Strength of the attraction of the Skyrme-type forces used in the heavy-ion reaction problems

Takahiro Wada
Research Institute for Fundamental Physics, Kyoto University, Kyoto 606, Japan

Shinichiro Yamaguchi and Hisashi Horiuchi
Department of Physics, Kyoto University, Kyoto 606, Japan
(Received 19 June 1989)

The internucleus potentials of the ^{16}O-^{16}O, ^{12}C-^{16}O, ^{12}C-^{12}C, and α-^{16}O systems are calculated by using the Skyrme-type forces in the framework of the canonical moving wave packet method. The volume integrals j_v of the calculated potentials are compared with those of the real parts of the optical potentials which fit the data very well in a wide energy range. Compared with experiments, the calculated values of $|j_v|$ are found to have too weak energy dependence and to be too small in the incident energy region lower than about 40 MeV/nucleon. Finite-range nuclear forces are constructed with least modification to the Skyrme-type forces such that they reproduce the same nuclear matter properties as the Skyrme-type forces, and the properties of the internucleus potentials are studied by these finite-range forces.

I. INTRODUCTION

In the microscopic studies of the heavy-ion reaction, such as the studies by the time-dependent Hartree-Fock (TDHF) method, the Vlasov equation, and the Vlasov-Uehling-Uhlenbeck equation, it is quite often the case to use the Skyrme-type force as the effective nuclear force.[1,2] The Skyrme-type force which we investigate in this paper is usually expressed in the form of the mean field $U(r)$ constructed from it as follows:

$$U(r)=\alpha\left[\frac{\rho(r)}{\rho_0}\right]+\beta\left[\frac{\rho(r)}{\rho_0}\right]^{\sigma+1}, \tag{1.1}$$

where $\rho(r)$ stands for the density of the system. ρ_0 is the saturation density of the nuclear matter and three parameters α, β, and σ are uniquely determined by requiring the reproduction of the three properties of the nuclear matter; the binding energy E per nucleon at $\rho=\rho_0$, the minimum condition of E at $\rho=\rho_0$, and the incompressibility $K=9\rho_0^2(\partial^2E/\partial\rho^2)_{\rho_0}$. The effective two-nucleon force $\hat{v}$, which gives the nucleon mean field $U(r)$ of Eq. (1.1), is expressed as

$$\hat{V}=\frac{1}{2}\sum_{i,j}\hat{v}_{i,j}\ ,$$

$$\hat{v}=\left[\frac{4}{3}\frac{\alpha}{\rho_0}\right]\delta(x_1-x_2) + \left[\frac{8}{3}\frac{1}{2+\sigma}\frac{\beta}{\rho_0^{\sigma+1}}\right]\left[\rho\left[\frac{x_1+x_2}{2}\right]\right]^{\sigma}\delta(x_1-x_2)\ . \tag{1.2}$$

Although the Skyrme-type forces are convenient to treat for the numerical computation, they are too much simplified. The two-body force parts of them are of zero range and therefore the mean field $U(r)$ of Eq. (1.1) has no explicit energy dependence except that through $\rho(r)$. If they are applied to nuclear matter the mean field has no momentum dependence at all and the effective nucleon mass m^* is equal to the free-nucleon mass.

Since the Skyrme-type forces are used in the study of the nucleus-nucleus collision processes, to investigate how adequate they are when used for the description of the nucleus-nucleus elastic scattering processes is important in addition to the study of their properties for the nuclear matter. The purpose of this paper is to report the results of our study of the properties of the elastic channel nucleus-nucleus potentials calculated by the Skyrme-type nuclear forces. The systems we investigate are ^{16}O-^{16}O, ^{16}O-^{12}C, ^{12}C-^{12}C, and α-^{16}O.

We will see that the elastic channel internucleus potentials calculated by the Skyrme-type forces have the volume integral values whose energy dependence is too weak and which are too small in the incident energy region lower than about 40 MeV/nucleon, when compared with experiments. The weakness of the attraction of the Skyrme-type forces will weaken the acceleration of the relative motion of two colliding nuclei in their initial stage of the collision, which will result, for example, in the reduction of the high-momentum component of the nucleon momentum distribution in the early stage of nuclear collision.

This paper is organized as follows. In Sec. II we explain briefly the canonical moving wave packet (CMWP) method[3,4] by which we calculate the internucleus potential. In Sec. III the internucleus potentials of the ^{16}O-^{16}O, ^{12}C-^{16}O, ^{12}C-^{12}C, and α-^{16}O systems are calculated by using the stiff and soft Skyrme-type forces and are compared with the real part of the optical potentials. In Sec. IV we construct the finite-range effective nuclear forces such that they reproduce the same nuclear matter properties as the Skyrme-type forces, and we study the internucleus potentials calculated by the use of these forces. Summary and discussions are given in Sec. V.

II. FORMULATION

We investigate three Skyrme-type forces in this paper which are shown in Table I. We denote by S_1, $S_{1/3}$, and $S_{1/6}$ the Skyrme-type forces with $\sigma=1$, $\frac{1}{3}$, and $\frac{1}{6}$, respectively. S_1 belongs to the stiff nuclear force while $S_{1/3}$ and $S_{1/6}$ belong to the soft nuclear force.

The microscopic theory by which we calculate the elastic channel internucleus potential is the canonical moving wave packet method (CMWP).[3,4] We will briefly explain the CMWP method. The many-body wave function of the system of two scattering nuclei A and B is expressed as follows:

$$\Psi(\mathbf{D},\mathbf{K})=\mathcal{A}[\Psi_A(\mathbf{D}_A,\mathbf{K}_A)\Psi_B(\mathbf{D}_B,\mathbf{K}_B)] ,$$
$$\Psi_\alpha(\mathbf{D}_\alpha,\mathbf{K}_\alpha)=e^{i\mathbf{K}_\alpha\cdot\mathbf{X}_\alpha}\Psi_\alpha(x_i-\mathbf{D}_\alpha) \quad (\alpha=A,B) ,$$
$$\mathbf{X}_\alpha=\frac{1}{N_\alpha}\sum_{i\in\alpha}x_i , \tag{2.1}$$
$$\mathbf{D}_A=\frac{N_B}{N_A+N_B}\mathbf{D}, \quad \mathbf{D}_B=-\frac{N_A}{N_A+N_B}\mathbf{D} ,$$
$$\mathbf{K}_B=-\mathbf{K}_A=\mathbf{K} ,$$

where N_α stands for the mass number of the nucleus α $(=A,B)$ and $\Psi_\alpha(x_i-\mathbf{D}_\alpha)$ is the harmonic-oscillator shell-model wave function of the nucleus α located around the spatial position $\mathbf{D}_\alpha$ and with the oscillator parameter ν_α $(=m\omega_\alpha/2\hbar)$. The wave number vector $\mathbf{K}$ is determined as a function of $\mathbf{D}$ by solving the following energy conservation relation:

$$\frac{\langle\Psi(\mathbf{D},\mathbf{K})|H|\Psi(\mathbf{D},\mathbf{K})\rangle}{\langle\Psi(\mathbf{D},\mathbf{K})|\Psi(\mathbf{D},\mathbf{K})\rangle}=E_r+\tfrac{3}{4}\hbar\omega_r+E_A+E_B ,$$
$$\omega_r=\frac{(N_A+N_B)\omega_A\omega_B}{(N_A\omega_A+N_B\omega_B)} , \tag{2.2}$$

where H is the total Hamiltonian, E_r the incident energy, and E_α $(\alpha=A,B)$ the binding energy of the nucleus α.

In contrast to Fliessbach's moving wave packet method,[5] in the canonical moving wave packet method, the canonical coordinate vector $\mathbf{R}$ and its conjugate canonical wave number vector $\mathbf{P}$ are introduced and are calculated from $\mathbf{D}$ and $\mathbf{K}$ as follows:

TABLE I. Force parameters of three Skyrme-type forces. Units for α, β, E, and K are MeV and the unit for ρ_0 is fm^{-3}.

	σ	α	β	ρ_0	E	K
S_1	1	-124.0	70.5	0.165	-16.0	378
$S_{1/3}$	1/3	-218.0	164.0	0.169	-15.8	235
$S_{1/6}$	1/6	-356.0	303.0	0.162	-15.9	200

$$\mathbf{R}=Y\mathbf{D}, \quad \mathbf{P}=Y\mathbf{K} ,$$
$$Y=\left[\frac{\partial}{\partial s}\ln N(s)\right]^{1/2} ,$$
$$N(s)=Ce^s\langle\Psi(\mathbf{D},\mathbf{K})|\Psi(\mathbf{D},\mathbf{K})\rangle , \tag{2.3}$$
$$s=\nu_r\mu_0\mathbf{D}^2+\frac{\mathbf{K}^2}{4\nu_r\mu_0} ,$$
$$\nu_r=\frac{m\omega_r}{2\hbar}, \quad \mu_0=\frac{N_AN_B}{N_A+N_B} ,$$

where the constant C is so chosen to ensure the asymptotic property of $N(s)$

$$N(s)\to e^s \quad (s\to\infty) . \tag{2.4}$$

Strictly speaking, the overlap matrix $\langle\Psi(\mathbf{D},\mathbf{K})|\Psi(\mathbf{D},\mathbf{K})\rangle$ reduces to a function of only s, only under the condition that both $\Psi_A(x_i)$ and $\Psi_B(x_i)$ are Elliott SU_3 scalar and have a common harmonic oscillator parameter $\nu_A=\nu_B$ as in the ^{16}O-^{16}O system.[4,6] However, although this condition is slightly violated in ^{16}O-^{12}C, ^{12}C-^{12}C, and α-^{16}O systems, the dependence of $\langle\Psi(\mathbf{D},\mathbf{K})|\Psi(\mathbf{D},\mathbf{K})\rangle$ on $\mathbf{D},\mathbf{K}$ is almost absorbed by that on s.

By using the relation $\mathbf{K}=\mathbf{K}(\mathbf{D})$ obtained by Eq. (2.2), we can express $\mathbf{P}$ as a function of $\mathbf{R}$, $\mathbf{P}=\mathbf{P}(\mathbf{R})$. This allows us to calculate the elastic channel internucleus potential $V(\mathbf{R})$ as

$$V(\mathbf{R})=E_r-\frac{\hbar^2}{2\mu}\mathbf{P}^2(\mathbf{R}) ,$$
$$\mu=\frac{N_AN_B}{N_A+N_B}m . \tag{2.5}$$

In this paper we only treat the case of the head-on collision, namely $\mathbf{K}\parallel\mathbf{D}$ and $\mathbf{P}\parallel\mathbf{R}$. A very important feature of the CMWP method is the existence of the Pauli-forbidden region in the phase space defined by

$$\frac{\hbar^2}{2\mu}\mathbf{P}^2+\frac{\mu\omega_r^2}{2}\mathbf{R}^2<N_{\mathrm{ald}}\hbar\omega_r , \tag{2.6}$$

where N_{ald} is the minimum number of the harmonic-oscillator quanta of the Pauli-allowed relative wave functions. This feature is exactly the same[7] as in the semiclassical treatment of the resonating group method (RGM) which we abbreviate as the RGM+WKB (Wentzel-Kramers-Brillouin) method.[8] In Ref. 4, it is shown that in the α-α system the real and imaginary parts of the internucleus potentials calculated by the CMWP method by using the realistic complex effective nuclear force (CEG) (Ref. 9) are close to those by the RGM+WKB method by using the same nuclear force. Later in this paper we will show that the ^{16}O-^{16}O potentials calculated by the CMWP method are close to those by the RGM + WKB method under the use of the Skyrme-type nuclear forces. The internucleus potentials calculated by the RGM + WKB method are expected to be very reliable since it has already been shown that the α-^{16}O (Ref. 10) and α-^{40}Ca (Ref. 11) real potentials calculated by the RGM + WKB method are surprisingly in good agreement with the real parts of the best fitting "unique" optical potentials by Michel *et al.*[12] for α-^{16}O and by Delbar *et al.*[13] for α-^{40}Ca, respectively.

When the effective nuclear force is given as

$$\hat{V}=\frac{1}{2}\sum_{i,j} t_0\delta(x_i-x_j)+\frac{1}{6}\sum_{i,j,k} t_3\delta(x_i-x_j)\delta(x_j-x_k)\ , \quad (2.7)$$

the nucleon mean field is calculated to be

$$U(r)=A\rho(r)+B\rho^2(r)\ ,$$
$$A=\tfrac{3}{4}t_0,\quad B=\tfrac{3}{16}t_3\ , \quad (2.8)$$

and the potential energy by the CMWP method is calculated to be

$$\frac{\langle\Psi(\mathbf{D},\mathbf{K})|\hat{V}|\Psi(\mathbf{D},\mathbf{K})\rangle}{\langle\Psi(\mathbf{D},\mathbf{K})|\Psi(\mathbf{D},\mathbf{K})\rangle}=\frac{A}{2}\int dr[\rho(r)]^2+\frac{B}{3}\int dr[\rho(r)]^3\ , \quad (2.9)$$

where the CMWP density $\rho(r)$ is given by

$$\rho(r)=\frac{\langle\Psi(\mathbf{D},\mathbf{K})|\sum_{i=1}^{N_A+N_B}\delta(x_i-r)|\Psi(\mathbf{D},\mathbf{K})\rangle}{\langle\Psi(\mathbf{D},\mathbf{K})|\Psi(\mathbf{D},\mathbf{K})\rangle}\ . \quad (2.10)$$

Therefore, for the Skyrme-type nuclear forces $\hat{V}$ of Eq. (1.2) which give the nucleon mean field $U(r)$ as in Eq. (1.1), the potential energy by the CMWP method is given as

$$\frac{\langle\Psi(\mathbf{D},\mathbf{K})|\hat{V}|\Psi(\mathbf{D},\mathbf{K})\rangle}{\langle\Psi(\mathbf{D},\mathbf{K})|\Psi(\mathbf{D},\mathbf{K})\rangle}=\frac{\alpha}{2}\int dr\left[\frac{\rho(r)}{\rho_0}\right]^2+\frac{\beta}{\sigma+2}\int dr\left[\frac{\rho(r)}{\rho_0}\right]^{\sigma+2}\ . \quad (2.11)$$

III. INTERNUCLEUS POTENTIALS BY THE SKYRME-TYPE FORCES AND COMPARISON WITH EXPERIMENTS

First we calculate the ^{16}O-^{16}O potential by using the force S_1 in Table I. In Fig. 1 the potentials calculated by CMWP and those by RGM + WKB are compared. The oscillator parameter ν of the ^{16}O shell-model wave function is so determined as to minimize the ^{16}O binding energy by the S_1 force. The determined values of ν and the binding energies are shown in Table II. At low energy such as $E_r=20$ MeV/nucleon, the potential by RGM + WKB is seen to be deeper than that by CMWP, but at higher energies the potentials by the two methods are close to each other. In Fig. 2 we compare the volume integrals j_v per nucleon pair of these potentials

$$j_v=\frac{4\pi}{N_AN_B}\int_0^{\infty}dr\cdot r^2V(r)\ . \quad (3.1)$$

We see that even at $E_r=20$ MeV/nucleon the values of j_v are not so much different between CMWP and RGM + WKB. This is because the difference of potentials $V(r)$

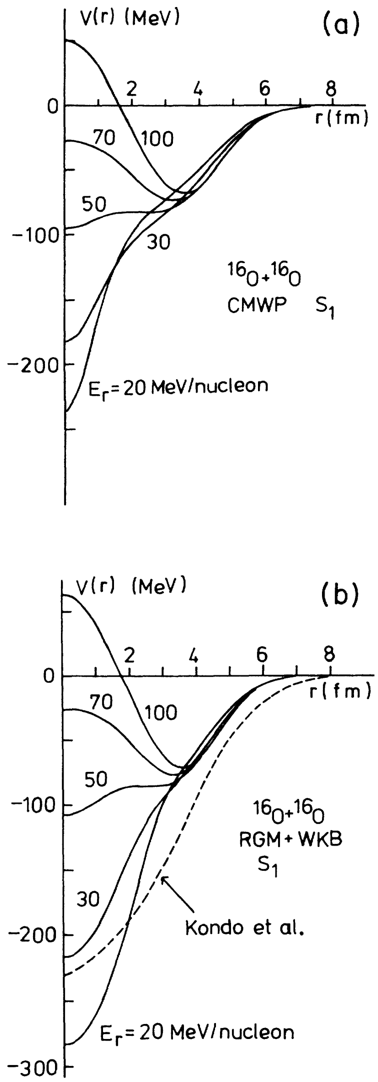

FIG. 1. The ^{16}O-^{16}O potentials by S_1 force at incident energies $E_r=20$, 30, 50, 70, and 100 MeV/nucleon. (a) shows the potentials calculated by the CMWP method while (b) by the RGM + WKB method. For the sake of comparison, in (b) we also display for the partial wave $J=18$ the real part of the ^{16}O-^{16}O optical potential by Kondo *et al.* which fits the data well in the low-energy resonance region ($E_r<10$ MeV/nucleon).

TABLE II. Oscillator parameter ν which minimizes the binding energy and the minimized binding energy E for each of ^{4}He, ^{12}C, and ^{16}O for each of S_1, $S_{1/3}$, and $S_{1/6}$ forces. Units for ν and E are fm^{-2} and MeV, respectively.

		S_1	$S_{1/3}$	$S_{1/6}$
^{4}He	ν	0.288	0.318	0.318
	E	−47.8	−53.3	−55.3
^{12}C	ν	0.219	0.224	0.223
	E	−110.9	−122.0	−124.9
^{16}O	ν	0.198	0.208	0.205
	E	−180.3	−192.0	−198.3

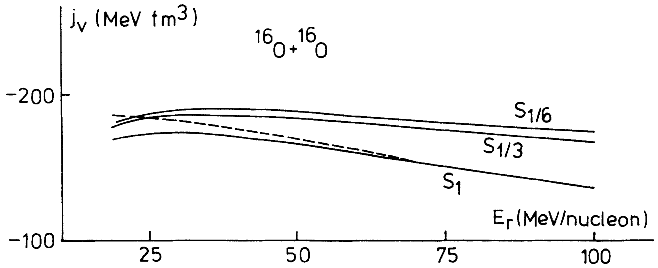

FIG. 2. Comparison of the volume integrals per nucleon pair j_v of the ^{16}O-^{16}O potentials calculated by the S_1, $S_{1/3}$, and $S_{1/6}$ forces. The dotted curve shows j_v by the S_1 force by the RGM + WKB method while solid curves show j_v by the CMWP method.

between CMWP and RGM + WKB at $E_r=20$ MeV/nucleon is almost restricted to the short-distance region of r. In Fig. 2 we also display j_v of the potentials calculated by CMWP by using the $S_{1/3}$ and $S_{1/6}$ forces. We note that j_v by $S_{1/3}$ and $S_{1/6}$ are very close to each other and that $|j_v|$ by any Skyrme-type force is smaller than 190 MeV fm^3.

Now we discuss the comparison with experiments. In contrast to the optical potentials of the light ions such as ^{3}He and α which are now believed to be determined "uniquely" for many systems,[14,13,12] those of the heavy ions are still full of ambiguities. However, recently in the case of lighter heavy ions such as ^{12}C and ^{16}O, the optical-model analyses seem to have determined almost unique optical potentials by fitting the characteristic nearside-farside interference cross sections in a very wide energy range.[15–17] These potentials have weak imaginary parts and smooth and nonstrong energy dependence in their real parts. In Fig. 3 we show j_v of the real parts of the ^{12}C-^{12}C and ^{12}C-^{16}O potentials presented by Brandan.[15] As is seen in the fact that the real parts of these Brandan potentials are close to the double folding potentials of Brandan and Satchler,[17] these potentials belong to the so-called deep potential group. In Fig. 3 we also display j_v of the real parts of the α-^{16}O potential by Michel *et al.*[12] and the α-^{40}Ca potential by Delbar *et al.*[13] which are known to be the unique α potentials and fit the data very well in a wide energy range. We note that the values of j_v of ^{12}C-^{12}C and ^{12}C-^{16}O are close to those of α-^{16}O and α-^{40}Ca in the low-energy region, $E_r \leq 25$ MeV/nucleon. Recently Kondo *et al.*[18] have presented a deep ^{16}O-^{16}O potential which can fit the low-energy data ($E_r < 10$ MeV/nucleon) exhibiting resonant behavior. Their potential is sufficiently deep to support wave functions with the proper number of radial nodes consistent with the Pauli principle,[7,19,20] and its j_v value is about -305 MeV fm^3 for the partial wave $J=18$ which is very close to the j_v values of the ^{12}C-^{12}C and ^{12}C-^{16}O Brandan potentials[15] in the low-energy region. It should be noticed that the $|j_v|$ values of the ^{16}O-^{16}O potentials by the Skyrme-type potentials given in Fig. 2 are far smaller than 300 MeV fm^3 in the low-energy region. For the sake of comparison, we display in Fig. 1(b) the ^{16}O-^{16}O potential by Kondo *et al.*[18] for $J=18$.

We calculated the ^{12}C-^{16}O and ^{12}C-^{12}C potentials by S_1, $S_{1/3}$, and $S_{1/6}$ forces by the CMWP method. The $0p_{3/2}$ closed-shell configuration was adopted to describe the ^{12}C nucleus. The oscillator parameter ν of the ^{12}C-^{12}C system is chosen to be the optimum ν values of a ^{12}C nucleus given in Table II, while in the ^{12}C-^{16}O system the oscillator parameters of ^{12}C and ^{16}O were chosen to have the same value, $\nu=0.205$ fm^{-2} for S_1 and $\nu=0.211$ fm^{-2} for both $S_{1/3}$ and $S_{1/6}$. In all the systems of ^{16}O-^{16}O, ^{12}C-^{16}O, and ^{12}C-^{12}C, the potentials by $S_{1/6}$ are deeper than those by $S_{1/3}$ which in turn are deeper than those by S_1. The j_v values of the potentials of the three systems calculated by the $S_{1/6}$ force are displayed in Fig. 3. We see that both in ^{12}C-^{16}O and ^{12}C-^{12}C systems, compared to $|j_v|$ values of the Brandan potentials,[15] those of the calculated potentials are small in the low-energy region, $E_r <$ about 50 MeV/nucleon and are very small in the region $E_r <$ about 25 MeV/nucleon.

Here we should recall that the theoretical potentials were calculated only for the head-on collision, namely for the S wave. It has been ascertained by many calculations that the theoretical potentials for higher partial waves are in general less attractive than the S-wave potential.[21] In the case of the RGM + WKB study of the ^{16}O-^{16}O potential, it was found that the $|j_v|$ values of the partial-wave-averaged-potential is smaller than that of the S-wave potential by more than 20 MeV fm^3 in the energy region $E_r < 40$ MeV/nucleon.[22] An even larger difference of the $|j_v|$ values was found in the RGM + WKB study of the α-^{16}O potential.[10] Therefore we should regard that the difference of the $|j_v|$ values between the Brandan potentials and the partial-wave-averaged potentials by the Skyrme-type forces is much larger by about or more than 20 MeV fm^3 than that between Brandan potentials and the S-wave potentials shown in Fig. 3.

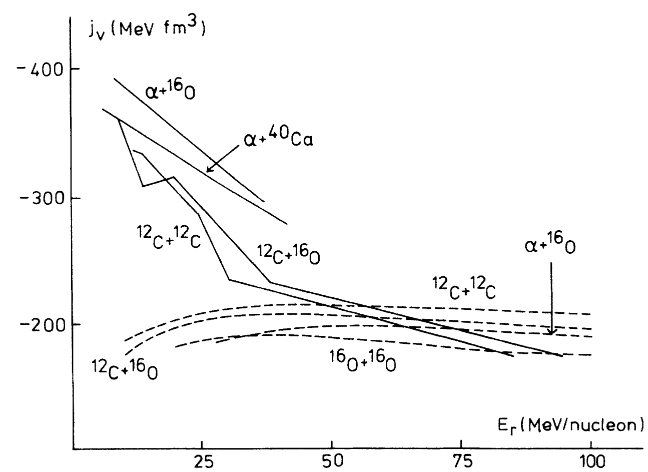

FIG. 3. Comparison of the calculated j_v values (dotted curves) with the j_v values of the real parts of the optical potentials which fit the data very well. The j_v values by the $S_{1/6}$ force are displayed here because the $|j_v|$ by $S_{1/6}$ are larger than those by S_1 and $S_{1/3}$.

In Fig. 4 we show the ^{12}C-^{16}O potentials $V(r)$ by S_1

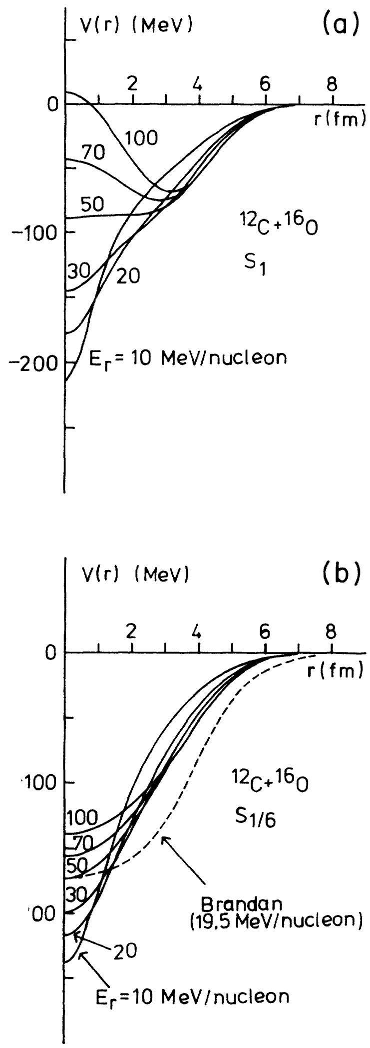

FIG. 4. The ^{12}C-^{16}O potentials by the S_1 force (a) and by the $S_{1/6}$ force (b), at incident energies $E_r=10$, 20, 30, 40, 50, 70, and 100 MeV/nucleon. For the sake of comparison, in (b) we also display the real part of the ^{12}C-^{16}O optical potential by Brandan at $E_r=19.5$ MeV/nucleon.

and by $S_{1/6}$. The potential $V(r)$ by $S_{1/3}$ is of intermediate character between the potentials by S_1 and $S_{1/6}$ and is closer to that by $S_{1/6}$. Just as the energy-dependent behavior of the ^{12}C-^{16}O potential by S_1 is similar to that of the ^{16}O-^{16}O potential by S_1, the energy-dependent behavior of the potentials of ^{16}O-^{16}O, ^{12}C-^{16}O, and ^{12}C-^{12}C systems is similar to one another when the same Skyrme-type force is used. For the sake of comparison, the real part of the ^{12}C-^{16}O Brandan potential[15] at $E_r=19.5$ MeV/nucleon is displayed in Fig. 4(b).

Since the optical potentials for the α particle are now safely regarded to be unique, we calculated the α-^{16}O potentials by S_1, $S_{1/3}$, and $S_{1/6}$ by the CMWP method and compared them with the real part of the α-^{16}O optical potential by Michel *et al.* The characteristic features of the energy dependence of the calculated α-^{16}O potentials are quite similar to those of ^{16}O-^{16}O, ^{12}C-^{16}O, and ^{12}C-^{12}C potentials when the same Skyrme-type forces are used. In Fig. 3, we display j_v of the α-^{16}O potential by $S_{1/6}$ whose absolute value is almost the same as $|j_v|$ by $S_{1/3}$ and is larger than that by S_1. We clearly see that the α-^{16}O potentials by the Skyrme-type forces are much less attractive than the "unique" optical potential by Michel *et al.* In Fig. 5, we compare the α-^{16}O potentials by S_1, $S_{1/3}$, and $S_{1/6}$ with the optical potential by Michel *et al.* at $E_r=36.5$ MeV/nucleon.

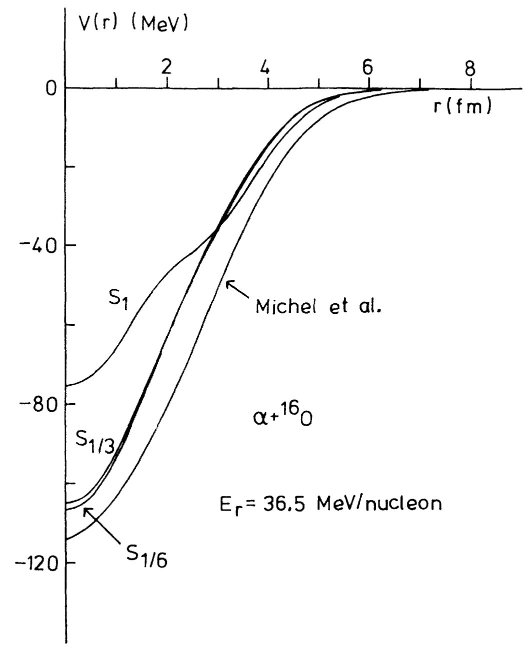

FIG. 5. Comparison of the calculated α-^{16}O potentials by the S_1, $S_{1/3}$, and $S_{1/6}$ forces with the real part of the optical potential by Michel *et al.* at $E_r=36.5$ MeV/nucleon.

IV. FINITE-RANGE EFFECTIVE FORCE GIVING THE SAME NUCLEAR MATTER PROPERTIES AS THE SKYRME-TYPE FORCE

As mentioned in Sec. I, the four parameters ρ_0, α, β, and σ of the Skyrme-type force are uniquely determined by requiring to reproduce the values of the following three quantities of nuclear matter: the value of the saturation density ρ_0 for which the binding energy per nucleon E is minimum, $(\partial E/\partial\rho)_{\rho=\rho_0=0}=0$, the value of E at $\rho=\rho_0$, and the value of the incompressibility, $K=\rho^2(\partial^2 E/\partial\rho^2)_{\rho=\rho_0}$. Needless to say, there are infinitely many effective nuclear forces which can reproduce these three quantities, ρ_0, $(E)_{\rho=\rho_0}$, and K. Below we construct, by making the least modification to a given Skyrme-type force, the finite-range force which reproduces the same values as the given Skyrme-type force to the three quantities of nuclear matter, ρ_0, $(E)_{\rho=\rho_0}$, and K.

We construct the effective force of the following form:

$$\hat{V}=\frac{1}{2}\sum_{i,j}\hat{v}_{i,j}\ ,$$

$$\hat{v}_{12}=V_0(1-m+mP_x)e^{-\mu(x_1-x_2)^2}$$
$$+b'\left[\rho\left[\frac{x_1+x_2}{2}\right]\right]^{\sigma'}\delta(x_1-x_2)\ . \tag{4.1}$$

Once a Skyrme-type force with four parameters, ρ_0, α, β, and σ, is given, we determine the five parameters in Eq. (4.1), V_0, m, μ, b', and σ', by the following conditions. First, we choose $\sigma'=\sigma$. Under this condition ($\sigma'=\sigma$), we require that the force of Eq. (4.1) and the given Skyrme-type force yield the same values to the three quantities of nuclear matter, ρ_0, $(E)_{\rho=\rho_0}$, and K. As is shown in the Appendix, these requirements determine the range parameter μ uniquely as a solution of the equation

$$H(\lambda_0)=\frac{3}{2}(1+\sigma)[G(\lambda_0)-F(\lambda_0)]\ ,$$
$$\lambda_0=\left[\frac{3\pi^2}{2}\rho_0\right]^{1/3}\Big/\sqrt{\mu}\ , \tag{4.2}$$

where

$$H(\lambda)=6-2\lambda^2-(6+4\lambda^2+\lambda^4)e^{-\lambda^2}\ ,$$
$$G(\lambda)=-2+\lambda^2+(2+\lambda^2)e^{-\lambda^2}\ ,$$
$$F(\lambda)=\sqrt{\pi}\lambda^3\mathrm{erf}(\lambda)+(2-3\lambda^2)+(-2+\lambda^2)e^{-\lambda^2}\ ,$$
$$\mathrm{erf}(x)=\frac{2}{\sqrt{\pi}}\int_0^x dt\ e^{-t^2}\ . \tag{4.3}$$

Among the remaining three parameters, V_0, m, and b', one parameter can be given an arbitrary value. We adopt the Majorana exchange mixture m as a free parameter. Once the value of m is fixed, the values of V_0 and b' are determined by the following relations:

$$V_0=\left[\frac{\mu}{\pi}\right]^{3/2}\frac{\alpha}{\rho_0}\left\{x_d+x_e\frac{6}{\lambda_0^6}\left[\left[1+\frac{1}{\sigma}\right]F(\lambda_0)-\frac{1}{\sigma}G(\lambda_0)\right]\right\}^{-1}\ ,$$
$$b'=\frac{8}{3(2+\sigma)}\frac{\beta}{\rho_0^{\sigma+1}}+\frac{x_e}{\sigma\rho_0^{\sigma}}V_0\left[\frac{\pi}{\mu}\right]^{3/2}\frac{8}{\lambda_0^6}[F(\lambda_0)-G(\lambda_0)]\ , \tag{4.4}$$

where

$$x_d=1-\frac{5}{4}m,\quad x_e=\frac{5}{4}m-\frac{1}{4}\ . \tag{4.5}$$

The case of $m=0.2$ is an exception to the preceding arguments. In this case we have no exchange potential both in the nucleon mean field and in the internucleus potential. For $m=0.2$, we can choose the range parameter μ freely, and the values of V_0 and b' are determined by Eq. (4.4) by putting $x_d=\frac{3}{4}$ and $x_e=0$.

For $\sigma'=\sigma=1$, there exists no solution of Eq. (4.2). Therefore we are forced to choose $m=0.2$ for $\sigma'=\sigma=1$. In Table III we show some sets of the parameters, μ, m, V_0, and b' for each of the three Skyrme-type forces S_1, $S_{1/3}$, and $S_{1/6}$. In this table we also give the value of the effective mass m^* of the nuclear matter at the Fermi momentum, and the value of the oscillator parameter ν of ^{16}O which minimizes the binding energy and that of the minimized binding energy E of ^{16}O for each finite-range force.

In Fig. 6 we display j_v of the ^{16}O-^{16}O potentials calculated by the finite-range forces of Eq. (4.1) with $\sigma'=\sigma=\frac{1}{3}$

TABLE III. Force parameters of the finite-range forces of Eq. (4.1) which reproduce the same nuclear matter properties as the Skyrme-type forces listed in the leftmost column. Here are also shown the values of the oscillator parameter ν of ^{16}O which minimize the binding energy and those of the minimized binding energy of ^{16}O. The unit for μ and $\nu_{^{16}\mathrm{O}}$ is fm^{-2} and that for V_0, $\rho_0^{\sigma'+1}b'$, and $E_{^{16}\mathrm{O}}$ is MeV.

	σ'	μ	m	V_0	$3/8(2+\sigma')\rho_0^{\sigma'+1}b'$	$(m^*/m)_{k_F}$	$\nu_{^{16}\mathrm{O}}$	$E_{^{16}\mathrm{O}}$
S_1	1	7.24	0.2	−351.65	70.5	1.0	0.190	−163.2
$S_{1/3}$	$\frac{1}{3}$	1.396	0.2	−508.4	164.0	1.0	0.135	−93.5
	$\frac{1}{3}$	1.396	0.35	−488.9	135.6	0.80	0.152	−114.7
	$\frac{1}{3}$	1.396	0.5	−470.9	109.2	0.68	0.172	−141.1
$S_{1/6}$	$\frac{1}{6}$	0.839	0.35	−345.1	211.1	0.70	0.109	−77.2
	$\frac{1}{6}$	0.839	0.45	−314.4	163.5	0.61	0.128	−99.7
	$\frac{1}{6}$	0.839	0.5	−301.0	142.7	0.58	0.139	−113.1

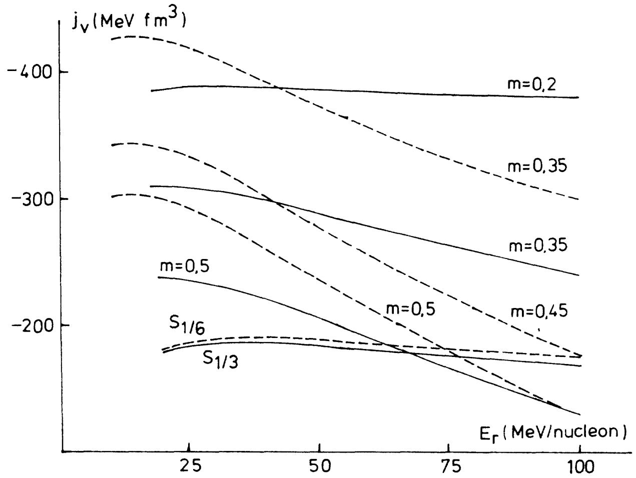

FIG. 6. The j_v values of the ^{16}O-^{16}O potentials calculated by the finite-range effective forces of Eq. (4.1). The solid curves show the j_v values by the forces with $\sigma'=\sigma=\frac{1}{3}$ and with $m=0.2$, 0.35, and 0.5, while the dotted curves by the forces with $\sigma'=\sigma=\frac{1}{6}$ and with $m=0.35$, 0.45, and 0.5. The force parameters of these finite-range forces are given in Table III. For the sake of comparison, we also display j_v values by $S_{1/3}$ and $S_{1/6}$ forces.

for $m=0.2$, 0.35, and 0.5, and with $\sigma'=\sigma=\frac{1}{16}$ for $m=0.35$, 0.45, and 0.5, whose force parameters are given in Table III. For the sake of comparison, j_v by $S_{1/3}$ and $S_{1/6}$ are also shown. From this figure we see that even if two effective nuclear forces reproduce the same nuclear matter properties, they can give very different internucleus potentials. What are the main factors that determine the j_v value, the strength of the attraction of the internucleus potential? We consider that an important factor is the radii or the densities of the colliding nuclei. By comparing Fig. 6 with the ν values of ^{16}O given in Tables II and III, we see that, within the group of the nuclear forces with the same σ value, the larger the ν value of ^{16}O is, the smaller the $|j_v|$ value of the ^{16}O-^{16}O potential is. That ν of ^{16}O is larger means that the radius of ^{16}O is smaller and the density of ^{16}O is higher. The smaller ^{16}O radius makes the range of the ^{16}O-^{16}O potential smaller, and higher ^{16}O density makes stronger the repulsive effect due to the density-dependent part of the nuclear force. Both of these result in the weakening of the strength of the attraction of the ^{16}O-^{16}O potential.

It is worth noticing that the radii of the scattering nuclei affect the value of j_v only through two factors, namely the antisymmetrization and the density dependence of nuclear force. This is because without these factors the internucleus potential $V(r)$ reduces to the simple double-folding potential of the density-independent two-nucleon force $v_{NN}(x_1-x_2)$,

$$V(r)=\int dx_1 dx_2 \rho_A(x_1)\rho_B(x_2)v_{NN}(x_1-x_2+r) , \quad (4.6)$$

which gives us j_v independent of any properties of scattering nuclei,

$$j_v=\frac{1}{N_A N_B}\int dr\, V(r) = \int dr\, v_{NN}(r) . \quad (4.7)$$

For $m\neq 0.2$ we get $x_e\neq 0$, which means the existence of the exchange potential. As is well known, the exchange potential gives rise to a prominent energy dependence of the internucleus potential, as long as the nuclear force is neither of the zero range nor close to the zero range, in such a way that the potential becomes less attractive at higher scattering energy. This feature is clearly seen in Fig. 6.

It may seem that the finite-range force with $\sigma=\frac{1}{6}$ and $m=0.45$ is recommended to be used since in Fig. 6 we see that j_v by this force is similar to j_v of Brandan's ^{12}C-^{16}O and ^{12}C-^{12}C potentials. However, as seen in Table III this force gives a rather smaller ν value and binding energy of ^{16}O than experiments since the observed value of ν is about 0.16 fm^{-2} and that of E is -128 MeV including Coulomb energy. It is desirable to construct an effective nuclear force which reproduces not only the nuclear matter properties but also the radii (or densities) and the binding energies of the colliding nuclei, and the attraction of the elastic channel potential between the two nuclei.

V. SUMMARY AND DISCUSSION

We calculated, by using the Skyrme-type forces of Eq. (1.1) [or Eq. (1.2)] with force parameters given in Table I, the internucleus potentials of the ^{16}O-^{16}O, ^{12}C-^{16}O, ^{12}C-^{12}C, and α-^{16}O systems. For calculating the potentials we used the canonical moving wave packet method (CMWP).[3,4] In the case of the ^{16}O-^{16}O system we compared the potentials calculated by the CMWP method with those calculated by the semiclassical treatment of the resonating group method [abbreviated as RGM + WKB (Ref. 8)], and found that both kinds of potentials are quite similar to each other, which is especially true when their volume integral values are compared. Attention was given to the fact that the RGM + WKB method succeeded to reproduce very well the real parts of the uniquely determined optical potentials of the α-^{16}O and α-^{40}Ca systems in a wide energy range.[10,11]

We compared the volume integrals j_v of the calculated potentials with those of the real parts of the optical potentials which fit the data very well. We found two characteristic features of the calculated potentials in comparison with experiments. First, the energy dependence of the volume integrals j_v of the calculated potentials is too weak, and second, $|j_v|$ of the calculated potentials are too small in the incident energy region lower than about 40 MeV/nucleon.

We consider that the weakness of the attraction of the Skyrme-type forces in the region of $E_r<40$ MeV/nucleon will give rise to the weaker acceleration of the relative motion of two colliding nuclei in their initial stage of the collision. This will affect, for instance, the nucleon momentum distribution in the early stage of the nuclear collision process in such a way that the high-momentum component is reduced. Actually, in Ref. 23, the study of the ^{16}O-^{16}O collision was made by solving the Vlasov equation[2] and the increase of the high-momentum component of the nucleon momentum distribution was found to be due to the acceleration of the internucleus relative motion.

We constructed the finite-range nuclear forces by making the least modification to any given Skyrme-type force such that they reproduce the same nuclear matter properties as the given Skyrme-type force. We found that the internucleus potentials calculated by thusly constructed finite-range nuclear forces can have j_v values very different from the j_v value of the potential calculated by the given Skyrme-type force. An important origin of the large difference of the j_v values was attributed to the difference of the values of the radii (or densities) of the colliding nuclei which were so determined as to give the minimum binding energies of colliding nuclei. The sensitivity of the j_v value to the radii (or densities) of the scattering nuclei was already noticed in Ref. 24.

Like the nucleon-nucleus optical potential, the nucleus-nucleus optical potential becomes less attractive as the incident energy gets higher. This is clearly shown in Fig. 3 in the energy dependence of the j_v values of the optical potentials which reproduce the experimental data very well. A main origin of this energy dependence is the energy dependence or momentum dependence of the exchange potential, which is attractive and decreases its strength as the energy gets higher because the antisymmetrization tends to loose its effect as the energy gets higher.[8,25] This behavior of the exchange potential is common to the nucleon-nucleus and nucleus-nucleus potential. Compared to the optical potentials fitting the data very well in a wide energy range, the internucleus potentials calculated by the Skyrme-type forces have volume integral values which are almost independent of incident energy. This is the second defect of the Skyrme-type forces. The finite-range forces we constructed have j_v values whose energy-dependence is full of variety, depending on the magnitudes of the force range and Majorana exchange mixture.

The "experimental" optical potentials contain the effects of inelastic channels (so-called polarization potential) which are important at low energies. Therefore the discrepancy between the theoretical and experimental volume integrals could partly be due to the neglect of the inelastic channels on the part of the theoretical potential. It is desirable to take into account the effects of inelastic channels to draw a definite conclusion about the adequacy of the effective nuclear force. We believe, however, that these effects are not sufficient to explain the large discrepancy ($\gtrsim 100$ MeV fm^3 at $E/A \sim 10$ MeV/nucleon) between the theoretical (j_v^{Skyrme}) and experimental (j_v^{exp}) volume integral.

That the nuclear force is of finite range is equivalent to that it is momentum dependent. The momentum dependence of the nuclear force has attracted much attention recently in intermediate energy heavy-ion collision studies since the momentum-dependence of the soft nuclear force can behave at high energy like the stiff nuclear force which is favorable for the reproduction of the observed sideward momentum flow phenomena.[26,27]

From the above-mentioned results of our studies, we find it desirable to construct the finite-range effective nuclear force not restricted to the one-range force as in the present preliminary study, such that it reproduces not only the nuclear matter properties but also the basic important properties of the internucleus potential, especially the proper strength of attraction and proper energy dependence. We learned from our present studies that in order to reproduce the proper strength of the attraction of the internucleus potential it is important for the effective nuclear force to reproduce the observed radii and binding energies of two colliding nuclei. The requirement of the reproduction of the energy dependence of the internucleus potential is twofold. First, the effective force should reproduce the energy dependence of the nucleon-nucleus optical potential, and second, it should reproduce the energy dependence of the optical potential between the two nuclei under consideration. The main parameters of the effective force which are responsible for the energy dependence of the internucleus potential are the magnitudes of the force range and the exchange mixture. We found in this paper that although the j_v values of the inter-nucleus potentials constructed by the Skyrme-type forces are similar to one another both in magnitude and in energy-dependence, those by the finite-range forces show the large difference in their energy dependence depending on with which Skyrme-type force the finite-range force shares the same nuclear matter properties.

APPENDIX

When we use the effective nuclear force of Eq. (4.1), the binding energy E per nucleon of the nuclear matter at the density ρ is given by

$$E=\frac{3}{5}\frac{\hbar^2}{2m}\left[\frac{3\pi^2}{2}\rho\right]^{2/3}+\frac{V_0}{2}\left[\frac{\pi}{\mu}\right]^{3/2}\rho\left[x_d+x_e\frac{6}{\lambda^6}F(\lambda)\right]+\frac{3}{8}b'\rho^{1+\sigma},\quad \lambda=\left[\frac{3\pi^2}{2}\rho\right]^{1/3}\Big/\sqrt{\mu}\,, \tag{A1}$$

where x_d and x_e are defined in Eq. (4.5) and $F(\lambda)$ is defined in Eq. (4.3). If we use the Skyrme-type force of Eq. (1.2), E is given by

$$E=\frac{3}{5}\frac{\hbar^2}{2m}\left[\frac{3\pi^2}{2}\rho\right]^{2/3}+\frac{\alpha}{2}\left[\frac{\rho}{\rho_0}\right]+\frac{\beta}{2+\sigma}\left[\frac{\rho}{\rho_0}\right]^{\sigma+1}. \tag{A2}$$

The condition that E takes its minimum value at $\rho=\rho_0$ is expressed as

$$0=\frac{2}{5}\frac{\hbar^2}{2m}\left[\frac{3\pi^2}{2}\rho_0\right]^{2/3}+\frac{V_0}{2}\left[\frac{\pi}{\mu}\right]^{3/2}\rho_0\left[x_d+x_e\frac{6}{\lambda_0^6}G(\lambda_0)\right]+\frac{3}{8}(1+\sigma)b'\rho_0^{1+\sigma},\quad \lambda_0=(\lambda)_{\rho=\rho_0}\,, \tag{A3}$$

while for the Skyrme-type force it is expressed as

$$0=\frac{2}{5}\frac{\hbar^2}{2m}\left[\frac{3\pi^2}{2}\rho_0\right]^{2/3}+\frac{\alpha}{2}+\frac{1+\sigma}{2+\sigma}\beta\,. \qquad \text{(A4)}$$

The incompressibility K is given by

$$K=-\frac{6}{5}\frac{\hbar^2}{2m}\left[\frac{3\pi^2}{2}\rho_0\right]^{2/3}+\frac{V_0}{2}\left[\frac{\pi}{\mu}\right]^{3/2}\rho_0 x_e\frac{36}{\lambda_0^6}H(\lambda_0)$$
$$+\frac{27}{8}\sigma(1+\sigma)b'\rho_0^{1+\sigma}\,, \qquad \text{(A5)}$$

while for the Skyrme-type force,

$$K=-\frac{6}{5}\frac{\hbar^2}{2m}\left[\frac{3\pi^2}{2}\rho_0\right]^{2/3}+9\frac{\sigma(1+\sigma)}{2+\sigma}\beta\,. \qquad \text{(A6)}$$

$G(\lambda)$ in Eq. (A3) and $H(\lambda)$ in Eq. (A5) are defined in Eq. (4.3).

From Eq. (A1)~Eq. (A4), we get Eq. (4.4). From Eqs. (A5) and (A6) we get

$$V_0\left[\frac{\pi}{\mu}\right]^{3/2}\rho_0 x_e\frac{2}{\lambda_0^6}H(\lambda_0)$$
$$=\sigma(1+\sigma)\left[\frac{\beta}{2+\sigma}-\frac{3}{8}\rho_0^{1+\sigma}b'\right]\,, \qquad \text{(A7)}$$

which together with Eq. (4.4) gives Eq. (4.2).

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