Off-shell behavior of relativistic *NN* effective interactions and charge symmetry breaking

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We examine in detail the suggestion of Iqbal *et al.* for calculating the class-four charge symmetry breaking amplitude in *n-p* scattering. By simplifying to a model problem, we show explicitly that the approximation scheme is unreliable if a phenomenological, effective nucleon-nucleon T matrix is used. Our results have wider implications for observables calculated in relativistic impulse approximation calculations. They reinforce the observation made in the literature that the procedure of fitting only positive energy matrix elements can lead to an *NN* interaction whose off-shell behavior is incorrect.

I. INTRODUCTION

Our understanding of the interaction between two nucleons is still far from complete. Explicit quark models of the short-distance interaction now compete with the more conventional heavy boson exchange models. It is in this context that the study of the symmetries of the *NN* interaction is potentially extremely important. In particular, one might hope that a study of, for example, the breaking of charge independence and charge symmetry may give us new insight in building a realistic picture of the nucleon-nucleon interaction.

With the development of appropriate technology for polarized beams and targets there has recently been considerable activity in studying the class-IV charge symmetry breaking (CSB) component of the *NN* force. That is the component which mixes spin singlet and triplet states in the *n-p* system. Experimentally this effect is most easily seen as a difference between the neutron and the proton analyzing powers in *n-p* elastic scattering $A_n(\theta)$ and $A_p(\theta)$, respectively. The TRIUMF group of Abegg *et al.* recently reported a nonzero result for $A_n(\theta)-A_p(\theta)$ at 478 MeV, and further work is underway at both TRIUMF and Indiana University Cyclotron Facility (IUCF).

Within the conventional description of the *NN* interaction the class-IV interaction arises from photon exchange, heavy meson mixing (predominantly ρ-ω mixing), and the neutron-proton mass difference in the meson exchange diagrams. The latter is our particular concern here because of a suggestion by Iqbal, Thaler, and Woloshyn (ITW).[1] Rather than starting with a potential model, they proposed to calculate the class-IV scattering amplitude arising from the neutron-proton mass difference directly from the relativistic, non-CSB, *NN* T matrix. [The mathematical details are outlined in Sec. III below—see Eqs. (3.1) and (3.2).]

The formal equivalence between the ITW procedure and the more common DWBA treatments was established by Gersten and Thomas.[2] However, that proof assumed that the exact, relativistic *NN* T matrix was used. In practice it is far easier to use a phenomenological T matrix parametrized by five Fermi amplitudes.[3] This is, of course, the same sort of effective interaction now often used in relativistic impulse approximation (RIA) calculations—e.g., in evaluating inelastic and quasielastic reactions on nuclei.

In this work we investigate the ITW approach in more detail, by taking what might appear to be a purely academic problem. That is, we investigate the reliability of using a phenomenological representation of the *NN* T matrix to evaluate just the first term in the ITW expansion [i.e., the first term in Eq. (3.1)]. Initially we had intended that this procedure might allow us to test *NN* potential models against pseudodata. [These are numbers obtained from the measured phase shifts via Eq. (4.1) (see Sec. IV).] This hope was soon shattered. In fact, we discovered that the partial wave amplitudes generated by this procedure did not respect off-shell unitarity. (In this sense our work closely parallels for the relativistic case the work of MacFarlane and Redish for the nonrelativistic *NN* effective T matrix.)

Further work revealed the detailed reason for the failure of the scheme. This is explained in detail in what follows, but let us summarize briefly. The CSB amplitude generated by the ITS procedure in our model problem involves matrix elements for virtual transitions which are unconstrained by the usual fitting procedures. They are thus totally unreliable. (These matrix elements involve transitions between positive and negative energy spinors, whereas the fitting procedure naturally includes only positive energy spinors.) Our results must also raise questions about the use of such effective *NN* T matrices in the Dirac DWIA calculations.

II. THE THEORETICAL BASIS

We begin with the assumption that nucleon-nucleon (*NN*) scattering can be described by the Bethe-Salpeter

(BS) equation (in terms of 16×16 matrices in spin space). Its operator form[4] is

$$M^0=I^0+I^0G^0M^0 \tag{2.1}$$

or

$$M=I+IGM \tag{2.2}$$

Equation (2.1) applies to the case of two equal mass ($\bar{m}$) particles while (2.2) applies to nucleons with their real masses $\bar{m}\pm\delta m$. In the ladder approximation we may have

$$I=I^0 . \tag{2.3}$$

Following Ref. 5 we introduce the following notation for the spinors with helicity λ and c.m. momentum $\mathbf{p}$:

$$u_{\lambda\epsilon}(\mathbf{p})=\begin{cases}u_\lambda(\mathbf{p}) & \text{for } \epsilon=1 \text{ (positive-energy spinor)},\\ v_\lambda(\mathbf{p}) & \text{for } \epsilon=-1 \text{ (negative-energy spinor)}.\end{cases} \tag{2.4}$$

The negative-energy spinors are related to the positive energy ones via

$$v_\lambda(\mathbf{p})=\gamma_5 u_\lambda(\mathbf{p}),\quad \bar{v}_\lambda(\mathbf{p})=\bar{u}_\lambda(\mathbf{p})\gamma_5 , \tag{2.5}$$

and Eqs. (2.4) and (2.5) can be replaced by (1 is a unit matrix)

$$\begin{aligned}u_{\lambda\epsilon}(\mathbf{p})&=[\tfrac{1}{2}(1+\epsilon)\mathbf{1}-\tfrac{1}{2}(1-\epsilon)\gamma_5]u_\lambda(\mathbf{p}) ,\\ \bar{u}_{\lambda\epsilon}(\mathbf{p})&=\bar{u}_\lambda(\mathbf{p})[\tfrac{1}{2}(1+\epsilon)\mathbf{1}+\tfrac{1}{2}(1-\epsilon)\gamma_5] .\end{aligned} \tag{2.6}$$

Equation (2.2) [or Eq. (2.1)] can be brought to a form similar to a Lippmann-Schwinger equation using the following procedure (see, e.g., Ref. 4):

$$M=W+WG^PM , \tag{2.7a}$$

$$W=I+I(G-G^P)W , \tag{2.7b}$$

with the property

$$\sum_{\lambda_1\lambda_2\lambda_3\lambda_4} u_{\lambda_3\epsilon_3}(\mathbf{p})u_{\lambda_4\epsilon_4}(-\mathbf{p})G^P\bar{u}_{\lambda_1\epsilon_1}(\mathbf{p})\bar{u}_{\lambda_2\epsilon_2}(-\mathbf{p}) =g(s,p)\mathbf{1}\delta_{\epsilon_1 1}\delta_{\epsilon_2 1}\delta_{\epsilon_3 1}\delta_{\epsilon_4 1} . \tag{2.8}$$

That is, G^P has projections only into positive-energy spinors, and 1 is the unit 16×16 matrix and s the total energy squared. In this case Eq. (2.7a) can be replaced by the coupled equations

$$\begin{aligned}\bar{u}_{\lambda_3\epsilon_3}(\mathbf{p}_f)\bar{u}_{\lambda_4\epsilon_4}(-\mathbf{p}_f)Mu_{\lambda_1\epsilon_1}(\mathbf{p}_i)u_{\lambda_2\epsilon_2}(-\mathbf{p}_i)&=\bar{u}_{\lambda_3\epsilon_3}(\mathbf{p}_f)\bar{u}_{\lambda_4\epsilon_4}(-\mathbf{p}_f)Wu_{\lambda_1\epsilon_1}(\mathbf{p}_i)u_{\lambda_2\epsilon_2}(-\mathbf{p}_i)\\ &+\sum_{\mu_1\mu_2}\int d^4k\,\bar{u}_{\lambda_3\epsilon_3}(\mathbf{p}_f)\bar{u}_{\lambda_4\epsilon_4}(-\mathbf{p}_f)Wu_{\mu_1 1}(\mathbf{k})u_{\mu_2 1}(-\mathbf{k})g(s,k)\bar{u}_{\mu_1 1}(\mathbf{k})\\ &\times\bar{u}_{\mu_2 1}(-\mathbf{k})Mu_{\lambda_1\epsilon_1}(\mathbf{p}_i)u_{\lambda_2\epsilon_2}(-\mathbf{p}_i) .\end{aligned} \tag{2.9}$$

Equation (2.9) has the merit that for positive-energy spinors ($\epsilon_1=\epsilon_2=\epsilon_3=\epsilon_4=1$) it is completely decoupled and with a proper choice of $g(s,k)$ (satisfying the two-body unitarity condition) can be three-dimensional (i.e., have the appearance of a Lippmann-Schwinger equation). In terms of the 16×16 matrices M and W let us define the matrix elements of the transition and potential matrices, respectively:

$$\langle\lambda_3\lambda_4\epsilon_3\epsilon_4\mathbf{p}_f|T|\lambda_1\lambda_2\epsilon_1\epsilon_2\mathbf{p}_i\rangle =\bar{u}_{\lambda_3\epsilon_3}(\mathbf{p}_f)\bar{u}_{\lambda_4\epsilon_4}(-\mathbf{p}_f)Mu_{\lambda_1\epsilon_1}(\mathbf{p}_i)\bar{u}_{\lambda_2\epsilon_2}(-\mathbf{p}_i) , \tag{2.10}$$

$$\langle\lambda_3\lambda_4\epsilon_3\epsilon_4\mathbf{p}_f|V|\lambda_1\lambda_2\epsilon_1\epsilon_2\mathbf{p}_i\rangle =\bar{u}_{\lambda_3\epsilon_3}(\mathbf{p}_f)\bar{u}_{\lambda_4\epsilon_4}(-\mathbf{p}_f)Wu_{\lambda_1\epsilon_1}(\mathbf{p}_i)u_{\lambda_2\epsilon_2}(-\mathbf{p}_i) . \tag{2.11}$$

Then Eq. (2.9) can be cast into an equivalent (for positive and negative energies) operator form of the Lippmann-Schwinger equation:

$$T=V+VG^PT . \tag{2.12}$$

In a similar way one can recast Eq. (2.1) into the form

$$T^0=V^0+V^0G^{0P}T^0 . \tag{2.13}$$

One has to remember that the physical amplitudes are defined for Eq. (2.13) if averaged mass positive-energy spinors are taken and for Eq. (2.12) if neutron and proton positive-energy spinors are taken. If we take matrix elements of either (2.12) or (2.13) between spinors with either masses or energies different from those appearing in the Green's function, the matrix elements are not on the energy shell and are related to virtual transitions. This is the case, for example, when negative-energy spinors are applied to Eq. (2.12), then Eq. (2.9) defines them in a unique way.

Let us consider here the small displacement from the average mass spinors to the neutron and proton spinors. In Appendix A we show that up to the first order in the mass difference

$$u^n_\lambda(\mathbf{p})=u^0_\lambda(\mathbf{p})-\eta\lambda v^0_\lambda(\mathbf{p}) , \tag{2.14a}$$

$$u^p_\lambda(\mathbf{p})=u^0_\lambda(\mathbf{p})+\eta\lambda v^0_\lambda(\mathbf{p}) , \tag{2.14b}$$

where u^n_λ, u^p_λ, and u^0_λ are neutron, proton, and average mass spinors, respectively, and

$$\eta=(p/E)(\delta m/m),\quad E=(p^2+m^2)^{1/2} . \tag{2.15}$$

Here we see that a small mass shift necessarily involves virtual transitions.

III. THE ITW APPROXIMATION

The original idea of Iqbal, Thaler, and Woloshyn[1] was that the approximate expression for Eqs. (2.12) and (2.13)

$$T \simeq T^0 + T^0(G^p - G^{0p})T^0 , \tag{3.1a}$$

or equivalently

$$M \simeq M^0 + M^0(G^p - G^{0p})M^0 , \tag{3.1b}$$

when evaluated between *np* states (initial and final), would yield the whole class-IV CSB contribution due to the nucleon mass difference (to order $\delta m/m$). Equation (3.1) can be obtained under the following assumptions:

(i) In Eq. (2.2) one-meson exchanges only are taken, and $I = I^0$—i.e., Eq. (2.3) is satisfied; and (3.2)

(ii) $W = I^0$ in Eq. (2.7a) and Eq. (2.9)—i.e., the corrections of Eq. (2.7b) are neglected.

In a previous paper of Gersten and Thomas[2] (denoted here as GT) the formal equivalence of Eq. (3.1) and the DWBA approximation for Eq. (2.12) were proved for the two potentials V^0 and $(V - V^0)$.

ITW also assumed that M^0 can be obtained by a five-operator fit to the five independent *NN* amplitudes:[3]

$$M^0 = F_s \mathbf{1}^{(1)} \otimes \mathbf{1}^{(2)} + F_v \gamma_\mu^{(1)} \otimes \gamma_\mu^{(2)} + F_T \sigma_{\mu\nu}^{(1)} \otimes \sigma_{\mu\nu}^{(2)} + F_A i\gamma_5^{(1)}\gamma_\mu^{(1)} \otimes i\gamma_5^{(2)}\gamma_\mu^{(2)} + F_p \gamma_5^{(1)} \otimes \gamma_5^{(2)} . \tag{3.3}$$

Here F_s, F_v, T_T, F_A, and F_p are the five independent Fermi amplitudes for *NN* scattering at a given energy. Equation (3.3) was inserted in Eq. (3.1b) and direct off-shell extrapolation was used. M^0 was chosen in such a way that it agreed with the physical values of Eq. (2.10) for on-shell momenta with $\epsilon_1 = \epsilon_2 = \epsilon_3 = \epsilon_4 = 1$—for which the phase shift analysis is known.

We will show that the assumption (3.3) is incorrect and should be regarded, at best, as a further approximation. Our next efforts will be devoted to evaluations of this approximation. First we note that in Eq. (3.1) different spinors are physical for the left-hand side and for the right-hand side, and according to Eq. (2.14) virtual transitions contribute. Explicitly, if we apply the *np* spinors to both sides of Eq. (3.1), then on the right-hand side we find the matrix elements of M^0 which up to the first order of $\delta m/m$ become

$$\begin{aligned}
\bar{u}^n_{\lambda_3}(\mathbf{p}_f)\bar{u}^p_{\lambda_4}(-\mathbf{p}_f)M^0 u^n_{\lambda_1}(\mathbf{p}_i)u^p_{\lambda_2}(-\mathbf{p}_i) &= [\bar{u}^0_{\lambda_3}(\mathbf{p}_f) - \eta\lambda_3 \bar{v}^0_{\lambda_3}(\mathbf{p}_f)][\bar{u}^0_{\lambda_4}(-\mathbf{p}_f) + \eta\lambda_4 \bar{v}^0_{\lambda_4}(-\mathbf{p}_f)] \\
&\quad \times M^0[u^0_{\lambda_1}(\mathbf{p}_i) - \eta\lambda_1 v^0_{\lambda_1}(\mathbf{p}_i)][u^0_{\lambda_2}(-\mathbf{p}_i) + \eta\lambda_2 v^0_{\lambda_2}(-\mathbf{p}_i)] \\
&= \langle \lambda_3\lambda_4 11\mathbf{p}_f | T | \lambda_1\lambda_2 11\mathbf{p}_i \rangle - \eta[\lambda_3\langle \lambda_3\lambda_4 - 11\mathbf{p}_f | T | \lambda_1\lambda_2 11\mathbf{p}_i \rangle \\
&\quad - \lambda_4\langle \lambda_3\lambda_4 1 - 1\mathbf{p}_f | T | \lambda_1\lambda_2 11\mathbf{p}_i \rangle + \lambda_1\langle \lambda_3\lambda_4 11\mathbf{p}_f | T | \lambda_1\lambda_2 - 11\mathbf{p}_i \rangle \\
&\quad - \lambda_2\langle \lambda_3\lambda_4 11\mathbf{p}_f | T | \lambda_1\lambda_2 1 - 1\mathbf{p}_i \rangle] .
\end{aligned} \tag{3.4}$$

That is, M^0 has to be known not only when sandwiched with positive-energy spinors (physical amplitudes) but also when sandwiched with one negative-energy spinor (virtual amplitudes). Therefore the assumption that M^0 can be represented by Eq. (3.3) involves an *arbitrary* extrapolation to virtual amplitudes. A different choice of five operators in order to represent M^0 as in Eq. (3.3) will, in general, lead to different results. Indeed, as we show in Appendix B different results are obtained if instead of (3.3) we choose the so-called five perturbative amplitudes (with different extrapolations to virtual negative-energy states). Therefore we see that Eq. (3.3) is only an approximation and we shall examine its validity. It will turn out that it is actually a very poor approximation. We shall use the results of the GT paper extensively. We therefore present the relevant ones here.

We define the six helicity amplitudes (with positive-energy spinors)

$$\begin{aligned}
&T_1 = \langle ++|M|++\rangle; \quad T_2 = \langle ++|M|--\rangle ; \\
&T_3 = \langle +-|M|+-\rangle; \quad T_4 = \langle +-|M|-+\rangle ; \\
&T_5 = \langle ++|M|+-\rangle; \quad T_6 = \langle ++|M|-+\rangle .
\end{aligned} \tag{3.5}$$

Denoting average mass spinors by

$$|\alpha\beta\rangle_0 = u^0_\alpha(\mathbf{p})u^0_\beta(-\mathbf{p}) \tag{3.6}$$

and physical mass spinors by

$$|\alpha\beta\rangle = u^n_\alpha(\mathbf{p})u^p_\beta(-\mathbf{p}) , \tag{3.7}$$

we write for the amplitudes and their partial wave projections

$$\begin{aligned}
&T_\lambda = \langle \gamma\delta|M|\alpha\beta\rangle; \quad T_\lambda(J) = \langle \gamma\delta|M^J|\alpha\beta\rangle , \\
&{}'T_\lambda = {}_0\langle \gamma\delta|M|\alpha\beta\rangle; \quad {}'T_\lambda(J) = {}_0\langle \gamma\delta|M^J|\alpha\beta\rangle , \\
&T'_\lambda = \langle \gamma\delta|M|\alpha\beta\rangle_0; \quad T'_\lambda(J) = \langle \gamma\delta|M^J|\alpha\beta\rangle_0 , \\
&{}'T'_\lambda = {}_0\langle \gamma\delta|M|\alpha\beta\rangle_0 ; \quad {}'T'_\lambda(J) = {}_0\langle \gamma\delta|M^J|\alpha\beta\rangle_0 , \\
&{}'T^{0\prime}_\lambda \equiv \phi_\lambda ;
\end{aligned} \tag{3.8}$$

here λ labels the four helicities $(\alpha\beta\gamma\delta)$, and J is the total angular momentum.

If we define the symbol $\underline{c}$ to mean "equal in terms of the contributions to the class-IV CSB amplitude f to first order in $\delta m/m$," then

$$f = i(T_5 + T_6) \equiv iT_{56} \tag{3.9}$$

and

$$T_{56}(J) \underline{\subseteq} T^0_{56}(J) + \{T^0_{12}(J) g^0 [T^0_{56}(J) - {}'T^0_{56}(J)]\} + [T^0_{56}(J) - T^{0\prime}_{56}(J)] g^0 T^0_{34}(J) \ . \tag{3.10}$$

Here $T_{12}(J) = T_1(J) - T_2(J)$ and $T_{34}(J) = T_3(J) - T_4(J)$ are, respectively, the singlet T^J and the uncoupled triplet T^{JJ} partial wave amplitudes. From this we recognize $[1 + g^0 T^0_{12}(J)]$ and $[1 + g^0 T^0_{34}(J)]$ as the Møller operators which generate the appropriate distorted waves from plane waves.

In this work we do not wish to calculate the whole of $T_{56}(J)$, which is necessary if one wants the full class-IV CSB amplitude due to the *n-p* mass difference. Instead we consider the apparently academic problem of calculating the matrix elements ${}'T^0_{56}(J)$ and $T^{0\prime}_{56}(J)$. According to GT [Eq. (50)] these have the form (for the J partial waves)

$$\begin{aligned} T^{0\prime}_{56}(J) &\underline{\subseteq} V'_{56}(J)[1 + g^0 T_{34}(J)] \ , \\ {}'T^0_{56}(J) &\underline{\subseteq} [1 + T_{12}(J) g^0]' V_{56}(J) \ . \end{aligned} \tag{3.11}$$

Our motivation in considering these unphysical amplitudes is simple. Given some model for the *NN* force one can readily calculate $V'_{56}(J)$ or ${}'V_{56}(J)$. Then ${}'T^0_{56}(J)$ or $T^{0\prime}_{56}(J)$ is easily evaluated in a semi-distorted-wave calculation. On the other hand, given an expansion for T^0 of the form given in Eq. (3.3), one can directly find ${}'T^0_{56}$ (or $T^{0\prime}_{56}$) by sandwiching the Dirac operator between appropriate spinors—equal (average) mass on one side and physical masses on the other. In terms of the usual five Fermi amplitudes of Eq. (3.3) we find

$$i'T^0_{56} \underline{\subseteq} \frac{ip^2 \delta m}{2E\bar{m}^2} \sin\theta(-\tfrac{1}{2}F_s - F_T - \tfrac{1}{2}F_p) \ , \tag{3.12}$$

and

$$iT^{0\prime}_{56} \underline{\subseteq} \frac{ip^2 \delta m}{2E\bar{m}^2} \sin\theta(-\tfrac{1}{2}F_s + F_v + F_A + \tfrac{1}{2}F_p) \ . \tag{3.13}$$

What makes the results given in Eqs. (3.12) and (3.13) particularly interesting is that the particular combination of Fermi amplitudes appearing there are directly related to certain combinations of non-CSB helicity amplitudes, namely

$$\frac{p^2}{\bar{m}^2}(-\tfrac{1}{2}F_s + F_v + F_A + \tfrac{1}{2}F_p) = \frac{\phi_3}{1+\cos\theta} - \frac{\phi_4}{1-\cos\theta} \ , \tag{3.14}$$

and

$$\frac{p^2}{\bar{m}^2}(-\tfrac{1}{2}F_s - F_T - \tfrac{1}{2}F_p) = \frac{\phi_3}{1+\cos\theta} + \frac{\phi_4}{1-\cos\theta} + \frac{2E}{m}\frac{\phi_5}{\sin\theta} \ . \tag{3.15}$$

The ϕ_λ can be evaluated using standard NN phase shifts.[6] Thus it seems as if we could use Eqs. (3.12)–(3.16) to generate pseudodata with which to compare the results of a given model [obtained using Eq. (3.11)]. We discuss some of these comparisons in detail in the next section.

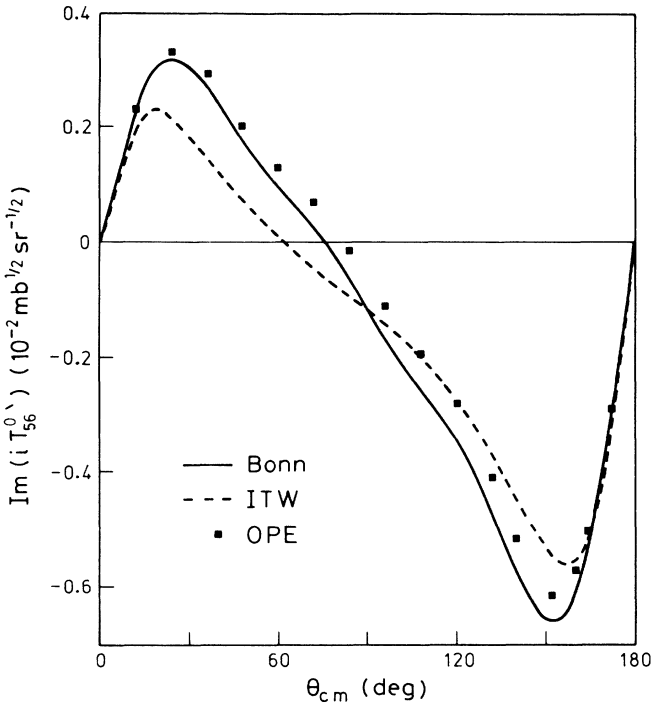

FIG. 1. The charge symmetry breaking amplitude Im($iT^{0\prime}_{56}$) as a function of the c.m. angle calculated (i) from the HM3A Bonn potential (solid line denoted "Bonn") in the DWBA, (ii) from the HM3A Bonn potential bar phase shifts in the ITW approximation (dashed line denoted "ITW"), (iii) from the HM3A Bonn potential where only pion exchange was used in the DWBA (solid squares, denoted "OPE").

IV. NUMERICAL RESULTS

In this section we examine the results of our formalism for the HM3A Bonn potential.[7] On the one hand we solve Eq. (3.11), and on the other hand the bar phase shifts of the same potential are used to generate Eqs. (3.12) and (3.13). This will allow us to make a direct comparison between the two results which should in principle be identical.

We do this in two ways: (i) We compare the $iT^{0\prime}_{56}$, $i'T^0_{56}$, and iT^0_{56} amplitudes at 200 MeV (see Figs. 1–6), and (ii)

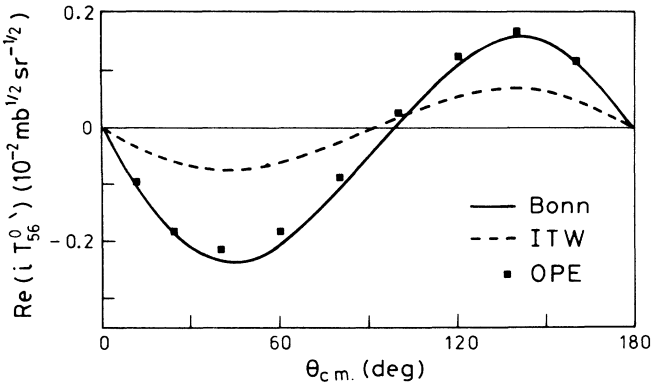

FIG. 2. As in Fig. 1 for Re($iT^{0\prime}_{56}$).

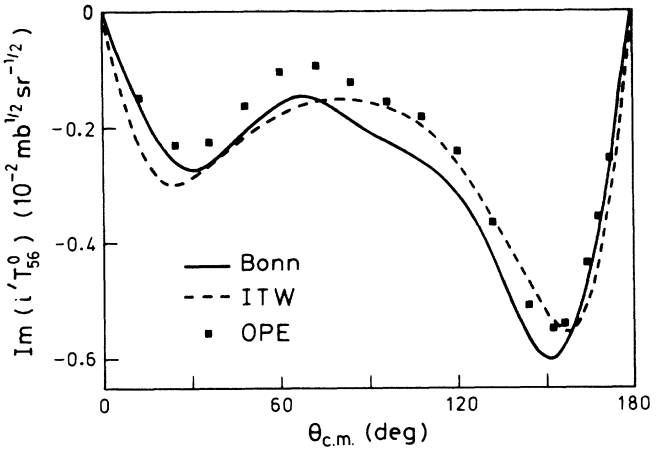

FIG. 3. As in Fig. 1 for Im($i'T^0_{56}$).

we also compare the partial wave expansions:

$$
\begin{aligned}
iT^{0\prime}_{56} &= \frac{i}{p}\sum (2J+1)T^{0\prime}_{56}(J)d^J_{10}(\theta) \\
&= \frac{i}{p}\sum (2J+1)\gamma'_J e^{i\bar{\delta}_{JJ}} d^J_{10}(\theta)\ , \\
i'T^0_{56} &= \frac{1}{p}\sum (2J+1)'T^0_{56}(J)d^J_{10}(\theta) \\
&= \frac{i}{p}\sum (2J+1)'\gamma_J e^{i\bar{\delta}_J} d^J_{10}(\theta)\ .
\end{aligned}
\tag{4.1}
$$

Here $T^{0\prime}_{56}(J)$ and $'T^0_{56}(J)$ are the solutions of Eqs. (3.11), $\bar{\delta}_{JJ}$ and $\bar{\delta}_J$ are the uncoupled triplet bar phase shifts and singlet bar phase shifts, respectively, and the $'\gamma_J,\gamma'_J$ parameters are real for real potentials.

Next we expand Eqs. (3.14) and (3.15) in a similar expansion to Eq. (4.1).[8] The appropriate formulas and procedures are given in Appendix C. In Figs 1–6 we show the real and imaginary parts of the amplitudes $iT^{0\prime}_{56}$, $i'T^0_{56}$, and iT^0_{56} which result from using the phase of the Bonn potential (in the figures it is denoted "Bonn") and the ITW approximation of Eqs. (3.14) and (3.15). In Tables I and II the phase of the partial wave expansions

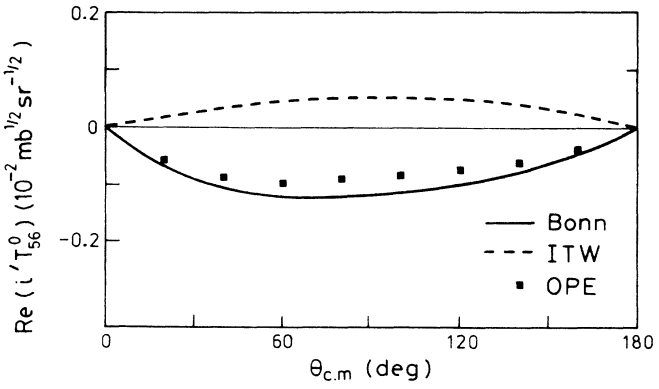

FIG. 4. As in Fig. 1 for Re($i'T^0_{56}$).

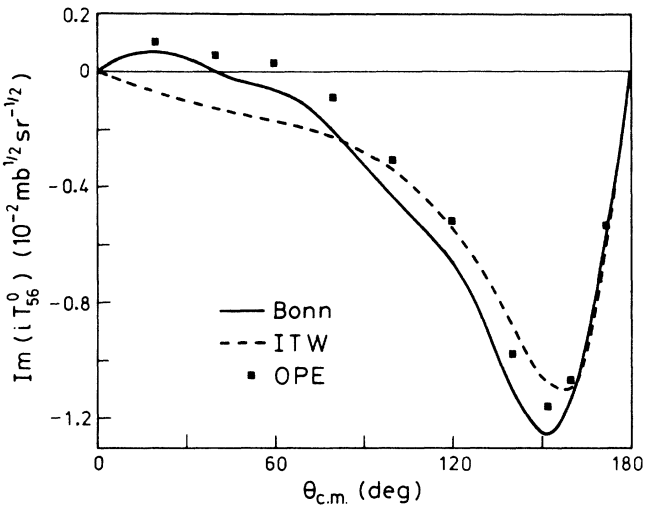

FIG. 5. As in Fig. 1 for Im(iT^0_{56}).

of the two approaches are compared for energies 200 and 325 MeV laboratory energy.

Clearly the ITW ansatz is of variable reliability. It is usually best at backward angles for the imaginary part of iT_{56}. This particular region is dominated by the pion pole.

V. DISCUSSION AND CONCLUSIONS

The failure of the procedure for generating pseudodata which was presented in Sec. III can be fairly easily understood. Seen in the context of the rather general formulation of the relativistic *NN* problem by Tjon and Wallace[9] it is actually not very surprising. Nevertheless we feel that this example provides a simple and therefore useful warning of the problems implicit to some extent in all direct reaction studies involving relativistic effective T matrices.

The essential ingredient in deriving Eqs. (3.12) and (3.13) is the assumption that the operator $M^{(0)}$, which we sandwich between $\langle\gamma\delta|$ and $|\alpha\beta\rangle_0$ to get $T^{0\prime}_{56}$, can be exactly expanded in terms of five invariant amplitudes. Of course $M^{(0)}$ is really a 16×16 matrix which Tjon and Wallace[9] have shown to be expressible in terms of 44 invariant amplitudes (after invoking parity and time-

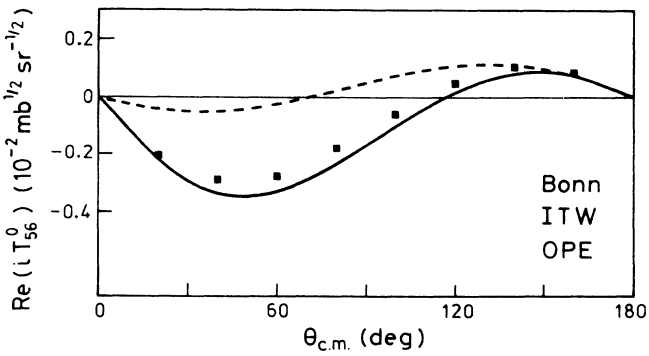

FIG. 6. As in Fig. 1 for Re(iT^0_{56}).

TABLE I. The γ_J parameters and bar phases (in degrees) for laboratory kinetic energy 200 MeV in three cases: (i) from the HM3A Bonn potential (denoted "Bonn") in the DWBA; (ii) from the HM3A Bonn potential bar phase shifts in the ITW approximation (denoted "ITW"); (iii) from the HM3A Bonn potential where only pion exchange was used in the DWBA (denoted "OPE," the bar phases are the same as for "Bonn").

J	γ'_J (Bonn)	γ'_J (ITW)	γ'_J (OPE)	$\bar{\delta}^{JJ}$ (Bonn)	$\bar{\delta}^{JJ}$ (ITW)
1	0.021 54	0.020 78	0.013 49	−20.30	−1.82
2	−0.037 51	−0.023 90	−0.035 70	28.59	16.28
3	0.005 06	0.005 10	0.004 72	−3.03	−0.25
4	−0.008 58	−0.007 87	−0.008 56	5.25	1.54
5	0.001 50	0.001 50	0.001 49	−0.81	−0.13
J	$'\gamma_J$ (Bonn)	$'\gamma_J$ (ITW)	$'\gamma_J$ (OPE)	$\bar{\delta}^{J}$ (Bonn)	$\bar{\delta}^{J}$ (ITW)
1	0.043 59	0.036 28	0.033 22	−22.62	11.38
2	−0.011 59	−0.007 30	−0.010 47	6.66	3.35
3	0.014 43	0.014 53	0.014 00	−3.80	0.84
4	−0.002 72	−0.002 62	−0.002 70	0.89	0.58
5	0.004 46	0.004 48	0.004 45	−1.23	−0.31

reversal invariance). It is only the 4×4 positive-energy submatrix which is expressible in terms of five amplitudes (in our case the Fermi amplitudes). When charge symmetry is good, and we can work with average-mass spinors $u(k,\lambda)$, to an excellent approximation, this 4×4 submatrix is all we need. In that case the five helicity amplitudes are related to the five invariant amplitudes and vice versa.

The subtlety in our case is that when charge symmetry is broken, by displacing m_n and m_p from $\bar{m}$, it is a different 4×4 positive-energy submatrix that we need—e.g., $\langle\gamma\delta|M^{(0)}|\alpha\beta\rangle_0$. The positive-energy proton and neutron spinors are in fact a linear combination of positive- *and* negative-energy spinors with respect to the average mass spinors. Indeed we have shown [see Eq. (3.4)] that $T^{0\prime}_{56}$ is entirely determined by $M^0(+-,++)$ and $M^0(-+,++)$ where the ± refer to the energy of average-mass spinors. These matrix elements are *completely independent* of the 4×4 submatrix $M^0(++,++)$ which the five Fermi amplitudes were constrained to fit. That is, the pseudodata are unconstrained by any data—or random.

From the limited point of view of trying to test potential models, this result is a disappointment. Clearly the procedure of ITW, while it is formally correct in some of its stages, is unreliable in practice whenever a phenomenological, relativistic T matrix is used for M^0. A similar point has been made by MacFarlane and Redish[11] in the

TABLE II. The γ_J parameters and bar phases (in degrees) for laboratory kinetic energy 325 MeV in three cases: (i) from the HM3A Bonn potential (denoted "Bonn") in the DWBA; (ii) from the HM3A Bonn potential bar phase shifts in the ITW approximation (denoted "ITW"); (iii) from the HM3A Bonn potential where only pion exchange was used in the DWBA (denotes "OPE," the bar phases are the same as for "Bonn").

J	γ'_J (Bonn)	γ'_J (ITW)	γ'_J (OPE)	$\bar{\delta}^{JJ}$ (Bonn)	$\bar{\delta}^{JJ}$ (ITW)
1	0.027 37	0.023 14	0.015 50	−26.80	−7.40
2	−0.046 58	−0.026 90	−0.043 36	29.46	15.25
3	0.008 15	0.007 92	0.007 09	−4.34	−0.02
4	−0.014 62	−0.012 87	−0.014 52	8.18	2.33
5	0.002 77	0.002 78	0.002 72	−1.31	−0.10
6	−0.005 38	−0.005 17	−0.005 38	2.53	0.42
J	$'\gamma_J$ (Bonn)	$'\gamma_J$ (ITW)	$'\gamma_J$ (OPE)	$\bar{\delta}^{J}$ (Bonn)	$\bar{\delta}^{J}$ (ITW)
1	0.050 07	0.038 52	0.034 36	−30.03	8.07
2	−0.016 31	−0.008 52	−0.013 98	8.48	0.08
3	0.022 20	0.020 73	0.020 85	−5.26	2.18
4	−0.004 65	−0.004 29	−0.004 56	1.51	0.80
5	0.008 21	0.008 27	0.008 15	−1.77	−0.22
6	−0.001 76	−0.001 73	−0.001 75	0.43	0.15

context of nonrelativistic effective interactions, like that of Love and Franey,[12] which also fail to ensure off-shell unitarity.

Of course, relativistic effective interactions have not been constructed to study CSB. Rather they are intended to be inserted into distorted-wave matrix elements involving bound and scattering solutions of the Dirac equation. Our simple example makes it clear that the reliability of such calculations could be highly variable. In particular any Dirac distorted-wave or bound-state wave function can be expanded in terms of a complete set of positive- *and* negative-energy spinors. Thus all distorted-wave matrix elements include an uncontrolled admixture of unreliable matrix elements.

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APPENDIX A: RELATIONS BETWEEN SPINORS OF DIFFERENT MASSES

Let us compare the spinor of particle A with the spinor of particle B. Using the orthogonality property of the four spinors $u_\lambda(\mathbf{p})$, $u_{-\lambda}(\mathbf{p})$, $v_\lambda(\mathbf{p})$, and $v_{-\lambda}(\mathbf{p})$, we can express

$$u^B_\lambda(\mathbf{p})=\alpha u^A_\lambda(\mathbf{p})+\beta u^A_{-\lambda}(\mathbf{p})+\gamma v^A_\lambda(\mathbf{p})+\delta v^A_{-\lambda}(\mathbf{p}) . \tag{A1}$$

where λ is the helicity,

$$u^A_\lambda(p)=\frac{1}{\sqrt{2m_A W_A}}\begin{bmatrix} W_A & \chi_\lambda \\ 2p\lambda & \chi_\lambda \end{bmatrix} , \tag{A2}$$

m_A is the mass of the particle A, $W_A=E_A+m_A$,

$E_A=(\mathbf{p}^2+m_A^2)^{1/2}$, $p=|\mathbf{p}|$,

$$\chi_\lambda=\begin{bmatrix} \frac{1}{2}+\lambda \\ \frac{1}{2}-\lambda \end{bmatrix} \text{ (Pauli spinor)} , \tag{A3}$$

$$v^A_\lambda(\mathbf{p})=-\gamma_5 u^A_\lambda(\mathbf{p})=\frac{1}{\sqrt{2m_A W_A}}\begin{bmatrix} 2p\lambda & \chi_\lambda \\ W_A & \chi_\lambda \end{bmatrix} , \tag{A4}$$

$$\begin{aligned} &\bar{u}^A_\lambda(\mathbf{p})u^A_{\lambda'}(\mathbf{p})=\delta_{\lambda\lambda'};\quad \bar{v}^A_\lambda(\mathbf{p})v_{\lambda'}(\mathbf{p})=-\delta_{\lambda\lambda'} , \\ &\bar{u}^A_\lambda(\mathbf{p})v^A_{\lambda'}(\mathbf{p})=0;\quad \bar{v}^A_{\lambda'}(\mathbf{p})u^A_{\lambda'}(\mathbf{p})=0,\bar{\chi}_\lambda\chi_{\lambda'}=\delta_{\lambda\lambda'} , \\ &\bar{u}^B_\lambda(\mathbf{p})u^B_\lambda(\mathbf{p})=\alpha^2-\gamma^2 \end{aligned} \tag{A5}$$

and similar definitions for particle B.

From Eqs. (A2)–(A5) we obtain for Eq. (A1)

$$\begin{aligned} &\beta=\delta=0 , \\ &\alpha=\bar{u}^A_\lambda(\mathbf{p})u^B_\lambda(\mathbf{p})=\tfrac{1}{2}(W_A W_B-p^2)/\sqrt{m_A m_B W_A W_B} , \\ &\gamma=-\bar{v}^A_\lambda(\mathbf{p})u^B_\lambda(\mathbf{p})=-p\lambda(W_B-W_A)/\sqrt{m_A m_B W_A W_B} . \end{aligned} \tag{A6}$$

One can check also that the overall normalization is correct:

$$\begin{aligned} \bar{u}^B_\lambda(\mathbf{p})u^B_\lambda(\mathbf{p})&=\alpha^2-\gamma^2 \\ &=\tfrac{1}{4}[(W_A W_B-p^2)^2-p^2(W_B-W_A)^2]/(m_A m_B W_A W_B) \\ &=\tfrac{1}{4}(W_A-p)(W_B+p)(W_A+p)(W_B-p)/(m_A m_B W_A W_B) \\ &=\tfrac{1}{4}(W_A^2-p^2)(W_B^2-p^2)/(m_A m_B W_A W_B)=1 \end{aligned} \tag{A7}$$

[as $W^2-p^2=(E+m)^2-p^2=E^2+2Em+m^2-p^2=2m(E+m)=2mW$]. Hence

$$u^B_\lambda(\mathbf{p})=[\tfrac{1}{2}(W_A W_B-p^2)u^A_\lambda(\mathbf{p})-p\lambda(W_B-W_A)v^A_\lambda(\mathbf{p})]/\sqrt{m_A m_B W_A W_B} . \tag{A8}$$

In the first order of the mass difference, $\delta m=m_B-m_A$, Eq. (A8) becomes

$$u^B_\lambda(\mathbf{p})\underline{\subset}u^A_\lambda(\mathbf{p})-\frac{p\lambda}{E}\frac{\delta m}{m}v^A_\lambda(\mathbf{p}) , \tag{A9}$$

where $m=(m_A+m_B)/2$, $E=(p^2+m^2)^{1/2}$. In a similar way,

$$v^B_\lambda(\mathbf{p})=[\tfrac{1}{2}(W_A W_B-p^2)v^A_\lambda(\mathbf{p})-p\lambda(W_B-W_A)u^A_\lambda(\mathbf{p})]/\sqrt{m_A m_B W_A W_B}\underline{\subset}v^A_\lambda(\mathbf{p})-\frac{p\lambda}{E}\frac{\delta m}{m}u^A_\lambda(\mathbf{p}) , \tag{A10}$$

consistent with (A4).

APPENDIX B: PERTURBATIVE INVARIANTS

A set of five independent amplitudes can be obtained if the following 16×16 matrices are used:[10]

$$\begin{aligned} M^0=&a_1(s,t)P_1+a_2(s,t)P_2 \\ &+a_3(s,t)P_3+a_4(s,t)P_4+a_5(s,t)P_5 , \end{aligned}$$

where

$$\begin{aligned} P_1&=1^{(1)}\otimes 1^{(2)} , \\ P_2&=\frac{i}{2}[\gamma^{(1)}_\mu\cdot(p_2+p_4)_\mu\otimes 1+1\otimes\gamma^{(2)}_\mu(p_1+p_3)_\mu] , \\ P_3&=-\gamma^{(1)}_\mu(p_2+p_4)_\mu\otimes\gamma^{(2)}_\mu(p_1+p_3)_\nu , \\ P_4&=\gamma^{(1)}_\mu\otimes\gamma^{(2)}_\mu , \\ P_5&=\gamma^{(1)}_5\otimes\gamma^{(2)}_5 , \end{aligned} \tag{B1}$$

TABLE III. The α_i^J coefficients of Eq. (C4).

J / i	1	2	3	4	5	6	7	8
1	−0.707 11							
2	0.707 11	−0.816 50						
3	−0.353 55	1.020 62	−0.866 03					
4	0.212 13	−0.612 37	1.212 44	−0.894 43				
5	−0.141 42	0.408 25	−0.808 29	1.341 64	−0.912 87			
6	0.101 02	−0.291 61	0.577 35	−0.958 31	1.434 51	−0.925 82		
7	−0.075 76	0.218 70	−0.433 01	0.718 74	−1.075 88	1.504 46	−0.935 41	
8	0.058 93	−0.170 10	0.336 79	−0.559 02	0.836 80	−1.170 13	1.559 02	−0.942 8
9	−0.047 14	0.136 08	−0.269 43	0.447 21	−0.669 44	0.936 11	−1.247 22	1.602 7

where p_1,p_2,p_3,p_4 are the four momenta of the incoming and outgoing particles, respectively, and s and t are the Mandelstam invariants.

The $a_1,\ldots,a_5$ amplitudes are the perturbative invariants. In terms of Eq. (3.4) we obtain

$$T^{0'}{}_{56}=(\eta p/m)\sin\theta(-a_1-ma_2+\tfrac{1}{2}sa_3+2a_4+a_5)\ . \quad \text{(B2)}$$

The perturbative invariants are related to the Fermi amplitudes of Eq. (3.3) by

$$F_s=a_1+\tfrac{1}{4}(u-s)a_2/m\ ,$$
$$F_v=-ma_2-\tfrac{1}{4}(u-s)a_3+a_4\ ,\ F_T=\tfrac{1}{4}ta_2/m\ , \quad \text{(B3)}$$
$$F_A=-\tfrac{1}{4}ta_3\ ,\ F_p=\tfrac{1}{4}(u-s)a_2/m+m^2a_3+a_5\ .$$

The inverse relation is obtained by simple substitutions:

$$a_1=F_1-(u-s)F_3/t\ ,$$
$$a_2=4mF_3/t\ ,$$
$$a_3=-4F_4/t\ , \quad \text{(B4)}$$
$$a_4=F_2+4m^2F_3/t-(u-s)F_4/t\ ,$$
$$a_5=-(u-s)F_3/t+4m^2F_4/t+F_5\ .$$

From Eqs. (B2) and (B4) we obtain

$$T^{0'}{}_{56}=(\eta p/m)\sin\theta[-F_s+2F_v+4m^2F_T/t-(s+t-u)F_A/t+F_P]\ , \quad \text{(B5)}$$

which is different from Eq. (3.13).

APPENDIX C: PARTIAL WAVE EXPANSIONS

Equations (3.14) and (3.15) are

$$iT^{0'}_{56\underline{\subseteq}}\frac{i\Delta M}{E}\sin\theta[\phi_3(\theta)/(1+\cos\theta)-\phi_4(\theta)/(1-\cos\theta)]\ ,$$
$$i'T^{0}{}_{56\underline{\subseteq}}=\frac{i}{2}\frac{\Delta M}{E}\{\sin\theta[\phi_3(\theta)/(1+\cos\theta)+\phi_4(\theta)/(1-\cos\theta)]+2E\phi_5(\theta)/M\}\ . \quad \text{(C1)}$$

The helicity amplitudes $\phi_3(\theta)$ and $\phi_4(\theta)$ have the expansions

$$\phi_3(\theta)=\frac{1}{p}\sum(2J+1)\phi_3^Jd^J_{11}(\theta)\ , \quad \text{(C2)}$$

$$\phi_4(\theta)=\frac{1}{p}\sum(2J+1)\phi_4^Jd^J_{-11}(\theta)\ , \quad \text{(C3)}$$

where ϕ_3^J and ϕ_4^J are obtained directly from phase shifts.[6]

In order to expand Eqs. (C1) in the form of Eq. (4.1) we need to find the expressions of (see Table III)

$$\sin\theta d^J_{11}(\theta)/(1+\cos\theta)=\sum_{i=1}^{J}\alpha_i^Jd^i_{10}(\theta)\ , \quad \text{(C4)}$$

$$\sin\theta d^J_{-11}(\theta)/(1-\cos\theta)=\sum_{i=1}^{J}\beta_i^Jd^i_{10}(\theta)\ . \quad \text{(C5)}$$

Using recurrence relations for the $d^J_{\lambda\mu}(\theta)$ functions we found the following recurrence relation for the α_i^J coefficients:

$$\alpha_i^{J+1}=-\frac{2J+1}{J^2(J+2)}\alpha_i^J+\frac{(J+1)(2J+1)}{J(J+2)}\times\left[\frac{(i^2-1)^{1/2}}{2i-1}\alpha_{i-1}^J+\frac{\sqrt{i(i+2)}}{2i+3}\alpha_{i+1}^J\right]-\frac{(J+1)(J^2-1)}{J^2(J+2)}\alpha_i^{J-1}\ , \quad \text{(C6)}$$

with initial values $\alpha_1^0=0$, $\alpha_1^1=-1/\sqrt{2}$, $\alpha_2^2=1/\sqrt{2}$, $\alpha_2^2=-(2/3)^{1/2}$, $\alpha_i^0=\alpha_0^J=0$, and $\alpha_i^J=0$ for $J>i$. The coefficients β_i^J of Eq. (C5) can be evaluated from the relation

$$\beta_i^J=(-1)^{i+J}\alpha_i^J\ . \quad \text{(C7)}$$

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