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Low energy theorem and polarization effects in $^{2}H(\gamma,n)^{1}H$

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Polarization effects in $^{2}H(\gamma,n)^{1}H$ are considered in the low photon energy domain. We discuss linear photon polarization, one-nucleon polarization, and vector target asymmetry.

INTRODUCTION

Low's soft photon theorem[1] provides a convenient way of studying model-independent features of radiative nuclear reactions given in terms of corresponding nonradiative ones. The object of interest for the theorem is the power-series expansion in the photon energy ω of the amplitude for the process under consideration. The theorem is based on the observation that the only singular terms are of the type A/ω and that those are given by the diagrams in which the photon is coupled to an external charged particle. The content of the theorem is that the terms of order ω^0 are completely determined by the gauge-invariance condition.

This theorem has been applied to the study of the total cross section and angular distribution of the deuteron photodisintegration. In contrast to earlier results,[2] in Refs. 3 and 4, a rather satisfactory description of the experimental data is achieved in a parameter-independent way.

Although the advantages and limitations of Low's theorem have been discussed in detail by Govaerts *et al.*,[3] for the sake of completeness we will summarize the results and the physical picture obtained in this approach:

(1) The total cross section is very well described for photon energies between 4 and 80 MeV. This cross section comes entirely from the modulo square of the ω^{-1} terms and it involves only the mass and electric charge of the particles, but not the magnetic moments.

(2) The angular distribution is reasonably described in the same energy range and for angles between 40° and 140°. In the forward and backward directions, the ω^{-1} terms vanish identically.

(3) In the forward and backward directions only the modulo square of the ω^0 terms contributes. This quantity is sensitive to the magnetic moments of the particles involved. Comparison with the experimental data shows discrepancies, particularly at low energies where Low's theorem predicts a monotonous increase of the cross section whereas the data seem to present a deep when the photon has an energy around 5 MeV.

(4) From the preceding discussion it is clear that what we mean in the present case by low energy is a photon energy below 80 MeV. From our analysis it appears that Low's theorem provides a good description of physical quantities whenever the spin and magnetic moments of the particles can be ignored.

(5) Below 4 MeV, the n-p final-state interactions are so huge in the magnetic transition that the Low's theorem is not applicable in a straightforward way.

Extension of this theorem to include nucleon polarization effects has been developed by Fearing[5] along the lines followed by Burnett and Kroll.[6] However, to our knowledge, no explicit calculations for the deuteron have been carried out and no comparison with the experimental data have been performed to test the reliability of the theorem in this context.

Our goal is to determine if Low's theorem has a practical content beyond classical electromagnetism, i.e., once the spin and the magnetic moments of the particles play a role in the determination of a physical quantity, as is the case for polarization observables.

Fearing's work can be easily extended to include deuteron polarization. The results can be summarized as follows:[17]

$$|M(\gamma+d\rightarrow n+p)|^2=Q|M^0(d\rightarrow n+p)|^2+0(\omega^0)\ . \qquad (1)$$

Thus the modulo square of the amplitude describing the deuteron photodisintegration—for arbitrary polarization of the particles involved—is expressed as an operator acting on the modulo square of the corresponding nonradiative amplitude. On the left-hand side of this equation we have the square of the invariant amplitude describing the radiative process $\gamma+d\rightarrow n+p$. This is clearly the quantity we want to know. M^0 stands for the on-shell nonradiative amplitude, that is the on-shell neutron-proton-deuteron vertex[3]

$$M^0(d\rightarrow n+p)\equiv \bar{U}(p,s)\left[A\not\eta+\frac{B}{2m}(p-p')\cdot\eta\right]\times U(p',s')\ . \qquad (2)$$

A and B are the dnp form factors with the three particles on shell. p, p', and d are the proton, neutron, and deuteron momentum four-vectors, respectively, and s, s', and η are the corresponding real polarization four-vectors, with

$$s^2=s'^2=\eta^2=-1, \quad s\cdot p=0, \quad s'\cdot p'=\eta\cdot d=0 \;;$$

$m(M)$ is the nucleon (deuteron) mass.

Q is an operator which, for real polarization four-vectors, is given by

$$Q=e^2F\left\{F+D_\mu(p)\frac{\partial}{\partial p_\mu}+\left[\kappa_p D(p)\cdot s\frac{p_\mu}{m^2}+(1+\kappa_p)E_\mu(p,s)\right]\right. \times\frac{\partial}{\partial s_\mu}+\kappa_n\left[D(p')\cdot s'\frac{p'_\mu}{m^2}+E_\mu(p',s')\right]\frac{\partial}{\partial s'_\mu}+\left.\frac{M}{2m}\mu_d E_\mu(d,\eta)\frac{\partial}{\partial \eta_\mu}\right\}, \tag{3}$$

where e is the proton electric charge, ϵ_μ is the real polarization four-vector of the photon, κ_p and κ_n are the proton and neutron anomalous magnetic moments, and μ_d is the deuteron magnetic moment[2,3]

$$F\equiv\frac{\epsilon\cdot d}{k\cdot d}-\frac{\epsilon\cdot p}{k\cdot p}\,,$$

$$D_\mu(p)\equiv\frac{\epsilon\cdot p}{k\cdot p}k_\mu-\epsilon_\mu\,,$$

$$E_\mu(p,s)\equiv\frac{\epsilon\cdot s}{k\cdot p}k_\mu-\frac{k\cdot s}{k\cdot p}\epsilon_\mu\,.$$

Of course when we sum over the deuteron and nucleon polarizations, Eq. (1) reduces to the Burnett and Kroll theorem. As far as the first two orders in the photon energy are concerned, all the information—Eq. (1)—is contained in the on-shell nonradiative amplitude. Note that the two form factors describing the on-shell *npd* vertex are closely related[2,8] to the *S*- and *D*-wave deuteron wave functions.

LINEAR PHOTON POLARIZATION

We consider a photon linearly polarized. In terms of the polarizations $\epsilon_\parallel$ and $\epsilon_\perp$ (in the Coulomb gauge, $\epsilon_0=0$) which are parallel and perpendicular to the reaction plane, respectively, we have

$$\epsilon=\cos\phi\epsilon_\parallel+\sin\phi\epsilon_\perp\,. \tag{4}$$

If we denote by $d\sigma_\parallel/d\Omega$ and $d\sigma_\perp/d\Omega$ the cross section for the polarizations $\epsilon_\parallel$, $\epsilon_\perp$, then the following relation for the azimuthal asymmetry $\Sigma(\theta)$ (Ref. 9) is obtained:

$$\left[\frac{d\sigma}{d\Omega}\right]\Sigma(\theta)=\frac{d\sigma_\parallel}{d\Omega}-\frac{d\sigma_\perp}{d\Omega} \tag{5}$$

with

$$\frac{d\sigma}{d\Omega}=\frac{d\sigma_\parallel}{d\Omega}+\frac{d\sigma_\perp}{d\Omega}\,.$$

The differential cross section and the asymmetry $\Sigma(\theta)$ are conventionally[9] parametrized as

$$\frac{d\sigma}{d\Omega}=a+b\sin^2\theta+C\cos\theta+d\sin^2\theta\cos\theta+e\sin^4\theta\quad\left[\frac{d\sigma}{d\Omega}\right]\Sigma(\theta)=f\sin^2\theta+g\sin^2\theta\cos\theta+h\sin^4\theta\,. \tag{6}$$

The coefficients $a,b,\ldots,e$ can be obtained by expanding in powers of v the corresponding cross section.[3(a)] In this way we obtain

$$a=\tfrac{1}{6}(\mu_p-\mu_n)^2,\quad b=t\left[\frac{v^2+3v^4}{k^2}m^2\right],\quad c=\frac{v}{3}t(\mu_p-\mu_n),\quad d=2t\frac{m^2}{k^2}v^3,\quad e=-3\frac{m^2}{k^2}v^4\,,$$

with

$$t=\frac{4\cdot 11\alpha}{16\pi}\frac{v}{Ek}\,,$$

where $\alpha=\frac{1}{137}$, $v(E)$ is the proton velocity (energy), and k is the photon c.m. energy. We will refer to those as Low's theorem prediction and denote them with a subindex L.

Using Eqs. (5) and (6) we can write

$$\left[\frac{d\sigma}{d\Omega}\right]\Sigma(\theta)=(a_\parallel-a_\perp)+(b_\parallel-b_\perp)\sin^2\theta+(C_\parallel-C_\perp)\cos\theta+(d_\parallel-d_\perp)\sin^2\theta\cos\theta+(e_\parallel-e_\perp)\sin^4\theta\,. \tag{7}$$

By symmetry arguments, it is easily seen that $a_{\parallel}=a_{\perp}$ and $C_{\parallel}=C_{\perp}$. On the other hand, we know that in Low's approximation only $\epsilon_{\parallel}$ contributes to the differential cross section. Indeed, the ϵ dependence of the differential cross section in Low's approximation is of the form[3(a)]

$$\left[\frac{d\cdot\epsilon}{d\cdot k}-\frac{p\cdot\epsilon}{p\cdot k}\right]^2 .$$

Since this is a Lorentz and gauge-invariant quantity we can evaluate it in any gauge and reference frame. Let us take the c.m. and the $\epsilon_0=0$ gauge:

$$\left[\frac{d\cdot\epsilon}{d\cdot k}-\frac{p\cdot\epsilon}{p\cdot k}\right]^2=\left[\frac{\mathbf{d}\cdot\boldsymbol{\epsilon}}{d\cdot k}-\frac{\mathbf{p}\cdot\boldsymbol{\epsilon}}{p\cdot k}\right]^2 ,$$

but $\mathbf{d}=-\mathbf{k}$, then $\mathbf{d}\cdot\boldsymbol{\epsilon}=0$. On the other hand, if we split $\boldsymbol{\epsilon}$ in $\boldsymbol{\epsilon}_{\parallel}$ and $\boldsymbol{\epsilon}_{\perp}$ then we see that in Low's approximation only $\boldsymbol{\epsilon}_{\parallel}$ contributes.

The preceding argument led us to conclude that $b_{\perp}=d_{\perp}=e_{\perp}=0$. Furthermore, since to this order in the photon energy expansion, Low's theorem predicts a vanishing differential cross section ($a=c=0$) we see that $d\sigma_{\perp}/d\Omega=0$. Then comparing (6) and (7) we conclude that

$$f=b_{\parallel}=b_L ,\quad g=d_{\parallel}=d_L ,\quad h=e_{\parallel}=h_L . \tag{8}$$

In Table I we show that value of the coefficients as measured by De Pascale *et al.*[10] and compare it with the value predicted by the low energy theorem. We see that although for photon energies below 30 MeV the relations $f=b_{\parallel}$ and $g=d_{\parallel}$ are well fulfilled, the actual values predicted by the low energy theorem are at variance with the experimental data. It is clear that the values obtained for f and e in this approach, very much as the differential cross section, are sensitive only to the electric charge and Born diagrams. Thus, departures from these predictions indicate the necessity to go beyond the Born diagrams.

NUCLEON POLARIZATION

Nuclear polarization effects in the differential cross section are obtained by applying the operator Q—Eq. (3)—to the square of the absolute value of the on-shell *npd* vertex given in Eq. (2). It is easily seen that

$$|M^0(d\rightarrow n+p)|^2 ,$$

after spin summation on one nucleon state is *independent* of the other nucleon polarization. Therefore, the low energy theorem predicts zero polarization effect in the case of one polarized nucleon. This is in complete disagreement with the experimental data.[11]

VECTOR DEUTERON POLARIZATION

Let us consider deuteron polarization in the $0y$ direction. Then in the rest frame of the deuteron, the polarization corresponding to ±1 and 0 are

$$\boldsymbol{\eta}^{\pm}\frac{1}{\sqrt{2}}\begin{bmatrix}1\pm i\\0\\1\mp i\end{bmatrix},\quad \boldsymbol{\eta}^0=\begin{bmatrix}0\\1\\0\end{bmatrix} .$$

TABLE I. The coefficients b,f,d,g,e,h as reported in Ref. 10 and as predicted by the low energy theorem (with subindex L).

ω	$\frac{b\times10^2}{\sigma_T}$	$\frac{f\times10^2}{\sigma_T}$	$\frac{b_L\times10^2}{\sigma_T}$	$\frac{d\times10^2}{\sigma_T}$	$\frac{g\times10^2}{\sigma_T}$	$\frac{d_L\times10^2}{\sigma_T}$	$\frac{e\times10^2}{\sigma_T}$	$\frac{h\times10^2}{\sigma_T}$	$\frac{e_L\times10^2}{\sigma_T}$
19.8	11.47±0.15	10.13±0.12	12.46	1.26±0.5	1.16±0.33	3.21	−0.88+0.7	1.297±0.2	−0.65
29.0	9.74±0.24	9.51±0.16	12.73	2.77±0.51	2.37±0.34	3.93	−0.22±0.56	−0.397±0.25	−0.99
38.6	10.6±0.52	8.38±0.58	13.0	2.55±0.75	2.89±0.33	4.55	−1.62±0.7	−0.79±0.75	−1.33
60.8	7.79±0.85	4.54+0.57	13.59	1.99±1.0	2.84±0.46	5.66	−1.75±0.65	0.371±1.0	−2.08

The differential cross sections associated with each of these polarizations are denoted by $d\sigma_\pm$ and $d\sigma_0$. None of them is vanishing in the soft photon limit. The quantity usually considered is the vector target asymmetry, defined as

$$T=\frac{d\sigma_+-d\sigma_-}{d\sigma_++d\sigma_0+d\sigma_-} .$$

Given that $(\eta^+)^*=\eta^-$ and that Low's amplitude $M^L=\eta^\mu M^L_\mu$ is real,[1,3] except for the polarization four-vector η^μ, it follows that $T=0$. There are no experimental data for this quantity at low energies. However, this result is in disagreement with conventional potential calculations.[12]

To summarize, we have considered polarization effects in the deuteron photodisintegration. The experimental results indicate that the low energy theorem is not capable of describing the physics of phenomena sensitive to the spin of the particles involved and that at least final-state interactions (which are not given by the low energy theorem[2,3] are necessary to get nonvanishing nucleon polarization and vector target asymmetry.

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