Off-shell effects from meson exchange in the nuclear optical potential

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First-order Kerman-McManus-Thaler nuclear optical potentials for intermediate energy protons are constructed from realistic meson-exchange models of the nucleon-nucleon interaction including off-shell effects. At 200 MeV the full model developed by the Bonn group is employed, while at 500 MeV a simplified version with delta resonance inelasticity is used. The resulting off-shell effects on the nuclear scattering spin observables are found to be substantial.

The first-order description of intermediate energy proton-nucleus optical potentials in the nonrelativistic form $t(q)\rho(q)$ is very appealing because both the physical nucleon-nucleon scattering amplitude t and the one-nucleon density ρ for the target nucleus can be obtained from other experiments. It is also convenient in that it is local and thus serves as a basis for a large body of phenomenological work. Apart from second-order correlation terms, there is continuing interest in theoretical corrections at the first-order level especially from relativistic dynamics,[1] medium modifications of t^2, and off-shell effects.[3] Off-shell t matrices generally introduce a nonlocal character to the optical potential. It is still not known whether the off-shell behavior of a modern boson-exchange model of the NN force can influence the N-nucleus spin observables in a manner that would affect the interpretation of recent results obtained from local treatments of relativistic dynamics or medium modifications. In the present paper we report off-shell effects from the full Bonn meson-exchange model at 200 MeV and from one of its derivatives at 500 MeV in the prediction of elastic proton-nucleus scattering through a first-order optical potential.

A previous study[3] was carried out with the Love Franey[4] fit to the NN t matrix in which off-shell effects enter only through the extrapolation of a "direct+exchange" parametrization of on-shell NN data. The resulting off-shell elements are without features (such as off-shell unitarity[5]) that would come naturally through the use of a realistic NN potential and the associated wave equation. The consequences for nuclear scattering are not known. It is important to have results from off-shell nonlocal calculations based on a realistic NN interaction so that the dominant first-order mechanism can be judged in a better light. Also, approximate expansions, which have been put forward to obtain an effective local form,[6] can then be tested for the ability to capture the dominant effect.

In the present work we deal directly with a nonlocal optical potential. Approximations are still made, however, in the derivation of the form employed. In the multiple scattering theory of Kerman, McManus, and Thaler (KMT),[7] the nonrelativistic first-order optical potential may be expressed as

$$U(\mathbf{k}',\mathbf{k})=\left[\frac{A-1}{A}\right]\left\langle \mathbf{k}',\phi_0\left|\sum_i^A t_i(\varepsilon)\right|\mathbf{k},\phi_0\right\rangle . \tag{1}$$

Here $\mathbf{k}'$ and $\mathbf{k}$ are the final and initial momenta in the frame of zero total nucleon-nucleus momentum, and $t_i(\varepsilon)$ is the free NN t-matrix operator at energy ε, which we fix at a value corresponding to free NN scattering at the beam energy. Because of the complexity of the full integration implied by (1), factorization approximations are usually made.[8] We use the method of optimum factorization,[3] which means that the t-matrix element is expanded as a Taylor series in the integration variable about a fixed value chosen so that the contribution of the second term in the expansion is minimized. In the present case, that contribution can be made zero if the expansion point corresponds to the initial momentum of the active target nucleon being $\mathbf{q}/2$. The result is[3]

$$U_{\text{off}}(\mathbf{k}',\mathbf{k})=\left[\frac{A-1}{A}\right]\eta(\mathbf{k}',\mathbf{k})\sum_i^A t_i\left[\mathbf{q},\frac{A+1}{A}\mathbf{K},\varepsilon\right]\rho_i(q) . \tag{2}$$

Here $\mathbf{K}=(\mathbf{k}'+\mathbf{k})/2$, $\mathbf{q}=\mathbf{k}'-\mathbf{k}$, and $\eta(\mathbf{k}',\mathbf{k})$ is the Möller factor, which transforms the t matrix from the two-nucleon c.m. frame to the nucleon-nucleus c.m. frame ignoring relativistic spin effects.[9]

The quantity $t(\mathbf{q},\mathcal{K},\varepsilon)$ obtained in (2) is the fully off-shell NN t matrix for scattering from a state with NN relative momentum $\boldsymbol{k}=(\mathcal{K}-\mathbf{q})/2$ to a state with NN relative momentum $\boldsymbol{k}'=(\mathcal{K}+\mathbf{q})/2$ where we have set $\mathcal{K}=(A+1)\mathbf{K}/A$. It is to be understood that the spin integrations in (1) (under the usual assumption of a spin-saturated target) have reduced the required t matrix to just the spin-independent component (corresponding to Wolfenstein amplitude A) and the spin-orbit component (corresponding to Wolfenstein amplitude C). The elastic nuclear scattering amplitude is generated from the momentum space solution of the integral equation $T=U+UG(E)T$, where U is given by (2) and the inverse propagator G^{-1} is taken to be the difference of relativistic energies. Further details of calculations of this type can be found in Ref. 3. Nearly all calculations previous

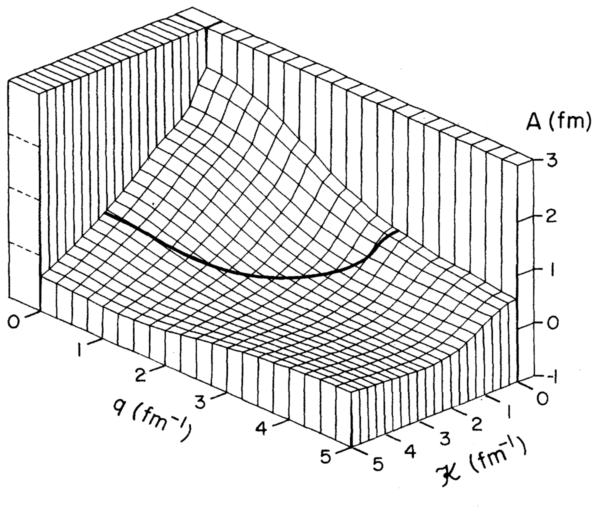

FIG. 1. The real part of the off-shell NN scattering amplitude A from the full Bonn interaction of Ref. 10 at 200 MeV laboratory energy as a function of $q=|\mathbf{k}'-\mathbf{k}|$ and $\mathcal{K}=|\mathbf{k}'+\mathbf{k}|$, with $\hat{\mathbf{q}}\cdot\hat{\mathcal{K}}=1$. The solid curve shows the on-shell amplitude.

to the last few years have ignored off-shell effects through the assumption that, for all values of $\mathcal{K}$, the two-body t matrix $t(\mathbf{q},\mathcal{K},\varepsilon)$ in (2) is equal to its on-shell value $t^{\text{on}}(\mathbf{q},\varepsilon)$. This means that the conditions $\mathbf{q}\cdot\mathcal{K}=0$ and $q^2+\mathcal{K}^2=4k_0^2$ are imposed on $t_i(\varepsilon)$, with k_0 being the on-shell relative momentum for NN scattering at energy ε.

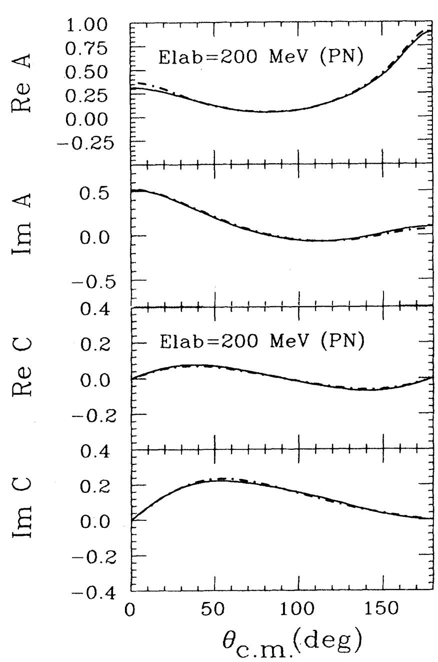

FIG. 2. Wolfenstein amplitudes for proton-neutron scattering derived from the full Bonn interaction of Ref. 10 (solid lines) and from the phase shift analysis of data from Ref. 12 (dot-dashed lines).

The result is the following local approximation

$$U_{\text{on}}(\mathbf{k}',\mathbf{k}')=\left[\frac{A-1}{A}\right]\eta(q,\varepsilon)\sum_i^A t_i^{\text{on}}(\mathbf{q},\varepsilon)\rho_i(q) \ . \qquad (3)$$

In this work we contrast the nuclear scattering observables obtained from U_{off} and U_{on}. An exact method for handling the Coulomb effects in momentum-space scattering, without problems in the higher angular momentum states that we require here, is not presently available. The prescription we employ here[3] is that the Coulomb distorted nuclear bar phase shifts are approximated by the pure nuclear phase shifts generated by Eqs. (2) and (3). The remaining effects which dominate the Coulomb-nuclear interference, namely, the multiplicative Coulomb partial wave S-matrix factor and the additive pure Coulomb scattering amplitude, are included in the standard way. This procedure is used in exactly the same way for U_{on} and U_{off} to examine nuclear off-shell effects.

At 200 MeV we employ for the NN t matrix the full Bonn interaction,[10] which contains relativistic kinematics, retarded meson propagators as given by time-ordered

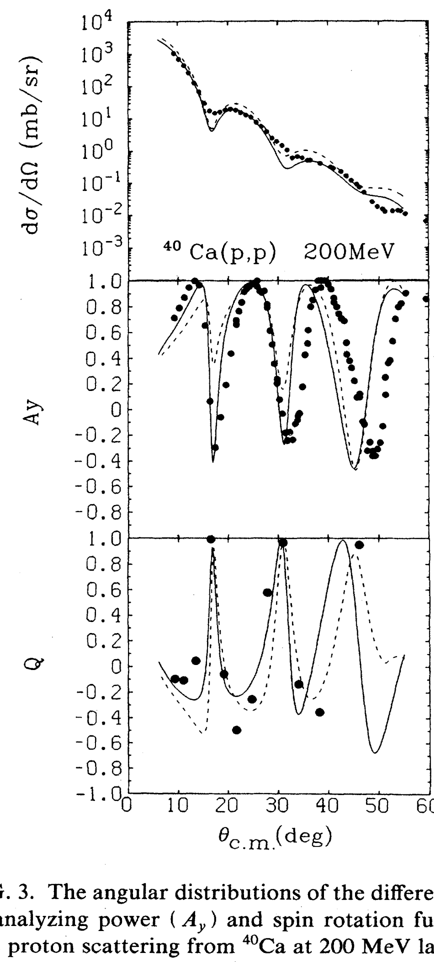

FIG. 3. The angular distributions of the differential cross section, analyzing power (A_y) and spin rotation function (Q) for elastic proton scattering from ^{40}Ca at 200 MeV laboratory energy. The calculations are with a first-order optical potential from the full Bonn interaction in the on-shell local approximation (dashed curve) and with off-shell nonlocal effects included (solid curve). The data are from Ref. 13.

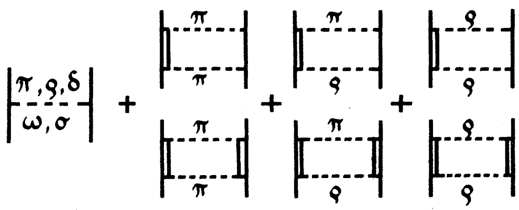

FIG. 4. The set of irreducible diagrams for the processes included in the extended Bonn model D52 from Ref. 16. A solid line represents a nucleon and a double line represents a Δ(1232) isobar.

perturbation theory, along with crossed and uncrossed meson exchanges with NN, $N\Delta$, and $\Delta\Delta$ intermediate states. Relativistic effects from negative-energy fermion states are excluded from this model. To obtain some insight into the character of the off-shell structure and the nonlocality that will be induced for the optical potential, we display in Fig. 1 the real part of the NN off-shell amplitude A. There is essentially no dependence upon the angular quantity $\hat{q}\cdot\hat{\mathcal{K}}$, and very weak dependence of this off-shell shape upon energy. These general features apply to the other amplitudes relevant to the optical potential and also to amplitudes from other NN models.[11] The on-shell amplitude is indicated by the curve in the figure. Since the nuclear density ρ is independent of $\mathcal{K}$ and falls off with q faster than the NN amplitude, the largest contribution to physical nuclear scattering comes from the region near the point ($q=0, \mathcal{K}=2k_0$) where both the NN and N-nucleus systems are on shell. The importance of nuclear scattering to off-shell intermediate states is governed mainly by the $\mathcal{K}$ dependence at low q. We note that the maximum of the NN amplitude occurs in the region where both momenta are small, rather than at the forward on-shell point. As shown in Fig. 2 for proton-neutron scattering, the relevant on-shell amplitudes reproduce well the amplitudes from the phase shift analysis of data.[12]

The results for p-nucleus scattering from ^{40}Ca at 200 Mev are shown in Fig. 3 along with the data.[13] To our knowledge, this is the first application to a many-nucleon system of the full model of the Bonn group (in contrast to the simplified one-boson-exchange parametrizations and localized versions that are available[10]). The off-shell

TABLE I. Meson parameters for the Bonn D52 model of the NN interaction. The columns contain the meson-baryon vertex, the coupling strength (with tensor to vector ratio in parentheses), the meson mass and the dipole form factor range.

	$g_\alpha^2/4\pi$ (f_α/g_α)	m_α (MeV)	Λ_α (MeV)
$NN\pi$	14.4	138.03	1400
ρ	0.7 (6.1)	769	1400
ω	20	782.6	1600
σ	7.7983	570	1280
δ	0.1838	983	2000
$N\Delta\pi$	0.224	138.03	1050
$N\Delta\rho$	20.45	769	1050

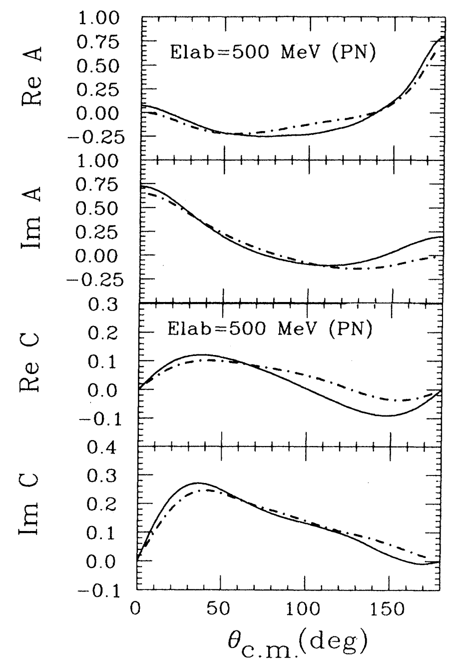

FIG. 5. Wolfenstein amplitudes for proton-neutron scattering derived from the extended Bonn model D52 of Ref. 16 (solid lines) and from the phase shift analysis of data from Ref. 12 (dot-dashed lines).

effects are quite noticeable in the spin observables and are larger than those found in Ref. 3 in this energy region with a Love-Franey type of representation of the NN t matrix. In particular the achievement of a uniform depth

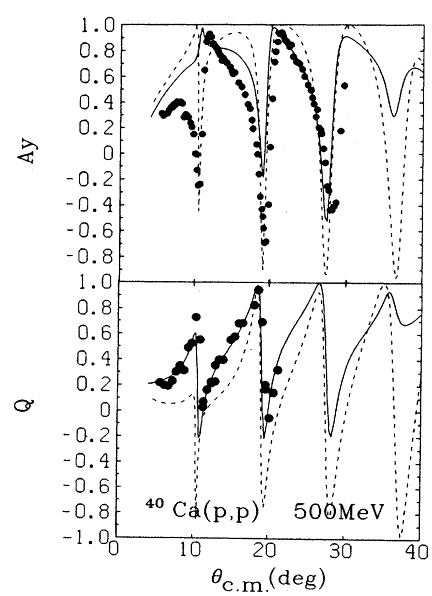

FIG. 6. A_y and Q for elastic proton scattering at 500 MeV. The on-shell local (dashed line) and off-shell nonlocal (solid line) calculations use the Bonn NN model D52 in the first-order optical potential. The data are from Ref. 17.

for the three minima in A_y has previously been difficult for nonrelativistic calculations based on a free t matrix in this energy range.[3,14] Relativistic effects and medium modifications (i.e., Pauli blocking and local field corrections that would convert t into a g matrix) would be required to play somewhat less of a role in the present circumstance.

For our p nucleus scattering calculation at 500 MeV we start from a recently developed extension of the Bonn meson exchange interaction above pion production threshold.[15] This extended model omits crossed meson exchanges but retains retardation in meson propagation. The retardation would make it necessary to include one-loop self-energy corrections to the baryon propagators in order to satisfy unitarity in the three-particle sector.[15] To avoid the difficulties of extracting an off-shell t matrix from three-body calculations of the type performed in Refs. 15, we use here a further simplified model in which the energy dependence of meson propagation is ignored to produce static propagators. Thus, pion production in this model is described only through the decay of the delta isobar with a width obtained consistently from the imaginary part of the one-pion loop diagram for the delta self-energy. This model (called D52) consists of the diagrams shown in Fig. 4, and the meson parameters shown in Table I result from a fit to NN data up to 1 GeV.[16] In the energy range appropriate to the present work, the description is an improvement over that obtained in Ref. 15. The relevant on-shell proton-neutron amplitudes are compared in Fig. 5 to the amplitudes from the phase shift analysis of data.[12]

For 500 MeV scattering from ^{40}Ca, the off-shell dependence has a strong effect on the spin observables while the only change in the cross section is that the diffraction minima become slightly shallower. In Fig. 6 we show the results for the spin observables A_y and Q along with the data.[17] These off-shell spin effects can change the qualitative shape of the angular distributions and are comparable in size to the spin effects obtained in relativistic treatments. Often, but not always, the description of data is improved as exemplified here by the observable Q. We note that detailed comparisons with data can be affected by the quality of the on-shell t matrix and also by Coulomb effects. This latter point is particularly relevant for the precise shape of the first interference minimum in A_y at 500 MeV for ^{40}Ca which is extremely sensitive to many details.[18] In Fig. 7 we display, for purely on-shell calculations, the difference obtained from use of the Bonn D52 model and the amplitudes from phase shift analysis

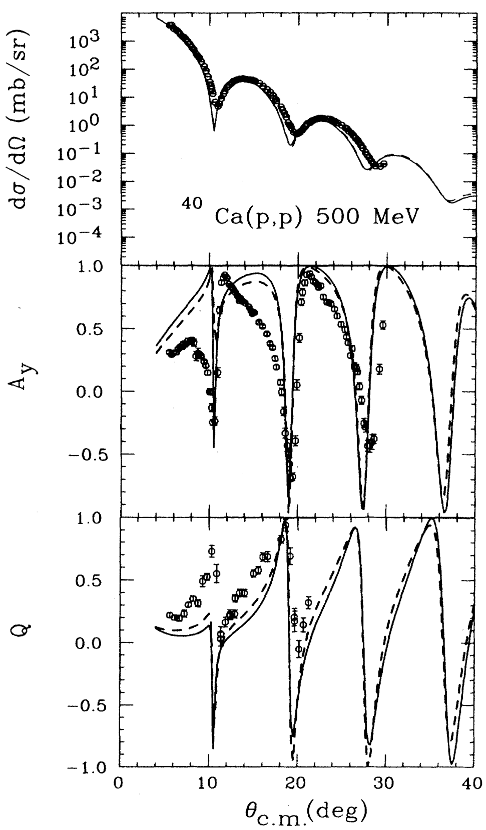

FIG. 7. The differential cross section, A_y, and Q for elastic proton scattering at 500 MeV. The data are from Ref. 17. The two calculations shown use the on-shell local approximation with a t matrix obtained from the Bonn model D52 (solid line) and from the phase shift analysis of data (dashed line) from Ref. 12.

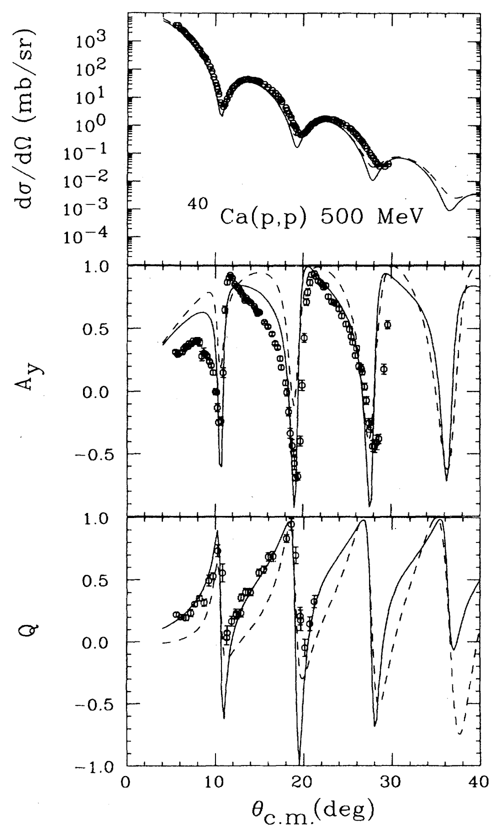

FIG. 8. Same as for Fig. 7, except here the off-shell (solid line) and on-shell (dashed line) calculations from the Bonn D52 t matrix are shown and all Coulomb contributions are omitted.

of data displayed previously in Fig. 5. The qualitative features of the on-shell results are preserved. With the removal of all Coulomb contributions from the calculations displayed in Fig. 6, the nuclear off-shell effects show through with essentially the same magnitude and qualitative behavior in the regions between the sharp diffractive minima as shown in Fig. 8. A precise description of spin observables at the sharp minima in this case remains a difficult task, and these results indicate that both off-shell effects and Coulomb effects have a sensitive bearing on the matter. Further complicating this task is the observation that, in a relativistic treatment, a 1% change in the Lorentz scalar component of the nuclear interaction can significantly change the qualitative behavior at the first diffractive minimum.[18]

The situation is less problematical for scattering from ^{16}O at the same energy. In Fig. 9 we display the obtained off-shell dependence at 500 MeV with the Coulomb effects included as described. No qualitative change in the angular distribution is produced and the overall size of the off-shell spin effects is quite comparable to that for ^{40}Ca. One would expect a significantly reduced role for Coulomb contributions for ^{16}O and this is confirmed by the results shown in Fig. 10 produced through the removal of all Coulomb contributions. The general conclusion that remains is that spin-dependent off-shell effects are sizable and cannot be safely ignored. There is no evidence that off-shell effects from meson exchange models of the present type can alone explain the data for spin observables within a nonrelativistic treatment.

In summary, we find off-shell effects and the resultant nonlocalities from a meson exchange model of the *NN* t matrix to be significant enough to affect the interpretation of results from local treatments of the dynamics. There is no obvious reason why such off-shell nonlocal behavior should be any less important in the relativistic impulse approximation since the dominant sector, produced by projection onto positive energy free Dirac spinors, is very similar in physical content to the first-order KMT calculations performed here. The nonlocal effect introduced by off-shell t matrices in this work is quite distinct from the nature of the effective "pair potential" that is introduced through a relativistic approach. With local t matrices, the latter potential is essentially local due to the extremely short range of the pair component of the propagator. Typical meson exchange models, at least at the formulation stage, imply both relativistic meson-nucleon dynamics and an off-shell momentum structure for the free *NN* t matrix in a related fashion. The present work suggests that a consistent approach to nuclear scattering may require a coordinated treatment of both of these features. This is not an easy task because, even with a truncation to eliminate negative energy nucleon states, retardation effects remain. A time-ordered perturbation approach, such as that used for the Bonn *NN* model, will in principle introduce many-body forces in the nuclear environment. This may well require a

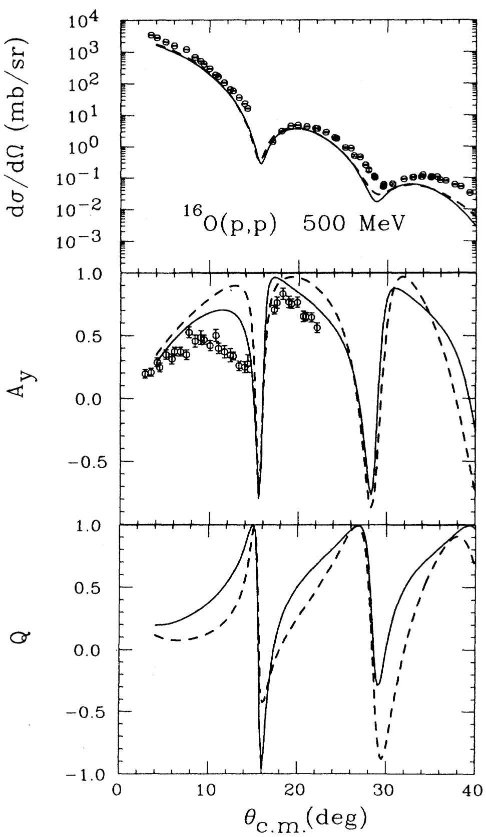

FIG. 9. The differential cross section, A_y, and Q for 500 MeV proton scattering from ^{16}O. The calculations shown are the off-shell (solid line) and on-shell (dashed line) results from the first-order optical potential. The data are from Ref. 19.

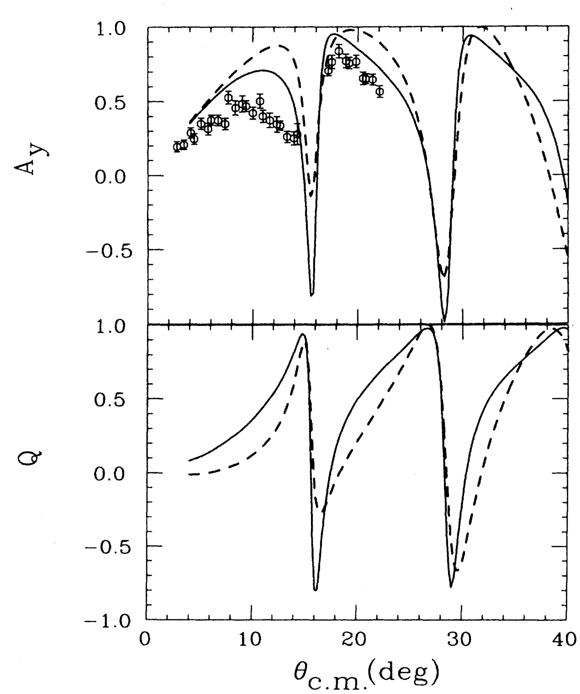

FIG. 10. Same as for Fig. 9 for A_y and Q except here both the off-shell (solid line) and on-shell (dashed line) calculations omit all Coulomb contributions.

reassessment of the higher-order terms in the multiple scattering expansion that underlies almost all approaches to the nuclear optical potential.

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