Instability of infinite nuclear matter in the $\sigma-\omega$ model

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We demonstrate that, within the relativistic σ-ω model, the uniform mean-field solution for infinite nuclear matter is unstable for certain values of the scalar mass. For scalar masses greater than about 690 MeV, there is a state of nuclear matter which has a spatially periodic density and is lower in energy than the uniform state. We also demonstrate that the periodic state of nuclear matter has a lower compressibility than the uniform state, and that the density of the periodic state can be interpreted in terms of alpha-particle clustering. Furthermore, the secondary effects of the instability of nuclear matter can be seen in relativistic calculations of finite nuclei and similar effects are present in electron scattering data.

I. INTRODUCTION

The relativistic mean-field model obtained from quantum hadrodynamics[1] (QHD) has been very successful in describing the ground state properties of a wide variety of nuclei.[2-4] It has also been extensively used as the basis for random-phase approximation (RPA) calculations of nuclear excited state properties[5] and for investigations of scattering processes.[6] An appealing feature of this model is the small number of free parameters. These parameters are the coupling constants of the meson fields (scalar and vector) and the mass of the scalar meson. Generally the coupling constants are fit to the saturation density and binding energy of infinite nuclear matter so that only one parameter remains which can be adjusted to reproduce the properties of finite nuclei. This procedure assumes that the ground state of the infinite system is uniform and may be described in terms of the plane wave solutions of a Dirac equation which includes the uniform meson mean fields.

As early as 1960, Overhauser[7] suggested that the ground state of nuclear matter is in fact not uniform, but instead contains a spatial density oscillation with a period that is determined by the Fermi momentum. In this paper we show that, in the relativistic mean-field model, the ground state of nuclear matter is not necessarily uniform, and that the amplitude of the periodic component of the ground state density is sensitive to the value of the scalar mass. In Sec. II, we review the basic relativistic model and the standard treatment of uniform nuclear matter. In Sec. III, we investigate the nonuniform solution for nuclear matter in the limit of one spatial dimension. In this limit, we are able to establish the connection between our exact calculations and the random phase approximation (RPA). We also demonstrate that in one dimension the periodic ground state has a significantly lower compressibility than the usual uniform state. In Sec. IV we use the RPA to define the region of instability of the uniform state in three space dimensions. We also demonstrate that in three dimensions the periodicity can be interpreted as alpha-particle clustering.

We emphasize that this study is in the context of mean-field theory only. Preliminary studies including vacuum effects have shown some evidence for a qualitatively similar instability of the uniform ground state. A more complete study of the Overhauser effect in the relativistic Hartree approximation (RHA) which includes vacuum polarization contributions is in progress.

II. WALECKA MODEL

Our starting point is the relativistic σ-ω model[1] (Walecka model) which is characterized by the following renormalizable Lagrangian density:

$$\mathcal{L}=\bar{\psi}[\gamma_\mu(i\partial^\mu-g_v V^\mu)-(M-g_s\Phi)]\psi +\tfrac{1}{2}(\partial_\mu\Phi\partial^\mu\Phi-m_s^2\Phi^2) -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}+\tfrac{1}{2}m_v^2V_\mu V^\mu+\delta\mathcal{L}\ , \tag{2.1}$$

where

$$F_{\mu\nu}=\partial_\mu V_\nu-\partial_\nu V_\mu\ . \tag{2.2}$$

This model Lagrangian includes the coupling of the nucleon field (ψ) to scalar (Φ) and vector (V) mesons and can easily be extended to include π and ρ mesons, the photon field, and nonlinear scalar self-couplings. In Eq. (2.1), $\delta\mathcal{L}$ is the renormalization counter term Lagrangian (see Ref. 1 for details).

We consider the mean-field approximation to this theory in which the quantum meson fields are replaced by their expectation values which are classical fields. In the rest frame where the baryon current vanishes, applying Lagrange's equation leads to the set of field equations

$$(\nabla^2 - m_s^2)\Phi(\mathbf{x}) = -g_s\langle\bar\psi\psi\rangle = -g_s\sum_\alpha^{\text{occ}}\bar\psi_\alpha(\mathbf{x})\psi_\alpha(\mathbf{x}) \equiv -g_s\rho_s(\mathbf{x}) ,$$

$$(\nabla^2 - m_v^2)V_0(\mathbf{x}) = -g_v\langle\psi^\dagger\psi\rangle = -g_v\sum_\alpha^{\text{occ}}\psi_\alpha^\dagger(\mathbf{x})\psi_\alpha(\mathbf{x}) \equiv -g_v\rho_B(\mathbf{x}) , \quad (2.3)$$

$$[-i\boldsymbol{\alpha}\cdot\nabla + g_v V_0(\mathbf{x}) + \beta(M - g_s\Phi(\mathbf{x}))]\psi_\alpha(\mathbf{x}) = \epsilon_\alpha\psi_\alpha(\mathbf{x})$$

which can be solved either for nuclear matter or finite (spherical) nuclei. In Eq. (2.3), α refers to a complete set of quantum numbers for an individual nucleon, ϵ_α is the nucleon eigenvalue, and the summations are over all occupied nucleon states. Tradiationally the coupling constants g_s and g_v are fixed by insisting that the model reproduce the binding energy and saturation density of nuclear matter. To do this, it is implicitly assumed that infinite nuclear matter is a spatially uniform system so that the meson fields and densities in Eq. (2.3) are simply constants. In this case the equations of motion are

$$g_s\Phi = \frac{g_s^2}{m_s^2}\rho_s, \quad g_v V_0 = \frac{g_v^2}{m_v^2}\rho_B ,$$
$$[-i\boldsymbol{\alpha}\cdot\nabla + g_v V_0 + \beta(M - g_s\Phi)]\psi_\alpha(\mathbf{x}) = \epsilon_\alpha\psi_\alpha(\mathbf{x}) . \quad (2.4)$$

The solutions to this Dirac equation are simply plane waves in which the energy and mass have been shifted by the meson mean fields:

$$\psi_{\mathbf{k}}(\mathbf{x}) = \left[\frac{E_{\mathbf{k}}^* + M^*}{2E_{\mathbf{k}}^*}\right]^{1/2}\begin{bmatrix}1\\ \dfrac{\sigma\cdot\mathbf{k}}{E_{\mathbf{k}}^* + M^*}\end{bmatrix}\chi e^{-i\mathbf{k}\cdot\mathbf{x}} , \quad (2.5)$$

where χ is a Pauli spinor, $\epsilon_{\mathbf{k}} = g_v V_0 + E_{\mathbf{k}}^*$, $E_{\mathbf{k}}^* \equiv (\mathbf{k}^2 + M^{*2})^{1/2}$, and $M^* \equiv M - g_s\Phi$. The ground state of infinite nuclear matter is obtained by filling all the nucleon momentum states up to the Fermi momentum, k_F, which corresponds to the saturation density. The scalar and baryon densities defined in Eq. (2.3) may now be written as

$$\rho_B = \frac{\gamma}{(2\pi)^3}\int_0^{k_F} d^3k\,\psi_{\mathbf{k}}^\dagger(\mathbf{x})\psi_{\mathbf{k}}(\mathbf{x}) = \frac{\gamma}{6\pi^2}k_F^3 ,$$
$$\rho_s = \frac{\gamma}{(2\pi)^3}\int_0^{k_F} d^3k\,\bar\psi_{\mathbf{k}}(\mathbf{x})\psi_{\mathbf{k}}(\mathbf{x}) \quad (2.6)$$
$$= \frac{\gamma}{(2\pi)^3}\int_0^{k_F}\frac{M^*}{(\mathbf{k}^2 + M^{*2})^{1/2}} ,$$

where γ represents the spin-isospin multiplicity ($\gamma = 2$ in one dimension and $\gamma = 4$ in three dimensions).

The ground state energy of this system is given by

$$\mathcal{E} = \frac{\gamma}{(2\pi)^3}\int_0^{k_F} d^3k\,\epsilon_{\mathbf{k}} - \tfrac{1}{2}m_v^2 V_0^2 + \tfrac{1}{2}m_s^2\Phi^2 , \quad (2.7)$$

and the corresponding binding energy per particle is

$$E_B/A \equiv \frac{\mathcal{E}}{\rho_B} - M . \quad (2.8)$$

The ratios, $(g_s/m_s)^2$ and $(g_v/m_v)^2$, are fixed by insisting that this energy is a minimum at $k_F = 1.3\ \text{fm}^{-1}$ with a value of $E_B/A = -15.75$ MeV (in some versions of QHD $k_F = 1.42\ \text{fm}^{-1}$ is used). Since the vector mass can be taken from the mass of the ω meson, this is sufficient to fix g_v; however, since there is no low lying scalar meson to determine the scalar mass, only the ratio of g_s to m_s is determined in uniform nuclear matter. The value of m_s is usually adjusted to reproduce some property of the nuclear surface.

III. ONE-DIMENSIONAL NUCLEAR MATTER

A. Random-phase approximation

Overhauser's claim[7] is that the spatially uniform state of infinite nuclear matter described in terms of a filled Fermi sea of plane wave states is typically not the lowest energy configuration. The true ground state (in one dimension) is characterized by a spatial density variation of the form

$$\rho_B(x) = \rho_B^0 + \rho_B^1\cos(qx) , \quad (3.1)$$

where q determines the period of the oscillation ($P = 2\pi/q$) and is related to the Fermi momentum by $q = 2k_F$. If this is true, one simply way to demonstrate this instability of the uniform state is to use the RPA to calculate the energies of the low lying excitations. It is a general property of the RPA that all of the excitation energies are real provided that it is based on the true (Hartree-Fock) ground state of the system. If there exists a state with lower energy than the assumed ground state, then at least one of the RPA modes will have an imaginary energy. (For more detail on the general RPA problem see Ref. 8, and on applications to the σ-ω model see Refs. 5 and 9.)

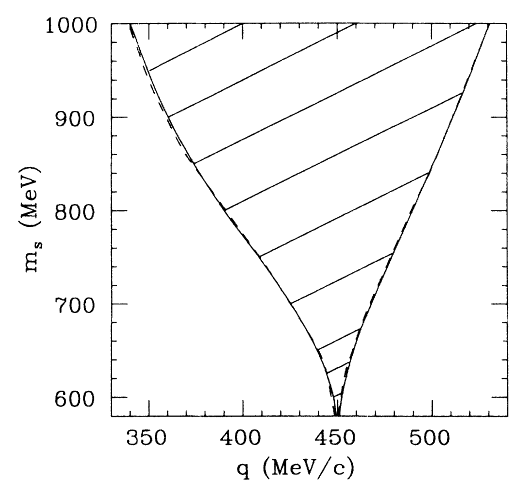

FIG. 1. Region of instability of uniform nuclear matter in one space dimension as a function of the scalar mass and the frequency. The solid curve and the shaded region mark the area in which the uniform state is not the lowest energy state of the system as determined in the RPA. The dashed curve shows the corresponding boundary determined from the 'exact' calculations.

With the ratios g_s/m_s and g_v/m_v fixed by insisting that uniform nuclear matter saturate at the empirical values of the binding energy per particle and the saturation density, we have mapped out the region of instability predicted by the RPA (see Fig. 1) as a function of the scalar mass m_s and the spatial frequency q. Consistent with Overhauser's suggestion, $q=2k_F$ defines the frequency at which the instability appears for the lowest scalar mass (or, alternatively, for the longest range of the scalar attraction).[10] For higher values of the scalar mass, periodic states characterized by a wide range of spatial frequencies lead to lower energies than the uniform state, but $q=2k_F$ should always give the state of lowest energy.

B. Bloch wave solution

In order to study the details of the periodic state, we need a self-consistent calculation of the nuclear wave functions and mean fields which does not rely on the assumption that infinite nuclear matter is uniform. To do this we use a somewhat more general form for the nuclear densities:

$$\rho_B(x)=\sum_{m=0}^{m_{\max}}\rho_B^m\cos(mqx)\ ,$$
$$\rho_s(x)=\sum_{m=0}^{m_{\max}}\rho_s^m\cos(mqx)\ . \quad (3.2)$$

Substituting these expressions into the first two equations in (2.3), we see that the meson mean fields have the same form as the densities,

$$\Phi(x)=\sum_{m=0}^{m_{\max}}\Phi^m\cos(mqx)\ ,$$
$$V_0(x)=\sum_{m=0}^{m_{\max}}V_0^m\cos(mqx)\ , \quad (3.3)$$

where

$$\Phi^m=\frac{g_s}{q^2+m_s^2}\rho_s^m \quad \text{and} \quad V_0^m=\frac{g_v}{q^2+m_v^2}\rho_B^m\ . \quad (3.4)$$

Due to the simple functional forms of the mean fields it is natural to express the nuclear wave functions in terms of a superposition of plane wave states:

$$\psi_k(x)=\sum_{n=-m_{\max}}^{m_{\max}}u_k^n e^{-i(k-nq)x}\ , \quad (3.5)$$

where the Dirac spinors, u_k^n, serve as the expansion coefficients. This type of wave function is familiar from solid state physics and is referred to as a Bloch wave. After substituting Eq. (3.5) into the Dirac equation in (2.3), we use the orthogonality of the plane waves to eliminate all of the spatial dependence. We are then left with a set of coupled algebraic equations involving the expansion functions u_k^n (and the eigenvalue ϵ_k) which can be solved by matrix inversion. It is then simple to show that the resulting nuclear wave functions lead to densities of the form given in Eq. (3.2). Self-consistency is achieved by first guessing the coefficients in the expansion of the densities, then solving for the wave functions, recalculating the densities, and iterating until the eigenvalues and the densities have converged. There is an approximation involved in the truncation of the summations in Eqs. (3.2), (3.3), and (3.5). A consistent treatment of the potentials and the wave functions would require that we allow $m_{\max}$ to be infinite, since the $m=1$ term in Eq. (3.3) is sufficient to induce all harmonics of q in the wave functions. Likewise, these higher harmonics will lead to all possible $\cos(mqx)$ terms in the potentials. In addition, the specific truncation scheme that we have employed is not unique. In practice, we find that the convergence with respect to our expansion is very rapid and that $m_{\max}=2$ is generally sufficient.

Using this 'exact' solution, we have again mapped out the region of instability of the uniform state as a function of m_s and q (see the dashed curve in Fig. 1). The close agreement of the exact results with those from the RPA demonstrates that the instability predicted by the RPA does in fact correspond to the presence of a periodic state of nuclear matter with a lower energy than the uniform state. (This correspondence will be important in three space dimensions where the exact calculation is more difficult.)

In order to investigate the details of the periodic state we concentrate on $q=2k_F$. Table I shows the binding energy per nucleon and ρ_B^1/ρ_B^0 (a measure of the degree of periodicity) as functions of the scalar mass. For low values of m_s, the uniform state of infinite nuclear matter is the ground state; but, as the scalar mass increases the ground state becomes increasingly periodic. Notice that at the highest value of m_s the binding energy per nucleon in the periodic state is 3.0 MeV greater than in the uniform state. Figure 2 shows the density corresponding to $m_s=1000$ MeV, where nuclear matter separates into nearly isolated regions of high density.

As is the case with any variational calculation, our solution method can only find the lowest energy state that is consistent with the assumed degrees of freedom. We can, however, extend the self-consistent solution to arbitrary values of ρ_B^1 by including an appropriate Lagrange multiplier. This allows us to demonstrate the general dependence of the energy on the degree of periodicity. Figure 3 shows the binding energy per nucleon versus ρ_B^1/ρ_B^0 for several values of the scalar mass. For

TABLE I. Predicted nuclear matter ground states (in one space dimensions), as a function of the scalar mass, using coupling constants that are adjusted to saturate uniform nuclear matter at $k_F=1.14$ fm^{-1} and $E_B/A=-15.75$ MeV (see Ref. 10).

m_s (MeV)	E_B/A (MeV)	ρ_B^1/ρ_B^0
500	−15.75	0.00
600	−15.752	0.05
700	−15.90	0.30
800	−16.51	0.53
900	−17.50	0.69
1000	−18.64	0.80

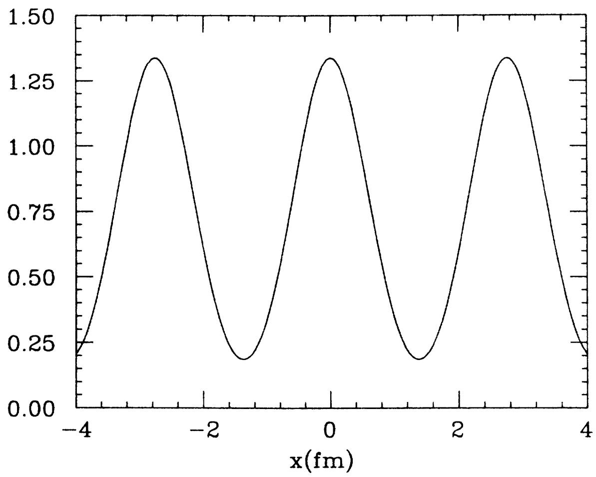

FIG. 2. Baryon density for infinite nuclear matter in one space dimension using a scalar mass of 1000 MeV.

$m_s = 510$ MeV, the uniform state ($\rho_B^1 = 0$) is a clear minimum. As m_s increases the binding energy curve takes on the familiar shape for a system having a broken symmetry, with the uniform state corresponding to a local maximum of the energy. In Fig. 3, the points are the values obtained directly from the exact calculations, and the curves are obtained by fitting to the functional form:

$$E_B/A = a(\rho_B^1)^2 + b(\rho_B^1)^4 . \tag{3.6}$$

The value of a is proportional to the square root of the energy difference between the periodic (energy minimum) and uniform (local energy maximum) states and is therefore sensitive to m_s, while the value of b is roughly independent of m_s.

Finally, for the higher scalar masses where the uniform state is an energy maximum, it is inappropriate to use the uniform state to adjust the coupling constants. Table II shows the values of the coupling constants obtained by insisting that the periodic state of infinite nuclear matter

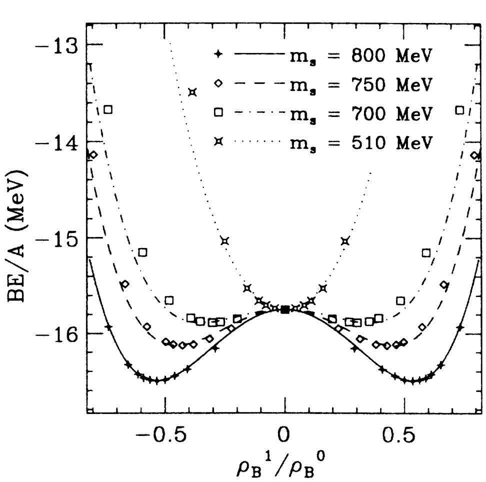

FIG. 3. Binding energy per nucleon as a function of the degree of periodicity of infinite nuclear matter.

TABLE II. Coupling constants obtained by fitting the uniform or periodic state of infinite nuclear matter (in one space dimension) to the empriical saturation point (with $m_s = 800$ MeV, see Ref. 10).

	Uniform	Periodic
E_B/A (MeV)	−15.75	−15.75
k_F (fm^{-1})	1.14	1.14
$(g_s/m_s)^2$	1.745	1.282
$(g_v/m_v)^2$	1.357	0.917
K (MeV)	361.9	252.9

reproduce the saturation data. Notice that both the scalar and vector coupling strengths are significantly reduced from the values obtained from uniform nuclear matter. Table II also shows the values of the compressibility for the uniform and periodic states. In the uniform state the compressibility is very high as is typical of the standard σ-ω model;[1] however, the compressibility in the periodic state is roughly 30% smaller. A reduction of this size in three dimensions would bring the Walecka model compressibility ($K = 550$ MeV) much nearer to the range of values that have been extracted from the data[11] ($K \approx 180-310$ MeV).

IV. THREE-DIMENSIONAL NUCLEAR MATTER

A. Random-phase approximation

Although calculations in one dimension are useful for investigating certain qualitative features of the problem, in order to draw any firm conclusions about the validity of the model it is important to consider the full three space dimensions. As we saw in Sec. III A, the most straightforward way to demonstrate the instability of uniform nuclear matter is through the RPA excited states. In Fig. 4, we show (as the solid line) the locus of points

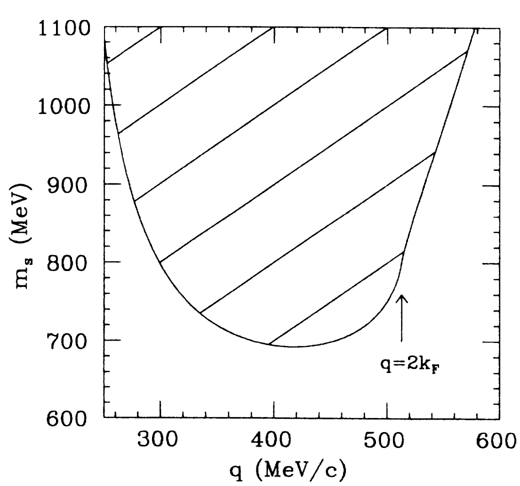

FIG. 4. Region of instability of uniform nuclear matter in three space dimensions as a function of the scalar mass and the frequency. The solid curve and the shaded region mark the area in which the uniform state is not the lowest energy state of the system as determined in the RPA.

for which the lowest RPA eigenvalue is at zero energy. In the unshaded region below this line all of the eigenvalues are real and the uniform state is the stable ground state of the system. In the shaded region above the line in Fig. 4 at least one of the RPA excited states is at an imaginary energy, which indicates that the uniform state is not the ground state of the system.

As in the one-dimensional (1D) calculation, the instability of the uniform state is present for values of the scalar mass that are close to the values that are typically used in the relativistic model. In contrast to the 1D results, the instability appears at the lowest value of m_s for $q \approx 1.6k_F$ rather than $q=2k_F$. In one dimension, $q=2k_F$ corresponded to the momentum transfer required to lift a nucleon from a state just below the Fermi surface at $p=-k_F$ to a state just above the Fermi surface at $p+q=+k_F$. In three dimensions, since the Fermi surface is a sphere in momentum space, the momentum transfer of $q=2k_F$ is not unique. Any value of q between 0 and $2k_F$ is sufficient to lift a nucleon from a state $\mathbf{p}$ ($|\mathbf{p}| \lesssim k_F$) just below the Fermi surface to a state $\mathbf{p}+\mathbf{q}(|\mathbf{p}+\mathbf{q}| \gtrsim k_F)$ just above the Fermi surface. The optimal value of q is determined by the number of states near the Fermi surface that can be connected by $\mathbf{q}$. With this in mind, $q \approx 1.6k_F$ is consistent with the 1D results. Notice also in Fig. 4, that there is a pronounced increase in m_s at $q=2k_F$. This supports the assertion that values of $q>2k_F$ are unfavorable since no pair of states near the Fermi surface can be connected by these larger momentum transfers.

B. Bloch wave solution

Just as in the one-dimensional calculations, in order to understand the details of the instability of the uniform state of nuclear matter, we need a self-consistent calculation of the nuclear wave functions and mean fields. Unfortunately, in three dimensions there is no obvious choice for the functional form of the periodic component of the nuclear density. Any combination of the following functions would be a reasonable guess for the additional terms in the density [as in Eq. (3.1)]:

$$\cos(qx),\quad \cos(qy),\quad \cos(qz),\quad \cos(qr),\quad j_0(qr),\ \ldots \tag{4.1}$$

In practice, the choice is made by the difficulty of the numerical solution for the wave functions in the presence of radially periodic meson mean fields. The most general form of the density for which we can obtain a solution for the wave functions is

$$\begin{aligned}\rho_B(\mathbf{x}) = \rho_B^0 &+ \rho_B^1[\cos(qx)+\cos(qy)+\cos(qz)] \\ &+ \rho_B^2[\cos(qx)\cos(qy)+\cos(qy)\cos(qz) \\ &\qquad + \cos(qz)\cos(qx)] \\ &+ \rho_B^3[\cos(qx)\cos(qy)\cos(qz)]\ .\end{aligned} \tag{4.2}$$

If this type of density modulation leads to a lower energy solution than the uniform state, then it is a clear indiction of the instability of the uniform ground state even though this solution may not be the absolute lowest energy configuration. In addition, if the region of instability is similar to that found in the preceeding section by way of the RPA, then we can expect the qualitative features of this state to be closely related to those of the true ground state.

As in Sec. III B, it is simple to show that the scalar and vector mean fields have the same form as the density in Eq. (4.2), and it is again possible to write the nucleon wave functions in terms of a superposition of plane wave states (Bloch wave):

$$\psi_{\mathbf{k}}(\mathbf{x}) = \sum_{n_x,n_y,n_z=-m_{\max}}^{m_{\max}} u_{\mathbf{k}}^{n_x,n_y,n_z} \exp -i[(k_x-n_xq)x+(k_y-n_yq)y+(k_z-n_zq)z]\ . \tag{4.3}$$

Substituting these forms [Eqs. (4.2)–(4.3)] into the Dirac equation in (2.3), again leads to a set of coupled algebraic equations for the Dirac spinors, $u_{\mathbf{k}}^{n_x,n_y,n_z}$, at each momentum $\mathbf{k}$. The self-consistent solution for the ground state is obtained iteratively as described in Sec. III B.

Table III shows the binding energy per nucleon and $3\rho_B^1/\rho_B^0$ (a measure of the periodicity[12]) as functions of the scalar mass using the RPA prediction for the momentum transfer, $q=1.6k_F=410$ MeV/c. The behavior is similar to that of the one-dimensional system. Below $m_s \approx 690$ MeV the uniform state is the ground state, while above this scalar mass the ground state becomes increasingly periodic and the binding energy per nucleon increases. By a scalar mass of about 1000 MeV, the periodic component of the ground state ($3\rho_B^1$) is as large as the uniform component (ρ_B^0).

In Fig. 5, we show a cross section (in the x-y plane) of the baryon density in the periodic state calculated using a scalar mass of 750 MeV. Keeping in mind the additional structure in the z direction, Fig. 5 shows that in the periodic state there are spherically symmetric regions of high density located on the corners of a cubic lattice. By

TABLE III. Predicted nuclear matter ground states (in three space dimensions), as a function of the scalar mass, using coupling constants that are adjusted to saturate uniform nuclear matter at $k_F=1.3\ \text{fm}^{-1}$ and $E_B/A=-15.75$ MeV.

m_s (MeV)	E_B/A (MeV)	$3\rho_B^1/\rho_B^0$
680	−15.75	0.00
700	−15.76	0.18
720	−15.78	0.28
750	−15.84	0.41
800	−16.04	0.57
900	−16.61	0.80
1000	−17.27	0.93

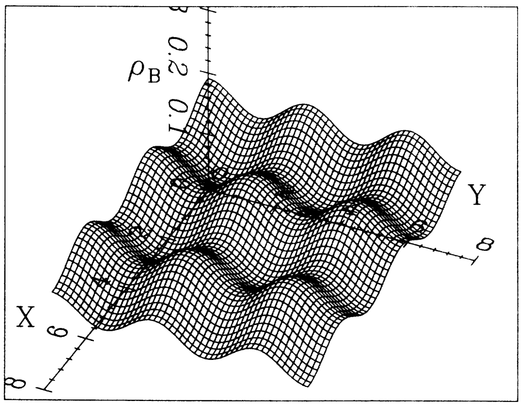

FIG. 5. Cross section (in the x-y plane) of the baryon density for infinite nuclear in three space dimensions using a scalar mass of 750 MeV.

integrating the baryon density over one of these regions we can show that each of the high density regions contains four nucleons (two protons and two neutrons). In effect, for the higher scalar masses, infinite nuclear matter crystallizes into distinct alpha-particle clusters.

Finally, in Table IV, we compare coupling constants appropriate to the uniform and periodic states of nuclear matter. In each case, the couplings are adjusted to saturate nuclear matter at the empirical values of k_F and E_B/A. As in the one-dimensional case both the scalar and vector couplings are reduced in the periodic state, although in three dimensions this reduction is relatively small (only about 2%). However, we cannot conclude that these changes are unimportant. Shepard *et al.*[5] have found that the lowest 3^- RPA excited state in ^{40}Ca appears at a complex energy when they use the standard couplings obtained from uniform nuclear matter. The complex eigenvalue indicates that the predicted ^{40}Ca ground state is unstable. As an *ad hoc* repair of this instability, they reduce the scalar coupling by roughly 2%. This reduction is comparable to the reduction found in the periodic state and is sufficient to bring the energy of the lowest 3^- state up to a positive excitation energy which is comparable to the experimental value. Table IV also contains a comparison of the nuclear matter compressibilities. Again, just as in the one-dimensional calculations, the compressibility of the periodic state is lower than that of the uniform state. This reduction is not sufficient to greatly improve the agreement with the experimental compressibility; however, it has been suggested[13] that the bulk of the discrepancy may be removed by extracting the compressibility from the pure breathing mode excitations rather than from the full monopole response.

TABLE IV. Coupling constants obtained by fitting the uniform or periodic state of infinite nuclear matter (in three space dimensions) to the empirical saturation point (with $m_s=750$ MeV).

	Uniform	Periodic
E_B/A (MeV)	−15.75	−15.75
k_F (fm^{-1})	1.3	1.3
g_s^2	226.0	221.6
g_v^2	188.5	184.4
K (MeV)	547.0	492.0

C. Spherical symmetry by way of large scale finite nuclei

At the beginning of Sec. IV B, we discussed the difficulty of considering spherically symmetric periodic solutions in nuclear matter. One way of avoiding the numerical problems is to approximate nuclear matter by a very large finite nucleus. If we can demonstrate that the structure of the nucleus is independent of the size of the system then it is a reasonable assumption that the structure would remain for an infinite system. Figure 6 shows the radial baryon density for three large "nuclei" using a scalar mass of 700 MeV. These solutions were obtained using standard techniques for finite nuclei (see, e.g., Ref. 2) assuming equal numbers of protons and neutrons with no Coulomb interaction (this simulates symmetric nuclear matter). No assumptions were made about the periodicity; the baryon density was obtained strictly from the self-consistent solution of the Hartree equations in a finite system. Clearly the density is highly periodic and the same structure is present in all three "nuclei." The only effect of adding additional nucleons is to add additional spherical shells. By reading off the spacing of the shells we can calculate the spatial frequency of the oscil-

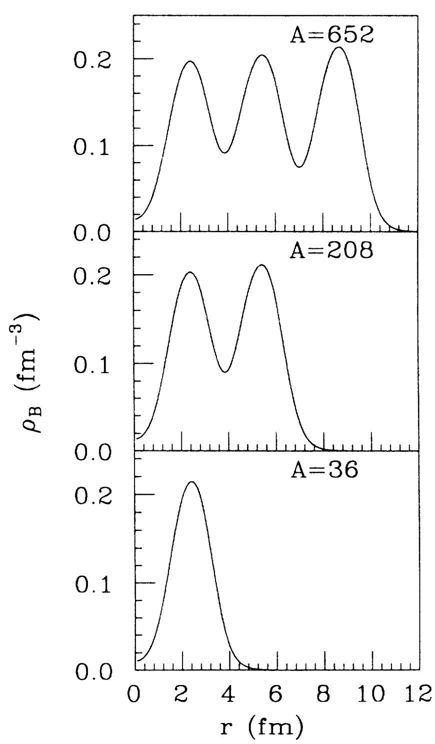

FIG. 6. Baryon density as a function of r for three large "nuclei." The scalar mass is fixed at 700 MeV.

latory structure:

$$P=\frac{2\pi}{q}=3.125\ \text{fm}\rightarrow q=2.01\ \text{fm}^{-1}=1.55k_F\ . \quad (4.4)$$

This value for q is very close to the result of $q=1.6k_F$ that we obtained in Sec. IV B.

The sensitivity of the spherical system to the value of the scalar mass is shown in Fig. 7 for $A=708$. The standard scalar mass (520 meV) leads to a relatively smooth density and a binding energy per nucleon of -13.0 MeV. The small scale structure in the interior region is present for all nuclei in the relativistic model and is often interpreted as arising from the shell structure of the least bound nucleons. In contrast, for a scalar mass of 700 MeV, the interior structure is greatly enhanced and the resulting periodic state is more tightly bound by 2 MeV per nucleon. In these finite nucleus calculations there is no sharp transition to the periodic state as was the case in Sec. IV C (see Table II). As the scalar mass is increased from 520 MeV, the small scale structure in the density is gradually enhanced and the spatial frequency of the structure approaches $q=1.6k_F$. The fact that the instability affects finite nuclei at lower scalar masses than in infinite nuclear matter can probably be attributed to the nuclear surface. In effect, the surface provides a source term for the periodic component of the density, which leads to an oscillatory structure away from the surface. This is similar to the Friedel oscillations induced by an external source in an electron gas.[14]

We demonstrate this connection to the Friedel oscillations by considering the self-consistent density change, $\delta\rho^{\text{ind}}$, induced in uniform nuclear matter by a density perturbation $\delta\rho$. For simplicity, we illustrate our method with scalar interactions only. The field equation [Eq. (2.3)] for the scalar field may be solved formally using the free scalar propagator D_0:

$$\Phi_0=D_0(-g_s\rho_s)\ . \quad (4.5)$$

The self-consistent response of the system may be included via linear response theory by replacing the bare propagator by its RPA counterpart:

$$D_0\rightarrow D=D_0+D_0\Pi_0 D\ , \quad (4.6)$$

where Π_0 is the free scalar-scalar polarization insertion. The *effective* scalar density change, $\delta\rho_s^{\text{eff}}$, may be expressed in terms of the density perturbation, $\delta\rho_s$, as

$$\delta\rho_s^{\text{eff}}=-\frac{1}{g_s}D_0^{-1}\delta\Phi=D_0^{-1}D\delta\rho_s\ . \quad (4.7)$$

Then, the *induced* density change due to the self-consistent response of the medium to $\delta\rho_s$ is

$$\delta\rho_s^{\text{ind}}=\delta\rho_s^{\text{eff}}-\delta\rho_s=(D_0^{-1}D-1)\delta\rho_s$$
$$=\Pi_0 D\delta\rho_s=\Pi_{\text{RPA}}D_0\delta\rho_s\ , \quad (4.8)$$

where $\Pi_{\text{RPA}}=\Pi_0+\Pi_0 D_0\Pi_{\text{RPA}}$ is the RPA polarization insertion.

This result is generalized to include the full dynamics of QHD-1 (see Ref. 1) by taking $\Pi_{\text{RPA}}\rightarrow\Pi_{\text{RPA}}^{ss}$, the scalar-scalar component of the full 5×5 QHD-1 RPA polarization insertion described by Chin.[9] Similarly, the induced baryon density change $\delta\rho_B^{\text{ind}}$ arising from a baryon density perturbation, $\delta\rho_B$, is

$$\delta\rho_B^{\text{ind}}=\Pi_{\text{RPA}}^{00}\Delta_0\delta\rho_B\ , \quad (4.9)$$

where Π_{RPA}^{00} is the timelike, timelike component of the full Π_{RPA} and Δ_0 is the q-dependent piece of the free vector propagator.

We have evaluated $\delta\rho_B^{\text{ind}}$ in QHD-1 assuming

$$\delta\rho_B(r)=a\delta(r-R)\ , \quad (4.10)$$

i.e., the density perturbation is a spherical shell at radius R, which is intended to crudely approximate the nuclear surface. Then $\delta\rho_B^{\text{ind}}$ is evaluated by Fourier transforming Eq. (4.9). The total baryon density ($\rho_B^{\text{tot}}=\rho_B+\delta\rho_B^{\text{ind}}$) is compared, in Fig. 8, with the baryon density for a hy-

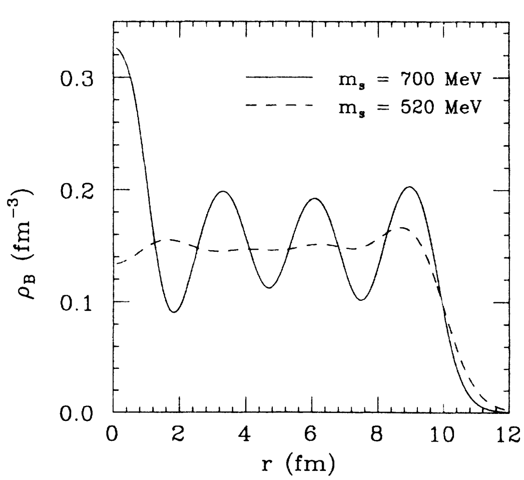

FIG. 7. Baryon density as a function of r for $A=708$. The solid curve is for $m_s=700$ MeV and the dashed curve is for $m_s=520$ MeV.

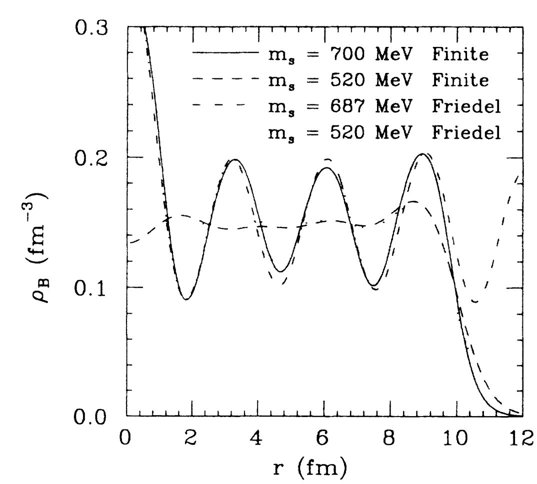

FIG. 8. Comparison of the baryon densities shown in Fig. 7 to the corresponding densities obtained from the Friedel oscillations in nuclear matter.

pothetical $T=0$ nucleus with $A=708$ obtained as discussed earlier. The linear response or nuclear Friedel calculations based on Eqs. (4.9) and (4.10) were done for scalar masses of 520 and 687 MeV, the latter being very close to the region of instability at $q=1.6k_F$ (see Fig. 4). The radius parameter R and the magnitude of the density perturbation a were adjusted to yield best fits to the exact finite baryon densities for each value of the scalar mass. As shown in Fig. 8, the strong dependence of the exact finite baryon densities on the scalar mass can be described with remarkable accuracy in terms of $\delta\rho_B^{\rm ind}(r)$. We may thus understand the oscillatory structure in ρ_B as arising from an admixture of the periodic state of nuclear matter induced by the perturbing influence of the nuclear surface.

It is important to point out that the spherically symmetric densities shown in Figs. 6–8, do not contradict our interpretation of the periodic state in terms of alpha-particle clustering. The spherical symmetry is an artifact of our initial assumptions and is consistent with the idea that all orientations of a nonsymmetric nucleus have equal energy and therefore we must average over the orientation. The central maximum in Fig. 7 contains exactly one alpha particle, and the fact that the subsequent shells have much lower maximum densities is consistent with the suggestion that these spherically symmetric shells are an orientation average of a collection of alpha particles distributed on a radial shell.

The small scale oscillatory structure shown in Fig. 7 (at 520 MeV) can also be seen in the experimental charge densities and transition densities obtained from electron scattering from closed shell nuclei (refer to Figs. 8 and 9 of Ref. 15). The periodic nature of these oscillations is particularly clear in the transition densities for exciting the first collective octupole vibration in each nucleus (Fig. 9 of Ref. 15). As aforementioned, the spatial frequency of the oscillations can be obtained from the observed period:[16]

$$P=\frac{2\pi}{q}=3.05\ \text{fm}\rightarrow q=2.06\ \text{fm}^{-1}=1.58k_F\ . \quad (4.11)$$

This value is in close agreement with the frequencies that we obtained both from the "exact" calculations with cubic symmetry (Sec. IV B) and the large scale finite nucleus calculations with spherical symmetry (Sec. IV C).

V. SUMMARY

The first and most obvious conclusion from this work is that the standard assumption that the mean-field ground state of infinite nuclear matter is spatially uniform and can be described in terms of plane wave nuclear eigenfunctions is not necessarily valid. We have shown that in the relativistic mean-field model the ground state of nuclear matter is not uniform for scalar masses greater than about 690 MeV, and that the nonuniformity can be understood in terms of alpha-particle clustering. When the attractive interaction is short ranged, the energy of the system is lowered by separating the nuclear matter into relatively isolated regions of high density (α clusters). Furthermore, there is some evidence for this periodic structure in electron scattering data. In addition, the oscillatory structure present in the densities obtained in relativistic Hartree calculations of closed shell nuclei is indirectly related to the periodic instability of nuclear matter. Our results also suggest that the presence of this periodic state may help to correct certain problems within the relativistic mean-field model, i.e., the predicted instability of the ^{40}Ca ground state and the high value for the predicted nuclear compressibility.

Finally, our most important conclusion is that it is inappropriate to use the uniform state of infinite nuclear matter to adjust the model parameters without at least considering the possibility of nonuniform solutions. In addition, since the saturation data are extracted from real finite nuclei where the nuclear surface is expected to induce some periodicity, it may be unwise to use uniform nuclear matter to fix the coupling constants even when the uniform state of nuclear matter is stable. We would suggest that the only reliable way to determine the model parameters is to adjust them so that the model reproduces the bulk propreties of real nuclei.

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