(^{6}Li,d) stripping into unbound states

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A new economical numerical method is presented for the correct zero-range evaluation of differential cross sections and angular correlation functions for α-transfer reactions into *high-lying unbound* (resonant) α-cluster states. The results for the 5_1^- (15.6 MeV) and 6_1^+ (16.3 MeV) states in ^{16}O occupied in the reaction ^{12}C(^{6}Li,d)^{16}O*(α)^{12}C support the suggested approach.

By now there exists an abundance of experimental data for various transfer reactions induced by heavy ions. Due to the pronounced cluster structures of the respective projectiles, (^{6}Li,d) and (^{7}Li,t) reactions play a particularly important role in this context.[1] As far as the reaction mechanism is concerned, angular correlation methods showed[2] that such an α transfer into virtual unbound (or resonant) α-cluster states of light nuclei is a direct one. Consequently, the distorted-wave method and the (α-d) angular correlation function should be suitable means for analyzing such reactions. This is readily appreciated by analogy to the rather similar single-particle transfer (d,p) reaction.

However, up to now there exists no general practical method for the evaluation of the respective breakup amplitude of α particles transferred into unbound resonance states. This unsatisfactory situation is mainly due to the necessity to account correctly for the boundary conditions in the three-particle exit channel.[3] The implications of such configurations are that the amplitude has to be regularized one way or the other.[4,5] Within the resonating group method (RGM), the simple procedure due to Huby and Mines[4] has been successfully applied to the analysis of the reaction ^{16}O(^{6}Li,d)^{20}Ne.[6] But it is only suitable for low-lying sharp resonances. Another method as put forward by Vincent and Fortune[7] evaluates the radial distorted-wave Born approximation (DWBA) integrals via contour integration in the complex plane. In its original formulation it works nicely for small values of the Coulomb parameters. But for heavy-ion reactions with their larger Coulomb parameters, application of this method is problematic.

When using such a code for heavy ions with the same steplength as for small projectiles, then numerical experiments show that the solution is "unstable" in the following sense. It turns out that changes in the initial parameters lead to different solutions, which is due to the singularity of the potential at the origin. (The potential for heavy ions contains terms proportional to r^{-n}; $n \geq 3$.) Possible ways out are the introduction of a hard core at the origin or the use of an exponential ensuring a reasonable behavior of the functions in the vicinity of the origin (see also Ref. 8). Experimenting with different steps shows that on a normal-size computer (like a CDC6600, say), an increase in the number of steps does not provide a remedy since it simply leads to computing times that exceed the limits available to the average user. This is largely due to the application of the method of Runge and Kutta, which yields such "unstable" results for the related Cauchy problem.[7] We are now looking for a more appropriate method that does not give rise to this deficiency and that converges faster than the traditional one so that it will be possible to apply the quite successful method of Vincent and Fortune as well to heavy-ion reactions.

The gist of the suggested modification is that we do *not* set out to solve the Coulomb equation for partial distorted waves via the method of Runge and Kutta, say. Instead we attempt to solve the corresponding boundary value problem. The motivation for this change is simple: In the latter case the asymptotic solution on the boundary is known so that there are no problems with its stability.

To clarify the notation, let us represent the wave number as usual by k, employ R_m for the cutoff radius in the standard DWBA, and take R and y as real and imaginary parts, respectively, of points r in the complex plane. The combination of Coulomb functions $H_l^{(\pm)}(\rho)=G_l(\rho) \pm iF_l(\rho)$ with $\rho=kr$ has the same meaning as in the conventional DWBA. From the traditional DWBA we know the function $H_l^{(\pm)}(\rho_1)$ at $\rho_1=kR_m$ where the value of R_m is chosen so as to provide a numerically stable solution. The expression for the well-known integral representation for $H_l^{(\pm)}(\rho_2)$ at $\rho_2=kR+iky$, which we want to evaluate, may be taken from the literature.[9] The well-established methods for solving boundary-value problems yield then the function $H_l^{(\pm)}(\rho)$ for all ρ with $\rho_1<\rho<\rho_2$.[10]

To describe the method let us assume that we want to evaluate for all r within a given interval a function $H=H(r)$, e.g., the above $H_l^{(\pm)}(\rho)$. We presume that we know H at the ends of the integration contour and that it satisfies the differential equation

$$\frac{d^2H}{dr^2}-pH=\frac{d^2H}{dr^2}+\left[1-\frac{2\gamma}{r}-\frac{l(l+1)}{r^2}\right], \quad H=0\ , \tag{1}$$

which does *not* contain first derivatives. We now make use of a special case of a general formula due to Bickley[11] which expresses the second-order central difference of the

function in terms of the even central differences of the second-order derivatives H'',

$$\delta^2 H = h^2(H'' + \tfrac{1}{12}\delta^2 H'' + \cdots) . \tag{2}$$

Denoting the integration steps by an index $n=0,1,2,\ldots,N$ and the steplength by h, this leads to the recurrence relation

$$H_{n+1}(1-p_{n+1}h^2/12) = (2+\tfrac{5}{6}p_n h^2)H_n - (1-p_{n-1}h^2/12)H_{n-1} , \tag{3}$$

connecting with each other the values of H at three different points $n-1$, n, and $n+1$ (Fox-Goodwin or Numerov method). In (3) the following abbreviations have been used:

$$H_n = H(r_1+nh) \quad \text{and} \quad p_n = p(r_1+nh) , \tag{4}$$

where r_1 denotes the first integration point. A special case of (3) is given by

$$H_2(1-p_2h^2/12) = (2+\tfrac{5}{6}p_1h^2)H_1 - (1-p_0h^2/12)H_0 , \tag{5}$$

where H_0, p_0, p_1, and p_2 are known quantities. We may cast this relation into the form

$$H_1 = a_1 H_2 + b_1 , \tag{6}$$

with

$$a_1 = (1-p^2h^2/12)/(2+\tfrac{5}{6}p_1h^2); \quad b_1 = H_0(1-p_0h^2/12)/(2+\tfrac{5}{6}p^1h^2) . \tag{7}$$

The general case of (6) (in which we do not yet know a_n and b_n) reads

$$H_n = a_n H_{n+1} + b_n , \tag{8}$$

and allows us to write (3) in the form

$$H_{n+1} - B_n H_n + C_n H_{n-1} = 0 , \tag{9}$$

with

$$B_n = (2+\tfrac{5}{6}p_n h^2)/(1-p_{n+1}h^2/12); \quad C_n = (1-p_{n-1}h^2/12)/(1-p_{n+1}h^2/12) . \tag{10}$$

This leads to

$$H_{n+1} - B_n H_n + C_n(a_{n-1}H_n + b_{n-1}) = 0 , \tag{11}$$

or

$$H_{n+1} + (C_n a_{n-1} - B_n)H_n + b_{n-1}C_n = 0 . \tag{12}$$

Thus we eventually arrive at a recurrence relation for the wanted coefficients a_n and b_n,

$$a_n = 1/(B_n - C_n a_{n-1}) , \quad b_n = b_{n-1}C_n/(B_n - C_n a_{n-1}) , \tag{13}$$

where a_1 and b_1 are already known from (7). Evaluating the coefficients a_n and b_n with $n=0,1,\ldots,N$, the desired function H is obtained as follows: If N is the number of steps exhausting the contour of interest, H_n is known and we may "return" to evaluate now the intermediate H_n by the aid of

$$H_{n-1} = a_{N-1}H_N + b_{N-1} , \tag{14}$$

etc. The accuracy of the procedure is of the order $\sim O(h^6)$.

The choices for ρ_1 and ρ_2 are guided by the following requirements: (1) To ensure a good accuracy in the evaluation of the usual radial DWBA integral, with the Coulomb functions, ρ_1 should be as small as possible; however, it should be large enough to be well beyond the range of the nuclear forces. (2) The real part of ρ_2, i.e., kR, should be as large as possible to ensure a fast convergence of the integral representation. (In the integral representation kR appears in an exponential under the integral sign thus explaining its role for the convergence.) Its imaginary part ky has to be large enough to take care of the convergence of the radial integral

$$\int_{\rho_1}^{\rho_2} \chi_{\mathrm{Li}}(r)\chi_{l_d}(r)F_\alpha(r)\cdot dr$$

of the standard DWBA along its contour in the complex plane.

In contrast to Ref. 7, we work with a fixed but arbitrary angle of inclination of the contour; see Fig. 1 (90° would also do, though the optimal angle turns out to be smaller). In addition to that, *all* functions (i.e., for d, $^6\mathrm{Li},\alpha$) in the radial integral are broken up into two parts corresponding to

$$\chi_l(\rho) = \frac{1}{2k}\exp(i\sigma_l)\cdot\left[H_l^{(-)} - \eta_l H_l^{(+)}\right] , \tag{15}$$

where σ_l denotes the Coulomb phase and η_l the element of the scattering matrix. Vincent and Fortune[7] use *two* contours, which is necessary when dealing with χ_l which is *not* symmetric with respect to the abscissa. However, $H_l^{(-)}$ and $H_l^{(+)}$ separately are symmetric functions. If we thus break up the respective functions into symmetric parts, it is sufficient to consider only a single contour.

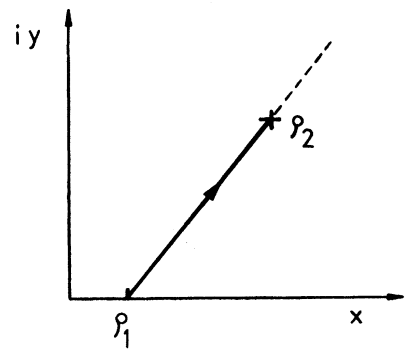

FIG. 1. The form of the chosen contour in the complex plane is illustrated. In contrast to Ref. 7, we follow the straight line from ρ_1 to ρ_2. It may have an arbitrary angle with the real axis.

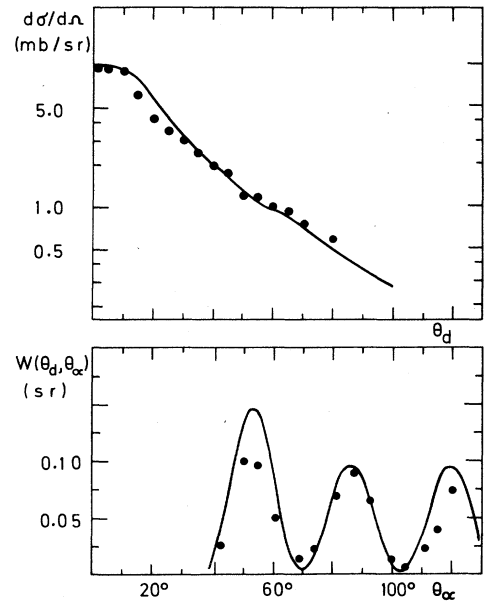

FIG. 2. *Top*: For the level $J^{\pi}=5^{-}$ (14.6 MeV in $^{16}O^{*}$) and for incident lithium ions with $E_{Li}=25.8$ MeV, the differential cross section of the transfer reaction $^{12}C(^{6}Li,d)^{16}O$ is plotted versus the scattering angle in the c.m. system. The experimental points have been taken from Ref. 12. *Bottom*: For the same state and for $E_{Li}=26$ MeV, the angular correlation function $W(\theta_d,\theta_\alpha)$ is given for $^{12}C(^{6}Li,d)^{16}O^{*}(\alpha)^{12}C$ at $\theta_d=9°$ in the lab system. The experimental points stem from Ref. 2.

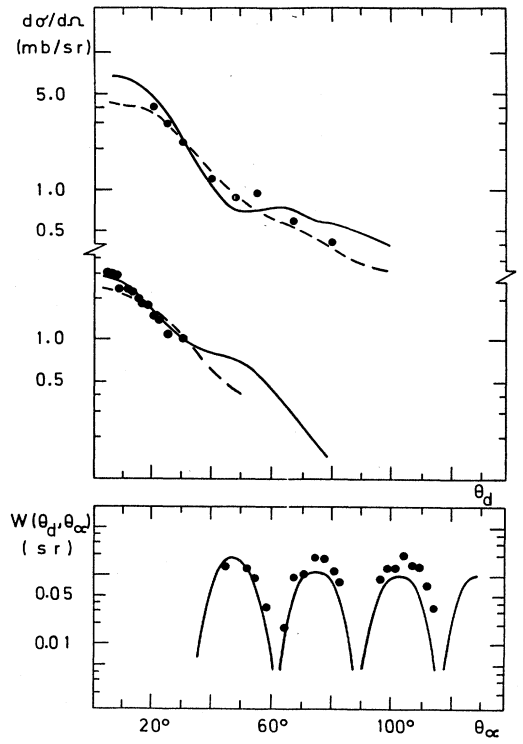

FIG. 3. *Top*: For the $J^{\pi}=6^{+}$ level at 16.3 MeV in $^{16}O^{*}$ and for incident ^{6}Li ions with $E_{Li}=28$ MeV, the differential cross section for the transfer reaction $^{12}C(^{6}Li,d)^{16}O$ is plotted versus the scattering angle in the c.m. system. Experimental points and broken curves stem from Ref. 15. *Middle*: Same as above, but for $E_{Li}=34$ MeV; experiment taken from Ref. 15. *Bottom*: For the same level and for $E_{Li}=34$ MeV, the angular correlation function $W(\theta_d,\theta_\alpha)$ is given for the reaction $^{12}C(^{6}Li,d)^{16}O^{*}(\alpha)^{12}C$. The experimental points have been adopted from Ref. 16. $\theta_d=10°$ in the lab system.

The precision of the computations is controlled by the adjustable parameters R_m, R, and y. The stability of the results for amplitude and cross section as accomplished by this method is of the order of 1% to 5%.

The suggested method for the solution of the Coulomb equation as based on the corresponding boundary-value problem may apparently also be used in the study of other reactions involving heavy ions. For the particular case of $(^{6}Li,d)$ reactions with an α transfer into high-lying resonance states with large decay widths, our method allows, to our knowledge, for the first time to perform correct zero-range calculations of differential cross sections $d\sigma/d\Omega$ and angular correlation functions $W(\theta_d,\theta_\alpha)$.

As an *example* we now consider the reaction $^{12}C(^{6}Li,d)^{16}O^{*}(\alpha)^{12}C$ involving the 5_1^- ($E^{*}=14.6$ MeV) and 6_1^+ ($E^{*}=16.3$ MeV) states in the nucleus $^{16}O^{*}$. In the case of the 5_1^- state there are unfortunately no experimental data for $d\sigma/d\Omega$ and $W(\theta_d,\theta_\alpha)$ at exactly the same energy of the incident ^{6}Li ions. So we take data referring to at least almost the same energies, i.e., $d\sigma/d\Omega$ as measured at $E_{Li}=25.8$ MeV (Ref. 12) and $W(\theta_d,\theta_\alpha)$ as determined at $E_{Li}=26$ MeV (Ref. 2).

Figures 2 and 3 contain the respective experimental data together with the theoretical curves. The optical-model parameters are taken to be the same ones for all energies; see Table I. From the work of Anantaraman *et al.*[17] and our own calculations it is seen that it is, in the cases under consideration, not sensible to vary these parameters with changing projective energies—this would only lead to rather small changes. For the evaluation of the wave functions of the α particles captured into the quasibound 5_1^- and 6_1^+ states, the effective surface potential with a hard core has been employed using the same parameters as in Ref. 18. (In Ref. 19 this phenome-

TABLE I. The optical model parameters used in the Li+C (Ref. 13) and d+O (Ref. 14) channels are given. The Coulomb radius is always 1.3 fm.

	V (MeV)	r_v (fm)	a_v (fm)	W_v (MeV)	W_s (MeV)	r_W (fm)	a_W (fm)
$^{6}Li+^{12}C$	245.0	1.2	0.800	12.5		1.75	1.000
$d+^{16}O$	101.4	1.0	0.717		8.75	1.58	0.625

nological effective surface potential has been shown to be in nice agreement with the RGM.) The calculations have been performed within the zero-range approximation to the interaction between α particle and deuteron $V_{\alpha d}$. As confirmed by an RGM analysis of the reaction $^{16}O(^{6}Li,d)^{20}Ne^{6}$, this is, apparently, sufficiently accurate for peripheral processes with small transferred momenta.[20] At first sight the use of finite-range DWBA appears more physical. However, in spite of the drastically increased sophistications, it does not incorporate antisymmetrization. In this respect it remains at the same level as the zero-range DWBA which is much simpler to use. In addition to that, the finite-range DWBA would introduce further uncertain parameters into the calculations without leading to reasonable improvements. In view of the small binding energies of α and d in the projectile and of the α particle in the metastable resonant state formed with the target nucleus, zero- and finite-range DWBA yield, for bound states with small binding energies, rather similar results. However, within the latter it is not so clear how to describe correctly resonant states. Hence, we resort to the simpler (less physical yet quite successful) zero-range DWBA.[20]

From Figs. 2 and 3 it is seen that the DWBA with the correctly evaluated zero-range amplitude (in which the nonresonant background has been neglected) describes rather nicely the characteristics of these direct α-transfer reactions that manifest themselves in the shapes of the measured differential cross section $d\sigma/d\Omega$ and in the angular correlation function $W(\theta_d,\theta_\alpha)$. The normalization of theory to experiment is an arbitrary one, i.e., no useful spectroscopic factors have been extracted.

Concluding we note that calculations of $d\sigma/d\Omega$ that take into account the microstructure of the α-cluster states *are* capable of providing reliable spectroscopic information, similarly as in the case of (d,p) reactions. For transfers into (resonant) *unbound states,* a practical prescription has been presented for evaluating this part of the reaction process. The good agreement of our calculations with experiment supports the proposed method.

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