# Boson effective charges for light Se, Kr, and Sr isotopes

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A consistent analysis of E2 transition probabilities for nine neutron-deficient even-even Se, Kr, and Sr isotopes has been performed, based on previous parametrizations for these nuclei within the proton-neutron interacting-boson model, from which boson effective charges  $e_{\pi}$  and  $e_{\nu}$  are deduced. Two sets of effective charges are obtained, one set having  $e_{\pi} \approx e_y$  for most of the nuclei studied, and the other set having  $e_x$  substantially larger than  $e_y$  for most of the nuclei. For both sets,  $e_x$  tends to decrease with increasing neutron number, while  $e<sub>y</sub>$  is roughly constant. More information on mixed-symmetry states is needed to determine which set is valid.

The interacting-boson model  $(IBM)^{1,2}$  has become a useful tool in considering the collective properties of nuclei. The neutron-proton (IBM-2) version of the model has previously been applied successfully to the light isotopes of Se, Kr, and  $Sr<sub>3</sub><sup>3-7</sup>$  with emphasis primarily on the description of energy levels. Sets of electromagnetic transition strengths have also been discussed for individutransition strengths have also been discussed for individual nuclei.<sup>8-11</sup> Extensive  $B(E2)$  data in this mass region are now available from the lifetime measurements and Coulomb excitation studies of several groups. The present work was motivated by questions as to how well the wave functions obtained from the previous IBM-2 calculations reproduce these  $B(E2)$  values and how the deduced effective neutron- and proton-boson charges in Se, Kr, and Sr nuclei vary as a function of neutron and proton number.

Level schemes and  $B(E2)$  values for nuclei in this mass region show rapid changes in deformation with decreasing neutron number, going away from the  $N=50$ closed shell. The Se, Kr, and Sr IBM-2 studies successfully reproduce the nuclear structure changes in the spectra for  $40 < N < 50$  nuclei, with smoothly varying parameters of the Hamiltonian. The results are less satisfactory for  $N \leq 40$ , apparently because of shape coexistence at low energies in these nuclei.<sup>12-14</sup>

For simplicity, most IBM-2 studies to date have assumed equal neutron- and proton-boson charges,  $e_{\pi} = e_{v}$ , that are independent of mass, within a given mass region. However, considerations of F-spin and mixed-symmetry states require  $e_{\pi} \neq e_{\nu}$  in heavier nuclei.<sup>15-19</sup> Also, a neutron-number dependence for the effective boson charges has been suggested for the Kr isotopes.<sup>20</sup> The data base is now large enough to test the mass dependence of  $e_{\pi}$  and  $e_{\nu}$  for the light isotopes of the  $Z = 34, 36$ isotope chains. The nucleus  $^{82}_{38}$ Sr is included in the study along with <sup>74-80</sup>Se and <sup>76-82</sup>Kr.

## I. INTRODUCTION **II. IBM-2 CALCULATIONS**

## A. Theory

The Hamiltonian employed for the Se and Kr calculations is given by Ref. 2:

$$
H = \epsilon \hat{n}_d + \kappa Q_\pi \cdot Q_\nu + \lambda M_{\pi\nu} \tag{1}
$$

where  $\hat{n}_d$  is the number operator for d bosons,  $n_d = n_{d\pi} + n_{d\nu}$ . The quadrupole operator is given by the usual expression

$$
Q_{\rho} = (s^{\dagger} \tilde{d} + d^{\dagger} s)_{\rho} + \chi_{\rho} (d^{\dagger} \tilde{d})_{\rho}^{(2)}, \ \rho = \pi \text{ or } \nu , \qquad (2)
$$

and, for simplicity, the Majorana operator is given by

$$
M_{\pi\nu} = \sum_{k=1,3} (d_{\nu}^{\dagger} d_{\pi}^{\dagger})^{(k)} \cdot (\tilde{d}_{\nu} \tilde{d}_{\pi})^{(k)} . \tag{3}
$$

The E2 transition operator employed in this study is defined as

$$
T^{(E2)} = e_{\pi} Q_{\pi} + e_{\nu} Q_{\nu} \tag{4}
$$

where  $e_{\pi}$  and  $e_{\nu}$  are the proton- and neutron-boson effective charges, and the quadrupole operators are given by Eq. (2). The reduced E2 transition rates  $J_i \rightarrow J_f$  are then given by

$$
B(E2; J_i \to J_f) = [\langle J_f || T^{(E2)} || J_i \rangle]^2 / (2J_i + 1) . \tag{5}
$$

Although, in principle, the boson numbers  $N_{\pi}$  and  $N_{\nu}$ can be treated as parameters, they are fixed here to be half the number of valence fermions and counted from the nearest closed she11. It was not clear for a long time which "closed shells" should be considered in counting bosons in the mass region around N and  $Z=40$ , as effects of the Z=40 subshell closure are well known in  $N \approx 50$ nuclei. With the large quadrupole deformations measured recently for  $Z \approx N \approx 40$  nuclei,<sup>21</sup> it is now eviden that no  $N=40$  spherical subshell closure exists in these

$N=$	40	42	44	46
$N_{v} =$	5	$\overline{\mathbf{4}}$	3	$\boldsymbol{2}$
$34Se^a$				
$\epsilon$	1.10	1.01	1.05	1.05
$\pmb{\kappa}$	$-0.14$	$-0.18$	$-0.22$	$-0.25$
$\chi_{\nu}$	0.28	0.495	0.71	0.92
$_{36}Kr^b$				
$\epsilon$	1.00	0.96	1.05	1.15
$\pmb{\kappa}$	$-0.16$	$-0.18$	$-0.18$	$-0.19$
$\chi_v$	0.28	0.495	0.71	0.925
$38$ Sr <sup>c</sup>				
$\epsilon$			0.98	
$\pmb{\kappa}$			$-0.16$	
$\chi_{v}$			0.71	

TABLE I. IBM-2 Hamiltonian parameters. The parameters  $\epsilon$  and  $\kappa$  are given in MeV.

 $N_{\pi}$  = 3,  $\chi_{\pi}$  = -1.2,  $\lambda$  = 0.2 MeV (*FS* = 0, *FK* = 0.1) (Ref. 6).

 ${}^{b}N_{\pi} = 4$ ,  $\chi_{\pi} = -1.127$ ,  $\lambda = 0.2$  MeV (*FS*=0, *FK*=0.1) (Ref. 4).

 $N_{\pi} = 4$ ,  $\chi_{\pi} = -1.127$ ,  $\pi = 0.2$  MeV ( $FS = 0$ ,  $FK = 0.17$  (Ref. 4).<br>  $N_{\pi} = 5$ ,  $\chi_{\pi} = -1.05$ ,  $\lambda = -0.4$  MeV ( $FS = 0$ ,  $FK = -0.2$ ) (Refs. 5 and 7).

nuclei. Most workers<sup>3–11</sup> have therefore chosen to coun both proton- and neutron-boson numbers from the major shell closures at 28 and 50.

The carefully adjusted Hamiltonian parameters of Refs. 4-7 are adopted intact for the present study and are given in Table I. (We note that changes in the Hamiltonian parameters  $\epsilon$ ,  $\kappa$ , and  $\lambda$  have little effect on transition rates between low-lying states, so long as the energy levels are well fitted. On the other hand, the parameters  $\chi_{\nu}$  and  $\chi_{\pi}$  do have an appreciable effect on transition rates.) Several of the previously studied Kr and Sr nuclei are not included here because of insufficient  $E2$  data. The nucleus  $^{72}$ Se has been fitted within the model, but it is not included in the present study. The intruder  $0^+$  level that is a common feature in this mass region occurs at a low energy for this nucleus and the configuration mixing between coexisting shapes distorts the transition rates in the ground-state band.<sup>12-14</sup> Evidence for this effect has also been found in  ${}^{70}$ Se.<sup>22</sup> An IBM-2+ configuration mixing treatment<sup>23</sup> of the nuclei  $^{70,72,74}$ Se has been report  $ed<sub>1</sub><sup>24</sup>$  in which the intruder levels are assumed to be due to a two-proton excitation across the  $Z=40$  subshell closure. While consideration of possible two-proton and/or two-neutron excitations in this mass region is of interest, it is beyond the scope of the present study.

The IBM-2 calculations have been performed using the computer codes NPBOS and NPBEM (Ref. 25) to obtain energy eigenvalues, wave functions, and E2 transition matrix elements.

#### B. Determination of effective charges

The two effective charges were determined by normalizing to the experimental values for  $B(E2;2_1^+\rightarrow 0_1^+)$  and

TABLE II. Boson effective charges and the experimental E2 data from which the charges were deduced.

	Set 1		Set 2		Experimental $B(E2)$ data employed		
	$e_{v}$ e b	$e_{\pi}$ e b	$e_{v}$ e b	$e_{\pi}$ e b	$2^+_1 \rightarrow 0^+_1$ $e^2b^2$	$2^{+}_{2} \rightarrow 0^{+}_{1}$ $e^2b^2$	$\rightarrow$ 2 <sup>+</sup> $4^{+}_{2}$ $e^2b^2$
$74$ Se	0.070	0.084	0.027	0.137	$0.077(2)^a$	$0.0015(4)^{b}$	
$76$ Se	0.082	0.085	0.015	0.154	$0.084(2)^{a}$	$0.0025(2)^c$	
$78$ Se	0.085	0.080	0.030	0.127	$0.067(2)^a$	$0.0014(4)^d$	
$80$ Se	0.119	0.057	0.033	0.116	$0.051(1)^{a}$	$0.0027(2)^e$	
$76$ Kr	0.088	0.098	0.043	0.142	$0.164(8)^a$		$0.0011(4)^c$
$78$ Kr	0.077	0.095	0.038	0.127	$0.12(1)^{a}$		$0.00065(16)^f$
${}^{80}\mathrm{Kr}$	0.083	0.075	0.045	0.103	$0.074(4)^a$	$0.0006(1)^e$	
${}^{82}$ Kr	0.092	0.059	0.067	0.074	$0.045(2)^{a}$	$0.00025(13)^g$	
${}^{82}Sr$	0.093	0.079	0.061	0.100	$0.103(4)^{a}$	$0.0003(1)^8$	

'Reference 26.

Reference 27.

'Reference 28.

Reference 29.

'Reference 30.

'Reference 31.

Reference 32.

	$B(E2;Ji\rightarrow Jf)(e2b2)$				
Transition	experiment	calc $1a$	calc $2^b$		
$2^+_1 \rightarrow 0^+_1$	$0.084(2)^c$	$0.084^d$	$0.084$ <sup>d</sup>		
$2^+_2 \rightarrow 0^+_1$	$0.0025(2)^e$	0.0025 <sup>d</sup>	$0.0025^{\circ}$		
$2^+_2 \rightarrow 2^+_1$	$0.082(6)^e$	0.079	0.058		
$4^+_1 \rightarrow 2^+_1$	$0.136(4)^e$	0.123	0.124		
$3^+_1 \rightarrow 2^+_1$	$< 0.025^{\text{e,f}}$	0.0038	0.0065		
$3^+_1 \rightarrow 2^+_2$	$< 0.8^{\rm e,f}$	0.095	0.106		
$3^+_1 \rightarrow 4^+_1$	$< 0.6^{\rm e,f}$	0.020	0.018		
$2^+_3 \rightarrow 0^+_1$	$0.00006^{+6e}_{-4}$	0.0000	0.0090		
$2^+_3 \rightarrow 0^+_2$	$0.010^{+10}_{-6}$	g	g		
$2^+_3 \rightarrow 2^+_1$	$0.0004(2)^e$	0.0077	0.0051		
$2^+_3 \rightarrow 2^+_2$	$< 0.011^{\text{e,f}}$	0.0031	0.0006		
$2^+_3 \rightarrow 4^+_1$	$0.006^{+6}_{-4}$ e	0.0009	0.0087		
$4^+_2 \rightarrow 2^+_1$	$< 0.0008$ <sup>e</sup>	0.0010	0.0047		
$4^+_2 \rightarrow 2^+_2$	$0.07^{+5}_{-2}$ <sup>e</sup>	0.059	0.058		
$4^+_2 \rightarrow 4^+_1$	$0.06(3)^e$	0.040	0.022		
$6^+_1 \rightarrow 4^+_1$	$0.16(3)^e$	0.134	0.128		
$Q2_1^+$ (e b)	$-0.34(7)^e$	$-0.36$	$-0.46$		

TABLE III. Experimental and calculated  $B(E2)$  values for <sup>76</sup>Se. The quadrupole moment for the first excited state is also given.

<sup>a</sup>Set 1:  $e_y = 0.082 e b$ ,  $e_\pi = 0.085 e b$ . <sup>b</sup>Set 2:  $e_v = 0.015 e b$ ,  $e_{\pi} = 0.154 e b$ . 'Reference 16. Normalized to experiment. 'Reference 28.  $E^{\text{f}}E2/M1$  ratio  $\delta$  not known. <sup>g</sup>The  $0_2^+$  level is outside of the model space.

either  $B(E2; 2^+_2 \rightarrow 0^+_1)$  or  $B(E2; 4^+_2 \rightarrow 2^+_1)$ . Because  $e_{\pi}$ and  $e_y$  are related to the square root of the  $B(E2)$ , there are two sets of solutions. Both sets are given in Table II, along with the E2 data from which they were extracted. The neutron and proton effective charges are roughly

equal for Set 1, in most cases. The Set 2 effective charges for most of the nuclei are rather different from the Set <sup>1</sup> values, but similar to each other, with the proton effective charge dominating. Although one would hope to discard one of the two sets of effective charges as physically un-

Transition			
	experiment	calc $1^a$	calc $2b$
$2^+_1 \rightarrow 0^+_1$	$0.12(1)^c$	0.120 <sup>d</sup>	0.120 <sup>d</sup>
$4^+_1 \rightarrow 2^+_1$	$0.18(2)^e$	0.176	0.179
$2^+_2 \rightarrow 0^+_1$	$0.0026(4)^e$	0.0024	0.0000
$2^+_2 \rightarrow 2^+_1$	$0.012(2)^e$	0.104	0.088
$3^+_1 \rightarrow 2^+_1$	$0.0062(6)^{f,g}$	0.0035	0.0003
$3^+_1 \rightarrow 2^+_2$	$0.15(3)^{f,g}$	0.126	0.132
$3^+_1 \rightarrow 4^+_1$	$< 0.03^{\text{e},\text{g}}$	0.025	0.022
$4^+_2 \rightarrow 2^+_1$	$0.00065(16)^e$	0.00065 <sup>d</sup>	$0.00065$ <sup>d</sup>
$4\overline{1}$ $\rightarrow$ $2\overline{1}$	$0.12(2)^e$	0.088	0.086
$4^{+}_{2} \rightarrow 4^{+}_{1}$	$0.055(14)^e$	0.054	0.041
$6^+_1 \rightarrow 4^+_1$	$0.20(4)^e$	0.193	0.195
$5^+_1 \rightarrow 4^+_1$	$0.005(2)^e$	0.0024	0.0008
$5^+_1 \rightarrow 3^+_1$	$0.15(5)^e$	0.109	0.116

TABLE IV. Experimental and calculated  $B(E2)$  values for <sup>78</sup>Kr.

<sup>a</sup>Set 1:  $e_v = 0.077 e b$ ,  $e_\pi = 0.095 e b$ .  $b$ Set 2:  $e_v = 0.038$  e b,  $e_\pi = 0.127$  e b.

'Reference 26.

Normalized to experiment.

'Reference 31.

'Reference 11.

 $E^2/M1$  ratio  $\delta$  not known.

reasonable, it is not clear that this can be done, because both sets appear to give about equally good results for the transitions not considered in the normalization.

#### C. Comparison of calculated  $B(E2)$  values with experiment

The calculated  $B(E2)$  values for the nuclei <sup>76</sup>Se and  $^{78}$ Kr are compared with experiment in Tables III and IV and discussed here, as examples. Initial states with spin  $> 6^+$  are not considered, since in this mass region they occur at energies for which the validity of the model is in question because of (2qp)  $g_{9/2}$  alignment. Transitions involving excited  $0^+$  states are not considered since the  $0^+_2$ levels for many of the nuclei studies are thought to be intruder states and thus outside of the present IBM model space.

Calculated  $B(E2)$  values are compared with experiment for fifteen  ${}^{76}$ Se transitions in Table III. For this nucleus, most of the transitions are well reproduced by both sets of calculated results. The calculated decay rates for the  $2_3^+$  states are rather different from the experimental rates for all the Se nuclei. The nature of these  $2^+_3$  states is not entirely clear, although they are characterized by Subber et  $al.^{33}$  as mixed-symmetry states. However, more than 60% of the decay strength for the  $2^{+}_{3}$  levels in <sup>76,78</sup>Se goes to the intruder  $0^+$  levels,<sup>28,31</sup> indicating that these  $2^+$  states have a substantial intruder- $2^+$  admixture, which is outside of the present IBM-2 model space.

The quadrupole moments for the first excited  $2^+$  states are known for  $74-80$ Se, and included in Table III for  $76$ Se. In all four nuclei, the "accepted" sign of the interference term  $P_3$  is positive,<sup>34</sup> which is consistent with the relative phases of the calculated matrix elements. The quadru pole moments for '<sup>6,78</sup>Se are better described by Set 1 than Set 2.

Table IV presents thirteen transitions for the nucleus  $78$ Kr. Both sets of results are in good agreement with experiment for the intraband transitions and neither set of calculations reproduces the comparatively small experimental  $B(E2; 2^+_2 \rightarrow 2^+_1)$ . In this particular nucleus, some of the weak transitions are poorly described by Set 2.

### III. DISCUSSION

#### A. Distinguishing between the two sets of solutions

The known  $B(E2)$  data do not appear to favor one set of boson effective charges over the other, although the quadrupole moments  $Q(2_1^+)$  in <sup>74,76,78</sup>Se support Set 1. It is important to find data that will determine which of the two sets is valid in this mass region.

The states that are described by the IBM-1 (which does not distinguish between neutron and proton bosons) correspond to the  $F$ -spin symmetric (or maximal  $F$  spin) states of the IBM-2, i.e., those states that are symmetric with respect to the interchange of neutron-boson and proton-boson degrees of freedom.<sup>2</sup> Actually, since the  $F$ spin symmetry of the IBM-2 Hamiltonian is broken (unless  $\chi_v = \chi_\tau$ ), most IBM-2 states will have nonmaximal F-spin (mixed-symmetry) admixtures in their wave functions. The separate dependence of the calculated  $E2$ 

transition probabilities on  $e_v$  and  $e_{\pi}$  comes from just these nonmaximal  $F$ -spin components. It is of interest to consider the  $F$ -spin symmetric limits of the IBM-2, which have been discussed by Van Isacker et al.<sup>35</sup>

In the *F*-spin symmetric U(5) limit of the IBM-2,  $35$ 

$$
B(E2; 2_1^+ \to 0_1^+) = (e_v N_v + e_\pi N_\pi)^2 / N \t{,}
$$

and

$$
B(E2; 2_M^+ \to 0_1^+) = (e_v - e_\pi)^2 N_v N_\pi / N \t{,}
$$
\t(7)

where the subscript  $M$  denotes the lowest mixedsymmetry  $2^+$  state. Other  $BE(2)$  expressions in the Fspin symmetric U(5) limit are similar, as are the corresponding expressions in the  $SU(3)$  and  $O(6)$  limits.<sup>35</sup> Thus, E2 transition rates between F-spin symmetric states are related to the sum of terms in  $e_y$  and  $e_\tau$ , while E2 transition rates between F-spin symmetric states and mixed-symmetry states are related to the difference between  $e_y$  and  $e_{\pi}$ .

In the present study, the Set <sup>1</sup> values of the quantity  $(e_v - e_\pi)$  are quite different from the Set 2 values, for most of the nuclei studied, while the values of  $(e_{v}N_{v}+e_{\pi}N_{\pi})$  are very similar for the two sets. Therefore, E2 transitions between mixed-symmetry states and members of the ground-state band should provide a sensitive test of the effective charges. Data on mixedsymmetry  $1^+$  and  $2^+$  states would be very useful in determining which of the two parameter sets is valid.

## B. Effective charge trends and comparison with other calculations

As seen in Table II, for both sets of boson effective charges,  $e_{\pi}$  tends to decrease with increasing neutron number, while  $e<sub>v</sub>$  is roughly constant, with deviations from the general trends for the heaviest nuclei.

Boson charges with  $e_v \neq e_\pi$  have been reported previously for Kr and Se isotopes. Meyer et al.<sup>10</sup> obtaine  $e_v = 2e_\pi = 0.107$  e b for the nucleus <sup>82</sup>Kr. Brüssermann et al.<sup>20</sup> were able to fit the  $2^+_1 \rightarrow 0^+_1$  transitions for Kr reasonably well with an effective charge ration that varies linearly between  $e_y/e_{\pi} = 0.5$  for <sup>82</sup>Kr and 1.25 for <sup>76</sup>Kr. Subber *et al.* <sup>33</sup> report  $e_y = 0.081$  *e b, e<sub>π</sub>* = 0.102 e b for <sup>76</sup>Se, and  $e_v = 0.06 e$  b,  $e_\pi = 0.102 e$  b for <sup>78</sup>Se.

The results of the present study are summarized in Fig. 1, which shows neutron effective charges versus proton effective charges. The error bars on  $e_y$  and  $e_\pi$  give the range within which one or the other of these parameters can be varied and the calculated results still stay within the reported uncertainties for those experimental  $B(E2)$ values employed for normalization.

Results from several studies for heavier nuclei are included in Fig. 1 for comparison. The  $144-150$ Nd results labeled (a), are from Scholten *et al*.<sup>36</sup> and include the effects of g-boson renormalization. The results (b) are from an F-spin study by Sala, Gelberg, and von Bren- $\tan\theta^{19}$  and constitute an overall fit to 36 rare-earth nuclei. The results (c) are from Bohle et al. as reported in Ref. 19, and were obtained from an SU(3)-limit study of



FIG. 1. Effective charge values,  $e_y$  vs  $e_\pi$ , in comparison with results from other studies. Set (a),  $^{142-150}$ Nd, includes the effect of g bosons (Ref. 36), (b) and (c) are rare-earth studies (Ref. 19), of g bosons (Ref. 36), (b) and (c) are rare-earth studies (Ref. 19)<br>(d) and (e) are the  $^{104-110}$ Pd results of Refs. 37 and 38, respec (d) and (e) are the  $^{104-110}$ Pd results of Refs. 37 and 38, respectively, (f) involves  $^{172-180}$ Hf and  $^{168-176}$ Yb (Ref. 35), and (g) intively, (f) involves  $1/2 - 1$ <br>volves  $190 - 196$  Pt and  $190,1$ volves  $^{190-196}$ Pt and  $^{190,192}$ Os (Ref. 35).

mixed-symmetry states in <sup>156</sup>Gd and neighboring nuclei<br>The <sup>104–110</sup>Pd results (d) of Ginocchio and Van Isacker<sup>3</sup> were extracted from pion-scattering data. In another cal-<br>culation for  $^{104-110}$ Pd, done by Saha *et al.*<sup>38</sup> and labeled (e), the  $E2$  operator was extended to include core polarization effects. Van Isacker et  $al$ .<sup>35</sup> obtained the results (f) for Hf and Yb isotopes from the analytic expression for  $B(E2; 2^+_1 \rightarrow 0^+_1)$  in the *F*-spin symmetric SU(3) limit. A similar procedure in the  $F$ -spin symmetric  $O(6)$  limit produced the results  $(g)$  for Os and Pt isotopes.<sup>35</sup> Effective charge results for the isotope chain  $148 - 154$ Sm<sup>18</sup> (not shown in Fig. 1) are consistent with the other rareearth results,  $(a)$  –  $(c)$ . Finally, we mention two more studies not shown on Fig. 1; one involves the  $N=84$  isotones<sup>16</sup> and is discussed below. The other is an  $O(6)$  limit study of Hg isotopes by Morrison,  $39$  resulting in  $e_v = 0.138$  and  $e_{\pi} = 0.239$ .

In the present study, for both the Se and Kr isotope chains, the ratio  $e_y/e_{\pi}$  increases with increasing neutron number (approaching the closed shell at  $N=50$ ), and  $e_v/e_{\pi} > 1$  for those nuclei with  $N_v \approx 2$ . Although one would expect  $e_v < e_\pi$  from shell-model considerations, Scholten et al.<sup>36</sup> have shown that  $e_v > e_\pi$  can result from the truncation of the IBM-2 space to  $s$  and  $d$  bosons only. Renormalization to include the effects of  $g$  bosons results in an average value of  $e_v/e_{\pi} = 0.8$  for the Nd isotopes, in contrast to  $e_y/e_{\pi} = 2.0$  before renormalization.<sup>36</sup> The gboson renormalization also leads to a mass dependence for the neodymium effective charges, shown in Fig. 1. Hamilton *et al.*<sup>16</sup> have found  $e_{\pi} = 0.12$  and  $e_{\nu} = 0.24$ , with a ratio  $e_v/e_{\pi} = 2.0$ , for the  $N=84$  isotones <sup>140</sup>Ba,  $^{142}$ Ce, and  $^{144}$ Nd (without g-boson renormalization).

Although the exact effective charges obtained in this study depend on which of the reported  $B(E2)$  data are employed for normalization purposes, the  $e_y$ ,  $e_\pi$  values extracted with different data sets are not very different from each other for the well-studied selenium and krypton isotopes. For example, normalizing to the new  $B(E2; 2_1^+ \rightarrow 0_1^+)$  value of Schwengner *et al.*, <sup>29</sup> instead of the value of Raman et al., <sup>26</sup> for the nucleus <sup>78</sup>Se result in  $(e_y, e_\pi) = (0.090, 0.087)$  e b and  $(0.035, 0.134)$  e b for Sets <sup>1</sup> and 2, respectively.

## IV. CONCLUSION

IBM-2 effective neutron- and proton-boson charges have been extracted from experimental E2 matrix elements for nine light Se, Kr, and Sr isotopes. Two different sets of solutions are found, one of which has nearly equal effective charges for most of the nuclei studied, while, for the other set, the proton charge dominates in most cases. Within the Se and Kr isotope chains, the proton effective charges tend to decrease with decreasing neutron-boson number, the variation being minimal near the center of the neutron shell and becoming more pronounced as the end of the neutron shell is approached. The neutron effective charges are roughly constant within an isotope chain, in general. The calculated  $B(E2)$ values resulting from the two sets of charges are very similar and in good agreement with experiment for most transitions. Calculated transition rates from mixedsymmetry states to members of the ground-state band are expected to be quite different for the two sets, since they depend on the difference  $(e_v - e_\pi)$ . Therefore, information on mixed-symmetry states is sought in order to determine which of the two sets of charges is valid. It appears that, in spite of various limitations of the IBM-2 approach in this mass region (shape coexistence at low spin, 2qp aligned configurations,  $\dots$ ), the simple concept of s and d bosons remains valid.

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