Effects of nuclear forces on neutrino opacities in hot nuclear matter

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Neutrino opacities are analyzed in terms of correlation functions for density, spin density, isospin density, and spin-isospin density operators. In a long-wavelength limit appropriate to the scattering of thermal neutrinos in most situations relevant to the supernova, the opacities coming from the neutral current interaction are given in terms of susceptibilities, $\partial\rho_i/\partial\mu_j$, where ρ_i are the various densities and μ_j the associated chemical potentials. These susceptibilities are calculated in the framework of a Skyrme interaction, one which has been used in calculations of the hot equation of state, supplemented by spin-dependent terms from other sources. There is a large reduction in the Gamow-Teller part of the cross sections in all of the domains of temperature, proton fraction, and density which were studied. Effects of the screened Coulomb force also cause a large reduction in the opacity arising from the Fermi interaction of the neutrino with protons, in some domains of density and temperature.

I. INTRODUCTION

In the interior of the supernova core, or in a hot, newly formed neutron star, the neutrinos, which play a key role in the dynamics, deleptonization, and early cooling of the object, have a free path much less than the size of the core. The opacities and the transport processes which govern the diffusion of neutrinos out of the core have been the subject of a number of papers in the last few years.[1–11] Nonetheless, the results are incomplete in two respects.

(1) The teatment of Fermi statistics for nucleons has been haphazard, and has been largely limited to consideration of cases in which particular species are completely degenerate or completely nondegenerate. The conditions that prevail, at different positions and in different eras, imply that from none to all three of the species, neutrons, protons, neutrinos, will be degenerate. (Electrons will be degenerate under almost all circumstances.) The degenerate and the nondegenerate limits are always more approachable analytically; but the reality is that much of the action will be in a domain which is transitional, for one species or another.

(2) There has been minimal attention to the effects of interactions on opacities, except for the work of Iwamoto and Pethick,[12] which addresses the degenerate case only, in the framework of Landau Fermi-liquid theory. In much of the density domain in which the neutrino opacity is needed, the strong interactions between nucleons are important. In addition, the neutrino wavelength will generally be greater than the interparticle spacing, so that coherent effects can be important.

The present paper is devoted to a systematization of the calculations of those opacities which derive from the neutral current interactions of neutrinos with nucleons, in the domains in which the neutrino wavelength is of the order of, or greater than, the internucleon spacing. We shall consider only neutrinos of energy no more than around $3k_BT$, i.e., energies in the thermal range. But, subject to these limitations, we present a systematic theory which includes the effects of coherence, nuclear interactions, and Fermi statistics. The results cannot be applied to the diffusion of electron neutrinos, in the early deleptonization era at least, both because of the high energies of the electron neutrinos involved in this case, and because of the importance of the charged current interactions. The results are applicable to the heat transfer by mu and tau neutrinos, and antineutrinos, on all time scales. Our approach is similar to that of Iwamoto and Pethick, as it applies to conditions of nucleon degeneracy, but it also can be applied to the nondegenerate case. Results will be given for nondegenerate and transitional regions of temperature and density in which the effects of interactions are still important.

In principle, the nuclear physics for the opacities could be worked out as accurately as the equation of state for hot matter which is used in building the star. In fact, in the long-wavelength limit many of the effects of the strong interactions are expressible in terms of thermodynamic quantities, such as the compressibility, the isospin polarizability, etc. What follows can be motivated from the following observations about scattering of neutrinos from a gas of neutrons, taking into account, in this example, only the Fermi part of the neutral current interaction:

(a) The differential cross section per unit volume for a neutrino of energy E_ν passing through this medium to make a transition with three momentum transfer,

$$|\mathbf{q}|=|\mathbf{p}_\nu-\mathbf{p}'_\nu|\approx 2E_\nu(1-\cos\theta)\ ,$$

is given by

$$\frac{1}{\mathcal{V}}\frac{d\sigma}{d\cos\theta}=\frac{2\pi G^2}{\mathcal{V}}\int d^3p'_\nu R\left[\sum_f\langle i|\rho(\mathbf{q})|f\rangle\langle f|\rho(-\mathbf{q})|i\rangle\right]_{\text{th.av.}}\delta(E_f-E_i-E_\nu+E'_\nu)\ , \tag{1.1}$$

where $\mathcal{V}$ is the volume, the subscript th.av. indicates thermal average,

$$R = \sum_{\text{spin}} \left| \bar{u}(p'_\nu)\gamma_0(1-\gamma_5)u(p_\nu) \right|^2 (4E_\nu E'_\nu)^{-1} ,$$

and

$$\rho(\mathbf{q}) = \int e^{i\mathbf{q}\cdot\mathbf{x}}\rho(\mathbf{x})d^3x \ . \tag{1.2}$$

Here $\rho(\mathbf{x})$ is the density operator for neutrons and G is the appropriate weak interaction strength for neutrino neutron interactions. The thermal average is to be taken over the states $|i\rangle$ of the hot nuclear medium. We note that the energy transfer to the nuclear matter will be of the order $\mathbf{q}\cdot\mathbf{p}/2M \ll E_\nu$, where $\mathbf{p}$ is a typical nucleon momentum. Therefore the energy delta function can be written approximately as $\delta(E_\nu - E_{\nu'})$ and taken out from under the sum in (1.1), giving

$$\mathcal{V}^{-1}\frac{d\sigma}{d\cos\theta} = \frac{G^2E_\nu^2}{8\pi}\rho_0(1+\cos\theta)S(\mathbf{q}) \ , \tag{1.3}$$

where $S(\mathbf{q})$, the static structure function, is defined as

$$S(\mathbf{q}) = (\rho_0\mathcal{V})^{-1}\frac{\mathrm{Tr}\{[\exp-\beta(H-\mu N)]\rho(\mathbf{q})\rho(-\mathbf{q})\}}{\mathrm{Tr}[\exp-\beta(H-\mu N)]}$$
$$\equiv (\rho_0\times\mathcal{V})^{-1}\langle\rho(\mathbf{q})\rho(-\mathbf{q})\rangle \tag{1.4}$$

and ρ_0 is the average nucleon density, and $\beta = T^{-1}$ (we take $k_B = 1$).

(b) In the case in which the neutrons form a noninteracting Fermi gas, the structure function has the familiar representation in terms of the neutron Fermi distribution, $n(\mathbf{p})$,

$$S(\mathbf{q}) = \rho_0^{-1}\int\frac{d^3p}{(2\pi)^3}n(\mathbf{p})[1-n(\mathbf{p}+\mathbf{q})] \ . \tag{1.5}$$

When $q <$ (the interparticle spacing)$^{-1}$, the expansion of $S(\mathbf{q})$ in powers of q should converge rapidly. We retain only the first two terms. At low temperatures S can be expanded in powers of T as well. We obtain

$$S(\mathbf{q}) = \frac{3|\mathbf{q}|}{4k_F} + \frac{3TM}{2k_F^2} \ , \tag{1.6}$$

where M is the mass of the nucleon and k_F is the nucleon Fermi momentum.

Thermal neutrinos have energy, and therefore typical momentum transfers $\mathbf{q}$, of order T, so that the relative importance of the two terms on the right-hand side of (1.6) will be determined by the ratios of the coefficients. For a neutrino of energy as high as $3T$, the second term dominates (because the nucleons are nonrelativistic).

(c) The second term on the right-hand side of (1.6), which takes into account the effects of nonzero temperature when $\mathbf{q}$ vanishes, is exactly the classical result for the average thermal density fluctuations of a fluid,[13] when we reexpress the coefficient of T in terms of the thermodynamic quantities obtained from the equation of state of a free Fermi gas,

$$\lim_{q\to 0} S(\mathbf{q}) = \rho_0^{-1}T\left[\frac{\partial^2F}{\partial\rho_0^2}\right] = \frac{T}{K_T} \ , \tag{1.7}$$

where $F(\rho_0, T)$ is the free-energy density and K_T the isothermal bulk modulus, $K_T = \partial p/\partial\rho_0$.

(d) The result (1.7) for the long-wavelength fluctuations, in terms of the compressibility, is a general one, true for all temperatures, valid in both quantum and classical cases, and in the presence of interactions.[14] The fact that the corrections for finite q are relatively unimportant in the scattering of thermal neutrinos, for the special case of noninteracting nucleons in the limit of high-nucleon degeneracy, suggests that the $q = 0$ limit for $S(\mathbf{q})$ will be adequate for the treatment of thermal neutrinos under all conditions. Certainly this approximation becomes better, rather than worse, as the temperature is raised, or the nucleon density lowered, so that the system becomes less degenerate.

In the present paper we shall take advantage of the dominance of the classical terms to examine neutrino opacities in domains of density and temperature in which degeneracy ranges from none to complete. In addition to the fluctuations of neutron density which determined the Fermi part of the scattering rate in the above example, we shall need to consider the isospin fluctuations in a medium of protons and neutrons; the spin fluctuations, in the presence of spin-dependent forces, in order to deal with the Gamow-Teller (GT) matrix elements; and the spin-isospin fluctuations as well.

II. THEORY OF MULTICOMPONENT FLUCTUATIONS

We take the density of free energy, F, to be a function of the separate densities for the species, ρ_i (in our case taken to be p, n, each with spin up and spin down), and the temperature. The chemical potentials of each species are determined by the solutions to

$$\frac{\partial F}{\partial\rho_i} = \mu_i \ . \tag{2.1}$$

The probability distribution of long-wavelength fluctuations is given by[15]

$$\text{Prob.}(\delta\rho) = \mathcal{N}\exp\left[-(2\times\mathcal{V})^{-1}\beta\sum_{i,j}A_{ij}\sum_{\mathbf{q}}\delta\rho_i(\mathbf{q})\delta\rho_j(-\mathbf{q})\right] , \tag{2.2}$$

where $\mathcal{N}$ is a normalizing factor, and

$$A_{ij} = \left.\frac{\partial^2F}{\partial\rho_i\partial\rho_j}\right|_T \ . \tag{2.3}$$

The average of $\delta\rho_i\delta\rho_j$ in this distribution is given by

$$\lim_{q\to 0}\langle\rho_i(q)\rho_j(-q)\rangle = T(A^{-1})_{ij}\times\mathcal{V} \ , \tag{2.4}$$

where we have omitted the δ symbols applied to the ρ's, which are redundant for $q\neq 0$, in the case of a uniform medium. The coefficients $(A^{-1})_{ij}$ are expressible in terms of the derivatives of the densities with respect to the chemical potentials

$$(A^{-1})_{ij} = \frac{\partial\rho_i}{\partial\mu_j} \ . \tag{2.5}$$

The same result is established quantum mechanically, by direct differentiation of $\langle \rho_i \rangle$ with respect to the chemical potentials,

$$\langle \rho_i(0)\rho_j(0)\rangle - \langle \rho_i(0)\rangle\langle \rho_j(0)\rangle = \beta^{-1}\frac{\partial}{\partial \mu_j}\langle \rho_i(0)\rangle = \beta^{-1}\frac{\partial \rho_i}{\partial \mu_j}\times \mathcal{V} , \quad (2.6)$$

where

$$\langle \rho_i(0)\rangle = \frac{\mathrm{Tr}\rho_i(0)\exp\left\{-\beta\left[H-\sum_i \mu_i\rho_i(0)\right]\right\}}{\mathrm{Tr}\exp\left\{-\beta\left[H-\sum_i \mu_i\rho_i(0)\right]\right\}} , \quad (2.7)$$

etc.

III. WEAK INTERACTION REACTION RATES

In a standard model of the weak interactions the term in the Lagrangian density which scatters neutrinos from nucleons is[12]

$$L_\omega = \frac{G}{2\sqrt{2}}[\bar{\psi}_\nu \gamma_u(1-\gamma_5)\psi_\nu] \times [a\psi_n^\dagger\gamma_\mu(1-\gamma_5)\psi_n + b\psi_p^\dagger\gamma_u\psi_p + c\psi_p^\dagger\gamma_u\gamma_5\psi_p] . \quad (3.1)$$

We shall take the nucleons to be nonrelativistic, and make the quasielastic approximation for the neutrino energies $E_\nu \approx E_\nu'$. The squared matrix element for scattering from the nuclear medium, summed over states of the medium, using closure, averaged over a thermal ensemble of initial states and summed over neutrino spins, is then given by

$$W = \left[\sum_{f,\mathrm{spin}} \langle p',f|M|p,i\rangle|^2\right]_{\mathrm{th.av.}} = W(\mathrm{Fermi}) + W(\mathrm{GT}) ,$$

where

$$W(\mathrm{Fermi}) = 32G^2E_\nu^2(1+\cos\theta)(a^2\langle \rho_n\rho_n\rangle_q + b^2\langle \rho_p\rho_p\rangle_q + 2ab\langle \rho_n\rho_p\rangle_q) \quad (3.2)$$

and

$$W(\mathrm{GT}) = \tfrac{32}{3}G^2E_\nu^2(3-\cos\theta)\sum_{i=1}^{3}[a^2\langle \rho_n^i\rho_n^i\rangle_q + 2ac\langle \rho_n^i\rho_p^i\rangle_q + c^2\langle \rho_p^i\rho_p^i\rangle_q] . \quad (3.3)$$

Here $\cos\theta$ is the neutrino scattering angle, $\cos\theta \approx (E_\nu^2-q^2)E_\nu^{-2}$, and the averages which enter are defined by

$$\langle \rho_n\rho_n\rangle_q = \left[\left\langle \int e^{i\mathbf{q}\cdot\mathbf{x}}n^\dagger(x)n(x)\int e^{-i\mathbf{q}\cdot\mathbf{x}'}n^\dagger(x')n(x')\right\rangle\right]_{\mathrm{th.av.}}$$
$$\langle \rho_n^i\rho_p^j\rangle_q = \left[\left\langle \int e^{-i\mathbf{q}\cdot\mathbf{x}}n^\dagger(x)\sigma_i n(x)\int e^{-i\mathbf{q}\cdot\mathbf{x}'}p^\dagger(x')\sigma_j p(x')\right\rangle\right]_{\mathrm{th.av.}} \quad (3.4)$$

etc. If we make the further approximation of taking the long-wavlength limit of the correlation functions, we can then integrate over the neutrino scattering angle to obtain

$$\Gamma = \mathrm{transitions/time} = (\pi\times\mathcal{V})^{-1}\times 32G^2E_\nu^2[\langle (b\rho_p+a\rho_n)(b\rho_p+a\rho_n)\rangle + 3\langle (c\rho_p^{(3)}+a\rho_n^{(3)})(c\rho_p^{(3)}+a\rho_n^{(3)})\rangle] . \quad (3.5)$$

In the last term on the right-hand side of (3.5) we have used rotational invariance, and the small q limit, to express the spin-density correlations which enter the Gamow-Teller terms as

$$\langle \rho^i\rho^j\rangle_{q\to 0} = \delta_{ij}\langle \rho^{(3)}\rho^{(3)}\rangle_{q\to 0} . \quad (3.6)$$

Note, however, that isospin invariance of the nuclear potential does not by itself provide any relation among the correlation functions in the case of unsymmetrical nuclear matter. Of the six independent correlation functions, for the neutral current operators, two combinations can be determined, in the long-wavelength limit, from an equation of state calculation. As in (1.7), the nucleon density correlation can be determined from the compressibility,

$$\langle (\rho_p+\rho_n)(\rho_p+\rho_n)\rangle_{q\to 0} = \rho T K_T^{-1}\times\mathcal{V} , \quad (3.7)$$

where K_T is the isothermal bulk modulus. It can also be shown that

$$\langle (\rho_p+\rho_n)(\rho_p-\rho_n)\rangle_{q\to 0} = \left[\frac{1-g'}{1+g'}\right]\frac{\partial\rho}{\partial\hat{\mu}}\times\mathcal{V} , \quad (3.8)$$

where $\hat{\mu} = \mu_n - \mu_p$, and the function g expresses the neutron-to-proton ratio along the equilibrium path $\rho_p = g(\rho_n)$.

The usual expressions for the cross section, e.g., on a single proton, may be recovered from (3.5) by making the replacements

$$\langle \rho_p\rho_p\rangle \to \mathcal{V}, \quad \langle \rho_n\rho_n\rangle \to 0, \quad \langle \rho_n\rho_p\rangle \to 0 ,$$

etc.

For the case of charged current reactions, which will be discussed in a later paper, the correlation functions which enter the cross-section formulae will be "off-diagonal," viz.,

$$\left\langle \left[\int n^{\dagger} p e^{i\mathbf{q}\cdot\mathbf{x}} \right] \left[\int p^{\dagger} n e^{-i\mathbf{q}\cdot\mathbf{x}} \right] \right\rangle ,$$

and an extended formalism will be necessary to deal with them, even in the long-wavelength limit.

IV. ANALYTIC MODELS

Before giving general results in the next sections, we work out some illustrative models to see the general effects at work. For analytic simplicity we consider a case in which, because of low density or high temperature, Boltzmann statistics are operative for both species.

(i) As a first example, consider a nondegenerate sea of neutrons interacting through a two-body potential,

$$V = V_1(r) + \boldsymbol{\sigma}^{(1)} \cdot \boldsymbol{\sigma}^{(2)} V_2(r) \ . \tag{4.1}$$

Let us consider states with possible density fluctuations in the z spin component. In the Hartree approximation the potential energy will then be of the form

$$U = \frac{a}{2}(\rho_\uparrow + \rho_\downarrow)^2 + \frac{b}{2}(\rho_\uparrow - \rho_\downarrow)^2 \ , \tag{4.2}$$

where the first term derives from V_1 and the second from V_2. (In our later numerical calculations, based on a Skyrme interaction, exchange and Fermi statistics will be taken into account.) The respective densities for spin up and for spin down are related to the chemical potentials by

$$\begin{aligned} \rho_\uparrow &= \{\exp T^{-1}[\mu_\uparrow - (a+b)\rho_\uparrow - (a-b)\rho_\downarrow]\} \\ &\quad \times \int \frac{d^3k}{(2\pi)^3} \exp(-k^2/2MT) \ , \\ \rho_\downarrow &= \{\exp T^{-1}[\mu_\downarrow - (a-b)\rho_\uparrow - (a+b)\rho_\downarrow]\} \\ &\quad \times \int \frac{d^3k}{(2\pi)^3} \exp(-k^2/2MT) \ . \end{aligned} \tag{4.3}$$

Differentiating, we obtain

$$\begin{aligned} \frac{\partial \mu_\uparrow}{\partial \rho_\uparrow} &= a + b + T\rho_\uparrow^{-1} \ , \\ \frac{\partial \mu_\uparrow}{\partial \rho_\downarrow} &= \frac{\partial \mu_\downarrow}{\partial \rho_\uparrow} = a - b \ , \\ \frac{\partial \mu_\downarrow}{\partial \rho_\downarrow} &= a + b + T\rho_\downarrow^{-1} \ . \end{aligned} \tag{4.4}$$

Solving for $\partial\rho_i/\partial u_j$, using (2.4), introducing the combination $\rho_3 = \rho_\uparrow - \rho_\downarrow$, and setting $\rho_3 = 0$ at the end of the calculation, we obtain

$$\begin{aligned} &\lim_{q\to 0}[(\mathcal{V})\rho]^{-1}\langle \rho(\mathbf{q})\rho(-\mathbf{q})\rangle_{q\to 0} = (1 + 2aT^{-1}\rho)^{-1} \ , \\ &\lim_{q\to 0}[(\mathcal{V})\rho]^{-1}\langle \rho_3(\mathbf{q})\rho_3(-\mathbf{q})\rangle_{q\to 0} = (1 + 2bT^{-1}\rho)^{-1} \ . \end{aligned} \tag{4.5}$$

Note that when the interactions are turned off, we obtain unity for the structure functions; a noninteracting Boltzman gas produces fluctuations that exactly give the results of incoherent scattering (inverse mean free path = cross section × density). The modifications in the density correlation function depend only on the spin independent part of the potential, and those of the spin-density correlation depend only on the spin-dependent term. Attractions give an enhancement to the correlation, and repulsions give suppression. The effect of the interaction is highest at the lowest temperature (but, of course, the temperature must be sufficiently high for Boltzman statistics to apply, in the above example).

(ii) As a second example, consider a system of neutrons and protons interacting through a combination if isospin independent forces and a "symmetry energy" term, as well as the screened Coulomb forces on the protons:

$$U_{\rm nuc} = \frac{a}{2}(\rho_p + \rho_n)^2 + \frac{b}{2}(\rho_p - \rho_n)^2 \ . \tag{4.6}$$

For the evaluation of the fluctuation energy arising from the nuclear effects alone, we follow the steps of (i), above (the only new element being that $\rho_p \neq \rho_n$), obtaining

$$\left.\frac{\partial \mu_p}{\partial \rho_p}\right|_{\rm nuc} = \left.\frac{\partial^2 F}{\partial \rho_p^2}\right|_{\rm nuc} = a + b + T\rho_p^{-1} \ , \tag{4.7a}$$

$$\frac{\partial^2 F}{\partial \rho_p \partial \rho_n} = a - b \ , \tag{4.7b}$$

$$\frac{\partial^2 F}{\partial \rho_n^2} = a + b + T\rho_n^{-1} \ . \tag{4.7c}$$

To this we add the Coulomb energy due to the fluctuations in proton charge density. If the electron motion is treated in the Born-Oppenheimer approximation to give the screened potential for the protons, we can use the standard results of a one-component plasma calculation. Since the electrons are very degenerate, the additional fluctuation free energy is just the fluctuation energy,[16]

$$\begin{aligned} \delta F_{\rm Coulomb} &= (2\times\mathcal{V})^{-1} \sum_q \frac{4\pi e^2}{q^2 \epsilon(q,0)} \delta\rho_p(\mathbf{q})\delta\rho_p(-\mathbf{q}) \ , \\ &\approx (2\times\mathcal{V})^{-1} \sum_q \frac{4\pi e^2}{q^2 + q_{\rm TF}^2} \delta\rho_p(\mathbf{q})\delta\rho_p(-\mathbf{q}) \ . \end{aligned} \tag{4.8}$$

The appropriate screening wave number, $q_{\rm TF}$, is that for a completely relativistic degenerate plasma, $q_{\rm TF}^2 = 4\times 3^{2/3}\pi^{1/3}\rho^{2/3}e^2$. In the domains of parameters which we shall consider, it turns out that q is often less than $q_{\rm TF}$. Accordingly (in our illustrative example), we take the long-wavelength limit of the Coulomb term, obtaining an additional term to be added to the right-hand side of (4.7a),

$$\left.\frac{\partial^2 F}{\partial \rho_p^2}\right|_{\rm Coulomb} = C \ , \tag{4.9}$$

where $C = 4\pi e^2/q_{\rm TF}^2$. Note that this addition is exactly equivalent to adding a relativistic electron kinetic energy correction to the energy density function; in the limit of long-wavelength fluctuations of proton density, the neutralization by electrons must be complete.

As before, we can solve for the three independent correlation functions. The diagonal terms are

$$(\mathcal{V}\times\rho_p)^{-1}\langle\rho_p(\mathbf{q})\rho_p(-\mathbf{q})\rangle_{\mathbf{q}\to 0} = \left[1+\beta C\rho_p+\beta(a+b)\rho_p-\frac{\beta^2(a-b)^2\rho_p\rho_n}{1+(a+b)\beta\rho_n}\right]^{-1},$$

$$(\mathcal{V}\times\rho_n)^{-1}\langle\rho_n(\mathbf{q})\rho_n(-\mathbf{q})\rangle_{\mathbf{q}\to 0} = \left[1+\beta(a+b)\rho_n-\frac{\beta^2(a-b)^2\rho_p\rho_n}{1+(a+b)\beta\rho_n+\beta C\rho_p}\right]^{-1}. \tag{4.10}$$

Looking at the neutron liquid structure factor, in the case of not-too-strong coupling we see that the influence of the isoscalar force, with coupling parameter a, will be to enhance the scattering in the case of attraction (the case for the potentials to be considered) and to suppress the scattering in the case of repulsion. The symmetry energy term, b, which is positive in fits to nuclei, tends to reduce the scattering. The Coulomb effects get fed in from the proton sector in the higher-order term. Looking at the structure factor for protons, we see the effect of the Coulomb term as a clear suppression of the proton part of the opacity. Finally, we observe that the cross term

$$\mathcal{V}^{-1}\langle\rho_n(q)\rho_p(-q)\rangle_{q\to 0} = \frac{\beta(b-a)\rho_n\rho_p}{1+\beta(a+b)(\rho_p+\rho_n)+4ab\beta^2\rho_p\rho_n}, \tag{4.11}$$

which vanishes in the noninteracting limit, can be quite significant, since the parameters a and b have different signs.

V. SKYRME POTENTIAL

For the most realistic calculation of the correlation functions for neutral current scattering, we use the models used in the equation of state calculations themselves. The two main approaches to the equation of state of neutron star matter have been: (1) variational calculations beginning from two-body potentials, as carried out extensively by Pandharipande;[17] and (2) Hartree calculations using a Skyrme interaction, as in the application to a wide variety of nuclear characteristics by Vautherin and Brink[18] and the application to hot nuclear matter by Lattimer and Ravenhall.[19] The latter theory has two essential features for our calculation of the thermal correlations: it is adapted to the treatment of excited states, and therefore calculations at finite temperature; and it is easy to apply.

The Skyrme Hamiltonian consists of a nonlinear function of the densities and kinetic energy densities of proton and neutron, with instructions that it is to be used in the Hartree approximation. The design of the Hamiltonian is such as to take into account the short-range distortions of the true wave function, through the potential terms which depend on the densities of kinetic energy. For nuclear matter the application of the Skyrme potential is particularly simple; the wave functions are of the form of plane waves. Since we are basing our approach on correlations, it is important to emphasize the distinction between the short-range correlations in the wave function, with a range of less than the interparticle spacing, which are lost in the Hartree approach, and the longer range correlations at finite temperature which are the subject of our calculations. The model cannot be used for any high-momentum-transfer physics, but should be as applicable to the calculation of the long-wavelength thermal fluctuations as it is to the equation of state.

Following the treatment of Ref. 19, Schrödinger equations for the single-particle wave functions for the two species are given in terms of constant self-consistent potentials

$$[(2M^*_{p,n})^{-1}k^2+V_{p,n}]\phi^{(k)}_{p,n}=\epsilon^{(k)}_{p,n}\phi^{(k)}_{p,n}, \tag{5.1}$$

where

$$(M^*_{p,n})^{-1}=M^{-1}+\tfrac{1}{2}(t_1+t_2)\rho+\tfrac{1}{8}(t_2-t_1)\rho_{p,n}, \tag{5.2}$$

and

$$V_{p,n}=\frac{\partial}{\partial\rho_{p,n}}U(\rho_p,\rho_n,\tau_p,\tau_n),$$
$$U=\frac{t_0}{2}\left[\rho^2\left[1+\frac{x_0}{2}\right]-(\rho_n^2+\rho_p^2)(x_0+\tfrac{1}{2})\right] +\tfrac{1}{4}(t_1+t_2)\tau\rho+\tfrac{1}{8}(t_2-t_1)(\rho_n\tau_n+\rho_p\tau_p) +\tfrac{1}{4}t_3\rho[\rho_n\rho_p+\lambda(\tfrac{1}{4}\rho^2-\rho_n\rho_p)]. \tag{5.3}$$

Here ρ's are densities and τ's are kinetic-energy densities times a factor $2M^*$. That is,

$$\tau_n=\sum_k\nabla\phi^{(k)}_n\cdot\nabla\phi^{(k)}_n,\quad \tau=\tau_n+\tau_p. \tag{5.4}$$

The parameters in U are given by $t_0=-1057$ MeV fm^3, $t_1=236$ MeV fm^5, $t_2=-100$ MeV fm^5, $x_0=0.288$ $t_3=14\,400$ MeV fm^6, and $\lambda=0.516$.

In the low to moderate density regime in which we will be working, the effective mass can be taken to be the free-nucleon mass, without important changes in the results. The quantities ρ and τ are now to be determined in terms of the chemical potentials by Fermi integrals, themselves dependent on the self-consistent potentials,

$$\rho_i=F(y_i),\quad \tau_i=G(y_i), \tag{5.5}$$

$$y_i=\beta[V_i(\rho_p,\rho_n,\tau_p,\tau_n)-\mu_i],\quad i=p,n, \tag{5.6}$$

where

$$F(y_i)=(2\pi^2)^{-2}(2MT)^{3/2}\int_0^\infty du\sqrt{u}\,[1+\exp(u+y)]^{-1}, \tag{5.7}$$

$$G(y_i)=(2\pi^2)^{-2}(2MT)^{5/2}\int_0^\infty du\,u^{3/2}[1+\exp(u+y)]^{-1}. \tag{5.8}$$

Differentiating the density functions with respect to the chemical potentials gives four linear equations for the quantities $\partial\rho_i/\partial\mu_j$,

$$T\frac{\partial\rho_i}{\partial\mu_j}=F'(y_i)\left[\delta_{ij}-\sum_{k=p,n}\frac{\partial V_i}{\partial\rho_k}\frac{\partial\rho_k}{\partial\mu_j}\right]-G'(y_i)\sum_{k=p,n}\frac{\partial V_i}{\partial\tau_k}\frac{\partial\rho_k}{\partial\mu_j}\ , \tag{5.9}$$

where we have used the relation

$$\frac{\partial\tau_i}{\partial\rho_i}=\frac{G'(y)}{F'(y)} \tag{5.10}$$

in deriving the last terms on the right-hand side. All of the coefficients of the $(\partial\rho/\partial\mu)$'s can be calculated as functions of proton and neutron density, and the system solved for the density correlation matrix (A^{-1}) and the correlations which enter the opacity formula, (3.5).

The treatment outlined above does not take into account the Coulomb interactions among the protons and electrons. In Sec. IV we saw that in the long-wavelength limit the proton density fluctuations are exactly neutralized by the electron density fluctuations. In this case the only modification called for, except for correlation energy and other terms higher order in e^2, was the addition of an electron kinetic-energy term, $U_e(\rho_p)$, to the energy functional. In the application, however, the Thomas-Fermi (TF) screening momentum will turn out to be comparable to the momentum transfer in a neutrino scattering, and the long-wavelength limit therefore inapplicable to the screening part of the problem; the nuclear phenomena are of comfortably shorter range. We can address this problem by going back to the finite q expression for Coulomb fluctuation energy, (4.8). If we follow the formalism through for finite q, but keeping only the q dependence from (4.8), it is equivalent to making the following replacements in (5.9):

$$\frac{\partial V_p}{\partial\rho_p}\to\frac{\partial V_p}{\partial\rho_p}+\frac{4\pi e^2}{\mathbf{q}^2+q_{\rm TF}^2}\ , \tag{5.11a}$$

$$\frac{\partial\rho_i}{\partial\mu_j}\to\mathcal{V}^{-1}\langle\rho_i(\mathbf{q})\rho_j(-\mathbf{q})\rangle\ . \tag{5.11b}$$

In our numerical calculations, we shall replace q on the right-hand side of (5.11b) by an estimate of an average value for momentum transfer, $q\approx 2T$. This estimate arises from the following considerations.

(1) It is a commonplace that the average energy in a Fermi distribution of massless particles at zero chemical potential is almost as large as $3T$. But the integrals which arise in the transport theory for neutrinos, to obtain the energy flux in response to a thermal gradient, are not those that define an average energy. In particular, because the neutrino cross sections go as the second power of energy, and go in the denominator of the flux integral, the power of E in that integral is reduced from three to one, and the dominance by large E's is correspondingly reduced.

(2) A neutrino of energy E scattering nearly elastically may have a momentum transfer of up to $2E$. However, because of the factor of $(1+\cos\theta)$ in the neutral current scattering cross section, the median momentum transfer in a collision is approximately E.

A shortcoming in the treatment presented so far is that it fails to provide the corrections to the Gamow-Teller terms. The authors of Ref. 18 did not have a reason to consider a configuration with excess spin in one direction, but it is necessary, in order to calculate the susceptibilities, to know the energy functional to quadratic order in the deviations of $\langle\rho_p^{(3)}\rangle$ and $\langle\rho_n^{(3)}\rangle$ from zero. In the Appendix, we go back to the local potential from which Vautherin and Brink calculated the functional of densities which served as the "potential" in the mean-field approach, and pick up the terms which result when spin is not neutralized. We obtain a result which has a rich dependence on spin (and which is teetering on the brink of a magnetic phase transition at nuclear densities). But the result makes little contact with what is thought to be known about the spin-dependent forces.

Instead, we use a spin-dependent term which is derived from the Fermi-liquid parameters estimated by Backman, Brown, and Niskanen, on the combined basis of pi and rho meson exchange and phenomenology. The forces arising from pi meson and rho meson exchanges have been extensively analyzed over the years. The coupling constants are known, and the forces are thought to supply the strongest spin-dependent components of the nuclear force. In the work of Ref. 20, the spin dependence, arising largely from pi and rho effects, is embodied in a term in the (quasiparticle) two-body interaction,

$$f(k,k')=\pi^2(2M^*k_F)^{-1}G'(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)\ , \tag{5.12}$$

where the value of G is approximately 1.7, if k_F and M^* are evaluated at the Fermi surface at saturation density. It is shown in the Appendix that this translates to an expression for the energy density in terms of the nucleon densities of

$$U_{\rm spin}(\rho)=\zeta(\rho_p^{(3)}-\rho_n^{(3)})^2\ , \tag{5.13}$$

where $\zeta=350$ MeV fm^3. In this case, as in the first example in Sec. IV, the combinations of densities which are spin-up–spin-down symmetrical and antisymmetrical effectively diagonalize the problem, since spin-up and spin-down states are equally occupied in the equilibrium state. When the equations which are to be solved for the 16 quantities, $\partial\rho_i/\partial\mu_j$, are separated into symmetrical and antisymmetrical parts, the symmetrical parts are as already given in (5.9), with V standing for the spin-independent potential only. The spin antisymmetrical equations are of the form (5.9) also, with the densities, $\rho_{n,p}$, replaced by $\rho_{n,p}^{(3)}$ with chemical potential differences, $\mu_{p,n}^{(3)}$, replacing $\mu_{p,n}$; and with the potentials replaced by those derived from the term (5.13) alone (using 5.3). We set $\rho_{p,n}^{(3)}=0$ after the differentiations in 5.9. The functions $F(y)$ and $G(y)$ are then given by the values used in the calculations of the Fermi correlations.

We have solved (5.9), and its analogue for the Gamow-Teller correlations, obtaining the correlation functions which enter the cross-section formulae (3.5). For the case of the functions $\langle\rho_n\rho_n\rangle$, $\langle\rho_p\rho_p\rangle$, $\langle\rho_n^{(3)}\rho_n^{(3)}\rangle$, $\langle\rho_p^{(3)}\rho_p^{(3)}\rangle$, which are nonvanishing in the absence of interactions, we present the results in terms of liquid structure factors (LSF's), S_n, S_p, $S_n^{(3)}$, $S_p^{(3)}$, where

$$S_n(0)=\rho_n^{-1}\mathcal{V}^{-1}\langle\rho_n(\mathbf{q})\rho_n(-\mathbf{q})\rangle_{\mathbf{q}\to 0}\ ,$$
$$S_n^{(3)}(0)=\rho_n^{-1}\mathcal{V}^{-1}\langle\rho_n^{(3)}(\mathbf{q})\rho_n^{(3)}(-\mathbf{q})\rangle_{\mathbf{q}\to 0}\ , \tag{5.14}$$

etc. The results are plotted, as functions of density, in Figs. 1 and 2 for a variety of values of temperature and proton ratio. The LSF's for the noninteracting case, which differ from unity only because of Fermi statistics, are plotted for comparison. For the case of the correlation functions, $\langle\rho_n\rho_p\rangle$ and $\langle\rho_n^{(3)}\rho_p^{(3)}\rangle$, which vanish for the noninteracting case but enter the cross-section formulae, we plot the dimensionless quantities

$$S_{np}(0)\equiv(\rho_n\rho_p)^{-1/2}(\mathcal{V})^{-1}\langle\rho_n(\mathbf{q})\rho_p(-\mathbf{q})\rangle_{\mathbf{q}\to 0}\ ,$$
$$S_{np}^{(3)}\equiv(\rho_n\rho_p)^{-1/2}(\mathcal{V})^{-1}\langle\rho_n^{(3)}(\mathbf{q})\rho_p^{(3)}(-\mathbf{q})\rangle_{\mathbf{q}\to 0}\ ,$$

in Fig. 3.

VI. DISCUSSION

The results of this work indicate a substantial modification, due to nuclear interactions, of the correlation functions which enter neutrino opacity formulae. Most striking are the reduction in the Gamow-Teller part of the cross sections due to repulsion in the spin-isospin channel, and the reduction in the Fermi cross sections on protons due to the Coulomb effects. Perhaps the particularly surprising aspect of our results, at first sight, is the persistence of significant interaction effects to densities of $\frac{1}{20}$ nuclear densities. This is the case because the importance of the interactions on the structure factor depends, roughly speaking, on the ratio of the interaction energy to kinetic energy, which falls off only at the rate (density)$^{1/3}$, as the density decreases, for the case of the short-range interactions, and not at all for the screened Coulomb interactions.

We emphasize again that the methods described here are limited to the case in which the momentum transfer to the neutrino, q, is sufficiently small, and that this will not be true in all of the domains important in the supernova problem, although it is true in some of them. How small is "sufficiently small?" Consideration of the free Fermi gas can be a guide. The degenerate case will be the

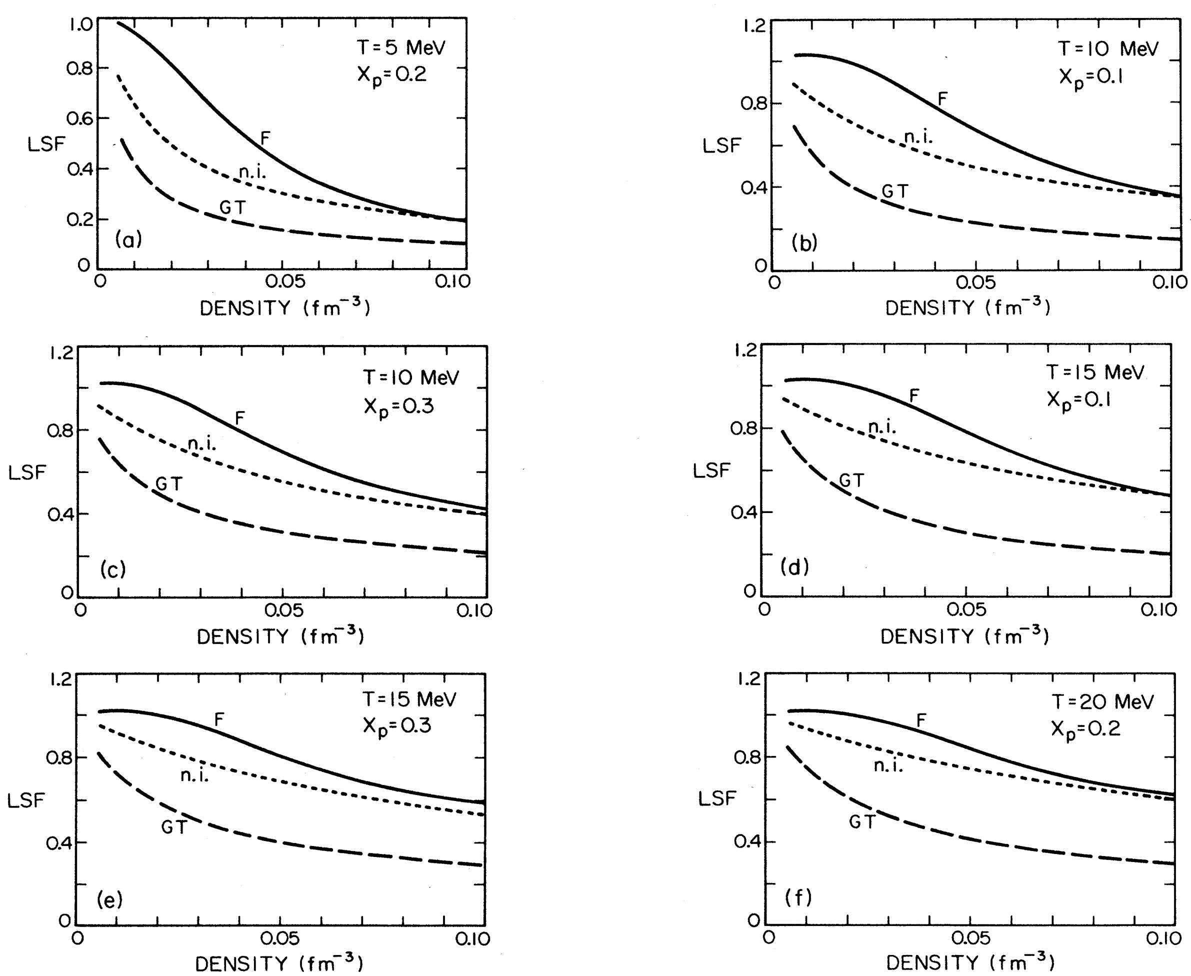

FIG. 1. Structure factors, $S_n, S_n^{(3)}$ (dashed line), for neutron density (Fermi) and spin-density (Gamow-Teller) correlations. The dotted line is for the noninteracting case. The abscissa is the total nucleon number density. $X_p=N_p/N$, the proton fraction. (a) $T=5$ MeV, $X_p=0.2$. (b) $T=10$ MeV, $X_p=0.1$. (c) $T=10$ MeV, $X_p=0.3$. (d) $T=15$ MeV, $X_p=0.1$. (e) $T=15$ MeV, $X_p=0.3$. (f) $T=20$ MeV, $X_p=0.2$.

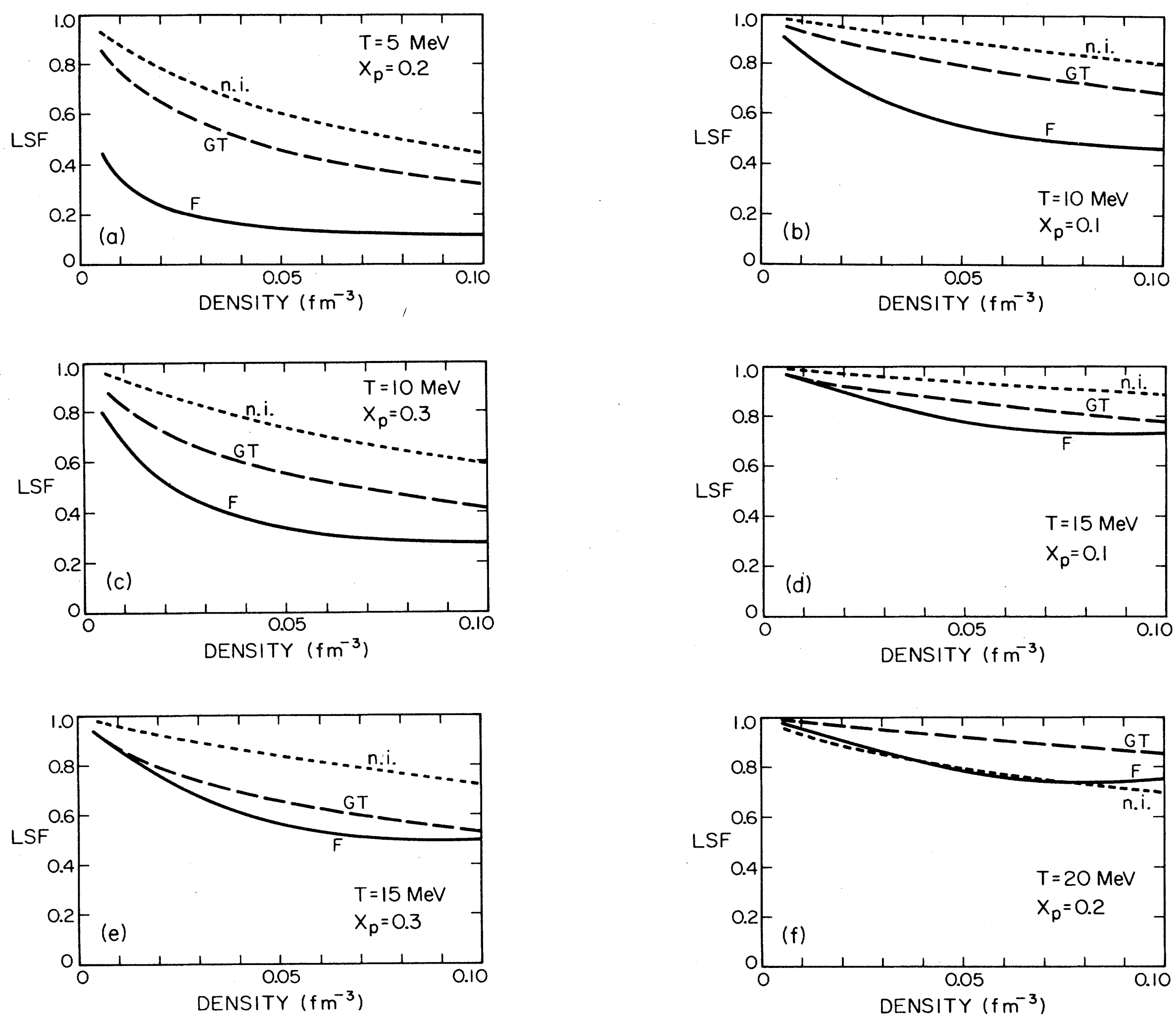

FIG. 2. Structure factors, $S_p, S_p^{(3)}$ (dashed line), for proton density and spin-density correlations. The dotted line is for the noninteracting case. (a) $T=5$ MeV, $X_p=0.2$. (b) $T=10$ MeV, $X_p=0.1$. (c) $T=10$ MeV, $X_p=0.3$. (d) $T=15$ MeV, $X_p=0.1$. (e) $T=15$ MeV, $X_p=0.3$. (f) $T=20$ MeV, $X_p=0.2$.

more restrictive. From (1.5) and (1.6) we can verify that (1) $q < k_F/3$ (i.e., less than the inverse interparticle spacing), and (2) $k_F \ll M$ (i.e., nonrelativistic nucleons), together insure the dominance of the thermal term, the calculation of which has been addressed in this paper. For an interacting system the new length which could enter

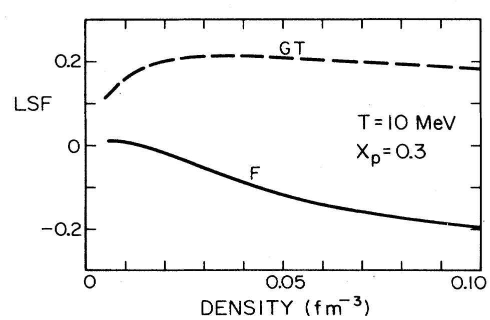

FIG. 3. Neutron-proton correlations, as measured by $S_{np}, S_{np}^{(3)}$ (dashed line). $T=10$ MeV, $X_p=0.3$.

the criterion is the correlation length. If the correlation length were comparable to, or larger than, the interparticle spacing, then the crition $q < k_F/3$ would be questionable for the interacting case, but, if not, we expect the Fermi gas criterion to be applicable. For the nearly nondegenerate case, which includes most of the temperature and density region of our calculations, the noninteracting limit of our results gives a structure factor of unity, which is the correct answer for a noninteracting system for values of q up to the region in which the single nucleon form factor starts to enter. For the interacting gas the scale of the q dependence should again be determined by a correlation length.

We believe it is premature to try to exhibit definitive numerical results for total opacities, because we have relied too heavily on the nuclear force model of Ref. 18, with an *ad hoc* modification of the spin-dependent terms, which are necessary for calculation of Gamow-Teller cross sections. We chose the model of Ref. 18 as the basis for our numerical estimates because of the fact that it is adapted to the study of the nondegenerate case, as well as the degenerate one, because the susceptibilities which enter into our density correlation functions are easily calculated in the model, and because the essential series of papers[19,21] on the equation of state of hot dense nuclear matter is based on this model. Other considerations can undoubtedly be brought to bear on specific terms in the development presented here, which is why the presentation of the results in the form of graphs of the individual correlation functions makes the most sense. The general formalism as presented in the present paper can be used with any approach to the nuclear physics which is capable of describing the excitations of hot nuclear matter.

APPENDIX

The Skyrme potential approach of Vautherin and Brink[18] is based on zero-range two- and three-body interactions, with phenomenologically determined parameters. The two-body part, omitting a term which does not contribute to infinite, constant-density matter, is given by

$$V(k,k')=t_0(1+x_0P_\sigma)+\tfrac{1}{2}t_1(k^2+k'^2)+t_2k\cdot k' . \qquad \text{(A1)}$$

The three-body operator is

$$V_3=t_3\delta(x_1-x_2)\delta(x_1-x_3) . \qquad \text{(A2)}$$

In (A1), P_σ is the spin-exchange operator, k and k' are relative momenta in initial and final states. The form (5.4), which depends only on the densities and kinetic-energy densities of the species, results from calculating the direct and exchange energies to first order in the potential. Consider, for example, the energy generated by the term t_0 on the right-hand side of (A1). Since the potential is zero range, the direct and exchange terms differ only in sign and through the traces of spin and isospin matrices, the result being

$$E_{t_0}=\frac{t_0}{4}[2\rho^2-\rho_p^2-\rho_n^2-(\rho_p^{(3)})^2-(\rho_n^{(3)})^2] . \qquad \text{(A3)}$$

In Ref. 18, these results are simplified by taking equal densities for spin up and for spin down, $\rho_p^{(3)}=\rho_n^{(3)}=0$; for our purposes we need the complete dependence. Similarly, the spin-exchange operator leads to the combination

$$E_{t_0x_0P_0}=\frac{t_0x_0}{4}[\rho^2+(\rho^{(3)})^2-2\rho_p^2-2\rho_n^2] . \qquad \text{(A4)}$$

It is only slightly more complicated to treat the three-body interaction and the momentum-dependent terms in the two-body interaction, in the presence of nonvanishing $\rho_n^{(3)}$ and $\rho_p^{(3)}$. We find

$$E_{k\text{ dependent}+\text{three body}}=\tfrac{1}{4}(t_1+t_2)\rho\tau+\tfrac{1}{8}(t_2-t_1)(\rho_n\tau_n+\rho_p\tau_p+\rho_n^{(3)}\tau_n^{(3)}+\rho_p^{(3)}\tau_p^{(3)})+\tfrac{1}{16}t_3\rho[\rho^2-(\rho^{(3)})^2-2(\rho_p^{(3)})^2-2(\rho_n^{(3)})^2] . \qquad \text{(A5)}$$

We comment on the connection between these forms, as used to calculate fluctuations, and the Fermi-liquid theory of nuclear matter, as discussed in Ref. 20. For the case of degenerate and symmetrical nuclear matter, the Fermi-liquid theory (FLT), using the Landau parameters recommended in Ref. 20, provides a way of directly determining the susceptibilities, $\partial\rho_i/\partial\mu_j$, which enter into the long-wavelength limit of the density correlation functions. We begin with the equations which define the FLT,[20]

$$\delta E=\sum_{q,i}\epsilon_i^{(0)}(q)\delta\rho_i(q)+\frac{1}{2}\sum_{q,q',i,j}f_{ij}(q,q')\delta\rho_i(q)\delta\rho_j(q') \qquad \text{(A6)}$$

and

$$\sum_{ij}\delta\rho_i(q)\delta\rho_j(q')f_{ij}(q,q')=\pi^2(2M^*k_F)^{-1}[\rho(\mathbf{q})\rho(\mathbf{q}')F+\rho_{3,0}(\mathbf{q})\rho_{3,0}(\mathbf{q}')F'+\rho_{0,3}(\mathbf{q})\rho_{0,3}(\mathbf{q}')G+\rho_{3,3}(\mathbf{q})\rho_{3,3}(\mathbf{q}')G'] , \qquad \text{(A7)}$$

where F, F', G, G' are functions of $\cos(\theta)=\hat{q}_1\cdot\hat{q}_2$, and the density combinations are defined, in terms of the notation introduced in (3.4), by

$$\begin{aligned}\rho_{3,0}&=\rho_p-\rho_n ,\\ \rho_{0,3}&=\rho_p^{(3)}+\rho_n^{(3)}=\rho_p^{\uparrow}-\rho_p^{\downarrow}+\rho_n^{\uparrow}-\rho_n^{\downarrow} ,\\ \rho_{3,3}&=\rho_p^{(3)}-\rho_n^{(3)} .\end{aligned} \qquad \text{(A8)}$$

We have, for example, the standard result for the bulk modulus,[22]

$$K_T=\rho\frac{\partial\mu}{\partial_\rho}=k_F^2(3M^*)^{-1}(1+F_0) , \qquad \text{(A9)}$$

where F_0 is the S-wave part of the function, μ is the chemical potential for nucleon number, and ρ the nucleon number density. The other three susceptibilities for the case of symmetric nuclear matter can be written in terms of F_0', G_0, and G_0' [coming from the $\tau_1\cdot\tau_2$, $\sigma_1\cdot\sigma_2$, and $(\tau_1\cdot\tau_2)(\sigma_1\cdot\sigma_2)$ parts of the interaction, respectively].

Note, however, that the form (A7) is based on an isospin invariant amplitude, in spite of its expression in terms of third components in isospin (the charge raising and lowering currents are not of interest in the application to neutral current phenomena). This form is not valid in the case of unsymmetrical matter, as is evidenced, for example, by the nonvanishing of the correlation functions, $\langle \rho_n \rho_p \rangle$ and $\langle \rho_n^{(3)} \rho_p^{(3)} \rangle$, in the calculations of the present paper. Thus we can derive Landau parameters to compare with those of Ref. 20 only for the special case of degenerate, symmetric matter.

We can calculate the S-wave Fermi-liquid parameters from the second-order variation of the energy calculated in the Skyrme approach,

$$\delta E = \frac{1}{2} \sum_{ij} \frac{\partial}{\partial \rho_i} \frac{\partial}{\partial \rho_j} [E(\rho,\tau)] \delta\rho_i \delta\rho_j + \sum_{ij} \left[\frac{\partial}{\partial \rho_i} \frac{\partial}{\partial \tau_j} E(\rho,\tau) \right] \delta\rho_i \delta\tau_j \ , \tag{A10}$$

where $E(\rho,\tau)$ is the sum of the terms on the right-hand side of (A3)–(A5). In the degenerate case, we can make the replacement $\tau_i \to k_F^2 \rho_i$. Comparing the result with (A7), we obtain

$$\begin{aligned} F_0 &= 2M^* k_F \pi^{-2} (\tfrac{3}{4} t_0 + \tfrac{3}{8} \rho t_3 + \tfrac{3}{8} k_F^2 t_1 + \tfrac{5}{8} k_F^2 t_2) = -0.12, \quad F_0' = 2M^* k_F \pi^{-2} \left[\left[-\frac{1}{4} - \frac{x_0}{2} \right] t_0 + D \right] = 0.64 \ , \\ G_0 &= 2M^* k_F \pi^{-2} \left[\left[-\frac{1}{4} + \frac{x_0}{2} \right] t_0 + D \right] = -1.1, \quad G_0' = 2M^* k_F \pi^{-2} [-\tfrac{1}{4} t_0 + D] = -0.33 \ , \end{aligned} \tag{A11}$$

where

$$D = \frac{k_F^2}{8}(t_2 - t_1) - \frac{\rho t_3}{16} \ .$$

The numerical values are for the case of $k = 1.25F^{-1}$.

The values of F_0 and F_0' are not in strong disagreement with those of Ref. 20. It should be borne in mind that the terms involving t_1, t_2, t_3 (and D) in (A11) are extremely density dependent. The interaction part of the symmetry energy (F_0' term), for example, will grow as the negative quantity D becomes less important at lower density. Likewise, the G_0 term would become positive; there is little significance in the fact that in the calculation of (A11) it overshot the value, -1, for which there would be a transition to a magnetized state. And the spin-isospin parameter G' would become positive at a slightly lower density. However, as explained in Sec. V, the spin-dependent terms of (5.13), with a strong repulsion in the spin-isospin channel, are much better based in physics.

[1] W. D. Arnett, Astrophys. J. **251**, 325 (1981).
[2] S. Bludman and K. Van Riper, Astrophys. J. **224**, 631 (1978).
[3] T. J. Mazurek, Nature (London) **252**, 287 (1974).
[4] K. Sato, Prog. Theor. Phys. **54**, 1135 (1975).
[5] H. A. Bethe, G. E. Brown, J. Applegate, and J. M. Lattimer, Nucl. Phys. **A324**, 487 (1979).
[6] R. F. Sawyer and A. Soni, Astrophys. J. **230**, 859 (1979).
[7] A. Burrows and J. M. Lattimer, Astrophys. J. **307**, 178 (1985).
[8] R. Mayle, J. R. Wilson, and D. M. Schramm, Astrophys. J. **318**, 288 (1987).
[9] L. J. Van den Horn and J. Cooperstein, Astrophys. J. **300**, 142 (1986).
[10] B. T. Goodwin and C. J. Pethick, Astrophys. J. **253**, 816 (1982).
[11] S. Bludman, L. Lichtenstadt, and G. Hayden, Astrophys. J. **261**, 661 (1982).
[12] N. Iwamoto and C. J. Pethick, Phys. Rev. D **25**, 313 (1982).
[13] R. F. Sawyer, Phys. Rev. D **11**, 2740 (1975).
[14] E. Feenberg, *Theory of Quantum Fluids* (Academic, New York, 1969), Sec. 1.6.
[15] L. D. Landau and E. M. Lifschitz, *Statistical Physics* (Pergamon, London, 1978), Chap. 12.
[16] S. Ichimaru, Rev. Mod. Phys. **54**, 1017 (1982), Eq. (3.80).
[17] P. J. Siemens and V. R. Pandharipande, Nucl. Phys. **A173**, 571 (1971).
[18] D. Vautherin and D. M. Brink, Phys. Lett. **32B**, 149 (1970).
[19] J. M. Lattimer and D. Ravenhall, Astrophys. J. **223**, 314 (1978).
[20] S. O. Backman, G. E. Brown, and J. A. Niskanen, Phys. Rep. **124**, 1 (1985).
[21] D. Q. Lamb, J. M. Lattimer, C. J. Pethick, and D. G. Ravenhall, Phys. Rev. Lett. **41**, 1623 (1978); Nucl. Phys. **A360**, 459 (1981); J. M. Lattimer, C. J. Pethick, D. G. Ravenhall, and D. Q. Lamb, *ibid.* **A432**, 646 (1985).
[22] J. W. Negele and H. Orland, *Quantum Many-Particle Systems* (Addison-Wesley, New York, 1988), Sec. 6.2.