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# Optical potential and the fusion barrier of two hot nuclei

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The finite-temperature self-consistent semiclassical calculation is carried out to determine the nucleon densities of some hot nuclei at thermal equilibrium with temperature $T$. These densities are applied, in the energy density formalism, to calculate the nucleus-nucleus optical potential and the corresponding fusion barrier between two hot nuclei. A moderate temperature dependence is found for the optical potential and fusion barrier.

## I. INTRODUCTION

The development of heavy-ion experiments makes is possible to create and study highly excited nuclei experimentally. These nuclei are often called hot nuclei since the intrinsic energy $E^*$ stored up in them during the collisions can be related with an intrinsic temperature $T$ via the level density formula.

It is noticed that at the initial stage of the collision, both the target and projectile nuclei have zero temperature. During the collision they get excited and obtain a temperature. It should be pointed out that the entire dynamical process of the collision and excitation is rather complicated and certainly beyond the content of the optical model. This situation is the same as in the transfer reaction: the incoming and outgoing particles are described by different optical potentials, while the transfer itself is a mechanism beyond the optical model. In the same spirit one should use in the entrance channel of a deep inelastic scattering a zero-temperature optical potential and in its exit channel a temperature-dependent optical potential. We intend in this paper to calculate and analyze self-consistently these temperature-dependent nucleus-nucleus optical potentials and the corresponding fusion barriers with the Skyrme-Hartree-Fock energy density which is easy to handle numerically.

The nucleus-nucleus optical potential, according to its energy density definition,[1] depends crucially on the nucleon densities of the colliding nuclei. The temperature dependence of the potential comes mainly from the temperature dependence of the nucleon densities. It is requisite to obtain the nucleon densities of hot nuclei in order to evaluate the potentials between these nuclei.

It is natural to assume the finite-temperature Hartree-Fock (FTHF) approach as a suitable one to deal with the static properties of a hot nucleus. This kind of calculation does exist in the literature.[2,3] But since the single-particle states that should be included in the calculation increase exponentially as the temperature increases, the FTHF approach is not very convenient in the practical applications. On the other hand, we have already observed the following two facts: (1) the self-consistent semiclassical (SCSC) approach at zero temperature, which excludes the shell effect, can reproduce very well the average properties and the collective behaviors of nuclei in a simpler way than the HF calculation, and (2) the previous FTHF calculations predicted that the shell effect gradually ceases to play a role as the temperature increases. When $T \gtrsim 2.0$ MeV, it is already possible to neglect the shell effect with great accuracy. Combining these two facts, we are confident that it is reasonable and feasible to substitute for the FTHF approach a finite-temperature self-consistent semiclassical (FTSCSC) one to determine the static properties of hot nuclei.

The FTSCSC calculation is carried out in Sec. II. Such calculations have been extensively studied in the past. Some authors assumed a temperature-independent local mean field, such as harmonic or Woods-Saxon potentials,[4,5] while others explicitly took into account the influence of the evaporated nucleon gas, and carried out a complicated two-phase self-consistent semiclassical calculation.[6-8] The prescription we adopt in this work is in between these two extremes. We neglect the influence of the gas phase while determining the mean field and nucleon densities self-consistently. The reasons are as follows. (1) If the influence of the nucleon gas phase is included, the FTSCSC calculation will be very involved. The merits of the semiclassical approximation cannot be fully explored and large-scale investigation of the temperature dependence of nucleus-nucleus optical potentials will certainly be out of the question. (2) We restrict our calculation to the temperature range below 6 MeV, since some previous calculations indicated that at low temperature the effect of the gas phase can approximately be neglected.[9] (3) If the temperature is very high, the nucleus will be unstable against the clustering and fragmentation, and such static approaches as FTHF and FTSCSC will no longer be suitable for this situation.

Sections III and IV are devoted to the calculation of nucleus-nucleus optical potentials and the corresponding fusion barriers with the use of hot nucleon densities determined by the present FTSCSC approach. We will follow the recent works of Tomasi *et al.*[10] and Rashdan *et al.*[11] in defining the optical potential as the difference of internal energies instead of the free energy as was adopted by the early work of Gahde and Stocker.[12]

## II. FTSCSC APPROACH AND STATIC PROPERTIES OF HOT NUCLEI

We assume the extended Skyrme force,[13] which unifies the conventional,[14] modified,[15] and generalized[16] ones, as the effective nucleon-nucleon interaction in this work:

$$V_{ij}=t_0(1+x_0P_\sigma)\delta(\mathbf{r})+\tfrac{1}{6}t_3(1+x_3p_\sigma)[\rho(\mathbf{R})]^\alpha\delta(\mathbf{r})+\tfrac{1}{2}t_1(1+x_1p_\sigma)[\delta(\mathbf{r})\mathbf{k}^2+\mathbf{k}'^2\delta(\mathbf{r})]$$
$$+\tfrac{1}{2}t_4(1+x_4p_\sigma)[\delta(\mathbf{r})\rho(\mathbf{R})\mathbf{k}^2+\mathbf{k}'^2\rho(\mathbf{R})\delta(\mathbf{r})]+t_2(1+x_2p_\sigma)\mathbf{k}'\cdot\delta(\mathbf{r})\mathbf{k}+t_5(1+x_5p_\sigma)\mathbf{k}'\cdot\rho(\mathbf{R})\delta(\mathbf{r})\mathbf{k}$$
$$+iW_0(\boldsymbol{\sigma}_i+\boldsymbol{\sigma}_j)\cdot\mathbf{k}'\cdot\delta(\mathbf{r})\mathbf{k}\ , \tag{1}$$

where $\mathbf{R}=\frac{1}{2}(\mathbf{r}_i+\mathbf{r}_j)$, $\mathbf{r}=\mathbf{r}_i-\mathbf{r}_j$ are the center-of-mass and relative coordinates, respectively. The relative momentum operators are $\mathbf{k}=1/2i(\nabla_i-\nabla_j)$, acting to the right and $\mathbf{k}'=-1/2i(\nabla_i-\nabla_j)$, acting to the left.

The temperature dependence of the effective interaction has been discussed by Lejeune *et al.*[17] and by Baldo *et al.*[18] Their calculations showed that when the temperature increases from zero to 10 MeV, the change in the effective interaction is approximately 5%. It is therefore unnecessary to include the effect of temperature on the effective interaction in the FTHF or FTSCSC calculations, since these calculations themselves are not expected to be better than 5%. From this argument we assume the parameters of the extended Skyrme force to be independent of the temperature. The energy density derived from this force has the same structure as in the zero-temperature case:

$$\mathcal{H}(\mathbf{r})=\frac{\hbar^2}{2m}(s_n\tau_n+s_p\tau_p)+(g_4+g_5\rho^\alpha)(\rho_n^2+\rho_p^2)+(g_6+g_7\rho^\alpha)\rho_n\rho_p+(g_8+g_9\rho)(\rho_n\Delta\rho_n+\rho_p\Delta\rho_p)$$
$$+(g_{10}+g_{11}\rho)(\rho_n\Delta\rho_p+\rho_p\Delta\rho_n)+g_{12}\rho((\nabla\rho_n)^2+(\nabla\rho_p)^2)+\tfrac{1}{2}W_0(\mathbf{J}\cdot\nabla\rho+\mathbf{J}_n\cdot\nabla\rho_n+\mathbf{J}_p\cdot\nabla\rho_p)\ , \tag{2}$$

where the nucleon effective mass is defined by

$$s_q=\frac{m}{m_q^*}=1+\frac{2m}{\hbar^2}[(g_0+g_1\rho)\rho_q+(g_2+g_3\rho)\rho_{-q}]\ . \tag{3}$$

For details see Ref. 19.

In the case of finite temperature, it is free energy $F$, not the internal energy $E$, which is minimized. The free energy density $\mathcal{F}$ is connected with energy density $\mathcal{H}$ and entropy density $\mathcal{S}$ through the relation

$$\mathcal{F}=\mathcal{H}-T\mathcal{S}\ .$$

We should therefore express the entropy density $\mathcal{S}_q$ as well as the kinetic energy density $\tau_q$ and the spin-orbit density $\mathbf{J}_q$ as functionals of nucleon density $\rho_q$ in order to obtain the free energy density functional $\mathcal{F}[\rho_n,\rho_p]$. The derivation of these functionals in the semiclassical approximation has been discussed by Brack *et al.*[6,7] We present here the final form of the free energy density functional:

$$\mathcal{F}[\rho_n,\rho_p]=T(y_n\rho_n+y_p\rho_p)-\frac{1}{3\pi^2}\left[\frac{2mT}{\hbar^2}\right]^{3/2}T(I_{3/2}(y_n)/s_n^{3/2}+I_{3/2}(y_p)/s_p^{3/2})$$
$$+\frac{1}{48}\frac{\hbar^2}{2m}\left[4a_ns_n\frac{(\nabla\rho_n)^2}{\rho_n}+(9a_n+7)\rho_n\frac{(\nabla s)^2}{s_n}+4(3a_n+5)\nabla\rho_n\cdot\nabla s_n+4a_p\rho_p\frac{(\nabla\rho_p)^2}{\rho_p}\right.$$
$$\left.+(9a_p+7)\rho_p\frac{(\nabla s_p)^2}{s_p}+4(3a_p+5)\nabla\rho_p\cdot\nabla s_p\right]+(g_4+g_5\rho^\alpha)(\rho_n^2+\rho_p^2)$$
$$+(g_6+g_7\rho^\alpha)\rho_n\rho_p+(g_8+g_9\rho)(\rho_n\Delta\rho_n+\rho_p\Delta\rho_p)+(g_{10}+g_{11}\rho)(\rho_n\Delta\rho_p+\rho_p\Delta\rho_n)$$
$$+g_{12}\rho((\nabla\rho_n)^2+(\nabla\rho_p)^2)-\frac{W_0^2}{8}\frac{2m}{\hbar^2}\left[(\nabla\rho+\nabla\rho_n)^2\frac{\rho_n}{s_n}+(\nabla\rho+\nabla\rho_p)^2\frac{\rho_p}{s_p}\right]\ . \tag{4}$$

$a_q$ and $b_q$ are two real numbers composed of the so-called Fermi-Dirac integrals $I_\nu(y_q)$:

$$a_q = I_{1/2}(y_q) I_{-3/2}(y_q) I^2_{-1/2}(y_q) ,$$

$$b_q = I^2_{1/2}(y_q) I_{-5/2}(y_q)/I^2_{-1/2}(y_q) ,$$

while $I_\nu(y_q)$ is defined as

$$I_\nu(y_q) = \int_0^\infty \frac{x^\nu}{1+\exp(x-y_q)} dx, \quad \nu > -1$$

$$I_{\nu-1}(y_q) = \frac{1}{\nu}\frac{d}{d\nu} I_\nu(y_q), \quad \nu < -1 . \qquad (5)$$

The Lagrangian functional $\mathcal{L}[\rho_n,\rho_p]$, from the stationary condition of which the nucleon density can be determined, is as follows:

$$\mathcal{L}[\rho_n,\rho_p] = \int (\mathcal{H}[\rho_n,\rho_p] - \mu_n\rho_n - \mu_p\rho_p) d\mathbf{r} , \qquad (6)$$

where $\mu_u$ and $\mu_p$ are nucleon chemical potentials connected with the following constraints:

$$N = \int \rho_n(\mathbf{r}) d\mathbf{r}, \quad Z = \int \rho_p(\mathbf{r}) d\mathbf{r} .$$

A trial density is utilized, as in the previous calculation of the nucleus-nucleus optical potentials,[10,11] to simplify the numerical work, although we are aware of the fact that the use of a trial density at high temperature is not very well founded.

From the nucleon densities thus determined, a series of nuclear static properties can be studied. For example, the charge distribution of a nucleus can be folded from the spatial distribution of protons and the intrinsic charge distribution of one proton:

$$\rho_c(\mathbf{r}) = \frac{1}{a_p\sqrt{\pi}}\frac{1}{r}\int_0^\infty s\left(e^{-(r-s/a_p)^2} - e^{-(r+s/a_p)^2}\right) \times \rho_p ds, \quad a_p = 0.65 \text{ fm} . \qquad (7)$$

The charge distribution of $^{120}$Sn is plotted in Fig. 1. It is seen that the distribution extends outwards as the temperature increases.

The root-mean-square radii of neutrons and protons can be computed from

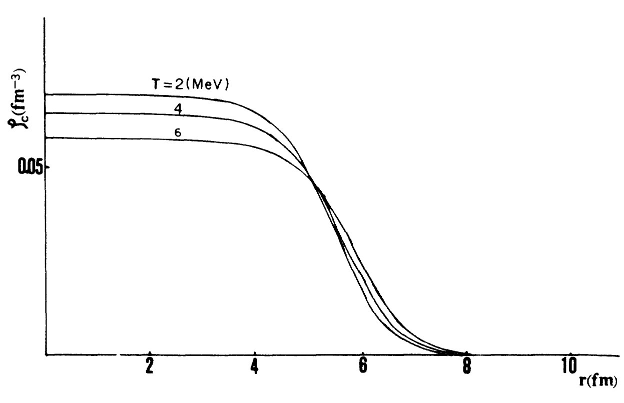


FIG. 1. The plot of charge distribution of $^{120}$Sn as a function of $r$.

$$y_n = \langle r_n^2 \rangle^{1/2} = \left[ \frac{\int y^2 \rho_n d\mathbf{r}}{\int \rho_n d\mathbf{r}} \right]^{1/2} , \qquad (8a)$$

$$y_p = \langle r_p^2 \rangle^{1/2} = \left[ \int y^2 \rho_p d\mathbf{r} \right]^{1/2} . \qquad (8b)$$

The rms radii of $^{20}$Zr, $^{120}$Sn, and $^{208}$Pb are drafted in Fig. 2. The higher the temperature, the greater the rms radii. This results from the outward shift of the nucleon distributions.

## III. THE OPTICAL POTENTIAL BETWEEN TWO HOT NUCLEI

At zero temperature, the nucleus-nucleus optical potential is defined as the energy difference of the composite

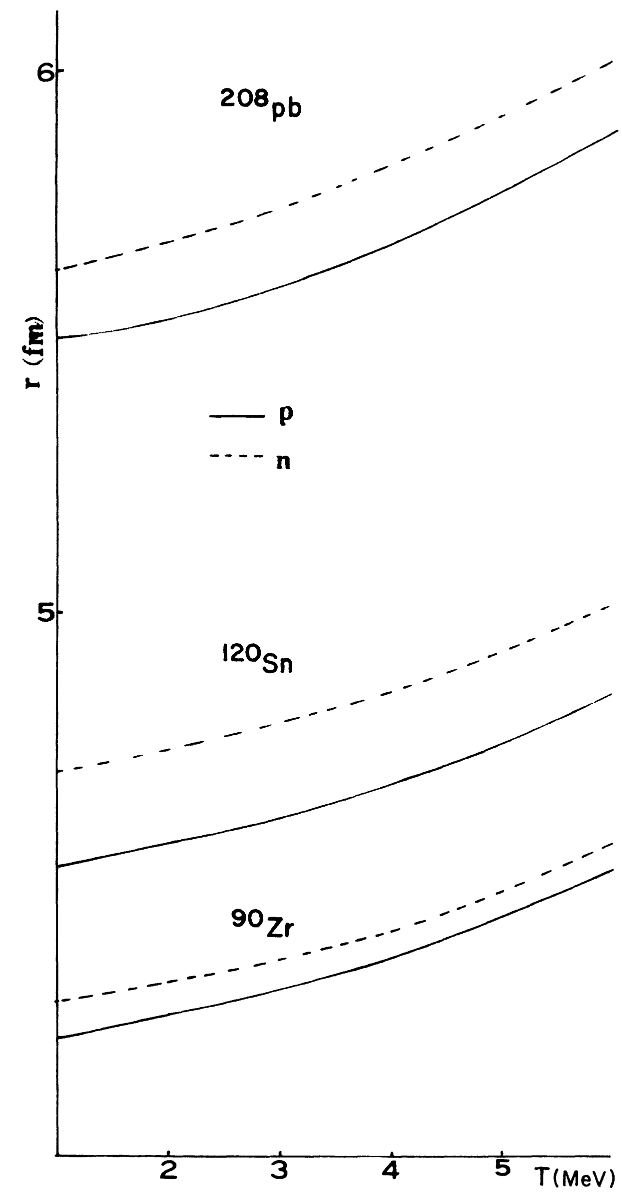


FIG. 2. The plot of neutron rms radii as a function of $T$.

system (target plus projectile nuclei) when two nuclei are separated by a distance $R$ and when the separation is infinite.

At finite temperature, we assume that the system has the same temperature at any separation $R$. This is quite a drastic and crude assumption, since the temperature is gradually acquired along with the dissipation of the incident kinetic energy during the collision. But as was pointed out in the Introduction, a fully dynamical treatment of this problem, which considers the variation of the temperature with $R$, is certainly very difficult and beyond the notation of optical potentials. The assumption of constant temperature was also adopted in the previous calculations.[10,11]

With this assumption, the optical potential is defined as the difference of total energy of the composite system calculated at $R$ and $T$ and that calculated at infinity and $T$:

$$V_n(R,T)=E(R,T)-E(\infty,T)$$
$$=\int \mathcal{H}(\mathbf{r})d\mathbf{r}-\left[\int \mathcal{H}_1(\mathbf{r})d\mathbf{r}+\int \mathcal{H}_2(\mathbf{r})d\mathbf{r}\right] . \quad (9)$$

The energy densities $\mathcal{H}_1$ and $\mathcal{H}_2$ of the target and the projectile nuclei are given by Eq. (2). Since the nucleon densities of these nuclei have been determined in Sec. II, it is straightforward to obtain $E(\infty,T)$. But to calculate $E(R,T)$, it is necessary first of all to get the nucleon density of the composite nucleus. Several approximations to this density have been proposed, the simplest of which is the sudden approximation where the nucleon density of the composite system is assumed to be the sum of the corresponding densities of the target and projectile nuclei:

$$\rho_q(\mathbf{r})=\rho_{1q}(\mathbf{r})+\rho_{2q}(\mathbf{r}) . \quad (10)$$

Generally speaking, the sudden approximation is reliable if and only if the collision time is much shorter than the relaxation time. If the former is longer than the latter, the adiabatic approximation would be much more suitable. For a definite collision, it is noticed that, in the entrance channel, up to the turning point, the shape of the two nuclei remains spherical to a good approximation. This justifies the use of the sudden approximation in the calculation of the potential for this stage of reaction. At the turning point and afterwards, large-scale deformation is induced and an adiabatic potential will be much more close to the actual situation.[10] A more realistic treatment is to combine the sudden potential with the adiabatic one in the whole process of reaction.[20]

Four systems $^{40}Ca+^{40}Ca$, $^{90}Zr+^{90}Zr$, $^{40}Ca+^{120}Sn$, and $^{208}Pb+^{208}Pb$ are considered at $T=2$, 4, and 6 MeV. The SKM Skyrme force is used in order to attain the self-consistency between the static and dynamical calculations. The results are depicted in Figs. 3–6. A moderate temperature dependence of the potentials, especially for the lighter systems, is seen in these figures. The higher the temperature, the deeper and broader the potentials, which is the result of outward shift of the nucleon densities as the temperature increases.

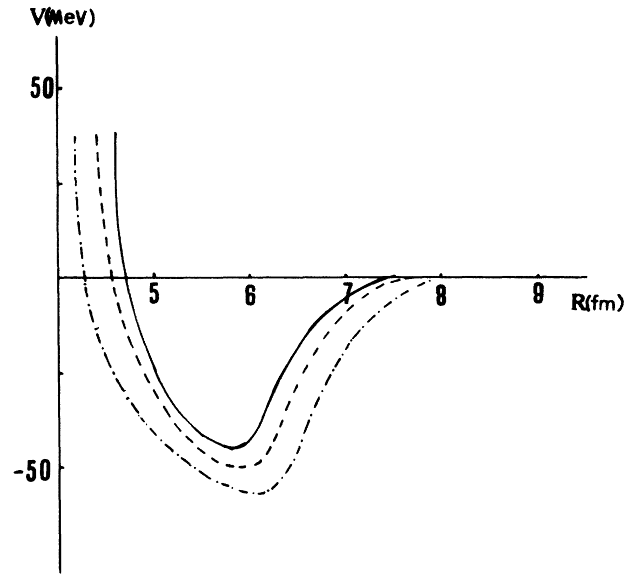


FIG. 3. The plot of the potential as a function of $R$. $^{40}Ca+^{40}Ca$, SKM, ——, $T=2.0$ MeV, — — —, $T=4.0$, MeV, —·—·—·, $T=6.0$ MeV.

## IV. THE FUSION BARRIERS AT FINITE TEMPERATURE

It is observed that some properties, such as the fusion barriers, are determined during the early stage of the reaction, i.e., before the system reaches the turning point. This hints at the reasonability of analyzing the fusion barriers from the sudden potentials.[10,21] But to calculate the fusion barrier of two hot nuclei, we should first evalu-

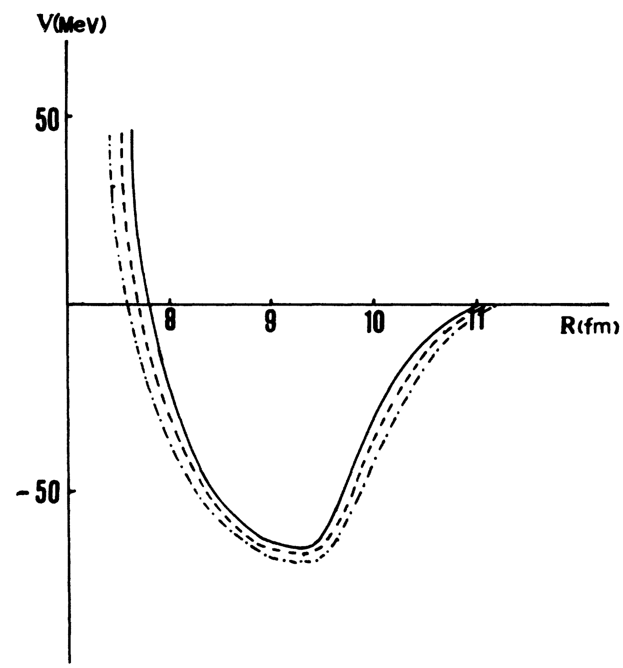


FIG. 4. Same as Fig. 3 but for $^{90}Zr+^{90}Zr$.

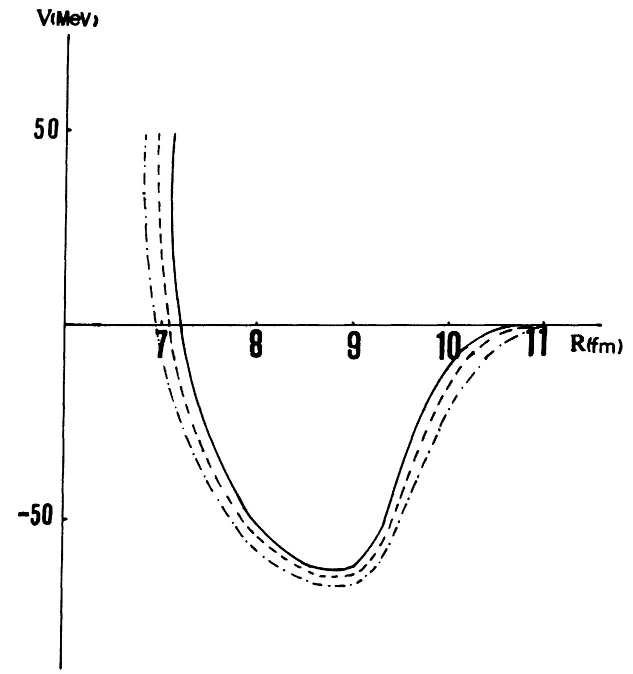


FIG. 5. Same as Fig. 3 but for $^{40}Ca+^{120}Sn$.

ate the Coulomb interaction between these nuclei which is defined as

$$V_c(R,T)=e^2\int \frac{\rho_{1c}(\mathbf{r}_1)\rho_{2c}(\mathbf{r}_2)}{|\mathbf{R}+\mathbf{r}_1-\mathbf{r}_2|}d\mathbf{r}_1 d\mathbf{r}_2 \ . \qquad (11)$$

This interaction is apparently temperature dependent since the charge distributions are temperature dependent

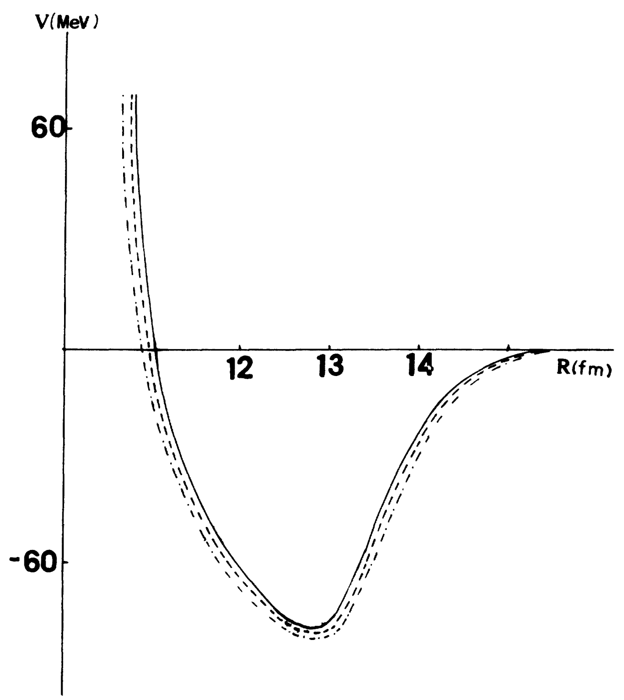


FIG. 6. Same as Fig. 3 but for $^{208}Pb+^{208}Pb$.

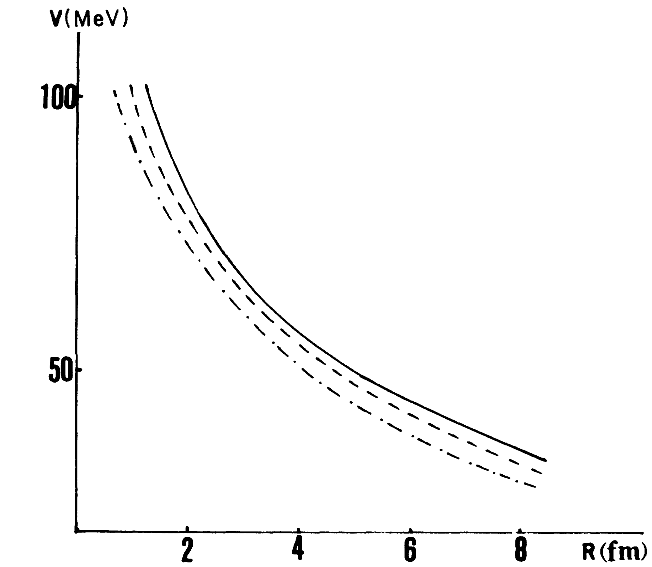


FIG. 7. The plot of the Coulomb interaction as a function of $R$. $^{40}Ca+^{40}Ca$, SKM, ——, $T=2.0$ MeV, — — —, $T=4.0$ MeV, —·—·—·, $T=6.0$ MeV.

(see Fig. 1). If the finiteness of protons is neglected, the charge distribution of a nucleus is equivalent to its proton density. The Coulomb interaction can then be computed from Eq. (11) with $\rho_{1c}$ and $\rho_{2c}$ replaced by $\rho_{1p}$ and $\rho_{2p}$. The resulting interactions are sketched in Figs. 7 and 8 for systems $^{40}Ca+^{40}Ca$ and $^{208}Pt+^{208}Pb$. The Coulomb interaction is slightly lowered as the temperature increases.

The total interaction between two hot nuclei is

$$V(R,T)=V_n(R,T)+V_c(R,T) \ . \qquad (12)$$

The fusion barrier happens at $R_B$ which is determined from the condition

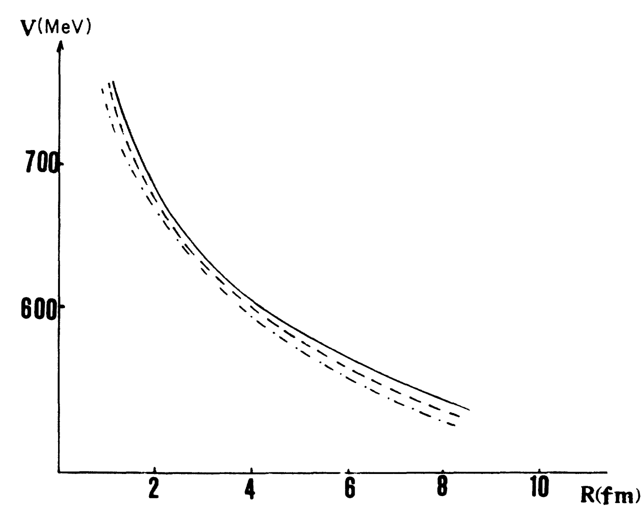


FIG. 8. Same as Fig. 7 but for $^{208}Pb+^{208}Pb$.

TABLE I. The temperature shift of fusion barriers, SKM.

| | $^{40}Ca+^{40}Ca$ | | $^{90}Zr+^{90}Zr$ | | $^{40}Ca+^{120}Sn$ | |
|---|---|---|---|---|---|---|
| $T$ (MeV) | $K_B$ (fm) | $V_B$ (MeV) | $R_B$ (fm) | $V_B$ (MeV) | $R_B$ (fm) | $V_B$ (MeV) |
| 1 | 9.39 | 41.60 | 12.47 | 183.4 | 11.39 | 109.3 |
| 2 | 9.50 | 40.30 | 12.67 | 186.7 | 11.51 | 106.3 |
| 3 | 9.61 | 38.63 | 12.90 | 183.9 | 11.56 | 104.7 |
| 4 | 9.72 | 36.70 | 13.03 | 181.3 | 11.60 | 103.1 |
| 5 | 9.83 | 33.48 | 13.39 | 178.6 | 11.66 | 101.5 |
| 6 | 9.91 | 32.63 | 13.77 | 174.3 | 11.73 | 99.7 |

$$\left.\frac{dV}{dR}\right|_{R_B}=0 \qquad (13)$$

and the corresponding height $V_B$ is given by

$$V_B=V(R_B,T)=V_n(R_B,T)+V_c(R_B,T) \ . \qquad (14)$$

The results are listed in Table I for three systems. As can be seen in the table the position of the barrier shifts outward while its height lowers somewhat as the temperature increases. This results from combining the effects of the nucleus-nucleus optical potential and the Coulomb interaction.

This self-consistent semiclassical approach to the energy density has been adopted by Nemeth *et al.*[22] and Dalili *et al.*[23] to study the fission barriers at finite temperature. In these works, the effect of the gas phase was also neglected.

## V. CONCLUSIONS

The semiclassical approach is currently one of the most frequently utilized approaches in nuclear physics. This approach has been extended from spherical nuclei to deformed nuclei[24,25] and from zero temperature to finite temperature.[4–8] The nucleon density determined by these approaches can be used to study the properties of nuclear giant resonances[18,24,25] and to calculate the nucleus-nucleus optical potentials as in this paper. The nucleus-nucleus optical potentials play a significant role in such processes as deep inelastic collisions and fusion reactions. A realistic nucleon-nucleon interaction such as the Reid soft-core potential was adopted by Faessler *et al.*[26,27] with local-density approximation to obtain both the real and imaginary parts of the potential. The nucleus-nucleus optical potential with Skyrme forces was calculated and analyzed in Refs. 1 and 28–30. The present work is a natural extension of the previous similar works from zero temperature to finite temperature. It should be pointed out that the assumption of constant temperature during the collision is not very realistic. If the potentials obtained in this work are applied to analyze the experimental data, a suitable treatment of the variation of temperature as was adopted in Ref. 10 is called for. This calculation is now in progress.

## ACKNOWLEDGMENTS

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