Local gauge invariance of nonlocal interactions

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The implication of invariance under local gauge transformation is investigated for electromagnetic interactions of nucleons and pions, which are particles extended in space time. It is shown that it is possible to derive electromagnetic interactions that are nonlocal and at the same time maintain local gauge invariance. Unlike the line current distribution assumed in the literature, the obtained current has a more flexible form that allows phenomenological analyses. It is proven that the current operators satisfy Ward-Takahashi equations for the nucleon, pion, and pion-nucleon vertex.

I. INTRODUCTION

In recent years field-theoretical approaches to nuclear systems have become popular, and point interactions are widely used for the description of nuclei. The quantum hadrodynamics, as it might be called, does not rest on firm foundations, however. Local theory integrates intermediate states of such high momentum that one cannot expect the description in terms only of hadronic degrees of freedom to be valid any longer. The nucleon is not a pointlike particle but is a composite particle with an inner structure. If we want to formulate consistent equations that contain only the contributions from ranges of momentum relevant to hadronic sizes, cutoffs or form factors should be included at interaction vertices. If we consider a nucleon making an electromagnetic transition from the state with four momentum p to the state with p', the form factors describing the electromagnetic structure of the nucleon depend not only on the momentum transfer squared $(p'-p)^2$ but also on p^2 and p'^2 since interacting nucleons go off the mass shell ($p^2 \neq -m^2$, $p'^2 \neq -m^2$, with m being the nucleon mass). That is, the most general electromagnetic vertex contains a sum of Lorentz invariants multiplied by form factors that depend on the virtual masses of all three legs. This fact has long been known[1-9] but information on the variation of the form factors with respect to the virtual nucleon masses is very limited both experimentally and theoretically. The usual treatment involves the assumption that the nucleons are on the mass shell, and that there remain two form factors F_1 and F_2, which can be known from experiment as functions of the virtual photon mass. The necessity of the off-shell form factors is recognized by Berends and West[4] concerning pion electroproduction, and it is shown that F_1 and F_2 alone are not sufficient to satisfy the constraints of gauge invariance, i.e., Ward-Takahashi identities.[10] Nyman[5] calculated the dependence on one of the nucleon masses using a sidewise dispersion-relation technique first developed by Bincer.[2] Off-mass-shell effects on nuclear interaction currents are discussed by Thakur.[6] Recently, Naus and Koch[9] investigated the half-off-shell form factors that emerge from radiative contributions in a local field theory.

The fact that the electromagnetic vertex of the nucleon depends on nucleon momentum unavoidably implies that the interaction is nonlocal in the nucleon-position space. In nonlocal interaction Lagrangians, the nucleon field operators $\psi, \bar{\psi}$, and the electromagnetic field A_μ do not appear at the same space-time point, but are taken at three different points over a finite region, its size being determined by the space-time extension of the nucleon. Namely, annihilation and creation of the nucleon are not limited to a point but to dimensions of the order of the nucleon size. The idea of nonlocal interactions is very old and was introduced to circumvent divergent terms in local theory of the electron, but after the success of the idea of renormalization nonlocal field theories become obsolete. Unlike the electron, however, one should take account of the fact that the matter distribution within hadrons is spatially extended. For a consistent description of hadron systems, nonlocality is one of the essential ingredients, and nonlocal theory for hadrons is called for.

A fundamental question concerning the electromagnetic interaction of extended objects is whether the invariance under the simultaneous transformations of the photon field

$$A_\mu(x) \to A_\mu(x) + \frac{\partial \Lambda(x)}{\partial x_\mu} , \tag{1.1}$$

and the matter field, e.g., the nucleon field,

$$\psi(x) \to e^{ie\Lambda(x)}\psi(x) , \tag{1.2}$$

is preserved. The gauge transformation of the second kind, (1.1) and (1.2), is also called *local* gauge transformation because the gauge function Λ is evaluated at the spacetime x, which is the position of the nucleon field. The local gauge invariance does not necessarily mean that the charge density associated with a nucleon is a point charge. Nishijima[11,12] emphasized that the Ward-Takahashi equations are valid both for elementary particles and for composite particles (i.e., extended particles). He showed that as long as the field operator satisfies the equal-time commutator with the charge density operator $j_0(z)$,

$$[\psi(x), j_0(z)]\delta(x_0 - z_0) = e\psi(x)\delta(x - z) , \tag{1.3}$$

with e being the electric charge, the Ward-Takahashi equation follows irrespectively of the existence of Lagrangians, and that the composite field operator satisfies (1.3). As is clear from his argument, the Ward-Takahashi equations rest upon the local gauge invariance, and it is expected that nonlocal interactions can be made gauge invariant. In fact, Chrétien and Peierls[13] proposed a gauge-invariant nonlocal interaction, and derived Bloch's gauge factor,[14] which makes a product of two wave functions of different argument gauge invariant. Bloch's gauge factor replaces the complicated current distribution inside particles by the simplest one, a line current distribution, which is of a very restricted form and is not useful for practical purposes. The local gauge invariance is not an academic problem but is of practical importance. There has been much controversy about the form factors to be used in evaluating meson exchange currents. Gross and Riska[15] showed that as long as electromagnetic vertex functions of nucleon and meson satisfy the Ward-Takahashi identities, there should be no constraints both on electromagnetic and hadronic form factors. The present author[16] also showed that the result of Gross and Riska can be derived using the standard minimal-substitution prescription that ensures local gauge invariance *a priori*. For a clearer understanding of the use of form factors in hadron interactions, it is desired that the implication of local gauge invariance for extended particles should be elucidated.

A consistent nonlocal field theory is yet to be established. The nonlocal interactions of the type investigated by Chrétien and Peierls did not succeed in removing divergences from field theory. Our aim of this paper is not to construct a quantum field theory but to study the physical implication of local gauge invariance of nonlocal interactions. The action function of Chrétien and Peierls is employed for this purpose. This paper is organized as follows: In Sec. II we modify the action principle developed by Chrétien and Peierls and derive a new electromagnetic interaction of nucleon. In Sec. III we consider an action principle for pion and for the pion-nucleon vertex. Section IV is devoted to a brief summary of this paper.

II. ELECTROMAGNETIC INTERACTION OF NUCLEON

A. Gauge-invariant action function

For a particle with a small but finite space-time extension, we assume the action function

$$I=-\int d^4x'd^4x\,\bar{\psi}(x')[\gamma\cdot\partial_{x'}g(x'-x)+mf(x'-x)]\psi(x)\ , \qquad (2.1)$$

which is nonlocal in the sense that the field operators $\bar{\psi}(x')$ and $\psi(x)$ are taken at two different points. The action function is an integral smeared out by scalar functions $f(x'-x)$ and $g(x'-x)$ depending only on the invariant distance between the two points, $(x'-x)^2$. The smearing function (also called as form function or averaging function) must fall off rapidly for $(x'-x)^2$ larger than r_0^2, where r_0 is the size of the particle.[17] Chrétien and Peierls[13] examined the case in which f and g are identical functions. Here we consider a more general case with $f\neq g$. The Lorentz invariance is preserved by this generalization. Local theory is recovered by replacing the smearing functions $f(x'-x)$ and $g(x'-x)$ by δ functions. The equation of motion becomes

$$\int d^4x[g(x'-x)\gamma\cdot\partial_x+mf(x'-x)]\psi(x)=0\ . \qquad (2.2)$$

In momentum space, we get

$$[i\gamma\cdot p\tilde{g}(p^2)+m\tilde{f}(p^2)]\psi(p)=0 \qquad (2.3)$$

with

$$f(x'-x)=\frac{1}{(2\pi)^4}\int d^4p e^{ip(x'-x)}\tilde{f}(p^2) \qquad (2.4)$$

and

$$g(x'-x)=\frac{1}{(2\pi)^4}\int d^4p e^{ip(x'-x)}\tilde{g}(p^2)\ . \qquad (2.5)$$

The Feynman propagator in momentum space is given by

$$\gamma\cdot\partial_{x'}(x'-x)+mf(x'-x)=\frac{1}{(2\pi)^4}\int d^4p e^{ip(x'-x)}S_F'^{-1}(p) \qquad (2.6)$$

with

$$S_F'(p)=[i\gamma\cdot p\tilde{g}(p^2)+m\tilde{f}(p^2)]^{-1}\ . \qquad (2.7)$$

Thus the effect of the smearing functions is to bring about a dressing of the nucleon. A prime means a dressed propagator of the nucleon. Sine the condition

$$(i\gamma\cdot p+m)S_F'(p)\to 1 \qquad (2.8)$$

is required for $i\gamma\cdot p+m\to 0$, we have two conditions,

$$\tilde{f}(-m^2)=\tilde{g}(-m^2) \qquad (2.9)$$

and

$$\tilde{g}(-m^2)+2m^2[\tilde{f}'(-m^2)-\tilde{g}'(-m^2)]=1\ , \qquad (2.10)$$

where

$$\tilde{f}'(-m^2)=\frac{d}{dp^2}\tilde{f}(p^2)\Big|_{p^2=-m^2}\ , \qquad (2.11)$$

$$\tilde{g}'(-m^2)=\frac{d}{dp^2}\tilde{g}(p^2)\Big|_{p^2=-m^2}\ . \qquad (2.12)$$

On account of (2.9), Eq. (2.3) admits the solutions of the free Dirac equation.

In the presence of an external electromagnetic field A_μ, we have to derive an action function which is invariant under the gauge transformations (1.1) and (1.2). According to Chrétien and Peierls, we find the gauge-invariant action function in the form

$$I^e=-\int d^4x'd^4x\,\bar{\psi}(x')e^{-ie\chi(x',x)}\times[\gamma\cdot\partial_{x'}g(x'-x)+mf(x'-x)]\psi(x)\ , \qquad (2.13)$$

where

$$\chi(x',x)=\int d^4z F_\mu(x'x:z)A_\mu(z) . \tag{2.14}$$

Since χ is transformed as

$$\chi(x',x)\to\chi(x',x)-\int d^4z\Lambda(z)\frac{\partial}{\partial z_\mu}F_\mu(x'x:z) , \tag{2.15}$$

gauge invariance requires that

$$\frac{\partial}{\partial z_\mu}F_\mu(x'x:z)=\delta(x'-z)-\delta(x-z) . \tag{2.16}$$

Only one example of F_μ is known. If one takes

$$F_\mu(x'x:z)=(x-x')_\mu\int_0^1 ds\,\delta((1-s)x'+sx-z) , \tag{2.17}$$

the gauge factor becomes

$$\chi(x',x)=(x-x')_\mu\int_0^1 ds\,A_\mu((1-s)x'+sx) , \tag{2.18}$$

in agreement with that proposed by Bloch,[14]

$$\chi(x',x)=\int_{x'}^{x}A_\mu(z)dz_\mu , \tag{2.19}$$

the integral to be taken over the straight line in space time joining x and x'. Bloch's gauge factor is a convenient devise of preserving gauge invariance, and yet it is far from being realistic in view of the fact that the current distribution inside the particle is presumably much more complicated than a line current. The purpose of this paper is to look for a more realistic form of the gauge factor.

Before presenting our electromagnetic interaction, it is important to note the two points: The first is that the action function (2.13) is by no means unique. We can freely add to it a term such that

$$I^e=-\int d^4x'd^4x\,\bar\psi(x')e^{-ie\chi(x',x)}[\gamma\cdot\partial_{x'}(x'-x)+mf(x'-x)]\psi(x) \\ +i\int d^4x'd^4xd^4z\,\bar\psi(x')e^{-ie\chi(x',x)}\Delta\Gamma_\mu(x'x:z)\psi(x)A_\mu(z) , \tag{2.20}$$

the added term being gauge invariant by itself insofar as the condition

$$\frac{\partial}{\partial z_\mu}\Delta\Gamma_\mu(x'x:z)=0 \tag{2.21}$$

is satisfied. The other point is that the electromagnetic current associated with the assumed action function satisfies the Ward-Takahashi identity. To see this we expand the exponential in the action function in powers of e,

$$I^e=I+i\int d^4x'd^4xd^4z\,\bar\psi(x')\Gamma_\mu(x'x:z)\psi(x)A_\mu(z)+\cdots . \tag{2.22}$$

The current operator becomes

$$\Gamma_\mu(x'x:z)=eF_\mu(x'x:z)[\gamma\cdot\partial_{x'}g(x'-x)+mf(x'-x)]+\Delta\Gamma_\mu(x'x:z) . \tag{2.23}$$

From the gauge-invariance conditions (2.16) and (2.21), one easily sees that

$$\frac{\partial}{\partial z_\mu}\Gamma_\mu(x'x:z)=e[\delta(x'-z)-\delta(x-z)][\gamma\cdot\partial_{x'}g(x'-x)+mf(x'-x)] . \tag{2.24}$$

We go over to momentum space using (2.6) and

$$\Gamma_\mu(x'x:z)=\frac{-i}{(2\pi)^8}\int d^4p'd^4p\,\exp[ip'(x'-z)+ip(z-x)]j_\mu(p',p) . \tag{2.25}$$

From Eq. (2.24) we get the Ward-Takahashi identity

$$(p'-p)_\mu j_\mu(p',p)=e[S_F'^{-1}(p')-S_F'^{-1}(p)] . \tag{2.26}$$

B. Local representation

In the integrands of Eqs. (2.4) and (2.5), we can replace p by the gradient $i\partial_x$ acting on the exponential, and hence $\tilde f(p^2)$ and $\tilde g(p^2)$ by $\tilde f(-\partial_x^2)$ and $\tilde g(-\partial_x^2)$, respectively. As a result, we can take them out from the integrands to obtain the representation

$$f(x'-x)=\tilde f(-\partial_x^2)\delta(x'-x) , \tag{2.27}$$

$$g(x'-x)=\tilde g(-\partial_x^2)\delta(x'-x) . \tag{2.28}$$

With these inserted into (2.1), I becomes

$$I=-\int d^4x'd^4x\,\bar\psi(x')[\tilde g(-\partial_x^2)\delta(x'-x)]\gamma\cdot\partial_x\psi(x) \\ -m\int d^4x'd^4x\,\bar\psi(x')[\tilde f(-\partial_x^2)\delta(x'-x)]\psi(x) . \tag{2.29}$$

Integrating by parts, we can convert the gradient acting on the δ function into the momentum operator acting on $\psi(x)$,

$$\partial_x\psi(x)=ip\psi(x) , \tag{2.30}$$

with the result

$$I=-\int d^4x\,\bar\psi(x)[i\gamma\cdot p\tilde g(p^2)+m\tilde f(p^2)]\psi(x) \\ =-\int d^4x\,\bar\psi(x)S_F'^{-1}(p)\psi(x) . \tag{2.31}$$

We use the same notation p both for the c number in momentum space and the gradient operator in position space.

The preceding procedure has converted a nonlocal action into a local, momentum-dependent form. A gauge invariant action function can be constructed straightforwardly from the local representation (2.31) by the minimal-substitution prescription

$$I^e = -\int d^4x\, \bar{\psi}(x) S_F'^{-1}(p-eA)\psi(x) , \tag{2.32}$$

where

$$\begin{aligned} S_F'^{-1}(p-eA) = {} & \tfrac{1}{2} i\gamma\cdot(p-eA)\tilde{g}((p-eA)^2) \\ & + \tfrac{1}{2}\tilde{g}((p-eA)^2) i\gamma\cdot(p-eA) \\ & + m\tilde{f}((p-eA)^2) . \end{aligned} \tag{2.33}$$

Owing to the identity

$$e^{-ie\Lambda(x)} S_F'^{-1}(p-eA(x)-e\partial_x\Lambda(x)) e^{ie\Lambda(x)} = S_F'^{-1}(p-eA) , \tag{2.34}$$

the gauge invariance of (2.32) is guaranteed.

The action function thus obtained is in conformity with (2.20) as we shall see soon. The forms of F_μ and $\Delta\Gamma_\mu$ can be found by comparing the first order terms between (2.20) and (2.32). To this end we use a formal Taylor expansion

$$\tilde{g}(p^2) = \sum_n c_n p^{2n} \tag{2.35}$$

and write

$$\tilde{g}((p-eA)^2) = \sum_n c_n (p^2 - ep\cdot A - eA\cdot p + e^2A^2)^n . \tag{2.36}$$

We make an expansion in terms of e and retain the linear term

$$-e\sum_n c_n [p^{2(n-1)}(p\cdot A + A\cdot p) + p^{2(n-2)}(p\cdot A + A\cdot p)p^2 + \cdots + (p\cdot A + A\cdot p)p^{2(n-1)}] . \tag{2.37}$$

The gradient operators standing on the left of A can be made the momentum operator acting on $\bar{\psi}(x)$,

$$i\partial_x \bar{\psi}(x) = \bar{\psi}(x)p' . \tag{2.38}$$

Therefore, (2.37) becomes

$$\begin{aligned} -e(p'+p)\cdot A \sum_n c_n [p'^{2(n-1)} + p'^{2(n-2)}p^2 + \cdots + p^{2(n-1)}] &= -e(p'+p)\cdot A \sum_n c_n \frac{p'^{2n}-p^{2n}}{p'^2-p^2} \\ &= -e(p'+p)\cdot A \frac{\tilde{g}(p'^2)-\tilde{g}(p^2)}{p'^2-p^2} . \end{aligned} \tag{2.39}$$

In going from the first to the second line we used (2.35). In exactly the same way we expand $\tilde{f}((p-eA)^2)$ with the linear term

$$-e(p'+p)\cdot A \frac{\tilde{f}(p'^2)-\tilde{f}(p^2)}{p'^2-p^2} . \tag{2.40}$$

Consequently, the linear term of (2.33) is given by

$$-\tfrac{1}{2} ie\gamma\cdot A[\tilde{g}(p'^2)+\tilde{g}(p^2)] - \tfrac{1}{2} ie\gamma\cdot(p'+p)(p'+p)\cdot A \frac{\tilde{g}(p'^2)-\tilde{g}(p^2)}{p'^2-p^2} - me(p'+p)\cdot A\frac{\tilde{f}(p'^2)-\tilde{f}(p^2)}{p'^2-p^2} , \tag{2.41}$$

and hence the linear term of (2.32) by

$$-\int d^4x\, d^4z\, \bar{\psi}(x) \left[\frac{\delta}{\delta A_\mu(z)} S_F'^{-1}(p-eA) \right]_{A\to 0} \psi(x) A_\mu(z) . \tag{2.42}$$

Thus the current operator in the local representation is given by the functional derivative

$$\begin{aligned} \left[\frac{\delta}{\delta A_\mu(z)} S_F'^{-1}(p-eA) \right]_{A\to 0} = {} & -e\frac{(p'+p)_\mu}{p'^2-p^2}[S_F'^{-1}(p') - S_F'^{-1}(p)]\delta(x-z) \\ & -\tfrac{1}{2} ie[\tilde{g}(p'^2)+\tilde{g}(p^2)] \left[\gamma_\mu - \frac{(p'+p)_\mu}{p'^2-p^2}\gamma\cdot(p'-p) \right] \delta(x-z) . \end{aligned} \tag{2.43}$$

We now return to nonlocal theory by inserting $\int d^4x' \delta(x'-x)$ into (2.42),

$$-\int d^4x' d^4x\, d^4z\, \bar{\psi}(x')\delta(x'-x) \left[\frac{\delta}{\delta A_\mu(z)} S_F'^{-1}(p-eA) \right]_{A\to 0} \psi(x) A_\mu(z) , \tag{2.44}$$

and restoring the gradient operator p' to that acting on $\bar{\psi}(x')$,

$$\bar{\psi}(x')p' = i\partial_{x'}\bar{\psi}(x') \ . \tag{2.45}$$

The electromagnetic interaction is identified with

$$\Gamma_\mu(x'x:z) = i\delta(x'-x)\left[\frac{\delta}{\delta A_\mu(z)} S_F'^{-1}(p-eA)\right]_{A\to 0} . \tag{2.46}$$

From Eq. (2.6), it follows that

$$\begin{aligned} S_F'^{-1}(p')\delta(x'-x)\delta(x-z) &= [S_F'^{-1}(-i\partial_{x'})\delta(x'-x)]\delta(x-z) \\ &= \delta(x-z)[\gamma\cdot\partial_{x'}g(x'-x)+mf(x'-x)] \ , \end{aligned} \tag{2.47}$$

$$\begin{aligned} S_F'^{-1}(p)\delta(x'-x)\delta(x-z) &= [S_F'^{-1}(i\partial_x)\delta(x'-x)]\delta(x'-z) \\ &= \delta(x'-z)[\gamma\cdot\partial_{x'}g(x'-x)+mf(x'-x)] \ . \end{aligned} \tag{2.48}$$

After some algebra we find

$$\begin{aligned} \Gamma_\mu(x'x:z) &= ie\frac{(p'+p)_\mu}{p'^2-p^2}[\delta(x'-z)-\delta(x-z)][\gamma\cdot\partial_{x'}g(x'-x)+mf(x'-x)] \\ &\quad + \tfrac{1}{2}e[\tilde{g}(p'^2)+\tilde{g}(p^2)]\left[\gamma_\mu - \frac{(p'+p)_\mu}{p'^2-p^2}\gamma\cdot(p'-p)\right]\delta(x'-z)\delta(x-z) \ . \end{aligned} \tag{2.49}$$

Consequently, it is found that (2.32) is indeed consistent with (2.20), and we get

$$F_\mu(x'x:z) = i\frac{(p'+p)_\mu}{p'^2-p^2}[\delta(x'-z)-\delta(x-z)] \tag{2.50}$$

and

$$\Delta\Gamma_\mu(x'x:z) = \tfrac{1}{2}e[\tilde{g}(p'^2)+\tilde{g}(p^2)]\left[\gamma_\mu - \frac{(p'+p)_\mu}{p'^2-p^2}\gamma\cdot(p'-p)\right]\delta(x'-z)\delta(x-z) \ . \tag{2.51}$$

It is straightforward to see that they satisfy the requirements (2.16) and (2.21) using

$$\frac{\partial}{\partial z_\mu}[\delta(x'-z)-\delta(x-z)] = -i(p'-p)_\mu[\delta(x'-z)-\delta(x-z)] \ , \tag{2.52}$$

$$\frac{\partial}{\partial z_\mu}\delta(x'-z)\delta(x-z) = -i(p'-p)_\mu\delta(x'-z)\delta(x-z) \ . \tag{2.53}$$

These can be proved as follows: We replace $\partial/\partial z_\mu$ by $\partial/\partial x'_\mu$ and $\partial/\partial x_\mu$. Integrations by parts put $\partial/\partial x'_\mu$ and $\partial/\partial x_\mu$ on to $\bar{\psi}(x')$ and $\psi(x)$, thereby making them the momentum operators p' and p, respectively.

C. Electromagnetic form factors of nucleon

In momentum space, the current operator (2.49) becomes

$$j_\mu(p',p) = e\frac{(p'+p)_\mu}{p'^2-p^2}\left[S_F'^{-1}(p')-S_F'^{-1}(p)\right] + \tfrac{1}{2}ie[\tilde{g}(p'^2)+\tilde{g}(p^2)]\left[\gamma_\mu - \frac{(p'+p)_\mu}{p'^2-p^2}\gamma\cdot(p'-p)\right] . \tag{2.54}$$

Rearranging terms leads to

$$j_\mu(p',p) = \tfrac{1}{2}ie[\tilde{g}(p'^2)+\tilde{g}(p^2)]\gamma_\mu + e(p'+p)_\mu\frac{\tilde{f}(p'^2)-\tilde{f}(p^2)}{p'^2-p^2} + \tfrac{1}{2}ie(p'+p)_\mu\gamma\cdot(p'+p)\frac{\tilde{g}(p'^2)-\tilde{g}(p^2)}{p'^2-p^2} \ , \tag{2.55}$$

which completely coincides with the longitudinal current proposed by Kusno.[8] He deduced the current by solving directly the Ward-Takahashi identity (2.26) under the condition that the current be free of kinematical singularities. Our method reveals that the longitudinal current follows from applying the minimal-substitution prescription to the inverse propagator of the nucleon. As is emphasized in Sec. II A, the current (2.54) is undetermined up to a term that is gauge invariant by itself. We can modify (2.54) in such a way that its divergence is unchanged,

$$\begin{aligned} j_\mu(p',p) &= e\left[F_0(p'+p)_\mu-(F_0-1)\frac{k\cdot(p'+p)}{k^2}k_\mu\right]\frac{S_F'^{-1}(p')-S_F'^{-1}(p)}{p'^2-p^2} \\ &\quad + \tfrac{1}{2}ieG_0[\tilde{g}(p'^2)+\tilde{g}(p^2)]\left[\gamma_\mu - \frac{(p'+p)_\mu}{p'^2-p^2}\gamma\cdot(p'-p)\right] , \end{aligned} \tag{2.56}$$

where F_0 and G_0 are arbitrary functions of p'^2, p^2, and $k^2=(p'-p)^2$. The electromagnetic vertex function of the nucleon has the most general form[16]

$$j_\mu(p',p)=iF_1\gamma_\mu-iF_2\sigma_{\mu\nu}k_\nu+F_3k_\mu+(i\gamma\cdot p'+m)(iF_4\gamma_\mu-iF_5\sigma_{\mu\nu}k_\nu+F_6k_\mu)$$
$$+(iF_7\gamma_\mu-iF_8\sigma_{\mu\nu}k_\nu+F_9k_\mu)(i\gamma\cdot p+m)+(i\gamma\cdot p'+m)(iF_{10}\gamma_\mu-iF_{11}\sigma_{\mu\nu}k_\nu+F_{12}k_\mu)(i\gamma\cdot p+m)\ . \tag{2.57}$$

The twelve form factors are functions of p'^2, p^2, and k^2. The current (2.56) has the form factors

$$F_1=\tfrac{1}{2}eF_0[\tilde{g}(p'^2)+\tilde{g}(p^2)]+\tfrac{1}{2}eF_0\frac{\tilde{g}(p'^2)-\tilde{g}(p^2)}{p'^2-p^2}(p'^2+p^2+2m^2)$$
$$+2m^2eF_0\frac{\tilde{f}(p'^2)-\tilde{f}(p^2)}{p'^2-p^2}-2m^2eF_0\frac{\tilde{g}(p'^2)-\tilde{g}(p^2)}{p'^2-p^2}\ , \tag{2.58}$$

$$F_2=F_4=F_7=-meF_0\frac{\tilde{f}(p'^2)-\tilde{f}(p^2)}{p'^2-p^2}+meF_0\frac{\tilde{g}(p'^2)-\tilde{g}(p^2)}{p'^2-p^2}\ , \tag{2.59}$$

$$F_3=-me(F_0-1)\frac{\tilde{f}(p'^2)-\tilde{f}(p^2)}{p'^2-p^2}+me(F_0-1)\frac{\tilde{g}(p'^2)-\tilde{g}(p^2)}{p'^2-p^2}\ , \tag{2.60}$$

$$F_5=-\tfrac{1}{2}eG_0\frac{\tilde{g}(p'^2)-\tilde{g}(p^2)}{p'^2-p^2}+e(G_0-F_0)\frac{\tilde{g}(p'^2)}{p'^2-p^2}\ , \tag{2.61}$$

$$F_6=-e(F_0-1)\frac{\tilde{g}(p'^2)}{k^2}\ , \tag{2.62}$$

$$F_8=-\tfrac{1}{2}eG_0\frac{\tilde{g}(p'^2)-\tilde{g}(p^2)}{p'^2-p^2}-e(G_0-F_0)\frac{\tilde{g}(p^2)}{p'^2-p^2}\ , \tag{2.63}$$

$$F_9=e(F_0-1)\frac{\tilde{g}(p^2)}{k^2}\ , \tag{2.64}$$

$$F_{10}=-eF_0\frac{\tilde{g}(p'^2)-\tilde{g}(p^2)}{p'^2-p^2}\ . \tag{2.65}$$

On the mass shell, we use the notation $\tilde{g}\equiv\tilde{g}(-m^2)$ and $\tilde{g}'\equiv\tilde{g}'(-m^2)$. From the condition (2.10), $\tilde{f}'\equiv\tilde{f}'(-m^2)$ is given by

$$\tilde{f}'=\tilde{g}'-\frac{\tilde{g}-1}{2m^2}\ . \tag{2.66}$$

In the limit of on-shell nucleon we obtain the relevant form factors,

$$F_1=eF_0\ , \tag{2.67}$$

$$F_2=eF_0\frac{\tilde{g}-1}{2m}\ , \tag{2.68}$$

where we have used (2.66). Since F_5 and F_8 contain kinematical singularities at $p'^2=p^2$, we have to be careful in taking the on-shell limit. First we put p on shell ($p^2=-m^2$ and $i\gamma\cdot p+m=0$) and take the limit $p'^2=-m^2$ and $i\gamma\cdot p'+m=0$. The F_8 term vanishes and the F_5 term gives the contribution

$$-ie(G_0-F_0)\frac{\tilde{g}(p'^2)}{p'^2+m^2}(i\gamma\cdot p'+m)\sigma_{\mu\nu}k_\nu\to-ie\frac{G_0-F_0}{2m}\tilde{g}\sigma_{\mu\nu}k_\nu\ . \tag{2.69}$$

(If we put p' on shell first, the F_5 term vanishes and the F_8 term gives the identical contribution as above.) Consequently, we obtain the standard form

$$j_\mu(p',p)=ieF_0\gamma_\mu-i\frac{\tilde{g}G_0-F_0}{2m}\sigma_{\mu\nu}k_\nu\ . \tag{2.70}$$

It turned out that $F_0(-m^2,-m^2,0)=1$ and $\tilde{g}G_0(-m^2,-m^2,0)$ is the total magnetic moment in units of the nuclear magneton. The nonlocality affects only the magnetic moment of on-shell nucleons.

In the limit of local theory, nonvanishing form factors are

$$F_1=eF_0\ , \tag{2.71}$$

$$F_5=-F_8=e\frac{G_0-F_0}{p'^2-p^2}\ , \tag{2.72}$$

$$F_6=-F_9=-e\frac{F_0-1}{k^2}\ , \tag{2.73}$$

and (2.56) is reduced to

$$j_\mu(p',p)=ie(F_0-1)\left[\gamma_\mu-\frac{k_\mu}{k^2}\gamma\cdot k\right]+ie\gamma_\mu-ie(G_0-F_0)\left[\frac{i\gamma\cdot p'+m}{p'^2-p^2}\sigma_{\mu\nu}k_\nu-\sigma_{\mu\nu}k_\nu\frac{i\gamma\cdot p+m}{p'^2-p^2}\right]\ . \tag{2.74}$$

This coincides with Gross and Riska,[15] and with Berends and West,[4] apart from the last term which is reduced to

$$-ie\frac{G_0-F_0}{2m}\sigma_{\mu\nu}k_\nu \tag{2.75}$$

for on-shell nucleons. The term proportional to k_μ arises from the Ward-Takahashi identity. In contrast to the line current implied by Bloch's gauge factor, our current has a freedom of choosing arbitrary form factors F_0 and G_0. Gauge invariance imposes no constraint on the form of F_0 and G_0.

III. EXTENDED PION-NUCLEON VERTEX

A. Electromagnetic interaction of pion

It is straightforward to extend the approach developed in Sec. II to a description of the electromagnetic interaction of charged mesons. The nonelectromagnetic action function is taken to be

$$I=-\int d^4y'd^4y\varphi_i(y')(-\partial_{y'}^2+\mu^2)f_\pi(y'-y)\varphi_j(y)\ , \tag{3.1}$$

where μ is the meson mass and $f_\pi(y'-y)$ is the smearing function. There is no need of introducing two smearing functions. The Feynman propagator is given by

$$(-\partial_{y'}^2+\mu^2)f_\pi(y'-y)=\frac{1}{(2\pi)^4}\int d^4q e^{iq(y'-y)}\Delta_F'^{-1}(q)\ , \tag{3.2}$$

with

$$\Delta_F'^{-1}(q)=(q^2+\mu^2)\tilde{f}_\pi(q^2)\ . \tag{3.3}$$

The condition

$$(q^2+\mu^2)\Delta_F'^{-1}(q)\rightarrow 1 \tag{3.4}$$

for $q^2+\mu^2\rightarrow 0$ leads to $\tilde{f}_\pi(-\mu^2)=1$.

Under the gauge transformation the meson field operator with isospin index i undergoes the rotation about the z axis in the isospin space,

$$\varphi_i\rightarrow R_{ij}(e\Lambda)\varphi_j\ , \tag{3.5}$$

where R is the rotation matrix and $e\Lambda$ is the rotation angle. The gauge-invariant action function is taken to be

$$I^e=-\int d^4y'd^4y\varphi_i(y')R_{ij}[-e\chi_\pi(y',y)]\times(-\partial_{y'}^2+\mu^2)f_\pi(y'-y)\varphi_j(y)\ , \tag{3.6}$$

where

$$\chi_\pi(y',y)=\int d^4zF_\mu^\pi(y'y:z)A_\mu(z)\ . \tag{3.7}$$

Gauge invariance is ensured by the condition

$$\frac{\partial}{\partial z_\mu}F_\mu^\pi(y'y:z)=\delta(y'-z)-\delta(y-z)\ . \tag{3.8}$$

Using expansion in powers of e,

$$R_{ij}[-e\chi_\pi(y',y)]=\delta_{ij}-e\epsilon_{3ij}\chi_\pi(y',y)+\cdots\ , \tag{3.9}$$

we find

$$I^e=I+i\int d^4y'd^4yd^4z\varphi_i(y')\Gamma_\mu^{\pi ij}(y'y:z)\times\varphi_j(y)A_\mu(z)+\cdots\ , \tag{3.10}$$

with the electromagnetic interaction

$$\Gamma_\mu^{\pi ij}(y'y:z)=-ie\epsilon_{3ij}F_\mu^\pi(y'y:z)(-\partial_{y'}^2+\mu^2)f_\pi(y'-y)\ . \tag{3.11}$$

It should be noted that we can add a term which is gauge invariant by itself. In momentum space,

$$\Gamma_\mu^{\pi ij}(y'y:z)=\frac{-i}{(2\pi)^8}\int d^4q'd^4q\exp[iq'(y'-z)+iq(z-y)]\times j_\mu^{\pi ij}(q',q)\ , \tag{3.12}$$

and again we get the Ward-Takahashi identity

$$(q'-q)_\mu j_\mu^{\pi ij}(q',q)=-ie\epsilon_{3ij}[\Delta_F'^{-1}(q')-\Delta_F'^{-1}(q)]\ . \tag{3.13}$$

Following the procedure of Sec. II, we write (3.1) in the local, momentum-dependent form, apply the minimal replacement, and obtain

$$j_\mu^{\pi ij}(q',q)=-ie\epsilon_{3ij}\Gamma_\mu^\pi(q',q) \tag{3.14}$$

with

$$\Gamma_\mu^\pi(q',q)=\frac{(q'+q)_\mu}{q'^2-q^2}[\Delta_F'^{-1}(q')-\Delta_F'^{-1}(q)]\ . \tag{3.15}$$

After adding a term

$$\Delta\Gamma_\mu^\pi(q',q)=(F_\pi-1)\left[(q'+q)_\mu-\frac{k\cdot(q'+q)}{k^2}k_\mu\right]\times\frac{\Delta_F'^{-1}(q')-\Delta_F'^{-1}(q)}{q'^2-q^2} \tag{3.16}$$

which is gauge invariant by itself, we obtain

$$\Gamma_\mu^\pi(q',q)=\left[F_\pi(q'+q)_\mu-(F_\pi-1)\frac{k\cdot(q'+q)}{k^2}k_\mu\right]\times\frac{\Delta_F'^{-1}(q')-\Delta_F'^{-1}(q)}{q'^2-q^2}\ . \tag{3.17}$$

The form factor F_π is an arbitrary function of q'^2, q^2, and $k^2=(q'-q)^2$.

B. Gauge-invariant pion-nucleon interaction

Let us consider an action function for the pion-nucleon interaction.

$$I_{\rm int}=\int d^4x'd^4xd^4y\bar\psi(x')\tau_i\Gamma(x'x:y)\psi(x)\varphi_i(y) \ , \qquad (3.18)$$

in which field operators are taken at three different points. When the external electromagnetic field is acting, we have to modify this action function in such a way that it is invariant under the gauge transformation. We propose the form

$$I^e_{\rm int}=\int d^4x'd^4xd^4y\mathcal{L}_{\rm int} \qquad (3.19)$$

with

$$\mathcal{L}_{\rm int}=\bar\psi(x')\exp[-ie_N\phi(x',y)]\tau_i\Gamma(x'x:y)\times\exp[ie_N\phi(x,y)]\psi(x)\varphi_i(y) \qquad (3.20)$$

and

$$\phi(x,y)=\int d^4zG_\mu(xy:z)A_\mu(z) \ . \qquad (3.21)$$

In investigating the pion-nucleon interaction we have to introduce the nucleon charge operator as

$$e_N=\tfrac{1}{2}e(1+\tau_3) \ . \qquad (3.22)$$

We now prove that under the condition

$$\frac{\partial}{\partial z_\mu}G_\mu(xy:z)=\delta(x-z)-\delta(y-z) \qquad (3.23)$$

the action function $I^e_{\rm int}$ is gauge invariant. The gauge transformation modifies $\phi(x,y)$ and $\phi(x',y)$ into

$$\phi'(x,y)=\phi(x,y)-\Lambda(x)+\Lambda(y) \ , \qquad (3.24)$$

$$\phi'(x',y)=\phi(x',y)-\Lambda(x')+\Lambda(y) \ , \qquad (3.25)$$

respectively, and $\mathcal{L}_{\rm int}$ into

$$\mathcal{L}'_{\rm int}=\bar\psi(x')\exp[-ie_N\phi'(x',y)-ie_N\Lambda(x')]\tau_i\Gamma(x'x:y)\exp[ie_N\phi'(x,y)+ie_N\Lambda(x)]\psi(x)R_{ij}(e\Lambda(y))\varphi_j(y) \ . \qquad (3.26)$$

Using (3.24) and (3.25), we find

$$\mathcal{L}'_{\rm int}=\bar\psi(x')\exp[-ie_N\phi(x',y)-ie_N\Lambda(y)]\tau_i\Gamma(x'x:y)\exp[ie_N\phi(x,y)+ie_N\Lambda(y)]\psi(x)R_{ij}(e\Lambda(y))\varphi_j(y) \ . \qquad (3.27)$$

Now we can make use of the formula

$$e^{-ie_N\Lambda(y)}\tau_ie^{ie_N\Lambda(y)}=R_{ik}(e\Lambda(y))\tau_k \ , \qquad (3.28)$$

and the orthogonality condition of the rotation matrix

$$R_{ik}(e\Lambda(y))R_{ij}(e\Lambda(y))=\delta_{kj} \ , \qquad (3.29)$$

to prove the required gauge invariance, $\mathcal{L}'_{\rm int}=\mathcal{L}_{\rm int}$.

As before we can add a term which is gauge invariant by itself,

$$I^e_{\rm int}=\int d^4x'd^4xd^4y\mathcal{L}_{\rm int}+\int d^4x'd^4xd^4yd^4z\bar\psi(x')\exp[-ie_N\phi(x',y)]M'^i_\mu(x'x:yz)\exp[ie_N\phi(x,y)]\psi(x)\varphi_i(y)A_\mu(z) \ . \qquad (3.30)$$

The added term is indeed gauge invariant under the condition

$$\frac{\partial}{\partial z_\mu}M'^i_\mu(x'x:yz)=0 \ . \qquad (3.31)$$

The proof goes as follows: First decompose the operator M'^i_μ into three parts,

$$M'^i_\mu=\delta_{3i}M'^{i(+)}_\mu-i\epsilon_{3ij}\tau_jM'^{i(-)}_\mu+\tau_iM'^{i(0)}_\mu \ . \qquad (3.32)$$

By the use of identities

$$e^{-ie_N\Lambda(y)}\delta_{3i}e^{ie_N\Lambda(y)}=R_{ij}(e\Lambda(y))\delta_{3j} \ , \qquad (3.33)$$

$$e^{-ie_N\Lambda(y)}\epsilon_{3ij}\tau_je^{ie_N\Lambda(y)}=R_{ik}(e\Lambda(y))\epsilon_{3kn}\tau_n \ , \qquad (3.34)$$

and (3.28), one immediately sees that

$$e^{-ie_N\Lambda(y)}M'^i_\mu e^{ie_N\Lambda(y)}=R_{ik}(e\Lambda(y))M'^k_\mu \ , \qquad (3.35)$$

and hence one is led to the gauge invariance of (3.30).

We again expand the action function in a power series of e,

$$I^e_{\rm int}=I_{\rm int}+\int d^4x'd^4x\,d^4y\,d^4z\bar\psi(x')\Delta M^i_\mu(x'x:yz)\psi(x)\varphi_i(y)A_\mu(z)+\cdots \ , \qquad (3.36)$$

where

$$\Delta M^i_\mu(x'x:yz) = -ie_N\tau_i G_\mu(x'y:z)\Gamma(x'x:y) + i\tau_i e_N G_\mu(xy:z)\Gamma(x'x:y) + M'^i_\mu(x'x:yz) \tag{3.37}$$

is the current operator for pion photoproduction. The four-dimensional divergence becomes

$$\frac{\partial}{\partial z_\mu}\Delta M^i_\mu(x'x:yz) = -ie_N\tau_i\delta(x'-z)\Gamma(x'x:y) + i\tau_i e_N\delta(x-z)\Gamma(x'x:y) - e\epsilon_{3ij}\tau_j\delta(y-z)\Gamma(x'x:y) , \tag{3.38}$$

where we have used (3.23), (3.31), and the identity

$$[e_N, \tau_i] = ie\epsilon_{3ij}\tau_j . \tag{3.39}$$

In momentum space, the pion-nucleon vertex function becomes

$$\Gamma(x'x:y) = \frac{-i}{(2\pi)^8}\int d^4p'd^4p \exp[ip'(x'-y)+ip(y-x)]\Gamma(q,p',p) , \tag{3.40}$$

where $q=p'-p$ is the pion momentum. The Fourier transform of ΔM^i_μ is defined by

$$\Delta M^i_\mu(x'x:yz) = \frac{-i}{(2\pi)^{12}}\int d^4p'd^4p\, d^4k \exp[ip'(x'-y)+ip(y-x)+ik(y-z)]\Delta M^i_\mu . \tag{3.41}$$

In momentum space, we suppress the arguments p', p, and k. The pion momentum is given by $q=p+k-p'$. From Eq. (3.38) we find the identity for ΔM^i_μ

$$k_\mu \Delta M^i_\mu = e_N\tau_i\Gamma(q,p'-k,p) - \tau_i e_N\Gamma(q,p',p+k) - ie\epsilon_{3ij}\tau_j\Gamma(q-k,p',p) . \tag{3.42}$$

In the previous paper[16] we deduced ΔM^i_μ from the most general form of the pion-nucleon vertex function using the minimal-substitution prescription, and proved that the obtained ΔM^i_μ indeed satisfies Eq. (3.38). The explicit form of G_μ can be found as

$$G_\mu(xy:z) = i\frac{(2p+i\partial_z)_\mu}{(p+i\partial_z)^2-p^2}\delta(x-z) - \frac{(2\partial_y-\partial_z)_\mu}{\partial_y^2-(\partial_y-\partial_z)^2}\delta(y-z) , \tag{3.43}$$

$$G_\mu(x'y:z) = i\frac{(2p'-i\partial_z)_\mu}{p'^2-(p'-i\partial_z)^2}\delta(x'-z) - \frac{(2\partial_y-\partial_z)_\mu}{\partial_y^2-(\partial_y-\partial_z)^2}\delta(y-z) . \tag{3.44}$$

It has been recognized[18–21] that at extended vertices additional extra currents are necessary to maintain gauge invariance. The current ΔM^i_μ is just that is required. If it were not for ΔM^i_μ, the Ward-Takahashi identity generalized to the photon-pion vertex[16] would be violated. For static nucleons ΔM^i_μ is reduced to that obtained in the previous paper.[21]

IV. SUMMARY

We have studied the general structure of electromagnetic vertex functions of extended particles such as nucleon and pion. Nonlocal interactions of the type studied by Chrétien and Peierls[13] are rewritten in the form of local, momentum-dependent interactions. Electromagnetic interactions are introduced by the gauge-invariant substitution of momentum operators. The resulting current operators are in conformity with the nonlocal interactions that are made gauge invariant by inserting gauge factors. In this way we obtained more realistic electromagnetic interactions of nucleon and pion instead of the line current introduced by Bloch.[14] It is proved that the obtained current operators satisfy Ward-Takahashi relations, as is required from the local gauge invariance. The line current is quite similar to the Sachs current[22–25] that is derived for isospin-dependent nuclear interactions. The line current preserves the continuity equation of charge and current densities, and yet it is a mathematical devise upon which it is difficult to put a physical interpretation. The approach presented here seems the most natural way of introducing current operators necessary for gauge invariance. In the previous paper,[26] we proposed a general prescription for constructing nuclear exchange currents. The result of this paper is along the same line. We have also considered the nonlocal pion-nucleon vertex function and obtained a gauge-invariant electromagnetic interaction.

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