Comparison between effective Hamiltonians in symmetry restoring theories: Intrinsic excitations in superfluid nuclei

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The comparison between effective Hamiltonians, constructed within the framework of symmetry restoring methods, is discussed for the case of intrinsic two-quasiparticle excitations in superfluid nuclei. Particularly, the meaning of currently adopted approximations, i.e., number symmetry restoring effective interactions and ideal quasiparticle representation in gauge space, is discussed in connection with an intrinsic symmetry breaking mechanism in the number variable.

I. INTRODUCTION

The microscopic description of intrinsic excitations in deformed systems has been the subject of an extended work. Many efforts have been devoted in the context of variational, projection, collective, and approximately realistic[1–4] theories. At the core of the question is the problem related to the definition of two-body interactions to be used for the description of nuclear properties in terms of intrinsic variables. Since the starting point of many attempts has been a particular choice for a deformed single particle mean field, thus making extreme approximations on the structure of the original two-body interaction, i.e., like in Nilsson's deformed field, the BCS quasiparticle mean field, etc., the models are always handicapped by the subsequent choice of the residual interactions. An example of this sort, namely, the difficulty posed by the search of a suitable residual interaction accordingly with the choice of a deformed mean field, is given by the treatment of dipole excitations in deformed nuclei,[5,6] where a variety of model interactions have been used. In fact, as shown in these references,[5,6] additional conditions which limit the structure of the residual forces beyond the constraints of the mean field approach have to be imposed in order to reproduce relevant observables. In realistic cases these additional conditions are not generally fulfilled and in consequence some ambiguities are not avoided in dealing with the construction of model residual interactions.[7] Out of the many aspects of the problem we shall focus our attention on the random phase approximation (RPA) treatment of intrinsic excitations once a deformed mean field has been adopted. A common feature of the currently used RPA method is the appearance of a zero energy mode. Therefore, perturbative corrections to the mean field+RPA treatment in a deformed basis become unfeasible. On the other hand, and in connection with the appearance of a zero energy mode, intrinsic symmetry breaking mechanisms can be invoked.[8] In this paper we aim at the discussion of formal equivalences between the construction of effective interactions and the treatment of intrinsic symmetry breaking in deformed systems at the level of the RPA method. Furthermore, we have restricted ourselves to the pairing force problem since it displays features which have been analyzed from different corners.[9–13] We shall concentrate our discussion on the RPA description of intrinsic excitations in superfluid systems by following (i) the effective interaction method of Pyatov *et al.*,[3] (ii) the intrinsic symmetry breaking mechanism proposed by Bohr and Mottelson,[8] and (iii) the ideal quasiparticle and boson representation of Suzuki *et al.*[13] For the sake of completeness we shall refer the reader to a series of papers[14] where another technique concerning the problem has been extensively studied. This technique, which is based on the Becchi, Rouet, Stora, and Tyutin (BRST) theory, aims at the foundation of the unified model of Bohr and Mottelson.[8] Because its more fundamental scope is somehow beyond our more modest objective which is, as we have said before, related to the discussion of the intersection between models (i)–(iii), we shall not discuss it here. However, we think that a careful study of the technique proposed in Ref. 14 would be important for further developments.

We have organized the context of this paper in the following order: Section II will be devoted to the analytical description of methods (i)–(iii) for the case of a separable monopole pairing force; in Sec. III we present the discussion of the theoretical results which have been obtained by the RPA treatment of the Hamiltonians corresponding to methods (i)–(iii). Finally some conclusions are drawn in Sec. IV. Details related to the basic algebraic approach, based on the standard BCS method, are given in Appendix A and useful expressions concerning the RPA formalism are presented in Appendix B.

II. FORMALISM

A. Effective symmetry restoring interactions

The method of Marshalek and Weneser[2] was used by Pyatov *et al.*[3] to reconstruct, in the boson approximation, the global rotational invariance of average mean fields. This formalism has also been adapted and applied to the pairing force problem[9] and to the microscopic description of intrinsic collective excitations in axially symmetric deformed single particle potentials.[5–7] In this subsection we shall briefly describe the main assumptions of the formalism as well as the main results of it. We shall refer the reader to Ref. 9 for some intermediate steps of the derivation.

The spontaneous symmetry breaking, for the case of the pairing force problem treated in the BCS theory (see Appendix A), is represented by the violation of the particle number conservation in the quasiparticle basis.[9] In order to restore this broken symmetry, additional terms should be added to the Hamiltonian H_{11}, which represents the average mean field in the BCS approximation. The structure of these terms, as a function of intrinsic variables, is not self-consistently defined. In fact, as we shall show below, it is obtained from an operational approach where the effective interaction is not directly related to the initial model Hamiltonian.

Following the procedure first suggested by Marshalek and Weneser[2] and lately particularized by Pyatov *et al.*[3] we shall evaluate the conmutator between H_{11} and the operator $\hat{H}_{2qp}$, which is the number of particle symmetry violating part of the number operator in the quasiparticle basis,[9] namely,

$$[H_{11},\hat{N}_{2qp}]=\hat{h} \ , \tag{1}$$

where by construction

$$\hat{h}=2\Delta\sum_j(\hat{P}_j^{\dagger}-\hat{P}_j) \ . \tag{2}$$

In Eq. (2) we have used the standard definition of the pairing gap parameter, Δ, and of the quasiparticle pair creation (annihilation) $\hat{P}_i^{\dagger}$ ($\hat{P}_i$) operators[9] [see Appendix A, Eqs. (A4)–(A6)].

The resulting effective Hamiltonian could therefore be written as

$$H_{\text{eff}}=H_{11}+\gamma\hat{h}^{\dagger}\hat{h} \ , \tag{3}$$

where the coupling constant γ is fixed by the condition

$$\langle\tilde{0}|[[H_{11},\hat{N}_{2qp}],\hat{N}_{2qp}]|\tilde{0}\rangle=(1/2\gamma) \ , \tag{4}$$

which is to be associated with the restoration of the global invariance of the Hamiltonian (3), i.e., conservation of the average number of particles. In fact, condition (4) guarantees that

$$\langle\tilde{0}|[H_{\text{eff}},\hat{N}_{2qp}]|\tilde{0}\rangle=0 \ , \tag{5}$$

and closes up the parametrization of the effective Hamiltonian (3) once the average mean field, deformed in the sense of a fixed particle number, is chosen as H_{11} in the BCS approximation. In Eqs. (4) and (5) the expectation values are taken on the BCS ground state, $|\tilde{0}\rangle$.

The resulting effective Hamiltonian, H_{eff}, Eq. (3), reads, in the notation given in Appendix A,

$$H_{\text{eff}}=H_{11}+g\sum_{jj'}(\hat{P}_j^{\dagger}\hat{P}_{j'}+\hat{P}_j\hat{P}_{j'}^{\dagger})-g\sum_{jj'}(\hat{P}_j^{\dagger}\hat{P}_{j'}^{\dagger}+\hat{P}_j\hat{P}_{j'}) \ , \tag{6}$$

where $g=4\Delta^2\gamma$, and $\gamma=-G/16\Delta^2$; G is the pairing coupling constant (see Appendix A). Although its structure differs from the one of the conventional BCS theory [see Eqs. (A2) and (A3)],

$$H(\text{BCS})=\sum_j E_j\hat{N}_j+\sum_{jj'}V_{jj'}\hat{P}_j^{\dagger}\hat{P}_{j'}+\sum_{jj'}W_{jj'}(\hat{P}_j^{\dagger}\hat{P}_{j'}^{\dagger}+\hat{P}_j\hat{P}_{j'}) \ , \tag{7}$$

it has some advantages which are more clearly illustrated by the associated RPA results.[9] In dealing with the problem of the structure of collective excitations, obtained via the RPA treatment of H_{eff} (see Appendix B), it has been shown that a zero frequency mode, which is to be associated with a collective rotation in gauge (or number) space, could be decoupled from the intrinsic spectrum and that it is represented by the action of the symmetry operator $\hat{N}_{2qp}$ upon the correlated ground state.[9] The remaining solutions of the RPA equation of motion are intrinsic excitations which do not differ appreciably from the ones obtained with the conventional Hamiltonian (7).[9] However, the above described procedure, at this level of the discussion, could not be supported by a more fundamental recipe or by the application of a more general symmetry restoring technique. This is shown in the next subsection.

B. Collective modes associated with intrinsic symmetry breaking

The quantal theory of nuclear collective modes has been discussed in detail by Bohr and Mottelson.[8] The same theory can be applied to the analysis of collective modes associated with intrinsic symmetry breaking[8] and we shall hereby show that the method which has been described in the preceding subsection does in fact result from Bohr and Mottelson's model. Let us start from the static quasiparticle potential, H_{11}, written in the BCS manner as

$$H_{11}=\sum_j E_j\hat{N}_j \ , \tag{8}$$

with

$$\hat{N}_j=\sum_m\alpha_{jm}^{\dagger}\alpha_{jm} \ ,$$

where we have used the standard BCS notation for the quasiparticle creation (annihilation), $\alpha_{jm}^{\dagger}$ (α_{jm}), operators as well as for the subindexes representing the quasiparticle states (j,m).[8] The structure of the field coupling can be obtained from the static potential, H_{11}, by employing the invariance of the total Hamiltonian, in the present case with reference to the number of particles symmetry. If we write the number operator, $\hat{N}$, in the quasiparticle basis we get

$$\hat{N}=\sum_j q_j(\hat{P}_j^{\dagger}+\hat{P}_j)+k_j\hat{N}_j+2\Omega_jV_j^2 \ , \tag{9}$$

with q_j, k_j, and $\hat{P}_j^{\dagger}$ ($\hat{P}_j$) defined like in Appendix A [Eq. (A3)]. Since H_{11} violates the number of particles symmetry, which should be an invariance of the total Hamiltonian, we can restore this invariance by including the effects of the collective field generated by a small rotation of the nucleus in gauge space. Let us summarize the basic assumptions which are related to this concept. The reader is referred to Ref. 8 for a detailed discussion. In a nucleus with open shells a condensate can be described in terms of a pair of particles coupled by the pairing interaction. The pair field can thus be expressed in terms of an approximately constant modulus and in terms of a phase, like

$$T(|\Delta N|=2)=T_2 e^{i\phi} , \tag{10}$$

where with $|\Delta N|=2$ we have represented a "transfer" quantum number. Since the condensate, in this context the BCS vacuum, does not possess a good (fixed) number of particles, different values of the phase $e^{i\phi}$ would produce a change in the number of quanta associated with the creation (or annihilation) of pairs in the condensate. As shown in Ref. 8, it means that the phase ϕ is playing the role of a variable which is conjugate to the number of particles; therefore for a given N we have

$$[N,\phi]=-i , \tag{11}$$

which is the canonical relationship which defines the gauge space of the variables N and ϕ. If the static pair deformation, generated by the action of the pair field T on the BCS vacuum, is larger than the zero point fluctuations, the motion of the nucleus in gauge space can be approximately separated into rotational and intrinsic components.[8]

In the present case we can define the angle variable ϕ in terms of quasiparticle pair creation and annihilation operators, namely,

$$\phi=\sum_j (s_j \hat{P}_j^\dagger + t_j \hat{P}_j) , \tag{12}$$

where the coefficients s_j and t_j should be determined from condition (11). After some algebra, conmutator algebra in the non-Abelian group with generators $\hat{P}_j^\dagger$ ($\hat{P}_j$), and $\hat{N}_j$ [see Eq. (A5)], we have obtained the expression

$$\phi=-i(G/4\Delta)\sum_j (\hat{P}_j^\dagger - \hat{P}_j) , \tag{13}$$

which, at leading order in the shell degeneracy Ω_j, fulfills condition (11). If we now write the conmutator (1) in terms of ϕ we obtain

$$[H_{11},\hat{N}]=if\phi , \tag{14}$$

with $f=8\Delta^2/G$. We can now argue, like in Bohr and Mottelson's model, that the same conmutator represents the angle gradient of a potential, $V(\phi)$, which obviously results from[8]

$$[H_{11},\hat{N}]=i\partial V(\phi)/\partial\phi , \tag{15}$$

and by comparing (14) and (15) we can finally write the formal identity

$$\partial V(\phi)/\partial\phi = f\phi . \tag{16}$$

In other words, the conmutator (1), which is the main element entering in the definition of the effective interaction H_{eff}, could be viewed, for a continuous symmetry breaking, as the angle gradient of the potential

$$V(\phi)=f\phi^2/2 , \tag{17}$$

which will restore the global invariance of the Hamiltonian when added to H_{11}. In this fashion,

$$H_{\text{eff}}=H_{11}-f\phi^2/2 \tag{18}$$

conmutes with the number operator. Then a connection between the results of Sec. II A, concerning the structure of the operator $\hat{h}$, Eq. (2), and the results of Sec. II B could be established by the following correspondence:

$$\hat{h} \Longrightarrow i\partial V(\phi)/\partial\phi ,$$

or

$$\hat{h} \Longrightarrow if\phi , \tag{19}$$

with the subsequent result

$$\begin{aligned} H_{\text{eff}} &= H_{11}+\gamma\hat{h}^\dagger\hat{h} \\ &= H_{11}+\gamma(-if\phi)(if\phi) \\ &= H_{11}+\gamma f^2\phi^2 = H_{11}-f\phi^2/2 , \end{aligned} \tag{20}$$

where the assignment $\gamma=-1/2f$ coincides with the value of γ fixed by Eq. (4).[9]

We can conclude our discussion on the symmetry restoring potential $V(\phi)$, by calculating the inertia parameter associated with it, namely,

$$D_\phi = 2\sum_j \frac{|\langle 2qp(j)|\partial V(\phi)/\partial\phi|0\rangle|^2}{(2E_j)^3} , \tag{21}$$

where $|2qp(j)\rangle$ denotes an uncorrelated state of two quasiparticles and $2E_j$ is the configuration energy of the pair. The expression for D_ϕ reads, for $V(\phi)$ given by Eq. (17),

$$D_\phi = \sum_j \frac{\Omega_j q_j^2}{E_j} , \tag{22}$$

which is exactly equal to the value that we have obtained by using Pyatov's prescription.[9]

In conclusion, the treatment of an intrinsic symmetry breaking in terms of a collective variable, ϕ, and the restoration of the global invariance of the Hamiltonian, leads to the expressions which have been obtained in the algebraic approach proposed by Pyatov;[9] as discussed in the preceding subsection.

C. Ideal boson quasiparticle representation

Another method of approximation, concerning the treatment of intrinsic excitations in superfluid systems, has been proposed in Ref. 13. This method, usually referred to as the ideal boson quasiparticle representation, is based on the doubling of quantum states which are taken as members of collective and intrinsic subspaces generated by conmuting operators, associated with collective and intrinsic number and gauge angle variables, respectively.[13] In this fashion, and in full analogy with the case of space rotations, collective and intrinsic number operators are introduced together with their canonical conjugates which play the role of orientation angles in gauge space. In this subsection we are going to discuss, in detail, the structure of the resulting Hamiltonian.

The starting point, for the treatment of the BCS pairing Hamiltonian as proposed by Suzuki *et al.*,[13] is given by the definition of the transformations

$$R_1=e^{i\pi\omega}$$

and (23)

$$R_2=e^{-iI\theta}\ ,$$

where the operators I and ω are the generators of a set of macroscopic (global) variables corresponding to microscopical ones associated to π, the number operator, and θ, the gauge angle, respectively. Furthermore, it is assumed that they are acting on the collective (I,ω) and intrinsic (π,θ) spaces and that

$$[I,\theta]=[I,\pi]=[\pi,\omega]=[\theta,\omega]=0\ ,$$
$$[I,\omega]=[\pi,\theta]=-i\ , \tag{24}$$

so that the total wave function of the system could be written as a product of rotational and intrinsic vibrational wave functions. In this product space the Hamiltonian could be represented by the sum of rotational and vibrational terms which can be obtained from the transformation of the original Hamiltonian H(BCS) (7) under the combined action of the rotations R_1 and R_2, namely,

$$\tilde{H}=(R_1R_2R_1)H(\text{BCS})(R_1R_2R_1)^{-1}\ . \tag{25}$$

The combined effect of these rotations upon H(BCS) amounts to the definition of a body fixed frame of reference where the intrinsic motion of the system does not depend explicitly upon a particular value of the number of particles.[13]

In order to do it we can define, like in the preceding subsection, the operators π and θ

$$\pi=\sum_j\beta_j\hat{\pi}_j\ ,$$
$$\theta=\sum_j m_j\hat{\theta}_j\ , \tag{26}$$

where

$$\hat{\pi}_j=\hat{P}_j^{\dagger}+\hat{P}_j\ ,$$

and

$$\hat{\theta}_j=-(1/2\Omega_j)(\hat{P}_j^{\dagger}-\hat{P}_j)\ , \tag{27}$$

the coefficients β_j and m_j of Eq. (26) are given by

$$\beta_j=U_jV_j\ ,$$

and by the condition

$$\sum_j m_j\beta_j=1\ .$$

A possible but not unique choice for the coefficients m_j is the following:

$$m_j=(G\Omega_j/\Delta)\ , \tag{28}$$

and we have adopted it in our calculations although, for the sake of convenience, we have carried out the operational transformations without taking the numerical values given by Eq. (28).

After some straightforward but somehow lengthy algebra we have obtained, for $\tilde{H}$, the expression

$$\tilde{H}=H(\text{BCS})+I\sum_j\tilde{b}_j\hat{\pi}_j-\sum_{jj'}\beta_j\tilde{b}_{j'}\hat{\pi}_j\hat{\pi}_{j'}\ , \tag{29}$$

where

$$\tilde{b}_j=E_jm_j/\Omega_j+\sum_{j'}2p_{jj'}m_{j'}\ ,$$

with

$$p_{jj'}=-(G/4)k_jk_{j'}\ ,$$

and

$$k_j=U_j^2-V_j^2\ .$$

The first term of $\tilde{H}$, Eq. (29), is the original H(BCS) pairing Hamiltonian; the second one could be interpreted as a coupling between rotational and vibrational degrees of freedom; the last term is an additional two-body interaction with coefficients which are dependent on the gauge angle weighting factors, m_j.

III. RPA TREATMENT OF THE EFFECTIVE HAMILTONIANS

In this section we shall discuss the structure of the RPA solutions which we have obtained from the above introduced model Hamiltonians. Since the essentials of the RPA method are presented in Appendix B we shall start from the corresponding expressions for the normal coordinates of Marshalek and Weneser[2] written in terms of the RPA phonons, namely,

$$\hat{\mathcal{P}}_\nu=(w_\nu/2)^{1/2}(\Gamma_\nu^{\dagger}+\Gamma_\nu)\ ,$$
$$\hat{\mathcal{L}}_\nu=-i(2w_\nu)^{-1/2}(\Gamma_\nu^{\dagger}-\Gamma_\nu)\ , \tag{30}$$

where $\Gamma_\nu^{\dagger}$ (Γ_ν) are the RPA phonon creation (annihilation) operators and w_ν are the RPA energies. By using the orthogonality conditions (B4) and the completeness relations of the RPA solutions we can write the quasiparticle pair creation (annihilation) operators in terms of RPA phonons, namely,

$$\hat{P}_j^{\dagger}=\Omega_j\sum_\nu(X_{j\nu}\Gamma_\nu^{\dagger}+Y_{j\nu}\Gamma_\nu)\ ,$$
$$\hat{P}_j=(\hat{P}_j^{\dagger})^{\dagger}\ , \tag{31}$$

where $X_{j\nu}$ and $Y_{j\nu}$ are forward and backward going amplitudes, respectively. Inserting in Eq. (31) the definitions given by Eq. (30) we can write

$$\hat{P}_j^{\dagger}=\Omega_j\sum_\nu(2w_\nu)^{-1/2}(X_{j\nu}+Y_{j\nu})\hat{\mathcal{P}}_\nu$$
$$+i(w_\nu/2)^{1/2}(X_{j\nu}-Y_{j\nu})\hat{\mathcal{L}}_\nu\ ,$$
$$\hat{P}_j=(\hat{P}_j^{\dagger})^{\dagger}\ . \tag{32}$$

With these values for $\hat{P}_j^{\dagger}(\hat{P}_j)$ we can now write the corresponding expressions for the Hamiltonians (6), (7), and (29), which are given by

TABLE I. RPA energies (w_ν) and matrix elements of the transfer operator $\hat{T}_2$ ($|\langle 0^+(w_\nu)|\hat{T}_2|0\rangle|^2$) corresponding to the Hamiltonians given by Eqs. (6), (7), and (29), respectively.

w_ν (MeV)			$\|\langle 0^+(w_\nu)\|\hat{T}_2\|0\rangle\|^2$		
BCS+RPA	$H_{\rm eff}$	$\tilde{H}$	BCS+RPA	$H_{\rm eff}$	$\tilde{H}$
2.841	2.873	2.841	0.216	0.059	0.231
2.989	3.303	2.989	0.185	0.168	0.071
3.571	3.811	3.571	0.010	0.054	0.078
4.032	4.071	4.032	0.206	0.335	0.148
		Total	0.617	0.616	0.528

$$H(\mathrm{BCS})=\sum_{\nu\omega}\left[\varepsilon_{\nu\omega}-\sum_{jj'}Gk_jk_{j'}(X_{j\nu}+Y_{j\nu})(X_{j'\omega}+Y_{j'\omega})(4w_\nu w_\omega)^{-1/2}\right]\hat{\mathcal{P}}_\nu\hat{\mathcal{P}}_\omega$$
$$+\sum_{\nu\omega}\left[\sigma_{\nu\omega}-\sum_{jj'}G\Omega_j\Omega_{j'}(X_{j\nu}-Y_{j\nu})(X_{j'\omega}-Y_{j'\omega})(4w_\nu w_\omega)^{1/2}/4\right]\hat{\mathcal{L}}_\nu\hat{\mathcal{L}}_\omega\ , \tag{33}$$

$$H_{\rm eff}=\sum_{\nu\omega}\varepsilon_{\nu\omega}\hat{\mathcal{P}}_\nu\hat{\mathcal{P}}_\omega+(\sigma_{\nu\omega}+\gamma h_\nu h_\omega)\hat{\mathcal{L}}_\nu\hat{\mathcal{L}}_\omega\ , \tag{34}$$

$$\tilde{H}=\sum_{\nu\omega}\left[\varepsilon_{\nu\omega}-\sum_{jj'}Gk_jk_{j'}(X_{j\nu}+Y_{j\nu})(X_{j'\omega}+Y_{j'\omega})(4w_\nu w_\omega)^{-1/2}-2\sum_{jj'}\beta_j\tilde{b}_{j'}(X_{j\nu}+Y_{j\nu})(X_{j'\omega}+Y_{j'\omega})(w_\nu w_\omega)^{-1/2}\right]\hat{\mathcal{P}}_\nu\hat{\mathcal{P}}_\omega$$
$$+\sum_{\nu\omega}\left[\sigma_{\nu\omega}-\sum_{jj'}G\Omega_j\Omega_{j'}(X_{j\nu}-Y_{j\nu})(X_{j'\omega}-Y_{j'\omega})(4w_\nu w_\omega)^{1/2}\right]\hat{\mathcal{L}}_\nu\hat{\mathcal{L}}_\omega\ , \tag{35}$$

where

$$\varepsilon_{\nu\omega}=(w_\nu w_\omega)^{-1/2}\sum_j\Omega_jE_j(X_{j\nu}+Y_{j\nu})(X_{j\omega}+Y_{j\omega})\ ,$$
$$\sigma_{\nu\omega}=(w_\nu w_\omega)^{1/2}\sum_j\Omega_jE_j(X_{j\nu}-Y_{j\nu})(X_{j\omega}-Y_{j\omega})\ , \tag{36}$$
$$h_\nu=(2\Delta^2)^{1/2}\frac{\sum_j 8E_j\Omega_j/(4E_j^2-w_\nu^2)}{\left[\sum_j 8E_j\Omega_j/(4E_j^2-w_\nu^2)^2\right]^{1/2}}\ .$$

From these results, Eqs. (33)–(35), we can observe that significant differences between the Hamiltonians appear at the RPA level of approximation, namely, (a) the zero energy mode has to be excluded from the RPA spectrum of H(BCS) but it is not automatically removed by the cancellation of the term proportional to $\hat{\mathcal{L}}_0^2$, (b) this cancellation is present in the case of $H_{\rm eff}$, where the coefficient $\sigma_{00}+\gamma h_0^2$ vanishes identically,[9] and (c) the transformed Hamiltonian $\tilde{H}$ shows the same features of H(BCS) except for the presence of an additional mass-like term which is dependent upon the gauge fixing coefficients $\tilde{b}_j$ but does not change the structure of the corresponding RPA dispersion relation. As shown in Appendix B the dispersion relations obtained with H(BCS) and $\tilde{H}$ do coincide.

In order to illustrate more clearly these differences we have solved the RPA equations, given in Appendix B, for each of the Hamiltonians. Numerical results, for the case of $N=14$ particles distributed in the single particle states $j\equiv(Nlj)=4d_{5/2}$, $4g_{7/2}$, $4s_{1/2}$, $5h_{11/2}$, and $4d_{3/2}$ with energies fixed at the values $\varepsilon_j=0.0$, 0.8, 2.4, 2.5, and 2.8 MeV, respectively, are shown in Table I, where the strengths associated with the two-particle transfer operator $\hat{T}_2=(2)^{-1/2}\sum_j[a_j^\dagger a_j^\dagger]_{00}$ are displayed. While the agreement between the results corresponding to H(BCS) and $H_{\rm eff}$ is fairly good, the result corresponding to $\tilde{H}$ shows a deviation which could be due to an incomplete decoupling of the zero energy mode from intrinsic excitations. We should remind the reader that the $w_\nu=0$ solution of the RPA equations, for the case of H(BCS), is usually ignored in numerical applications due to the unrenormalizability of the model which leads to divergent forward and backward going amplitudes.[15] On the other hand this $w_\nu=0$ solution is automatically decoupled in the case of the RPA treatment of $H_{\rm eff}$,[9] while the same root remains undetermined for the case of the RPA treatment of $\tilde{H}$.[13]

IV. CONCLUSIONS

We have discussed in this article the intersection between different techniques which aim at the description of intrinsic excitations in deformed systems, particularly for the pairing force problem in the quasiparticle (BCS) basis. We have shown that the symmetry restoring method proposed by Pyatov[3] could be interpreted in terms of the intrinsic symmetry breaking model of Bohr and Mottelson.[8] Another technique based on rotations in gauge (or number) space, due to Suzuki *et al.*,[13] was compared with the above-mentioned symmetry restoring method and it was found that the doubling of variables does not suffice for a complete decoupling of the zero energy mode, as illustrated by the results concerning matrix elements for two-particle transfer operators in a reduced single particle space.

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APPENDIX A

We write the pairing Hamiltonian[15]

$$H=\sum_{j,m}\varepsilon_j a^\dagger_{jm}a_{jm} - G\sum_{jj',mm'>0} a^\dagger_{jm}a^\dagger_{j\bar{m}}a_{\bar{j}'m'}a_{j'm'} \ , \quad \text{(A1)}$$

in a standard notation. The BCS transformation to the quasiparticle basis $\alpha^\dagger_{jm}$ (α_{jm}) leads to the transformed Hamiltonian

$$H=H_0+H_{11}+H_{(22+40)}+H_{31}+H_{\rm res} \ , \quad \text{(A2)}$$

where

$$\begin{aligned}
H_0&=-(\Delta^2/G)+\sum_j 2\Omega_j V_j^2(\varepsilon_j-\lambda-GV_j^2/2) \ ,\\
H_{11}&=\sum_j E_j\hat{N}_j \ ,\\
H_{(22+40)}&=\sum_{jj'}V_{jj'}\hat{P}^\dagger_j\hat{P}_{j'}+W_{jj'}(\hat{P}^\dagger_j\hat{P}^\dagger_{j'}+\hat{P}_j\hat{P}_{j'}) \ , \quad \text{(A3)}\\
H_{\partial i}&=\sum_{jj'}m_{jj'}(\hat{P}^\dagger_j\hat{N}_{j'}+\hat{N}_{j'}\hat{P}_j) \ ,\\
H_{\rm res}&=-(G/4)\sum_{jj'}q_jq_{j'}:\hat{N}_j\hat{N}_{j'}: \ ,
\end{aligned}$$

with

$$\begin{aligned}
\Omega_j&=j+\tfrac{1}{2} \ ,\\
V_{jj'}&=-G(U_j^2U_{j'}^2+V_j^2V_{j'}^2) \ ,\\
W_{jj'}&=(G/2)(U_j^2V_{j'}^2+V_j^2U_{j'}^2) \ ,\\
m_{jj'}&=(G/2)k_jq_{j'} \ ,\\
k_j&=U_j^2-V_j^2 \ ,\\
q_j&=2U_jV_j \ .
\end{aligned}$$

The operators

$$\begin{aligned}
\hat{N}_j&=\sum_m \alpha^\dagger_{jm}\alpha_{jm} \ ,\\
\hat{P}^\dagger_j&=\sum_{m>0}\alpha^\dagger_{jm}\alpha^\dagger_{j\bar{m}} \ , \quad \text{(A4)}\\
\hat{P}_j&=(\hat{P}^\dagger_j)^\dagger
\end{aligned}$$

obey the conmutation relations

$$\begin{aligned}
[\hat{P}_j,\hat{P}^\dagger_{j'}]&=\delta_{jj'}(\Omega_j-\hat{N}_j) \ ,\\
[\hat{N}_j,\hat{P}^\dagger_{j'}]&=\delta_{jj'}2\hat{P}^\dagger_j \ ,
\end{aligned} \quad \text{(A5)}$$

and in this representation the number operator $\hat{N}$ can be written as

$$\begin{aligned}
\hat{N}&=\sum_{j,m}a^\dagger_{jm}a_{jm}\\
&=\sum_j q_j(\hat{P}^\dagger_j+\hat{P}_j)+\sum_j k_j\hat{N}_j+\sum_j 2\Omega_jV_j^2\\
&=\hat{N}_{2qp}+\hat{N}_{11}+\hat{N}_0 \ . \quad \text{(A6)}
\end{aligned}$$

APPENDIX B

The RPA phonons can be defined like

$$\Gamma^\dagger_\nu=\sum_j(X_{j\nu}\hat{P}^\dagger_j-Y_{j\nu}\hat{P}_j) \ , \quad \text{(B1)}$$

where the index ν denotes the corresponding RPA eigenfrequency w_ν, which is the solution of the equation of motion

$$[H,\Gamma^\dagger_\nu]=w_\nu\Gamma^\dagger_\nu \ . \quad \text{(B2)}$$

The forward and backward going amplitudes, $X_{j\nu}$ and $Y_{j\nu}$, respectively, can be determined from the set of equations which are subsequently given by (B2); for the Hamiltonians which have been considered in the text we have obtained the following results:

(i) H(BCS) [cf. Eq. (7)]:

$$\begin{aligned}
X_{j\nu}&=\frac{\Lambda_\nu}{(2E_j-w_\nu)}(a_\nu k_j-b_\nu) \ ,\\
Y_{j\nu}&=\frac{\Lambda_\nu}{(2E_j+w_\nu)}(a_\nu k_j+b_\nu) \ ,
\end{aligned} \quad \text{(B3)}$$

where

$$\begin{aligned}
a_\nu&=Gw_\nu\sum_j\frac{\Omega_jk_j}{4E_j^2-w_\nu^2} \ ,\\
b_\nu&=-1+G\sum_j\frac{\Omega_jk_j^22E_j}{4E_j^2-w_\nu^2} \ ,\\
k_j&=U_j^2-V_j^2 \ ,
\end{aligned}$$

and Λ_ν is fixed by the condition

$$\sum_j(X_{j\nu}^2-Y_{j\nu}^2)\Omega_j=1 \ . \quad \text{(B4)}$$

The energies w_ν are given by the roots of the secular equation:

$$\left[1-G\sum_j\frac{\Omega_jk_j^22E_j}{4E_j^2-w_\nu^2}\right]\left[1-G\sum_j\frac{\Omega_j2E_j}{4E_j^2-w_\nu^2}\right]-\left[Gw_\nu\sum_j\frac{\Omega_jk_j}{4E_j^2-w_\nu^2}\right]^2=0 \ . \quad \text{(B5)}$$

(ii) $H_{\rm eff}$ [cf. Eq. (6)]:

$$X_{j\nu}=\frac{\tilde{\Lambda}_\nu}{(2E_j-w_\nu)}\ ,\qquad Y_{j\nu}=-\frac{\tilde{\Lambda}_\nu}{(2E_j+w_\nu)}\ , \tag{B6}$$

with $\tilde{\Lambda}_\nu$ fixed by condition (B4). The dispersion relation for this case reads

$$w_\nu^2 f(w_\nu)=0\ , \tag{B7}$$

with

$$f(w_\nu)=(G/2)\sum_j \frac{\Omega_j}{E_j(4E_j^2-w_\nu^2)}\ .$$

(iii) $\tilde{H}$ [cf. Eq. (29)]:

$$X_{j\nu}=\frac{1}{2E_j-w_\nu}[GB_\nu+(G/2)k_jA_\nu+\beta_jC_\nu+\tilde{b}_jD_\nu]\ ,$$
$$Y_{j\nu}=\frac{1}{2E_j+w_\nu}[-GB_\nu+(G/2)k_jA_\nu+\beta_jC_\nu+\tilde{b}_jD_\nu]\ , \tag{B8}$$

where

$$A_\nu=\sum_j k_j\Omega_j(X_{j\nu}+Y_{j\nu})\ ,$$
$$B_\nu=\sum_j \Omega_j(X_{j\nu}-Y_{j\nu})/2\ ,$$
$$C_\nu=\sum_j \tilde{b}_j\Omega_j(X_{j\nu}+Y_{j\nu})\ ,$$
$$D_\nu=\sum_j \beta_j\Omega_j(X_{j\nu}+Y_{j\nu})\ .$$

The quantities A_ν, B_ν, C_ν, and D_ν are solutions of the homogeneous system of equations

$$\begin{aligned}
&B_\nu(w_\nu^2GS/2)+A_\nu(Gw_\nu S_1/2)+C_\nu(w_\nu\Delta S/2)+D_\nu(w_\nu S_3)=0\ ,\\
&B_\nu(2w_\nu GS_1)+A_\nu[(w_\nu^2-4\Delta^2)GS/2]+C_\nu(2\Delta S_1)+D_\nu(4S_5)=0\ ,\\
&B_\nu(2w_\nu GS_3)+A_\nu(2GS_5)+C_\nu(-1+2\Delta S_3)+D_\nu(4S_6)=0\ ,\\
&B_\nu(w_\nu G\Delta S)+A_\nu(G\Delta S_1)+C_\nu(\Delta^2 S)+D_\nu(-1+2\Delta S_3)=0\ ,
\end{aligned} \tag{B9}$$

where we have defined

$$\begin{aligned}
S&=\sum_j \Omega_j/E_jD_{j\nu}\ ,\\
S_1&=\sum_j \Omega_jk_j/D_{j\nu}\ ,\\
S_3&=\sum_j \Omega_j\tilde{b}_j/D_{j\nu}\ ,\\
S_5&=\sum_j \Omega_jk_jE_j\tilde{b}_j/D_{j\nu}\ ,\\
S_6&=\sum_j \Omega_j\tilde{b}_j^2E_j/D_{j\nu}\ ,
\end{aligned} \tag{B10}$$

with

$$D_{j\nu}=(2E_j)^2-w_\nu^2\ .$$

The corresponding secular equation reads

$$w_\nu^2G^2[(w_\nu^2-4\Delta^2)S^2/4-S_1^2]=0\ . \tag{B11}$$

It should be noted that Eq. (B11) does in fact coincide with Eq. (B5) as it can easily be verified by replacing the sums entering in (B5) in terms of the definitions (B10) and using the BCS gap equation: $1=G\sum_j\Omega_j/2E_j$.[15]

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