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Coupling between the β and γ vibration in tungsten nuclei

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The quadrupole collectivity in ^{182,184}W has been studied by means of heavy-ion Coulomb excitation with ^{58}Ni and ^{136}Xe projectiles; the deexcitation γ rays were measured in γ-p-p triple coincidence experiments. These data were used to determine, model-independently, both the static and transition matrix elements for the low-lying states in these nuclei with a semiclassical Coulomb-excitation least-squares search code. The results for the quadrupole moments of the second 2^+ states and the $E2$ transitions between the γ band and the ground-state band in these nuclei do not support the prediction of a strong mixing between the β- and γ-vibrational bands in tungsten nuclei made by the pairing-plus-quadrupole model of Kumar and Baranger.

The β- and γ-vibrational bands are nearly degenerate around ^{182,184}W making these nuclei ideal for probing the interaction between these important collective degrees of freedom in nuclei. A perturbation treatment of the coupling between the various collective degrees of freedom (rotation, β vibration, and γ vibration) is inadequate to reproduce the collective properties in the shape transitional nuclei, a region of nuclear shape change from spherical to deformed or prolate to oblate. The first attempt at a more exact treatment of such couplings was accomplished by Kumar and Baranger[1] who solved Bohr's Hamiltonian using the pairing-plus-quadrupole model. Kumar and Baranger predicted some unusual features, such as a prolate-oblate shape transition in Os-Pt nuclei and a strong mixing between the β and γ bands in the more strongly deformed rotational W nuclei. The former has been established[2-5] by the measurement of the quadrupole moments of the first 2^+ states in Coulomb excitation experiments. A notable consequence of the latter is the reduction of the quadrupole moments for the 2_2^+ and 2_3^+ states and the predicted deviation from the Alaga rule for the decay of the β and γ bands. For example, the predicted quadrupole moment of the 2_2^+ state of ^{182}W is only 8% of what it would be if it were a pure $K=2$, γ-band member of a prolate nucleus. Most of the supportive experimental evidence[6-10] for this strong mixing in W nuclei comes from the electromagnetic properties of the 2_2^+ and 2_3^+ states and the level energies of the lower members of the γ and β bands.

Conflicting experimental evidence exists regarding the band mixing in W nuclei. The known $E2$ transitions between the 2_2^+ and 2_3^+ states and the ground-state band can also be well reproduced by means of a three-band-mixing calculation[11] (mixing of the ground-state, γ, and β bands), indicating a rather weak mixing between the β and γ bands. Moreover, the recent theoretical studies of W nuclei using phenomenological models, such as the general collective model by Hess, Maruhn, and Greiner[12] and the interacting-boson approximation model by Duval and Barrett,[13] suggest weak mixing between the β and γ bands. The present work was stimulated by the need for a systematic study of the $E2$ transitions between the β, γ, and ground-state bands.

Note that the electromagnetic properties are a sensitive and unambiguous probe of the collective degrees of freedom in contrast to the level energies which depend on both the collective and single-particle degrees of freedom. Coulomb excitation is the ideal experimental tool for a quantitative study of the electromagnetic properties of collective states due to its selectivity for populating low-lying collective bands. Also, its reaction mechanism is well understood. Recent advances in the field of heavy-ion Coulomb excitation, particularly the development of efficient detector systems and of sophisticated semiclassical Coulomb-excitation–deexcitation least-squares search codes, have been made at both Rochester[14] and Gesellschaft für Schwerionforschung Darmstadt m.b.H.[15] As described in a review article by Cline,[16] they allow a model-independent extraction, from heavy-ion induced Coulomb excitation data, of almost the complete set of $E2$ matrix elements for the low-lying states in nuclei.

The present Coulomb excitation experiments were performed by bombarding ^{182,184}W with 561-MeV ^{136}Xe ions, from the SuperHILAC at Lawrence Berkeley Laboratory, and 235-MeV ^{58}Ni ions from the Tandem accelerator at Brookhaven National Laboratory. The deexcitation γ rays were detected by an array of eight Ge detectors (Compton-suppressed Ge's were used in the Berkeley experiment) in triple coincidence with the detection of both the scattered projectile and recoiling target nuclei, in kinematic coincidence, by a pair of position-sensitive avalanche counters (PPAC's). The angular coverage of the PPAC's is $8° \leq \theta \leq 78°$ plus $104° \leq \theta \leq 162°$ and $-35° \leq \phi \leq 35°$ plus $145° \leq \phi \leq 215°$. This extensive data set was collected to ensure that the $E2$ matrix elements for these nuclei could be extracted in a model-independent analysis. The details of the experimental setup and the results together with the analysis will be presented in a later publication. This report concentrates on the results regarding the weak coupling be-

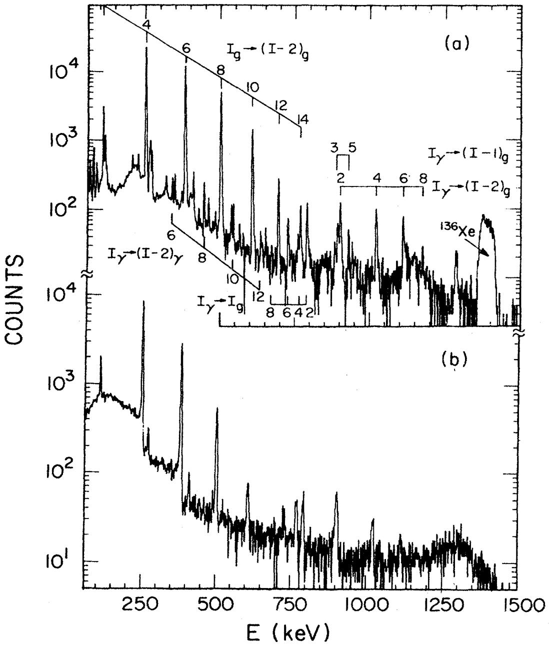

FIG. 1. The Doppler-shift corrected γ-ray spectrum of ^{184}W with (a) ^{136}Xe at $E=561$ MeV integrated over the laboratory angle between 54° and 74° and (b) ^{58}Ni at $E=235$ MeV integrated over the laboratory angle between 106° and 150°.

tween the β- and γ-vibration degrees of freedom in W nuclei.

Typical coincidence Doppler-corrected γ-ray spectra are shown in Fig. 1. The states with spin up to 16^+ (14^+) of the ground-state band and 8^+ (12^+) of the γ band for ^{182}W (^{184}W) were populated in the present experiments. The level schemes resulting from this work are shown in Fig. 2. The members of the β band were not populated with detectable strength. However, the β band was included in the analysis to extract the $E2$ matrix elements for the ground state and γ bands using either known[6] or assumed $E2$ matrix elements for the coupling to the β band. The uncertainty in $E2$ matrix elements to the β band was considered in the errors assigned to the extracted matrix elements. The data were partitioned into four scattering angle subdivisions for the ^{136}Xe experiment and five for the ^{58}Ni experiment to exploit the dependence of Coulomb excitation on impact parameter in addition to the dependence on the atomic number ($Z=28$ and 54). This procedure generated a total of ~ 1600 independent data points for ^{182}W and ~ 1900 for ^{184}W. These were used with the semiclassical Coulomb-excitation–deexcitation least-squares search code, GOSIA,[14] to extract 35 $E2$ and 4 $M1$ matrix elements for ^{182}W and similarly for ^{184}W. This code has been used successfully in a number of other cases.[18–22] For each nucleus, the minimization and multiple tests of the best set of matrix elements required about 30 central processing unit (CPU) hours on a supercomputer, an ETA10, and the error calculation required an additional 20 CPU hours. The normalized χ^2 are 2.30 and 1.62 for ^{182}W and ^{184}W, respectively.

The in-band transition and diagonal $E2$ matrix elements up to the highest-spin states observed in these experiments, for both the ground state and γ bands, were extracted. These results can be reasonably well correlated using the triaxial rigid-rotor model with a γ value of 11° and 14° for ^{182}W and ^{184}W, respectively. This is consistent with the expectation that these nuclei are good rotors. These results will be discussed in a later publication.

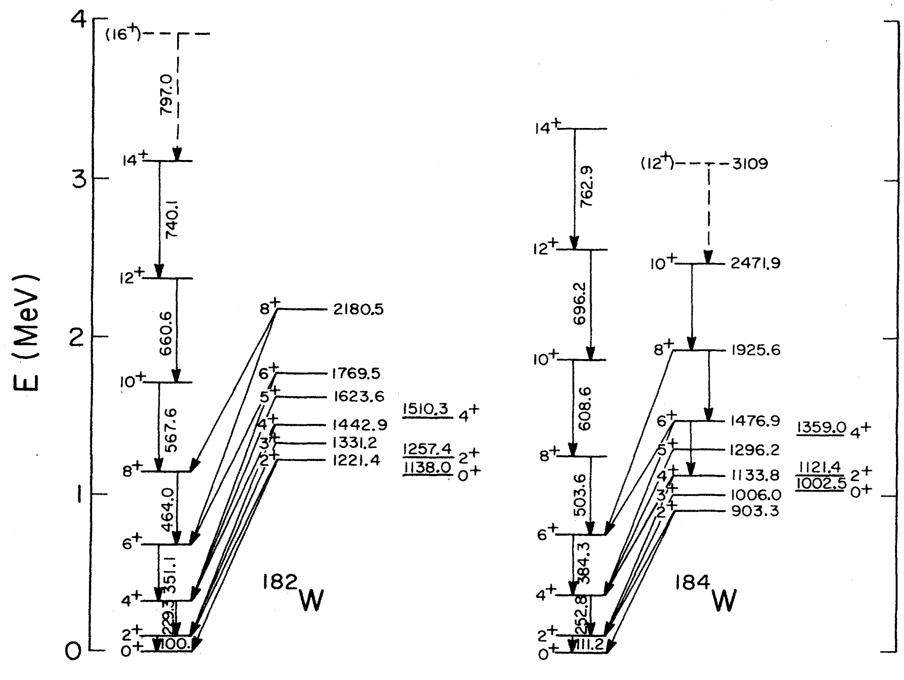

FIG. 2 The level schemes for ^{182,184}W.

The interband transitions are the best probe of the interaction between the bands. The "Mikhailov plot"[23] is a very useful method of presenting the interband transition matrix elements in the perturbation limit for the rotation-vibration coupling. Figure 3 shows such a plot for the $E2$ transition matrix elements between the γ and ground-state bands for ^{182,184}W.

For ^{184}W, the data are well correlated by the functional form[24]

$$\sqrt{B(E2,I_\gamma \rightarrow I_g)}/\langle I_\gamma 22-2\,|\,I_g 0\rangle$$
$$=\sqrt{2}\{M_1-M_2[I_g(I_g+1)-I_\gamma(I_\gamma+1)]\}\,, \quad (1)$$

where the spin-dependent term is a first-order correction due to the coupling between rotation and γ vibration. The M_1 and M_2 are related to the intrinsic matrix element by

$$M_1=\langle K_f\,|\,M(E2)\,|\,K_i\rangle+4(K_i+1)M_2\,. \quad (2)$$

The values of parameters M_1 and M_2 from this analysis are 0.272 ± 0.010 eb and -0.0043 ± 0.0006 eb, respectively. The M_2 determines the reduced amplitude[24] of the mixing between two bands, $\epsilon_2=-(9.3\pm1.4)\times10^{-4}$, assuming the intrinsic $E2$ moment $Q_0=5.99$ eb. The mixing amplitude determines the coupling matrix element,[24] $\langle\gamma\,|\,h_2\,|\,g\rangle=-0.84\pm0.12$ keV. Note that the off-diagonal interaction matrix element is related to the coupling matrix element by

$$\langle I,K_f\,|\,H\,|\,I,K_i\rangle=s\langle K_f\,|\,h_2\,|\,K_i\rangle\,, \quad (3)$$

where $s=[2(I-1)I(I+1)(I+2)]^{1/2}$ for $|\Delta K|=2$ and $s=I(I+1)$ for $\Delta K=0$. The coupling matrix element, $\langle\beta\,|\,h_2\,|\,\gamma\rangle$, between the β and γ bands, from the three-band-mixing calculation (ground-state, γ, and β bands) fitted to our $E2$ data, sets an upper limit of about 1 keV for lower-spin members and diminishes for higher-spin states in ^{184}W.

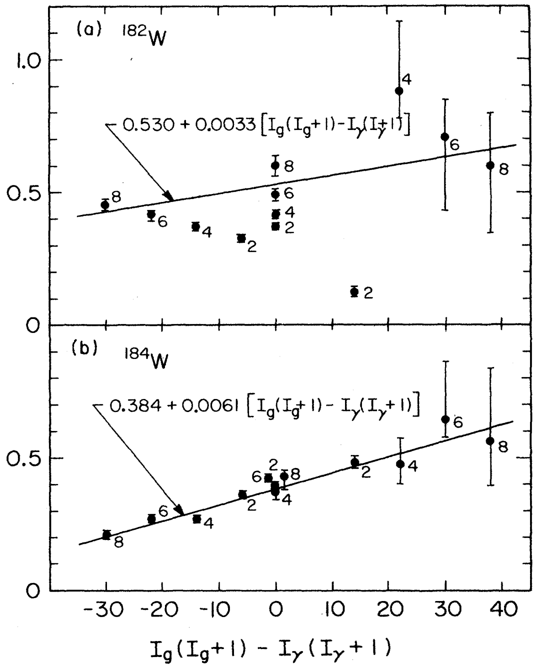

FIG. 3. The "Mikhailov plot" for the $E2$ transitions between the γ and ground-state bands for (a) ^{182}W and (b) ^{184}W. The data points are labeled by I_γ. The solid lines are the best linear fits to the data. The data associated with the 2^+ and 4^+ states of the γ band were excluded in the fit for ^{182}W. See text for details.

The data for ^{182}W show a more complicated pattern than those for ^{184}W, which suggests that the γ band is perturbed by other collective degrees of freedom in addition to the rotation degree of freedom. However, the data are approximately correlated by Eq. (1) if one excludes the data associated with the 2^+ and 4^+ states of the γ band. The values of parameters M_1 and M_2 from such analysis are 0.375 ± 0.026 eb and -0.0023 ± 0.0010 eb, respectively. The M_2 determines a reduced amplitude $\epsilon_2=-(4.8\pm2.2)\times10^{-4}$, assuming the intrinsic $E2$ moment $Q_0=6.15$ eb. The mixing amplitude determines a coupling matrix element $\langle\gamma\,|\,h_2\,|\,g\rangle=-0.59\pm0.27$ keV. This set of parameters was used in the following band-mixing calculation except for the value of M_1 used for the 2^+ states, which is 23% lower than the above value.

The unusual behavior of the 2^+ and 4^+ states in ^{182}W can be understood in terms of mixing between the almost degenerate states of the β and γ bands. It has been reported[11] that a small admixture ($\sim$13% in probability) of the 2_β^+ in the 2_γ^+ would explain the decay pattern of the 2_2^+ to the ground-state band. A three-band-mixing calculation similar to that of Ref. 11 for the 4^+ states shows that our measured $E2$ matrix elements between the γ and ground-state bands suggest a strong mixing for states between the β and γ in ^{182}W, i.e., $\sim$40% for the probability of the 4_β^+ admixture in the 4_γ^+. This corresponds to a coupling matrix element of $\langle\beta\,|\,h_2\,|\,\gamma\rangle\sim-1.5$ keV, which is consistent in magnitude with the 1.18 keV extracted from the analysis[7] of level energies. However, the sign is determined from our $E2$ data. The strong mixing is largely due to the close spacing in the unperturbed basis. This trend does not persist to higher spin states, where our measured $E2$ data are consistent with $\langle\beta\,|\,h_2\,|\,\gamma\rangle$ close to zero. The extracted $\langle\beta\,|\,h_2\,|\,\gamma\rangle$ for the 4^+ states has the opposite sign of the $+1.75$ keV reported for the 2^+ states.[11] The phenomenological analysis of $E2$ matrix elements of the 2^+ states in W nuclei, in the pairing-plus-quadrupole model,[1] indicates that the coupling matrix element $\langle\beta\,|\,h_2\,|\,\gamma\rangle$ is on the order of 5 keV, which is $\sim$4 times larger than the experimental values. These measured values suggest that the coupling between β- and γ-vibration in W nuclei is rather weak. The spin dependence of the coupling matrix element between the β and γ bands suggests that mixing of additional configurations may be important.

The mixing between the β and γ bands will have a dramatic effect on the observed quadrupole moments of their 2^+ states since the unperturbed values, whose absolute values are 2.32 eb for ^{182}W and 2.26 eb for ^{184}W in the axial symmetrical model, are the same in magnitude

TABLE I. Static matrix elements of the 2_1^+ and 2_2^+ states in ^{182,184}W. The quadrupole moment is related to the static matrix elements by $Q(I) = (16\pi/5)^{1/2} \begin{pmatrix} I & 2 & I \\ -I & 0 & I \end{pmatrix} \langle I \| E2 \| I \rangle$.

I	$\langle I \| E2 \| I \rangle$ (eb)			
	^{182}W		^{184}W	
	This work	Others[a]	This work	Others
2_1^+	$-2.00^{+0.04}_{-0.08}$	1.98 ± 0.28	$-1.98^{+0.06}_{-0.04}$	-2.16 ± 0.22[a]
2_2^+	$+1.94^{+0.10}_{-0.04}$		$+2.36^{+0.11}_{-0.05}$	$+0.13 \pm 0.52$[b]

[a] Reference 17.

[b] Reference 10.

but opposite in sign in this simple model assumption. Table I lists the measured quadrupole moments for the first and second 2^+ states of ^{182,184}W from this work. No anomaly was observed for the quadrupole moment of the second 2^+ state of ^{184}W in contrast to an early report.[10] There is an indication of mixing between the 2^+ states of the β and γ bands in ^{182}W, where the quadrupole moment of the 2_2^+ is lower than would be expected if it were a member of a pure $K=2$, γ band of a prolate nucleus. However, the measured value still accounts for ~80% of the expected value in contrast to 8% predicted by the pairing-plus-quadrupole model. The measured value is consistent with the report of a small β-band admixture mentioned earlier.[11]

The mixing between the β and γ bands indicated in the 2^+ and 4^+ states of ^{182}W is due to the near degeneracy of these states and does not persist to higher spin states. That is, there is no strong global coupling between β and γ vibration in W nuclei. The fact that the $E2$ data between the γ and ground-state bands are correlated well by introducing the coupling between rotation and vibration alone, indicates the weakness of the perturbation induced by the coupling between the β and γ vibrations. These findings do not support the prediction of strong coupling between β and γ vibration in W nuclei in the pairing-plus-quadrupole model calculations of Kumar and Baranger and point to a deficiency in the anharmonic terms in the kinetic energy functions of Bohr's collective Hamiltonian used in their model.

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