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ΛN-ΣN coupling in $^3_\Lambda$H

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Using separable potential equations to model the $T=0$, $J=\frac{1}{2}$, ΛNN three-body system, we explore the effects of ΛN-ΣN coupling in the ground state of the hypertriton. Explicit ΛN-ΣN coupling in the hyperon-nucleon interaction is shown to play a significant role, even in this lightly bound system. When the ΣN channel is formally eliminated, the dispersive energy dependence of the resulting ΛN effective interaction is repulsive whereas the resulting ΛNN three-body force is attractive.

The hypertriton ($^3_\Lambda$H) plays as important a role in hypernuclear physics as does the deuteron in conventional (nonstrange) nuclear physics, because[1] neither the ΛN nor ΣN (spin singlet or spin triplet) interactions possess sufficient strength to support a bound state. It is the ground state of the ΛNN system ($J^\pi=\frac{1}{2}^+$) that must be used to constrain our models of the hyperon-nucleon (YN) force, which are not well determined by the sparse data base for low-energy ΛN and ΣN scattering and reactions. Because the hypertriton is loosely bound:[2]

$$B_\Lambda(^3_\Lambda\mathrm{H})=B(^3_\Lambda\mathrm{H})-B(^2\mathrm{H}) \simeq 0.13\pm0.05\ \mathrm{MeV}\,,$$

one expects this molecularlike system to be most sensitive to the long-range aspects of the ΛN interaction. However, there is no one-pion-exchange mechanism allowed in the ΛN interaction in first order, because the $\Lambda(T=0)$ and $N(T=\frac{1}{2})$ cannot exchange a $T=1$ pion.[1] The longest-range part of the potential is due to the exchange of either two pions or a kaon. The shorter-range $\bar{K}$-exchange potential does admit a tensor force component; however, it is largely canceled by the K^*-exchange potential. (The π-exchange and ρ-exchange tensor forces in the NN interaction do not cancel so completely, because their masses are quite different.) Thus, tensor force effects in the ΛN interaction are anticipated to be somewhat smaller than those found in the NN interaction.

In contrast, ΛN-ΣN coupling effects[1,3,4] are expected to be much more important in hypernuclear physics than are NN-$N\Delta$ coupling effects in conventional nuclear physics. The $m_\Sigma-m_\Lambda$ mass difference is only 75 MeV, and the width of the Σ is small compared to that of the Δ, because it lies below the K^--p threshold. "Freezing out" or formally eliminating the Σ channel from the problem leads one to (ΛNN) three-body forces,[5] a subject of current interest in the nonstrange sector.[6]

Few $^3_\Lambda$H calculations for models that include ΛN-ΣN coupling have been published.[7,8] Since the work of Dabrowski and Fedorynska[8] with the simple Wycech[9] model, improved separable potential representations[10] of the YN interaction have appeared. We wish to report here initial results of new separable potential three-body calculations that pertain to: (i) The dispersive effect from embedding the ΛN-ΣN potential in a three-body system, which reduces the $^3_\Lambda$H binding and (ii) the three-body force effect due to ΣNN coupling to ΛNN states, which increases the $^3_\Lambda$H binding. The $\Lambda N(^3S_1$-$^3D_1)$ tensor force effect in $^3_\Lambda$H will be discussed elsewhere.

We partial wave expand the interactions in momentum space as

$$\langle \mathbf{p}|V|\mathbf{p}'\rangle=\sum_{nll'}\langle\hat{\mathbf{p}}|nl\rangle V^n_{ll'}(p,p')\langle nl'|\hat{\mathbf{p}}'\rangle\,, \tag{1}$$

where $n=\{sjt\}\sim$ total spin s, total angular momentum j, and total isospin t for the two-body system. For separable potentials we can write

$$V^n_{ll'}(p,p')=\langle p|g_{nl}\rangle C^n_{ll'}\langle g_{nl'}|p'\rangle\,, \tag{2}$$

which in matrix form becomes

$$V^n(p,p')=\langle p|V|p'\rangle\,, \tag{3}$$

where $V=|g_n\rangle C^n\langle g_n|$ and $[|g_n\rangle]_{ll'}=\delta_{ll'}|g_{nl}\rangle$. Tensor forces are included by admitting $C^n_{ll'}\neq0$, $l\neq l'$. Coupling between ΛN and ΣN channels is included by replacing l by $\{\zeta,l\}$, where ζ specifies the mass eigenstate of the hyperon. The corresponding scattering amplitude has a similar partial-wave expansion with the partial-wave amplitude given by

$$t^n(E)=|g_n\rangle\tau^n(E)\langle g_n|\,, \tag{4}$$

where

$$\tau^n(E)=C^n[I-G_0(E)C^n]^{-1}\,, \tag{5}$$

and

$$[G_0(E)]_{ll'}=\delta_{ll'}\langle g_{nl}|G_l(E)|g_{nl}\rangle\,. \tag{6}$$

The thresholds are included by writing

$$\langle p|G_l(E)|p\rangle = \begin{cases}(E-p^2/m_N-2m_N)^{-1} & \text{for } NN\,, \quad (7a)\\ (E-p^2/2\mu_{YN}-m_N-m_Y)^{-1} & \text{for } YN\,, \quad (7b)\end{cases}$$

where $\mu_{YN}=m_N m_Y/(m_N+m_Y)$. In this way, scattering for NN and YN systems including both a tensor force and ΛN-ΣN channel coupling can be included. For the YN system with both, we have a 4×4 matrix representation of $t^n(E)$ and $\tau^n(E)$. For the YNN system, we must embed the above amplitudes in a three-body Hilbert space. In spectator notation, the two-particle amplitude in the three-body space becomes

$$\langle \mathbf{q}_\gamma \mathbf{p}_\gamma | T_\gamma(E) | \mathbf{p}'_\gamma \mathbf{q}'_\gamma\rangle = \delta(\mathbf{q}_\gamma - \mathbf{q}'_\gamma)\langle \mathbf{p}_\gamma | t_\gamma(E-\varepsilon_\gamma)|\mathbf{p}'_\gamma\rangle\,, \quad (8)$$

where $\varepsilon_\gamma = m_\gamma + q_\gamma^2/2\mu_\gamma$ is the energy of the spectator particle including its rest mass m_γ. Here μ_γ is the reduced mass of the spectator and the interacting pair.

The three-body Alt-Grassberger-Sandhas (AGS) equations[11] can be generalized to allow for the additional degree of freedom corresponding to each particle being in more than one mass eigenstate. For the YNN system of interest here, one obtains four equations (before partial-wave expansion) which can be written in matrix form as

$$\begin{pmatrix} X_\Lambda \\ X_\Sigma \end{pmatrix} = \begin{pmatrix} Z_\Lambda & 0 \\ 0 & Z_\Sigma \end{pmatrix} \begin{pmatrix} \tau_{\Lambda\Lambda} & \tau_{\Lambda\Sigma} \\ \tau_{\Sigma\Lambda} & \tau_{\Sigma\Sigma} \end{pmatrix} \begin{pmatrix} X_\Lambda \\ X_\Sigma \end{pmatrix}, \quad (9)$$

where

$$Z_Y = \begin{pmatrix} 0 & Z_{Y,N} \\ Z_{N,Y} & Z_{N,N} \end{pmatrix}, \quad Y=\Lambda,\Sigma \quad (10)$$

are the input Born amplitudes corresponding to single-baryon exchange and

$$\tau_{YY} = \begin{pmatrix} \tau_{YY}^{Y} & 0 \\ 0 & \tau_{YY}^{N} \end{pmatrix}, \quad Y=\Lambda,\Sigma \quad (11)$$

$$\tau_{\Lambda\Sigma} = \begin{pmatrix} 0 & 0 \\ 0 & \tau_{\Lambda\Sigma}^{N} \end{pmatrix}. \quad (12)$$

The superscript on the $\tau_{\Lambda\Sigma}^N$ identifies the spectator particle. Note that for $\tau_{\Lambda\Sigma}=0$ there is no coupling of the ΛN and ΣN channels. Partial-wave expansion preserves the matrix structure of the equations, but the size of the matrix is determined by the number of three-body channels for a given (J,T).

For the ΛNN system there are two types of diagrams that contribute when ΛN-ΣN coupling is included. In Fig. 1 we see the ΛN-ΣN coupling diagram that contributes to the two-body ΛN amplitude. Note that such terms have the form

$$V_{\Lambda N} - \frac{|V_{XN}|^2}{\Delta E}\,,$$

where ΔE corresponds to the ΛN-ΣN energy difference, if one looks at a perturbation expansion. Embedded in the three-body problem, ΔE is augmented by the binding energy of the three-body system. Thus, the effective ΛN force becomes energy dependent and we expect it to be weaker (because ΔE is larger) when embedded in a few-body system. This is referred to in the literature as the dispersive three-body force effect.[6,12] It is effectively repulsive. It is exactly this dispersive effect (energy dependence) that also makes the NN tensor force less effective in binding the triton than it is in binding the deuteron. (The ΔE in the second-order term becomes larger because the triton is bound by about 8 MeV compared to about 2 MeV for the deuteron.)

In Fig. 2 we see that the ΛN-ΣN potential coupling on the left-hand side contributes to two types of diagrams on the right-hand side when the equations are properly summed. The first is the standard connected diagram contribution to the $^3_\Lambda$H binding energy coming from iteration of the ΛN two-body amplitude. The second, which occurs only when ΛN-ΣN coupling is included, corresponds to a true three-body force. (Note that ΛN-ΣN coupling also contributes to the first, but only in the generation of the $\tau_{\Lambda N}$, as in Fig. 1.) To evaluate the contribution due to this effective three-body force, we need to exclude the contribution of the second diagram on the right-hand side of Fig. 2 when including the ΛN-ΣN coupling. This is achieved by calculating τ^N with the ΛN-ΣN coupling but including only the diagonal terms in the three-body equations. In this way we estimate the contribution of the effective three-body force to the $^3_\Lambda$H binding energy.

To verify the validity of our equations and code, we first checked the calculation of Dabrowski and Fedorynska.[8] Although their NN model parameters correspond to a binding energy of approximately $B(^2\mathrm{H})=2.42$ MeV, the values of $B_\Lambda(^3_\Lambda\mathrm{H})$ quoted were reproduced for their $^3S_1(NN)$ potential parameters as well as for a $^3S_1(NN)$ potential[13] that gave a more realistic 2.23-MeV deuteron binding.

To explore the ΛN-ΣN coupling discussed above, we chose the Stepien-Rudzka and Wycech[10] model. The parameters of the model potentials are quoted in Table I along with those for the equivalent one-channel approximation (a single-channel ΛN potential having the same scattering length and effective range as the two-channel ΛN-ΣN potential) and the spin-triplet NN interactions.[13] Note that our potential strengths are related to those in the table by $C_{ij} = -4\pi\lambda_{ij}/2\mu_{ij}$. We report here only $^3_\Lambda$H

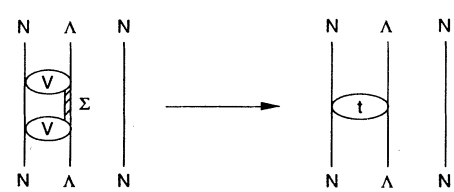

FIG. 1. A schematic representation of the contribution to the ΛN two-body t matrix arising from ΛN-ΣN coupling.

TABLE I. YN and NN separable potential parameters; C_{ij} in the text is $-4\pi\lambda_{ij}/2\mu_{ij}$; λ are in fm^{-3} and β are in fm.

Channel	$\lambda_{\Lambda N}$	λ_{XN}	$\lambda_{\Sigma N}$	β_Λ	β_Σ
$(YN)^{S=1}$	0.5298	−0.6777	0.9871	1.60	2.00
$(YN)^{S=1}$	0.3262			1.7251	
$(YN)^{S=0}$	0.7251	1.097	0.8916	1.18	1.44
$(YN)^{S=0}$	0.0952			1.2011	
Channel	λ_{NN}			β_N	
$(NN)^{S=1}$	0.3815			1.4056	

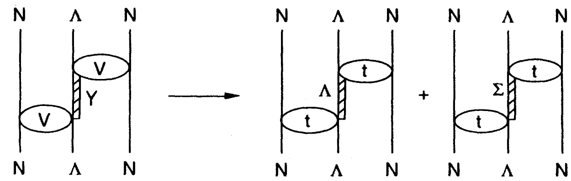

FIG. 2. A schematic representation of the two separate contributions to the hypertriton binding due to ΛN-ΣN coupling. The Λ diagram is the standard contribution, while the Σ diagram corresponds to an effective three-body force.

results for a central NN force, although we obtain qualitatively similar results for a $^3S_1$-$^3D_1$ NN interaction. Beginning with the full ΛN-ΣN interaction, we calculate a $^3_\Lambda$H binding energy of 2.63 MeV or a lambda separation energy $B_\Lambda(^3_\Lambda\mathrm{H})$ of 0.40 MeV. (These are large compared to experiment, because we have neglected the tensor-force nature of the NN interaction.) If we replace the ΛN-ΣN interactions by their one-channel approximations, then the corresponding binding energy and lambda separation energy are 2.37 and 0.14 MeV, respectively. Clearly, including explicit ΛN-ΣN coupling increases the binding energy of the system. This is to be expected from consideration of a coupled oscillator system, where coupling pushes the lower state (here the ΛNN system) down and the higher state (here the ΣNN system) up, and the fact that, if one increases the size of the Hilbert space, then the variational bound on the binding energy should increase. However, to understand the roles of the diagrams in Figs. 1 and 2 in this binding-energy enhancement, we have also turned off the ΣNN diagram as described above. The results are $B(^3_\Lambda\mathrm{H})=2.31$ and $B_\Lambda(^3_\Lambda\mathrm{H})=0.08$ MeV. That is, we verify that the dispersive energy dependence of the ΛN-ΣN interaction leads to a reduction in the $^3_\Lambda$H binding energy (2.31 vs 2.37 MeV); both interactions have the same scattering length and effective range but the true three-body force terms in the ΛN-ΣN coupled-channel calculation have been turned off. One should note that the $A=3$ system differs from the $A=4$ and $A=5$ Λ hypernuclei, where the excitation energies of the nuclear core states are larger (e.g., the $T=1$, $S=0$ excited states of the alpha core in $^5_\Lambda$He lie more than 40 MeV up in the spectrum) than the 2-MeV separation between the d^* system and the deuteron, which will suppress ΛN-ΣN coupling effects. That is, a ΛN-ΣN two-channel potential model may yield less binding in heavier systems (and nuclear matter) than the corresponding one-channel ΛN effective potential model. This is the essence of the $A=4$ study in Ref. 14, where it is argued that suppression of ΛN-ΣN coupling due to these effects lowers the 0^+ and 1^+ state binding relative to simple ΛN effective interaction models.

Restated, a static approximation $V^{st}_{\Lambda N}$ to the energy-dependent ΛN potential,

$$V^{\mathrm{eff}}_{\Lambda N}=V_{\Lambda N}+|V_{XN}|^2/(H_{\Sigma N}-E)\,, \tag{13}$$

binds the hypertriton more than $V^{\mathrm{eff}}_{\Lambda N}$ when both models yield the same ΛN scattering length and effective range. (A similar result holds for an NN-NΔ force model in the triton.[12]) At the same time, we see that for the hypertriton the true three-body force effect of Fig. 2 is attractive and much larger than the dispersive effect. Now that one can hope to carry out Faddeev calculations for the hypertriton with the available meson theoretic potential models,[15,16] one may be able to answer the intriguing question of whether the $^3_\Lambda$H is bound only because there is a ΛNN three-body force.

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