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Separable multichannel model for the nucleon-nucleon interaction with Δ isobars

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A general separable NN interaction model containing all possible couplings to $N\Delta$ and $\Delta\Delta$ channels is developed and used to describe elastic and inelastic two-nucleon scattering up to 1 GeV. We start out from the separable Graz-II potential for the elastic NN interaction and extend it to include inelastic channels. Quantitative fits to the phenomenological phase parameters in the $^1D_2$ and $^3F_3$ NN states are obtained between 0 and 1000 MeV. The resulting separable potentials are of low rank and therefore very convenient for applications to few-body calculations.

I. INTRODUCTION

Over recent years much data on nuclear reactions at intermediate energies has been accumulated. Theoretical interpretation requires evidence on the NN interaction also above the pion-production threshold as well as in the energy domain of Δ excitation. Several models describing the NN interaction up to the GeV region have already been developed within various frameworks.[1] For practical investigations of reactions involving intermediate-energy nucleons above all separable coupled-channels potentials have been very useful.[2,3] They are of particular interest with respect to applications to few-nucleon problems, such as $\pi-d$, $N-d$, etc. For the elastic NN interaction separable potentials could be refined not only to produce all (elastic) NN data but also to provide realistic off-shell properties. Such a model was constructed by the Graz group.[4] We refer to it as the Graz-II potential, which has been applied in many investigations of few-body problems, see, e.g., Refs. 5 and 6. The Graz-II potential remedied various shortcomings of earlier phenomenological separable potentials by providing an overall good fit to the available NN data and by modeling an off-shell behavior typical of realistic meson-exchange models.[7] As a result it turned out to be rather successful in describing observables of few-nucleon reactions. It seemed appropriate to extend the validity of this model to intermediate energies. The description of the inelastic NN interaction could be accomplished by including additional channels for the $N\Delta$ and $\Delta\Delta$ interactions. The elastic part should remain unaltered.

In the present paper we put forward our approach in the $^1D_2$ and $^3F_3$ NN partial waves. These states have attracted considerable attention in the past, because their resonating behavior hinted to possible dibaryon formation; also their predominant role for describing the $\pi-d$ system has become evident. Our model will already be applicable to investigate this issue in various respects.

In Sec. II the coupled-channel formalism for general separable interactions is outlined. Our model is presented in Sec. III; there we also discuss the results for $^1D_2$ and $^3F_3$. Section IV contains our conclusions.

II. MULTICHANNEL FORMALISM

We aim at a rather general formalism allowing us to couple the NN, $N\Delta$, and $\Delta\Delta$ channels. We like to include all possible partial-wave states occurring in these channels. For each state with fixed total angular momentum we in general assume rank-N separable interactions. Our formalism thus represents an extension of the single-channel problem with rank-N separable potentials relevant for the elastic NN interaction.[8] Let us denote by $|JMTM_TLSPc\rangle$ the eigenstates of the total angular momentum $\mathbf{J}^2$, z-component J_z, isospin $\mathbf{T}^2$, z-component T_z, angular momentum $\mathbf{L}^2$, spin $\mathbf{S}^2$, and parity P of a two-body system in channel c; we abbreviate by $\alpha=(JMTM_TP)$ the ensemble of conserved quantum numbers J,T,P with z-components M,M_T and we denote by $\beta=(L,S,c)$. Performing a partial-wave expansion for the interaction V one arrives at the potential components (for details of the formalism outlined below we refer to Ref. 9)

$$V^{\alpha}_{\beta\beta'}=\langle \alpha\beta|V|\alpha\beta'\rangle \; . \tag{1}$$

Each $V^{\alpha}_{\beta\beta'}$ represents the interaction for the transition from a particular, possibly coupled, angular momentum state in channel c' to a corresponding state in channel c. For the problem we have in mind, e.g., the transition from $^1D_2$ (NN) to $^5S_2$ $(N\Delta)$.

Suppressing from now on the index α we assume the interaction $V_{\beta\beta'}$ in the following separable form

$$V_{\beta\beta'}=|\mathbf{f}\rangle\Delta_\beta\Lambda\Delta_{\beta'}\langle\mathbf{f}| \; . \tag{2}$$

Here $|\mathbf{f}\rangle$ denotes the array of form factors, the matrix Λ contains the potential strengths and Δ_β projects on state β. The form (2) allows to foresee an arbitrary number N_β (resp. $N_{\beta'}$) of separable terms in each channel partial-wave state. By conservation laws Λ is symmetric.

We solve the coupled system of Lippmann-Schwinger equations for the transition operators $T_{\beta\beta'}(z)$,

$$T_{\beta\beta'}(z)=V_{\beta\beta'}+\sum_{\gamma} V_{\beta\gamma}G_{0,\gamma}(z)T_{\gamma\beta'}(z) \; , \tag{3}$$

and obtain the solution

$$T_{\beta\beta'}(z)=|\mathbf{f}\rangle\Delta_\beta(\mathbf{1}-\Lambda\mathbf{G}(z))^{-1}\Lambda\Delta_{\beta'}\langle\mathbf{f}| \ . \tag{4}$$

Here the matrix

$$\mathbf{G}(z)=\sum_\gamma \Delta_\gamma\langle\mathbf{f}|G_{0,\gamma}(z)|\mathbf{f}\rangle\Delta_\gamma \tag{5}$$

contains integrals of the separable form factors over the free channel resolvents

$$G_{0,\gamma}(E+i0)=\frac{1}{E-H_{0,\gamma}+i0} \ . \tag{6}$$

Since we like to calculate NN phase parameters (elastic phase shifts and inelasticities) we have to establish the connection between the transition operator T and the (multichannel) S matrix. Their partial-wave components are interrelated by

$$S_{\beta\beta'}(E)=\delta_{\beta\beta'}-2\pi i\sqrt{\rho_\beta(E)\rho_{\beta'}(E)}T_{\beta\beta'}(E+i0) \ , \tag{7}$$

where $\rho_\beta(E)=\mu_\beta k_\beta/\hbar^2$ with μ_β the reduced mass in state β. The energy is given by

$$E=\frac{\hbar^2k_\beta^2}{2\mu_\beta}+\Delta m_\beta \ , \tag{8}$$

where Δm_β is the threshold energy for a certain channel c; in particular, $\Delta m_\beta=0$, $m_\Delta-m_N$, $2(m_\Delta-m_N)$ for NN, $N\Delta$, and $\Delta\Delta$, respectively.

In parametrizing the S matrix we follow Ref. 10 to extract the phase parameters in the first channel, in our case the NN channel, which is coupled to all the remaining ones. We write the unitary S matrix as

$$S=\begin{bmatrix} S_1 & S_3^\dagger \\ S_3 & S_2 \end{bmatrix} . \tag{9}$$

Here S_1 contains the information on transitions in channel 1, S_2 corresponds to transitions within the other channels, and S_3 governs transitions from the first channel into all the other ones. Obviously S_1 is no longer unitary, when other channels are open. Next one defines

$$K_4=i(1-S_1)(1+S_1)^{-1} \ , \tag{10}$$

which is a symmetric but complex matrix. If S_1 is unitary (no inelastic channels open), Eq. (10) represents the Cayley transformation, and in this case it yields a real K_4.

From the real and imaginary parts of K_4 one can calculate the elastic phase shifts and inelasticities in channel 1. For uncoupled angular momentum states and conserved spin in channel 1, K_4 is a complex number and one has

$$\tan\bar{\delta}=\mathrm{Re}K_4, \quad \tan^2\bar{\rho}=\mathrm{Im}K_4 \ , \tag{11}$$

where $\bar{\delta}$ and $\bar{\rho}$ represent the bar phase parameters (corresponding to the Stapp parametrization of the elastic S matrix[11]).

For the NN system, S_1 and thus K_4 are at most two-dimensional (two angular momentum states). In this case the complex matrix K_4 is parametrized by six phase parameters. Their expression can be found, e.g., in Ref. 10.

TABLE I. Parameters for our separable potential in the $^1D_2$ $(NN)-{}^5S_2$ $(N\Delta)$ coupled channels.

$\beta_1=2.522\,189$	(fm^{-1})
$\beta_1'=1.652\,067$	(fm^{-1})
$\gamma_1=-0.267$	(fm^0)
$\Lambda_{11}=934\,200.2$	(MeV fm^{-5})
$\Lambda_{12}=89\,793.92$	(MeV fm^{-3})
$\Lambda_{22}=1\,509.428$	(MeV fm^{-1})
$\beta_2=1.705\,767$	(fm^{-1})

III. SEPARABLE NN POTENTIAL WITH Δ DEGREES OF FREEDOM

A general multichannel separable potential model for NN, $N\Delta$, and $\Delta\Delta$ would have to take into account all possible states given by angular momentum and spin coupling (cf. Tables I–IV in Ref. 9). For practical purposes, however, inclusion of all states, especially from the $N\Delta$ and $\Delta\Delta$ channels is neither necessary nor useful. In the construction of a phenomenological interaction one has to avoid a proliferation of open parameters because of the limited data base. Once the interaction in the NN sector is fixed, it usually suffices to add one or two of the most prominent angular momentum states from the other channels to fit the inelastic phase parameters.

Our aim is to extend the Graz-II potential beyond the inelastic threshold. We keep the part designed for the elastic interaction fixed and add the Δ channels. In the first instance we consider the $^1D_2$ and $^3F_3$ partial waves, because the inelasticities show a pronounced behavior in these states;[12] this has led people to argue about dibaryon resonances. Recent analyses of elastic and breakup $\pi-d$ scattering have furthermore shown the predominant role of just these $N\Delta$ states for describing the corresponding data.[13]

For both NN angular momentum states $^1D_2$ and $^3F_3$ there are four $N\Delta$ states to couple with. From former studies[14,15] it became clear that above the inelastic threshold only the $N\Delta$ states with the lowest angular momentum barrier contribute significantly. Therefore we foresee the couplings $^1D_2$ (NN)-$^5S_2$ $(N\Delta)$ and $^3F_3$ (NN)-$^5P_3$ $(N\Delta)$. The Graz-II potential consists in the NN sector (channel 1) of a single-term separable potential with form factors

$$f_{1l}(k_1)=\frac{k_1^l}{k_1^2+\beta_{1l}^2}+\gamma_{1l}\frac{k_1^{l+2}}{(k_1^l+\beta_{1l}'^2)^{l+2}} \ . \tag{12}$$

TABLE II. Parameters for our separable potential in the $^3F_3$ $(NN)-{}^5P_3$ $(N\Delta)$ coupled channels.

$\beta_1=1.952\,411$	(fm^{-1})
$\beta_1'=4.652\,381$	(fm^{-1})
$\gamma_1=134.957$	(fm^0)
$\Lambda_{11}=736\,819.9$	(MeV fm^{-5})
$\Lambda_{12}=5\,663\,629$	(MeV fm^{-3})
$\Lambda_{22}=-130\,995\,900$	(MeV fm^{-1})
$\beta_2=9.718\,343$	(fm^{-1})

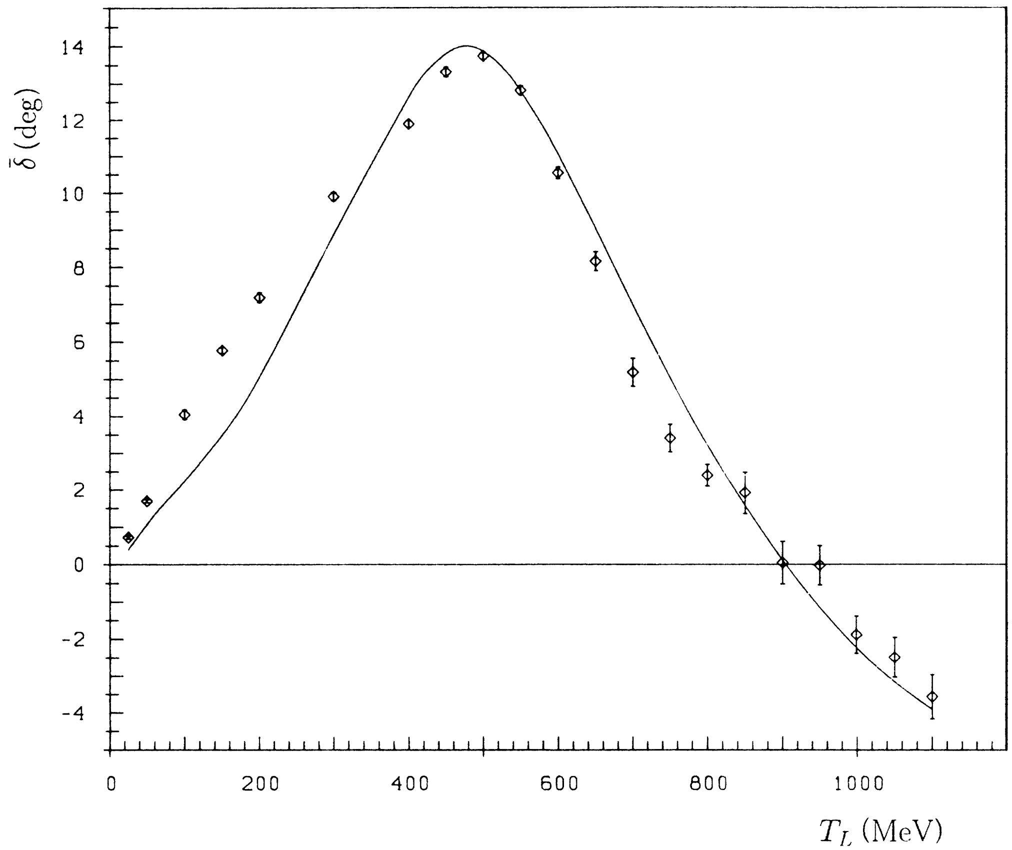

FIG. 1. $^1D_2$ phase shift $\bar{\delta}$ for NN scattering as a function of the laboratory kinetic energy.

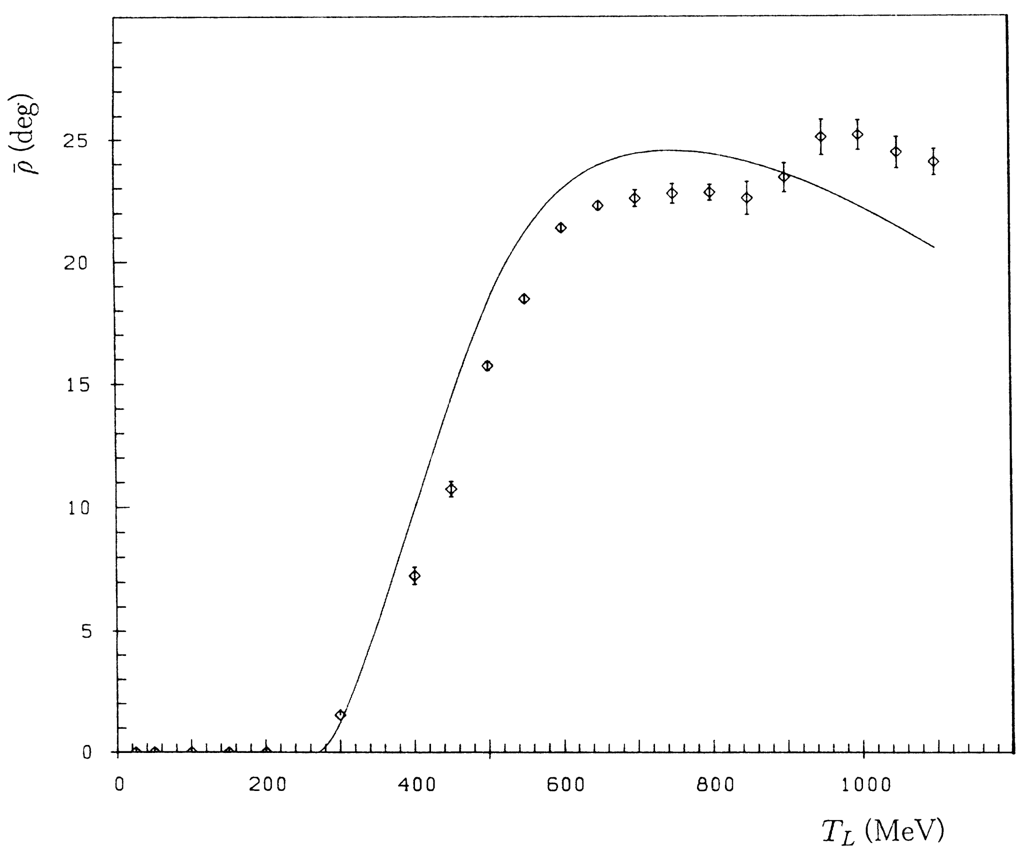

FIG. 2. $^1D_2$ inelasticity parameter $\bar{\rho}$ as a function of the laboratory kinetic energy.

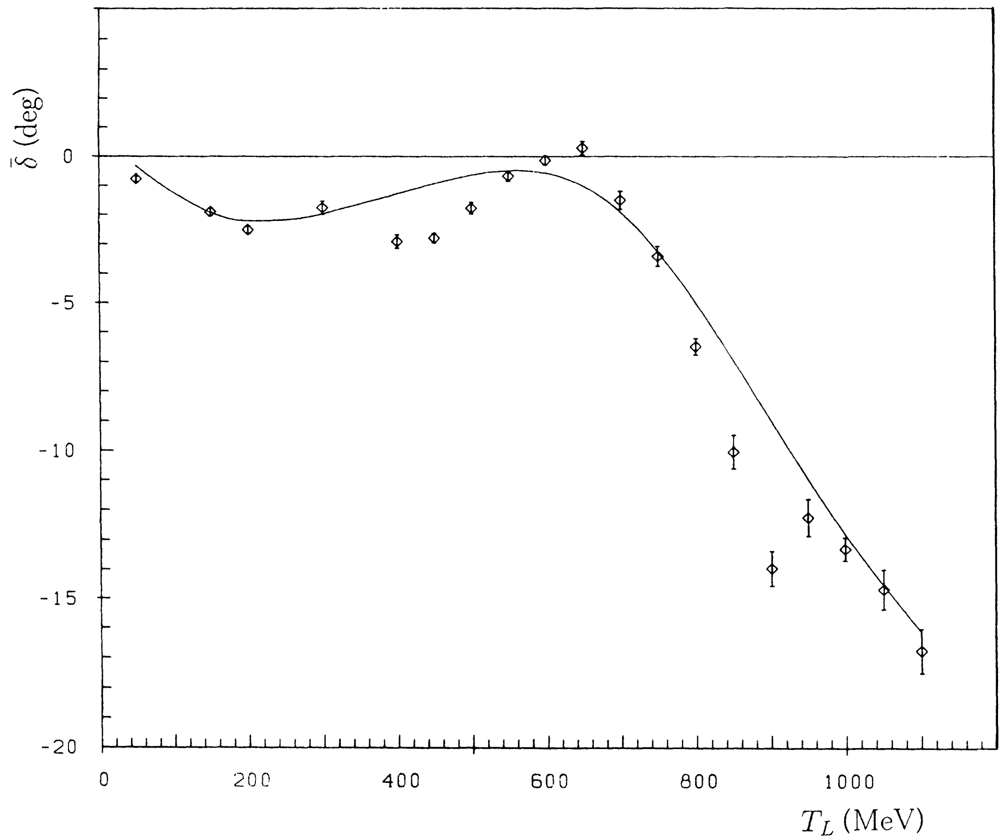

FIG. 3. $^3F_3$ phase shift $\bar{\delta}$ for NN scattering as a function of the laboratory kinetic energy.

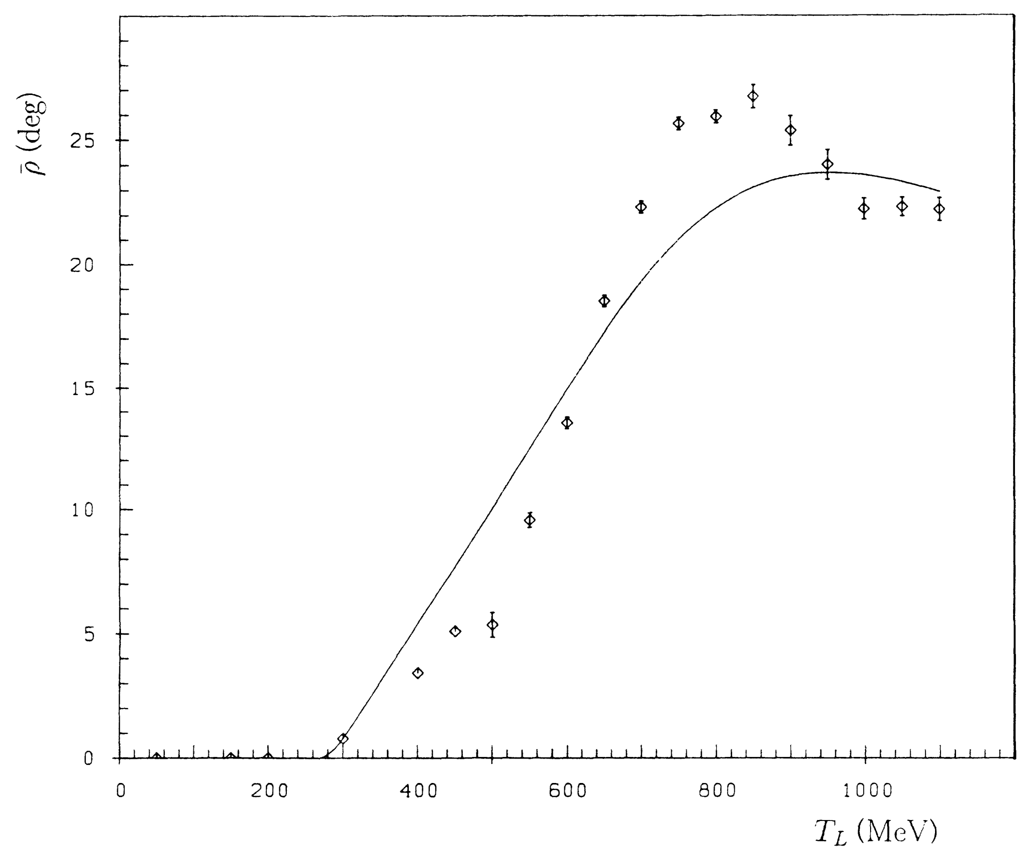

FIG. 4. $^3F_3$ inelasticity parameter $\bar{\rho}$ as a function of the laboratory kinetic energy.

The three open parameters occurring in Eq. (12) for each state, $^1D_2$ and $^3F_3$, were determined in Ref. 4 from the elastic phase shifts and then kept fixed.

For the $N\Delta$ states (channel 2) we add again a single-term separable potential with form factors

$$f_{2l}(k_2)=\frac{k_2^{l+2}}{(k_2^2+\beta_{2l}^2)^{l+2}} . \tag{13}$$

Following VerWest[2] we also introduce a complex Δ mass to effectively include the pion-production channel, whose threshold lies below the one for Δ excitation:

$$m_\Delta=1232-i\frac{\Gamma(T_l)}{2} . \tag{14}$$

The decay width depends on the laboratory kinetic energy in the same way as in Ref. 2. The magnitude of the parameter Γ_0 involved was conveniently chosen to be $\Gamma_0=250$ MeV, independently of the partial-wave state.

In each coupled partial-wave state we are thus left with four residual open parameters, three coupling strengths Λ_{ij}, and the inverse range β_2. They are determined from a least squares fit to the phenomenological phase shift $\bar{\delta}$ and inelasticity $\bar{\rho}$ between 300 MeV$\leq T_L \leq$1100 MeV. The numerical values of the potential parameters are quoted in Tables I and II for the $^1D_2-{}^5S_2$ and $^3F_3-{}^5P_3$ states, respectively.

The quality of the fit to the phase parameters of Ref. 12 is demonstrated in Figs. 1–4. It is evident that a satisfactory description is achieved up to $T_L \approx 1100$ MeV for both the phase shifts and the inelasticity parameters.

IV. CONCLUSION

We have presented a general coupled-channels formalism for arbitrary rank-N separable interactions in each channel (angular momentum) state. We have applied it to construct a separable potential model for the intermediate-energy NN problem. For the elastic NN interaction our model is based on the Graz-II separable potential, which yielded a good description of all elastic NN data and provided a reasonable off-shell behavior. We have extended this potential by adding additional channels allowing to introduce Δ degrees of freedom.

In the present paper we have considered the $^1D_2$ and $^3F_3$ partial wave states, which are most prominent in the investigation of the inelastic NN interaction. Our model couples $^1D_2$ (NN) to $^5S_2$ $(N\Delta)$ and $^3F_3$ (NN) to $^5P_3$ $(N\Delta)$ with a single-term separable potential for each state. To include the effect of pion production we have assumed a complex Δ mass. This allows our model to describe the associated increase of the inelasticity parameter $\bar{\rho}$ already below the $N\Delta$ threshold and involves the width parameter Γ_0. At the expense of lacking a unitary coupling to the $\pi-d$ channel, we could thereby make our simple separable model to provide a realistic reproduction of both the phase shifts and inelasticities. The open potential parameters have been determined by a fit to the phenomenological phases of the latest analysis.[12]

Our model provides a good description of the elastic phase shifts and inelasticities up to $T_L \approx 1100$ MeV. It extends the Graz-II potential to the inelastic domain leaving the elastic part unaltered. Thus it should be of good use to continue the investigation of few-body reactions toward higher energies, where the inelastic NN interaction comes into play.

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