the sums run over occupied positive-energy states. ($\boldsymbol{\alpha}$ is the usual Dirac matrix.) There are equations analogous to (2.3) and (2.4) for the other vector mesons, with appropriate isospin factors. In principle, there are small vacuum corrections for the isovector mesons, which we have not included. Each single-particle spinor $U_\alpha(\mathbf{x})$ satisfies a Dirac equation:

$$hU_\alpha(\mathbf{x}) = \epsilon_\alpha U_\alpha(\mathbf{x}) , \tag{2.5}$$

where the single-particle Dirac Hamiltonian h is

$$h \equiv \{\boldsymbol{\alpha}\cdot[-i\nabla - g_v\mathbf{V}(\mathbf{x})] + \beta[M - g_s\phi_0(\mathbf{x})] + g_v V_0(\mathbf{x})\} , \tag{2.6}$$

and where we have suppressed the rho and photon fields. Equations (2.2)–(2.6) are nonlinear, coupled partial differential equations, which must be solved self-consistently.

The source densities on the right-hand sides of Eqs. (2.2)–(2.4) include sums over occupied positive-energy states (valence and core) and contributions from the Dirac sea. In principle, one could explicitly do the sums over negative-energy wave functions in the finite system, but this is technically very difficult because of the subtractions needed to renormalize. Instead, we include the renormalized Dirac sea sums using derivative expansion techniques.[26] Details of the wave functions are most important near the Fermi surface, so it should be reasonable to include the positive-energy wave functions explicitly while treating the Dirac sea using derivative expansions. The derivative expansions converge rapidly for $\Delta\rho_B^{\text{vac}}(\mathbf{x})$ and $\Delta\rho_s^{\text{vac}}(\mathbf{x})$, because the inverse surface thickness, which dominates the derivatives, is small compared to the relevant mass M^*.[26] The convergence of $\Delta\mathbf{J}_B^{\text{vac}}(\mathbf{x})$ is not obviously as rapid; we discuss it further in the next section.

The one-loop (Hartree) field equations (2.2)–(2.4), including the derivative corrections, can be derived using the effective action formalism, as in Ref. 26. (Details on the derivation are given in the Appendix.) For each density, we keep only the leading derivative correction; this is known to be a good approximation for the scalar density and baryon density.[26,27] The vacuum densities $\Delta\rho_s^{\text{vac}}(\mathbf{x})$, $\Delta\rho_B^{\text{vac}}(\mathbf{x})$, and $\Delta\mathbf{J}_B^{\text{vac}}(\mathbf{x})$, which appear on the right-hand sides of the meson equations (2.2)–(2.4), are given by

$$\begin{aligned}\Delta\rho_s^{\text{vac}}(\mathbf{x}) = &-\frac{1}{\pi^2}\left[M^{*3}\ln\left[\frac{M^*}{M}\right] + \tfrac{1}{3}M^3 - \tfrac{3}{2}M^2M^* + 3MM^{*2} - \tfrac{11}{6}M^{*3}\right] \\ &-\frac{1}{4\pi^2}\left[2\ln\left[\frac{M^*}{M}\right]\nabla^2(g_s\phi) - \frac{1}{M^*}(\nabla g_s\phi)^2\right] - \frac{1}{12\pi^2}\frac{1}{M^*}g_v^2F_{\mu\nu}F^{\mu\nu} ,\end{aligned} \tag{2.7}$$

$$(\Delta J_B^{\text{vac}})^\mu(\mathbf{x}) = \frac{1}{3\pi^2}\partial_\nu\left[\ln\left[\frac{M^*}{M}\right]g_vF^{\nu\mu}\right] , \tag{2.8}$$

where $(\Delta J_B^{\text{vac}})^\mu(\mathbf{x}) \equiv \{\Delta\rho_B^{\text{vac}}(\mathbf{x}), \Delta\mathbf{J}_B^{\text{vac}}(\mathbf{x})\}$. The meson fields and $M^* \equiv M - g_s\phi$ are functions of $\mathbf{x}$, and $F_{\mu\nu} \equiv \partial_\mu V_\nu - \partial_\nu V_\mu$ can be simplified since $V_\mu(\mathbf{x})$ is independent of time. Taking into account the symmetries of the meson fields in the odd-A problem,[2] these expressions reduce to

$$\begin{aligned}\Delta\rho_s^{\text{vac}}(\mathbf{x}) = &-\frac{1}{\pi^2}\left[M^{*3}\ln\left[\frac{M^*}{M}\right] + \tfrac{1}{3}M^3 - \tfrac{3}{2}M^2M^* + 3MM^{*2} - \tfrac{11}{6}M^{*3}\right] \\ &-\frac{1}{4\pi^2}\left[2\ln\left[\frac{M^*}{M}\right]\nabla^2(g_s\phi) - \frac{1}{M^*}(\nabla g_s\phi)^2\right] + \frac{1}{6\pi^2}\frac{1}{M^*}[(\nabla g_v V_0)^2 - (\nabla g_v V_\varphi \hat{\varphi})^2] ,\end{aligned} \tag{2.9}$$

$$\Delta\rho_B^{\text{vac}}(\mathbf{x}) = -\frac{1}{3\pi^2}\nabla\cdot\left[\ln\left[\frac{M^*}{M}\right]\nabla g_v V_0\right] , \tag{2.10}$$

$$\Delta\mathbf{J}_B^{\text{vac}}(\mathbf{x}) = -\frac{1}{3\pi^2}\nabla\cdot\left[\ln\left[\frac{M^*}{M}\right]\nabla g_v V_\varphi \hat{\varphi}\right] , \tag{2.11}$$

where the fields are functions of r and θ only and we have defined V_φ through $\mathbf{V}(\mathbf{x}) \equiv V_\varphi(r,\theta)\hat{\varphi}$. (Note that $\nabla\hat{\varphi} \neq 0$.)

Let us summarize how the different mean-field approximations listed in Table I treat these vacuum densities. The RHA/DE includes all of the terms from (2.9)–(2.11). The local density approximation (RHA/EP), which does not include terms with derivates, only contributes a nonzero scalar density [the first line of (2.9)]. In this case, the baryon density and current have no direct contribution from vacuum polarization. Finally, MFT calculations do not include *any* of the "vac" sources in Eqs. (2.9)–(2.11).

In the RHA/RPA approach, the core and vacuum response to the valence nucleon is calculated using a polarization insertion evaluated in nuclear matter. The valence state wave function is evaluated in the fields of the nearby spherical nucleus. In the RHA/DE we are calculating the linear response of the core to the valence nucleon in two different ways: The positive-energy core response is included exactly through solving the Dirac equation, while the negative-energy response is included through the derivative expansion. We note that tachyon poles observed in the nuclear matter RHA response at momentum transfers ~ 3 GeV do not affect the present calculations. Retaining only the first term in the derivative expansion is equivalent to dropping the q^4 and higher terms in the vacuum polarization (see the Appendix). In this approximation, there is no tachyon pole in the RPA response at any momentum transfer.

In any of the approximations, we solve the Hartree equations by expanding the mean fields and source densities in Legendre polynomials of order L up to $L_{\max}$, and the nucleon orbits in terms of spherical spin-angle functions. After substituting the expansions into Eqs. (2.2)–(2.6), the problem is reduced to a system of coupled ordinary differential equations. The Dirac equations are solved by fourth-order Runge-Kutta integration and the meson equations are solved by an iterative procedure, in which the meson fields are determined at each iteration using radial Green's function integration with source densities that depend on the fields from the previous iteration. The radial functions are fitted by cubic splines, from which the radial derivatives needed in Eqs. (2.9)–(2.11) are read off. The latter procedure is somewhat more stable under iteration than other forms of numerical differentiation. Angular derivatives (e.g., derivatives of Legendre polynomials) are performed analytically, while angular integrals are evaluated numerically, with Gaussian integration. More details on the solution method and additional formulas can be found in Ref. 2.

We use elastic magnetic scattering of electrons from nuclei, which probes ground-state nuclear currents, to compare Hartree predictions to experiment. Following the discussion in Ref. 29, we introduce an effective electromagnetic current operator, to be used with relativistic Hartree wave functions:

$$\hat{J}^{\mu}(x) \doteq \bar{\psi}(x)\gamma^{\mu}Q\psi(x) + \frac{1}{2M}\partial_{\nu}[\bar{\psi}(x)\lambda\sigma^{\mu\nu}\psi(x)] \;, \qquad (2.12)$$

where the field operators are in the Heisenberg representation, and

$$\begin{aligned} Q &\equiv \tfrac{1}{2}(1+\tau_3) \;, \\ \lambda &\equiv \lambda_p \tfrac{1}{2}(1+\tau_3) + \lambda_n \tfrac{1}{2}(1-\tau_3) \end{aligned} \qquad (2.13)$$

are the charge and anomalous magnetic moment operators. The momentum dependence of the single-nucleon form factors is folded into the nuclear form factors at the end of the calculation. The idea is that the nuclear structure in mean-field models is dominated by neutral mesons while the single-nucleon structure, at least within QHD, is predominantly determined by charged mesons, which are beyond the scope of the Hartree approximation. Thus, it is reasonable to decouple these contributions.

The current operator in (2.12) is for the full many-body system, not just the valence nucleon. In some RPA discussions (e.g., Ref. 25), core polarization effects are discussed as a modification of the vertex for the valence nucleon (i.e., a renormalization of the nucleon form factor). In Ref. 16, the current operator itself was modified to incorporate the effects of the core response (RHA/RPA), and matrix elements of the modified current were taken in the undisturbed single-particle valence state. However, many-body matrix elements of the current operator (2.12) contain the *same* effects (up to $1/A$ corrections) as the single-particle matrix element of the valence nucleon with a modified vertex calculated in an RPA; that is to say, both methods include the screening of the valence current by the nuclear medium. In a full self-consistent RHA calculation, the renormalization due to the Dirac sea appears as the contribution to the current from the deformed Dirac sea, which is calculated here using a derivative expansion. One can argue that when (isoscalar) vacuum polarization is included, the experimental single-nucleon form factor should be adjusted for the screening by the Dirac sea at zero density, so that scattering from a free proton gives the experimental result. This effect, however, modifies the single-nucleon form factor by less than 5% for $q < 3\ \text{fm}^{-1}$ and produces negligible changes in the form factors presented here.

The transverse elastic form factor for a nuclear state $|J_i\rangle$ is given by[30]

$$F_T^2(q) = \frac{f_{\text{sn}}^2(q) f_{\text{c.m.}}^2(q)}{2J_i+1} \sum_{J=1,3,\ldots}^{2J_i} |\langle J_i \| \hat{T}_J^{\text{mag}}(q) \| J_i \rangle|^2 \;, \qquad (2.14)$$

where q denotes the magnitude of the three-momentum transfer, f_{sn} is the single-nucleon form factor (for simplicity the functional forms of F_1 and F_2 are taken to be the same), and $f_{\text{c.m.}}$ is a center-of-mass correction factor. The transverse magnetic multipole operators are defined in terms of the Schrödinger-picture nuclear current density operator $\hat{J}^{\mu}(\mathbf{x})$ by

$$\hat{T}_{JM}^{\text{mag}}(q) \equiv \int d^3x\, j_J(qx) \mathbf{Y}_{JJ_1}^{M}(\Omega)\cdot\hat{\mathbf{J}}(\mathbf{x}) \;, \qquad (2.15)$$

where j_J is a spherical Bessel function, $\mathbf{Y}_{JJ_1}^{M}$ is a standard vector spherical harmonic,[31] and $\hat{J}^{\mu}(\mathbf{x}) \equiv (\hat{\rho}(\mathbf{x}), \hat{\mathbf{J}}(\mathbf{x}))$. Once elastic magnetic form factors have been computed, the ground-state magnetic dipole moment μ follows from the $q \to 0$ limit:

$$\mu = \lim_{q\to 0} \left[-i \left[\frac{2M}{q} \right] \left[\frac{6\pi J_i}{(J_i+1)(2J_i+1)} \right]^{1/2} \langle J_i \| \hat{T}_{J=1}^{\text{mag}}(q) \| J_i \rangle \right] . \qquad (2.16)$$

Only the $M1$ multipole contributes to the magnetic moment.

As indicated by the notation, we assume that the ground states are approximately eigenstates of total $\mathbf{J}$, despite the deformation. Because we do not project states of good J, we restrict ourselves to elastic magnetic scattering from nuclei near closed shells that do not exhibit large core deformations.

In the MFT approximation to the nuclear ground state, the elastic matrix element of this current is given by

$$\langle J_i|\hat{\mathbf{J}}(\mathbf{x})|J_i\rangle = \sum_{\alpha}^{\text{occ}} U_\alpha^\dagger(\mathbf{x}) Q \boldsymbol{\alpha} U_\alpha(\mathbf{x}) + \frac{1}{2M}\nabla\times\sum_{\alpha}^{\text{occ}} U_\alpha^\dagger(\mathbf{x})\lambda\beta\boldsymbol{\Sigma} U_\alpha(\mathbf{x}) , \quad (2.17)$$

where the $U_\alpha(\mathbf{x})$ are the self-consistent (positive-energy) single-particle solutions for the finite nucleus, $\boldsymbol{\alpha}$ and β are the usual Dirac matrices, and

$$\boldsymbol{\Sigma} = \begin{bmatrix} \boldsymbol{\sigma} & 0 \\ 0 & \boldsymbol{\sigma} \end{bmatrix} .$$

Thus the calculation of nuclear matrix elements reduces to a sum of single-particle matrix elements. The first term in (2.17) is the Dirac convection current and the second term is the anomalous current. We emphasize that by using *self-consistent* Hartree wave functions in Eq. (2.17), we incorporate the same physics in the current as in the RPA approach to odd-A nuclei.[14] Explicit expressions for the reduced matrix elements of

$$\tilde{b}_J(q) \equiv -i\langle J_i \| \hat{T}_J^{\text{mag}}(q) \| J_i\rangle$$

are given in Ref. 2. [Additional details on calculating the form factors (single-nucleon form factors, center-of-mass correction, etc.) can be found in Refs. 1 and 29.]

In the RHA/DE, we must add to (2.17) the contribution to the current from the Dirac sea, $\Delta\mathbf{J}_B^{\text{vac}}(\mathbf{x})$, which is purely isoscalar in the present treatment. Although we use an extended model that has ρ mesons and photons, the isovector convection current induced in the Dirac sea is small, and is not included here. Because the anomalous current is predominantly isovector, we have not considered vacuum contributions to this part of the current. Note, however, that positive-energy core nucleon contributions to the anomalous current are included.

For some nuclei, such as ^{209}Bi, J_i is greater than the value of $L_{\max}$ (the highest order of the Legendre polynomials in the angular expansions) that is used to obtain the self-consistent Hartree solution. However, terms in the expansion up to $J=J_i$ are needed to incorporate all of the core corrections. In these cases, the Hartree equations are iterated to self-consistency using $L_{\max}$, and then the meson equations are solved one final time for each J up to J_i to determine the self-consistent source densities.

III. RESULTS AND DISCUSSION

In this section, we present calculations for magnetic moments, convection currents, and elastic magnetic form factors for RHA models with and without vacuum polarization currents. The RHA/EP and RHA/DE parameter sets are given in Table II. These RHA models predict identical nuclear matter properties (such as M^*/M), since for constant fields the one-loop results depend only on the ratio of meson couplings to masses and derivative corrections vanish. The scalar masses were determined by fitting the charge radius of ^{40}Ca. The set used in Ref. 2 for RHA calculations, RHA/EP, is from Ref. 1 while the set used for derivative expansion calculations in the present work, RHA/DE, is from Ref. 27.

Isoscalar magnetic moments for light nuclei near closed shells are given in Table III. It is evident that there are essentially no differences between the predictions of the different RHA approximations. As usual, the valence moment is enhanced with respect to the Schmidt moment, but less so than for MFT calculations,[2] because M/M^* is closer to one. The full self-consistent predictions are close to the Schmidt moments, as expected from the general considerations discussed earlier. The contribution of the vacuum polarization current of the magnetic moments is found to be negligible. (A simple RPA local density approximation would predict zero contribution, but the spatial variations of the fields in the full finite nucleus calculation lead to a very small contribution to the moments.)

Several calculations of the $A=15$, $J=1$ baryon (isoscalar) current in momentum space are shown in Fig. 1. (Calculations of the current in other nuclei lead to qualitatively similar results.) The "valence only" curve results if the core and the Dirac sea are not allowed to deform; only the valence wave function contributes. (This is accomplished by restricting the angular expansion to $L_{\max}=0$.) The "nonrelativistic" curve is generated by setting the lower component of the valence wave function to its free space value; it provides a measure of pure M^* effects. The valence curve lies above it for all values of $q \le 3$ fm^{-1}; this enhancement follows naturally from the reduced value of M^*. The other two curves are from a full self-consistent calculation, including the derivative

TABLE II. Parameters used for self-consistent calculations. MFT and RHA/EP parameters are from Ref. 35; RHA/DE parameters are from Ref. 27.

Model	g_s^2	m_s	g_v^2	m_v	g_ρ^2	m_ρ	M^*/M
MFT	109.6	520	190.4	783	65.2	770	0.54
RHA/EP	54.3	458	102.8	783	65.2	770	0.73
RHA/DE	78.3	550	102.8	783	65.2	770	0.73

TABLE III. Isoscalar magnetic moments of light nuclei for two RHA self-consistent models (see Tables I and II) with no Coulomb or ρ interaction.

			RHA/EP		RHA/DE		
A	Orbital	Schmidt	Valence	Full	Valence	Full	Expt.
15	$1p\frac{1}{2}$	0.187	0.258	0.189	0.259	0.191	0.218
17	$1d\frac{5}{2}$	1.440	1.51	1.44	1.52	1.44	1.414
39	$1d\frac{3}{2}$	0.636	0.82	0.65	0.82	0.65	0.706
41	$1f\frac{7}{2}$	1.940	2.11	1.95	2.12	1.95	1.918

corrections. The difference is that the dashed curve does not include the contribution from the vacuum polarization current. This curve is very similar to the full result in an RHA/EP calculation.

The isoscalar magnetic moment is proportional to the slope of the $J=1$ current near the origin, so Fig. 1 is consistent with Table I. The dashed curve is suppressed with respect to the valence curve for low q, but becomes greater somewhat beyond 1 fm^{-1}. This enhancement comes from the response of the occupied nuclear core. This is the same behavior observed in MFT calculations,[2] but on a smaller scale. At $q \sim 1$ fm^{-1} the vacuum polarization current starts to play an important role. It reduces the self-consistent current below that of the valence current, and for $q \geq 2$ fm^{-1} it cancels the response of the core. This result is consistent with the calculated RPA response in nuclear matter in the RHA: There is essentially no enhancement when vacuum polarization is included, while the MFT response implies large enhancements at nuclear matter density.[15]

As noted earlier, keeping just the leading order in the derivative expansion is known to be a good approximation to the scalar and baryon densities in spherical nuclei.[26] This conclusion is supported by explicit calculation of the next-to-leading-order term; the analogous calculation for the baryon current is feasible but quite messy. Instead, we have checked the convergence of the derivative expansion for the baryon current by studying the vacuum current predicted in an RPA approach, using successive approximations to the polarization insertion (see the Appendix), and a local density approximation to generate the vacuum current from the valence current. The result for the nuclei considered here is that the leading term in the expansion is an excellent approximation up to about 1 fm^{-1} and the correction from the next term is 10% or less for $q \leq 3$ fm^{-1}.

If we consider elastic magnetic scattering as a probe of the current, we find that the vacuum polarization current is a relatively small correction to the valence current. This is evident in Fig. 2, which shows the elastic magnetic form factor for ^{15}N, using the currents from Fig. 1 plus an MFT calculation from Ref. 19. The only appreciable difference between the valence only and full RHA/DE

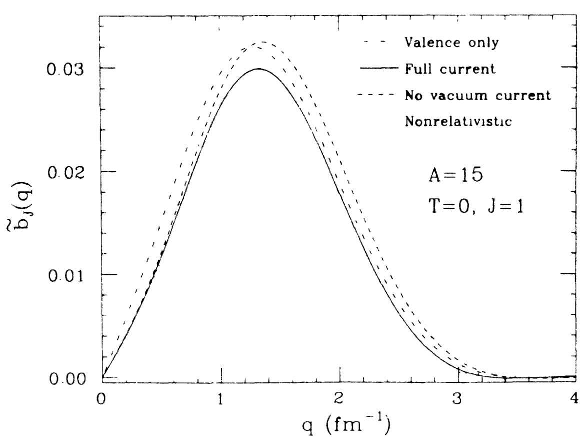

FIG. 1. The isoscalar, $J=1$ convection current for $A=15$, in momentum space, calculated in the RHA/DE model with no Coulomb or ρ interaction. The curves are valence only (dotted-dashed), nonrelativistic (dotted), full self-consistent current (solid), and self-consistent current with vacuum polarization current omitted (dashed).

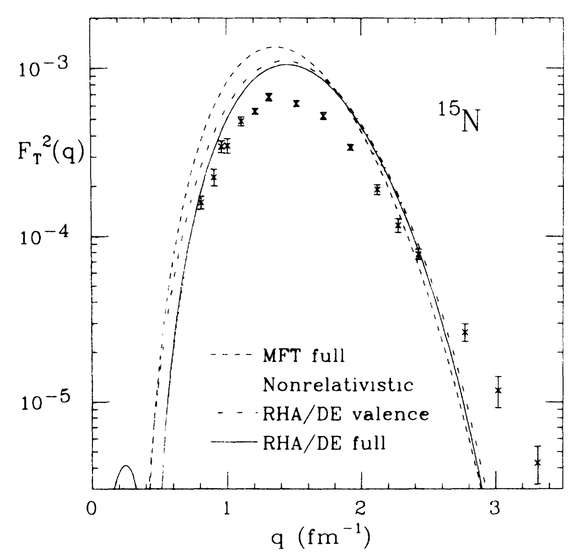

FIG. 2. Elastic magnetic form factor for ^{15}N in the RHA/DE model. The dotted-dashed curve is valence only and the solid curve is the full self-consistent result. For comparison, the dotted curve shows a Schrödinger-equivalent calculation (nonrelativistic), and the dashed curve shows the full self-consistent Hartree calculation in the MFT (taken from Ref. 2). Experimental data are from Ref. 32.

form factors is at low q, and this difference is due to the contribution from the positive-energy core states, not the vacuum polarization current. The change in the wave functions because of the increased M^*/M is the most important difference between the MFT and RHA/DE calculations. Based on a comparison with the data, the RHA/DE calculation is superior but still lies above the simplest nonrelativistic calculation and well above the data.

What about RPA calculations of odd-A nuclei? As noted earlier, the principal difference between RPA and fully self-consistent calculations is in how the valence wave function is treated. In RPA calculations, the valence orbital is determined in a spherical calculation and it remains fixed thereafter. In contrast, complete self-consistency requires that the valence orbital be treated like any other occupied state; in particular, it will develop new wave function components. These changes in the valence wave function can significantly affect the current, especially for higher multipoles, as illustrated by the ^{209}Bi form factors in Fig. 3. The solid curve is a full self-consistent calculation in the RHA/DE approximation. By fixing the valence orbital to have only one component, as in RPA calculations, the dotted curve ("full′") is generated. For low q, the full′ curve is indistinguishable from the solid curve, but for $q \geq 1$ fm^{-1}, it is close to the valence curve, as expected from RHA/RPA calculations.[33] It begins to deviate from the solid curve as the higher multipoles become important and the valence wave function mixing enhances both Dirac and anomalous currents. (With greater wave function mixing, one should worry more about the need for angular momentum projection.)

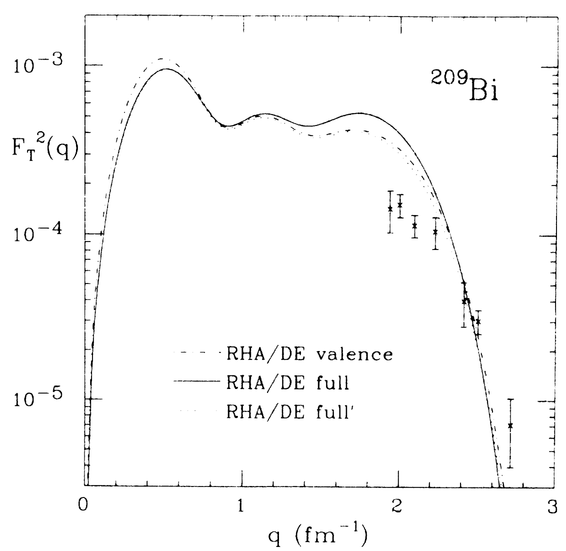

FIG. 3. Elastic magnetic form factor for ^{209}Bi in the RHA/DE model. The dotted-dashed curve is valence only and the solid curve is the full self-consistent result. The dotted curve, labeled "full′," is calculated by allowing all single-particle wave functions except the valence orbital to mix (see text).

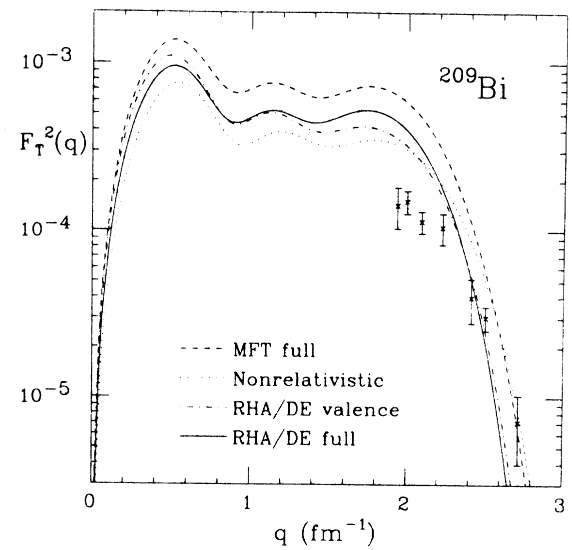

FIG. 4. Elastic magnetic form factor for ^{209}Bi in the RHA/DE model. The dotted-dashed curve is valence only and the solid curve is the full self-consistent result. For comparison, the dotted curve shows a Schrödinger-equivalent calculation (nonrelativistic), and the dashed curve shows the full self-consistent Hartree calculation in the MFT (taken from Ref. 2).

Upon comparing to nonrelativistic and MFT curves in Fig. 4, we see that the same observations made for ^{15}N can be applied to ^{209}Bi. Specifically, differences between MFT, RHA, and nonrelativistic calculations are to a large degree determined by the M^* values. Contributions from the vacuum polarization current are relatively small compared to other details, such as mixing in the valence wave function.

One could argue that the one-loop (RHA) model provides a superior description of relativistic currents.[15] In the first place, the enhancement of the pure valence current is less than for MFT models because M^*/M is closer to one. Secondly, the nuclear matter response in the RHA is very different from the MFT; no additional enhancement is predicted for intermediate q in the RHA while the MFT is strongly enhanced and even unstable at higher densities. The RHA core response still enhances the valence current, but the vacuum polarization current from the deformation of the Dirac sea cancels the enhancement. Thus, in a self-consistent calculation derivative corrections are essential for a theoretically correct result, since without these corrections there is no vacuum current. In practice, however, these contributions at intermediate momentum transfers are small compared to other corrections and to the discrepancy with experiments.

IV. SUMMARY

We have studied vacuum polarization currents in self-consistent Hartree calculations of finite nuclei, including leading-order derivative corrections. As was found in spherical nuclei for the scalar and baryon densities, the derivative expansion for the current converges rapidly and the leading term provides a good approximation for momentum transfers up to 3 fm^{-1}. The vacuum polarization current does not affect isoscalar magnetic moments, which are found to be close to the Schmidt predictions in all models. At intermediate momentum transfers, this current tends to cancel the current from the deformed occupied core, so the net contribution from core polarization is small. This result is consistent with the RPA response in nuclear matter. However, wave function mixing for the valence state, which arises in the self-consistent solution but not in RPA approximations, significantly affects both Dirac and anomalous currents.

As discussed in Ref. 15, vacuum polarization has major effects on the nuclear response at intermediate q, where the RHA response is very different from the MFT response. RHA electromagnetic currents yield elastic magnetic scattering predictions closer to nonrelativistic single-particle predictions and to the data than MFT currents, because of the core response and because M^*/M is closer to one. At higher densities or with nonlinear meson couplings the differences between MFT and RHA currents will become even more pronounced. As a result, the RHA can be said to provide a better phenomenology for currents. On the other hand, the RHA predictions are still well above the data and RHA models have not been able to reproduce quantitatively all of the nuclear structure successes of nonlinear MFT models.[6,28]

The bottom line is that we must go beyond mean-field approximations to adequately describe electromagnetic currents, particularly as we push to higher momentum transfers. Meson-exchange currents and many-body corrections known to be important nonrelativistically must be incorporated as well as a more sophisticated treatment of the ground state, isovector currents (pions), and single-nucleon form factors. Finally, we must address the questions of whether the RHA mean-field approximation and its extensions incorporate the correct physics and whether many-body physics can be disentangled from vacuum effects beyond the mean-field level.

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APPENDIX: THE DERIVATIVE EXPANSION AND VACUUM DENSITIES

In this Appendix, we sketch the derivation of the leading derivative expansion corrections to the scalar and baryon densities and the baryon current given in Eqs. (2.9)–(2.11). We use the effective-action formalism as in Ref. 26, where most of the results are stated but without details of the derivation. The one-loop effective action for the Walecka model is[26]

$$\Gamma=\int d^4x\left[\tfrac{1}{2}\partial_\mu\phi\partial^\mu\phi-\tfrac{1}{2}m_s^2\phi^2-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}+\tfrac{1}{2}m_v^2V_\mu V^\mu+\text{counterterms}\right]+\Gamma_{\text{occ}}-i\hbar\,\text{Tr}\ln(i\partial\!\!\!/-M^*-g_v V\!\!\!\!/\,)\ , \tag{A1}$$

where the trace is over spatial and internal variables. Here Γ_{occ} is the contribution from the occupied states and the final term is the contribution from the Dirac sea. (These two terms could be combined by introducing a chemical potential.) Field equations for the scalar and vector mesons are derived from the effective action by extremizing with respect to the meson fields. For time-independent background meson fields the effective action is proportional to the energy.

We focus on the final term in Eq. (A1), and look for an expansion in derivatives of the ϕ and V^μ fields, which will take the form

$$\begin{aligned}\Gamma'&\equiv -i\hbar\,\text{Tr}\ln(i\partial\!\!\!/-M^*-g_v V\!\!\!\!/\,)\\&=\int d^4x\left[-U_{\text{eff}}(\phi)+\tfrac{1}{2}Z_{1s}(\phi)\partial_\mu\phi\partial^\mu\phi+\tfrac{1}{2}Z_{2s}(\phi)(\Box\phi)^2+\tfrac{1}{4}Z_{1v}(\phi)F_{\mu\nu}F^{\mu\nu}+\tfrac{1}{2}Z_{2v}(\phi)(\partial_\alpha F^{\alpha\mu})(\partial^\beta F_{\beta\mu})+O(M^{*-4})\right]\ .\end{aligned} \tag{A2}$$

As noted in Ref. 26, gauge invariance implies that the expansion in derivatives of the background meson fields is also an expansion in inverse powers of $M^*=M-g_s\phi$. The effective potential $U_{\text{eff}}(\phi)$, found by evaluating the Tr ln with *constant* fields, is well known[1]:

$$U_{\text{eff}}(\phi)=-\frac{1}{8\pi^2}\left[2M^{*4}\ln\frac{M^*}{M}+2g_sM^3\phi-7g_s^2M^2\phi^2+\tfrac{26}{3}g_s^3M\phi^3-\tfrac{25}{6}g_s^4\phi^4\right]\ . \tag{A3}$$

There are a variety of systematic procedures that can be applied to determine the Z functions (see Ref. 26 and references therein). Here we will take advantage of the expressions in Ref. 19 for renormalized one-loop inverse propagators for this theory, which we can relate very simply to the one-loop Z functions.

First, expand Γ' about the constant field configurations ϕ_0 and V_0^μ by substituting $\phi=\phi_0+\tilde{\phi}$ and $V^\mu=V_0^\mu+\tilde{V}^\mu$ into (A2) and keeping terms to second order in the fluctuation fields. The result is

$$\Gamma'=\int d^4x \left[-U_{\rm eff}(\phi_0)-\frac{\partial U}{\partial\phi}\bigg|_{\phi_0}\tilde{\phi}-\frac{1}{2}\frac{\partial^2 U}{\partial\phi^2}\bigg|_{\phi_0}\tilde{\phi}^2+\tfrac{1}{2}Z_{1s}(\phi_0)\partial_\mu\tilde{\phi}\partial^\mu\tilde{\phi}+\tfrac{1}{2}Z_{2s}(\phi_0)(\Box\tilde{\phi})^2 \right.$$
$$\left. +\tfrac{1}{4}Z_{1v}(\phi_0)\tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu}+\tfrac{1}{2}Z_{2v}(\phi_0)(\partial_\alpha\tilde{F}^{\alpha\mu})(\partial^\beta\tilde{F}_{\beta\mu})+\cdots \right], \tag{A4}$$

where $\tilde{F}_{\mu\nu}=\partial_\mu\tilde{V}_\nu-\partial_\nu\tilde{V}_\mu$. Since Γ is the generator of one-particle-irreducible Green's functions, $\partial^2\Gamma/\partial\tilde{\phi}(x)\partial\tilde{\phi}(y)$ evaluated at ϕ_0 (and V_0) is the inverse scalar-meson propagator in the presence of these constant background fields. The inverse vector-meson propagator follows similarly. These functional derivatives applied to (A4) will give, after transforming to momentum space, the desired Z functions times powers of the four-momentum squared, q^2. But the contribution from Γ' to the inverse propagators at one loop are the (renormalized) one-loop polarization insertions, evaluated with propagators at M^*. (One can see this directly by explicitly expanding the Tr ln in powers of $\tilde{\phi}$ and $\tilde{V}$.) Therefore, we can expand the renormalized momentum space polarization insertions $\Pi_s^R(q^2)$ and $\Pi_v^R(q^2)$ from Ref. 19 in powers of q^2, and the coefficients (up to constant factors) are the Z's. We follow the notation and definitions of the polarization insertions from Ref. 19. Then

$$\Pi_s^R(q^2)=\frac{3g_s^2}{2\pi^2}\left[M^2+3M^{*2}-4M^*M-\tfrac{1}{6}q^2-\int_0^1 d\alpha[M^{*2}-\alpha(1-\alpha)q^2]\ln\left[\frac{M^{*2}-\alpha(1-\alpha)q^2}{M^2}\right]\right]$$
$$=\text{const}+\left[\frac{g_s^2}{2\pi^2}\ln\left[\frac{M^*}{M}\right]\right]q^2-\left[\frac{g_s^2}{40\pi^2}\frac{1}{M^{*2}}\right]q^4+\cdots, \tag{A5}$$

where we assume the isospin degeneracy of the vacuum is 2. $Z_{1s}(\phi_0)$ and $Z_{2s}(\phi_0)$ are the coefficients of q^2 and q^4, respectively, in the Taylor expansion of $-\Pi_s^R(q^2)$ about $q^2=0$. Thus,

$$Z_{1s}(\phi_0)=-\frac{g_s^2}{2\pi^2}\ln\left[\frac{M^*}{M}\right],$$
$$Z_{2s}(\phi_0)=\frac{g_s^2}{40\pi^2}\frac{1}{M^{*2}}. \tag{A6}$$

The vector meson requires somewhat more care because of the Lorentz indices, but the result is that $Z_{1v}(\phi_0)$ and $Z_{2v}(\phi_0)$ are the coefficients of q^2 and q^4, respectively, in the Taylor expansion of $\Pi_v^R(q^2)$ about $q^2=0$.

$$\Pi_v^R(q^2)=\frac{g_v^2q^2}{\pi^2}\int_0^1 d\alpha\,\alpha(1-\alpha)\ln\left[\frac{M^{*2}-\alpha(1-\alpha)q^2}{M^2}\right]$$
$$=\left[\frac{g_v^2}{3\pi^2}\ln\left[\frac{M^*}{M}\right]\right]q^2-\left[\frac{g_v^2}{30\pi^2}\frac{1}{M^{*2}}\right]q^4+\cdots. \tag{A7}$$

Thus

$$Z_{1v}(\phi_0)=\frac{g_v^2}{3\pi^2}\ln\left[\frac{M^*}{M}\right],$$
$$Z_{2v}(\phi_0)=-\frac{g_v^2}{30\pi^2}\frac{1}{M^{*2}}. \tag{A8}$$

Finally, we obtain the expressions in (2.9)–(2.11) from

$$\Delta\rho_s^{\rm vac}=\frac{\delta\Gamma'}{\delta(g_s\phi)}=-\frac{\delta\Gamma'}{\delta M^*},$$
$$(\Delta\mathbf{J}_B^{\rm vac})^\mu=-\frac{\delta\Gamma'}{\delta(g_v V_\mu)}, \tag{A9}$$

keeping only the leading corrections. (Note: Beyond leading order, derivative corrections will induce contributions to the meson propagators as well as to the densities.) The expansion in (A7) is used in a local density approximation to test the convergence of the derivative expansion for the vacuum polarization current.

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