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Chiral repulsion in the pion-nucleus optical potential

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It is shown that there exist additional sizable sources of repulsion in the S-wave pion-nucleus optical potential of chiral origin. It is found that $(b_0)_{\rm chiral} \simeq -0.011 m_\pi^{-1}$ and $({\rm Re}B_0)_{\rm chiral} \simeq -0.023 m_\pi^{-4}$ using cloudy bag form factors. Taking medium enhancement into account, this repulsion seems to be enough to offset both Galilean attraction ($\propto \rho$) and the attraction in ${\rm Re}B_0$ ($\propto \rho^2$) from Δ-hole pairs.

Fits to pionic-atom data have traditionally been described in terms of the parameter sets characterizing the complex pion-nucleus optical potential. In particular, the "standard"[1] fitting scenario to which we have become accustomed, by virtue of its long-term success, has been the focus of a considerable theoretical effort (especially in the case of the S-wave parameters) to establish its basis in the microscopic theory. This effort has met with mixed success.

On the one hand, it was shown lately[2] that a calculation of the rescattering contribution to the second-order S-wave pion-nucleus optical potential (i.e., the rescattering contribution to the usual Pauli term in the S-wave pion-nucleus optical potential,[2] $V^{\rm Pauli} = -4\pi b_0^{(2)}\rho$) in a model of nuclear-medium polarization[2,3] including nucleon-hole pair contributions as well as those of isobar-hole pairs, produced a value for the effective b_0 [with[4] $b_0 = b_0^{(1)} + (b_0^{(2)})_{\rm eff}$] in excellent agreement ($b_0 \simeq -0.03 m_\pi^{-1}$) with this fitting scenario which takes ${\rm Re}B_0 = 0$.

On the other hand, there are the recent calculations of García-Recio, Oset, and Salcedo[5] for ${\rm Re}B_0$ which find the dominant contribution from Δ-hole excitation appreciable and attractive [in particular, $({\rm Re}B_0)_{\Delta\text{-h}} = 0.055 m_\pi^{-4}$]. Furthermore, García-Recio *et al.*[5] claim that off-shell effects could inflate this result by $\sim 100\%$.[5] [This is consistent with my finding in a simple adaptation (which neglects nucleon recoil) of Riska's Δ-hole model[3] to the present situation that $({\rm Re}B_0)_{\Delta\text{-h}} = 0.049 m_\pi^{-4}$; note that both scattering and absorbing vertices in this model have the same monopole form-factor dependence. When only the absorbing vertices are governed by this dependence, one finds this estimate nearly doubled.] The final correction for short-range nucleon-nucleon correlations made in the usual manner[5,6] yields the result $({\rm Re}B_0)_{\Delta\text{-h;corr}} \approx \frac{1}{2}({\rm Re}B_0)_{\Delta\text{-h}}$, which is still indicative of an appreciably *attractive* parameter. Since these calculations[5] embody all the conventional wisdom on the subject, the microscopic theory would appear to be in gross disagreement with the standard scenario. Less narrowly stated, the general trend of pionic-atom fits which finds ${\rm Re}B_0 \approx 0$ or negative (and repulsive) is completely at odds with this apparent (and disconcerting) result of the microscopic theory. The purpose of this paper is to draw attention to additional sizable sources of repulsion in the S-wave optical potential of chiral origin. While the density dependence of the resulting distribution of repulsive strength differs somewhat from the standard scenario, the sum total of repulsion appears to be consistent with the demands of a successful fit.

Earlier calculations[7,8] of the chiral contribution to the pion-nucleus optical potential consisted in the evaluation of the static isoscalar spin-scalar part of the two-body scattering amplitude (see Fig. 1 of Ref. 7) using the two-body density of the Fermi-gas model, a formal process which rather obscures the precise perturbative origin of such a contribution if viewed in the usual Fermi-gas model of the microscopic theory. As an indication that chiral corrections are more usefully discussed in the latter approach I transform the earlier[7] result for $b_0^{(2')}$, whose partial contributions are given by Eqs. (7a) and (7b) of Ref. 7, in the large A limit,[9]

$$\begin{aligned} b_0^{(2')} &= (1/m_\pi)(f^2/4\pi)^2 A (16_\pi^2 R_0^6/3)^{-1}\Omega \int d\mathbf{r}\,\theta(R_0 - r)[j_1(k_F R_0 x)/(k_F R_0 x)]^2[-\tfrac{81}{2} + 54(1 - m_\pi R_0 x/8)]Y_0(m_\pi R_0 x) \\ &= \{6/[(2\pi)^3 m_\pi^2 k_F^3]\}(f^2/4\pi)^2 \int\int d\mathbf{k}\,d\boldsymbol{l}\,\theta(k_F - k)\theta(k_F - |\mathbf{k} + \boldsymbol{l}|)l^2/(l^2 + m_\pi^2)^2 . \end{aligned} \tag{1}$$

It is straightforward to identify the corresponding contribution to the S-wave optical potential $V^{(2')}$,

$$2m_\pi V^{(2')} = -[3/(\pi^2 m_\pi^2 k_F^3)](f^2/4\pi)^2 \int\int d\mathbf{k}\,d\boldsymbol{l}\,\theta(k_F - k)\theta(k_F - |\mathbf{k} + \boldsymbol{l}|)l^2/(l^2 + m_\pi^2)^2 , \tag{2}$$

with the set of Feynman graphs of Figs. 1(b)–1(d) which comprise the lowest-order chiral corrections of the "Pauli" variety [see the two graphs of Fig. 1(a)]. While the resulting contribution is small and attractive ($b_0^{(2')} \simeq 0.003 m_\pi^{-1}$), it will be almost immediately clear that there is no basis for the claim that chiral corrections to the optical potential are generally small.

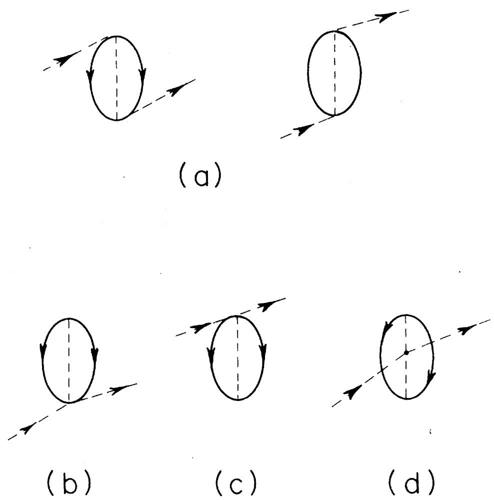

FIG. 1. "Pauli" graphs in the Fermi-gas model. (a) consists of the usual set of second order in b_0 and b_1; (b)–(d) produce the small chiral correction to (a) discussed in Refs. 6 and 7.

Keeping in mind[8,10] that the $\pi N\Delta$ and $3\pi N\Delta$ couplings bear the exact same relationship to each other as do the ordinary πNN and $3\pi NN$ couplings, one may consider the chiral corrections in the analogous situation where the hole-hole loop of the Pauli graph is replaced by a Δ-hole loop (see Fig. 2). In this case I find

$$2m_\pi V^{(2\Delta)} = -4\pi b_0^{(2\Delta)}\rho \,, \tag{3}$$

with

$$b_0^{(2\Delta)} = -(8/9\pi m_\pi^2)(f^2/4\pi)(f^{*2}/4\pi) \times \int_0^\infty dq\, q^4 |u(q)|^2 [\omega_q^2+\omega_q(\omega_q+\omega_\Delta)] \times [(\omega_q+\omega_\Delta)^2\omega_q^4]^{-1} \,, \tag{4}$$

where[3] $\omega_\Delta = 2.1 m_\pi$, $f^{*2}/4\pi = 0.35$. While Pauli corrections are insensitive[11] to the $\pi N\Delta$ form factors $u(q)$, because of Fermi surface restrictions, it is readily apparent, in the case of Δ-hole chiral corrections, that the presence of such a form factor is indispensable for finiteness. For $q < 2k_F$ or $q < 4m_\pi$, one finds comparable contributions to $b_0^{(2\Delta)}$ using either the monopole form factor (preferred by Ref. 3), with $u_{\text{monopole}}(q) = (\Lambda^2 - m_\pi^2)/(\Lambda^2 + q^2 - m_\pi^2)$, or the appropriate cloudy bag form factor,[12] with $u_{\text{CB}}(q) = 3j_1(qR/\hbar c)/(qR/\hbar c)$, where typically, $R = 0.8$ fm. Note that since the mass scale parameter Λ of the monopole form factor is customarily[3] taken to have the value $\Lambda = 1200$ MeV, the monopole form factor has, unfortunately, considerable strength [and hence, the integrand in Eq. (3) as well] up to $q = 10m_\pi$, and is thus less credible here than the cloudy bag form factor which drops off rapidly beyond $q = 4m_\pi$. The recent success[13] of the cloudy bag model in connection with the calculation of the S-wave πN phase shifts is certainly a rather compelling argument for the "conservative" choice of $u(q) = u_{\text{CB}}(q)$ in the calculations reported here. In fact, using the cloudy bag form factor, I find $b_0^{(2\Delta)} = -0.011 m_\pi^{-1}$ with a cutoff (q_c) at $q_c = 5m_\pi$; advancing the cutoff to $q_c = 6m_\pi$ only increases this result by approximately 10%. On the other hand, with the monopole form factor, I find $b_0^{(2\Delta)} = -0.012 m_\pi^{-1}$ for $q_c = 4m_\pi$, with $b_0^{(2\Delta)}$ swelling to $-0.034 m_\pi^{-1}$ for $q_c = 10m_\pi$. With medium enhancement reasonably estimated[8] by a factor of 2, the resulting repulsion is more than enough to compensate for the usually neglected induced Galilean attraction given (at zero pion kinetic energy) by

$$K(r) = -4\pi (b_0^{(\text{Gal})})^{\text{threshold}}_{\text{equivalent}}\rho \,, \tag{5}$$

with

$$(b_0^{(\text{Gal})})^{\text{threshold}}_{\text{equivalent}} \simeq (m_\pi/M)^2 (c_0 m_\pi^2)\, \tfrac{3}{5}\, (k_F/m_\pi)^2 = 0.0114 m_\pi^{-1} \,, \tag{6}$$

obtained by linearizing the density dependence of the customary expression.[14] [Note that the scattering length correction to the conventional Δ-hole calculation with the Galilean coupling, $f^*(m_\pi/M)\mathbf{p}_N \cdot \mathbf{S}T_\alpha\phi_\alpha$, yields the result $b_0^{(\text{Gal})} \simeq 0.009 m_\pi^{-1}$.] Making use of the linearizing rela-

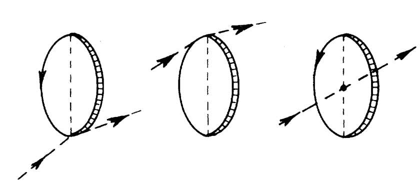

FIG. 2. Chiral corrections involving the Δ-hole loop analogous to those of Figs. 1(b)–1(d).

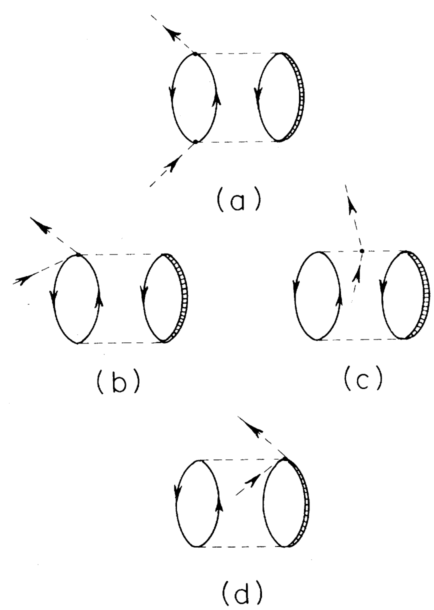

FIG. 3. Graphs contributing to ReB_0 in the Fermi-gas model. (a) is a principal contributor to a large unwanted attraction. The chiral corrections (b)–(d) (these should be doubled) are sizable and repulsive.

tion between coefficients Δb_0 of ρ and corresponding coefficients ΔB_0 of ρ^2 often remarked on[15] in discussions dealing with the uniform model of the optical potential, which here takes the form

$$\Delta B_0 m_\pi^4 = (2k_F^3/3\pi^2 m_\pi^3)^{-1}\Delta b_0 m_\pi \simeq 2.13(\Delta b_0 m_\pi)\,, \tag{7}$$

for $k_F = 1.35\ \mathrm{fm}^{-1}$, I find that the *uncompensated* repulsion obtained above is already $\sim 50\%$ of the size of the unwanted attraction in the dispersive parameter $\mathrm{Re}B_0$.

The remaining needed repulsion is to be found in analogous chiral corrections [see the representative graphs of Figs. 3(b)–3(d)] to the usual $\mathrm{Re}B_0$ [see Fig. 3(a)]; I find the correction to good approximation to be given by the expression

$$(\mathrm{Re}B_0)_{\mathrm{chiral}} = -(128/9m_\pi^4)(f^{*2}/4\pi)(f^2/4\pi)^2[2\omega_\Delta/(\omega_\Delta^2 - m_\pi^2)] \times \int_0^\infty dq\,|u_{\mathrm{CB}}(q)|^4[q^6/(q^2+m_\pi^2)^3][1-\theta(2k_F - q)(1-q/2k_F)^2(1+q/4k_F)]\,, \tag{8}$$

with $(\mathrm{Re}B_0)_{\mathrm{chiral}} = 0.023 m_\pi^{-4}$. This result when coupled with my earlier findings vis à vis $b_0^{(2\Delta)}$ would appear to indicate that the microscopic theory is able to provide as much repulsion as the conventional fit to experiment requires; I leave additional details and refinements dealing with nucleon-nucleon correlations, etc., for publication elsewhere.

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[4]$b_0^{(1)}$ denotes the isoscalar scattering length contribution to $(b_0)_{\mathrm{eff}}$ usually taken to be small. For example, $b_0^{(1)} = -0.005 m_\pi^{-1}$ in Ref. 3.

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[9]A is the nuclear mass number, k_F the Fermi momentum which is given by $k_F = 1.35\ \mathrm{fm}^{-1}$, and $x = r/R_0$, where the parameter R_0 is the nuclear radius with the usual A dependence, $R_0 = r_0 A^{1/3}$. Ω is the nuclear volume and $Y_0(x) = e^{-x}/x$.

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[11]The numerical value for the Pauli contribution to b_0 [see Fig. 1(a)], $b_0^{(2)}$ (in the notation of Ref. 3), is $-0.014 m_\pi^{-1}$. With the inclusion of monopole form factors, this result changes only slightly ($-0.0135 m_\pi^{-1}$). The analogous cloudy bag calculation [see Ref. 12] requires input from S-wave πN scattering with $\lambda_1 = 0$, $\lambda_2 \simeq m_\pi^2/16\pi f_\pi^2$. (These parameter values are very close to those of the scattering length Hamiltonian [R. Rockmore, Phys. Lett. B **205**, 179 (1988)].) I find $(b_0^{(2)})_{\mathrm{cloudy\ bag}} = -0.0117 m_\pi^{-1}$ using the appropriate cloudy bag form factor $F(q)$, where

$$F(q) = N_s^2 \int_0^R r^2 dr [j_0^2(\omega_s r/R) + j_1^2(\omega_s r/R)]\,,$$

with $N_s^2 = \omega_s/[j_0^2(\omega_s) R^3(\omega_s - 1)]$, $\omega_s R = 2.043$, and $R = 0.8$ fm.

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