Collective modes in hot and dense matter

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We propose a model for a relativistic many-body system at finite temperature in the framework of thermo field dynamics, which is a real-time formalism of finite-temperature field theory. Our model contains the scalar (σ) and the vector (ω) mesons as well as the Dirac nucleon. The full propagator and self-energy for each particle are presented in terms of spectral representations. The Feynman rules for a perturbation expansion are shown. They are applied to the study of collective modes in hot and dense matter within the random-phase approximation. The dispersion relations of the longitudinal and transverse collective modes in the meson branch are calculated. We also estimate the effective meson mass which is defined as the energy needed to create one meson at rest in extreme matter. The effects of vacuum fluctuations are also examined. They contribute a fair amount to the collective modes through the effective nucleon mass.

I. INTRODUCTION

It is very interesting to investigate matter under extreme conditions. In future experiments of very energetic heavy-ion collisions, very hot and dense hadronic matter and/or quark droplets will be produced and give us much information about new phases of matter.[1,2] We can also obtain a lot of knowledge of superdense matter by detailed observations of astronomical phenomena such as neutron stars, cooling of supernovas, etc.[3]

The investigation of matter with densities greater than a few times normal nuclear density and/or at temperatures beyond about 100 MeV should be based on a reliable relativistic model instead of a static potential approach in the conventional nonrelativistic many-body theory. It is required in such a relativistic model that the properties of normal nuclear matter at zero temperature be well reproduced, and that the explicit mesonic and (anti) nucleonic degrees of freedom can be easily handled. Walecka's relativistic field theory for hadronic matter[4] does satisfy such requirements, and hence it can be used to study properties of extreme matter. His model (the relativistic σ-ω model) contains neutral scalar σ mesons and neutral vector ω mesons: they are responsible for the observed short-range repulsive and long-range attractive force between two nucleons in the static limit. Recently this model has been applied to various fields of nuclear physics with great success.[5]

In this paper, we extend Walecka's theory at zero temperature to the one at finite temperature using the framework of thermo field dynamics (TFD), and study collective modes in hot and dense hadronic matter. The Hartree-Fock calculation of such a system is also performed and will be reported elsewhere.[6]

TFD is a real-time formalism of the statistical field theory first proposed by Leplae, Mancini, and Umezawa[7] in 1974 and then settled by Takahashi and Umezawa.[8] It is very powerful for describing many-body systems at finite temperature.[9,10] Since the Hilbert space is doubled in TFD, each field operator has two independent components as the thermal doublet. Correspondingly, the Green's functions, self-energies, etc. are expressed by the thermal matrices. By virtue of the extension of the Hilbert space, one can avoid some troubles which appear in a naive real-time formalism.[11] Moreover, because the Gell-Mann-Low formula and the Wick's theorem for the perturbation expansion are available in TFD, the usual perturbation theory at zero temperature can be extended quite naturally to finite temperature.

This paper is organized as follows. In Sec. II we review thermo field dynamics and show an example of the scalar field in order to illustrate the content of TFD.

In Sec. III the relativistic σ-ω model at finite temperature in TFD is summarized. We present the Feynman rules for the perturbation theory and the spectral representations for the full propagator and proper self-energy of each field. The propagator and self-energy are given in terms of the thermal 2×2 matrices and can be expressed in quite simple forms when we introduce the matrices $U_B(p_0)$ and $U_F(p_0)$ for boson and fermion fields respectively, where p_0 is the energy of a particle. These two matrices characterize the canonical transformations appearing in TFD. One can immediately see that our model is a natural extension of the σ-ω model of Walecka: the correspondence between the two is easily found.

In Sec. IV the collective modes in hot (up to a few hundred MeV) and dense (up to about 2 times normal nuclear density) matter are studied within the random-phase approximation (RPA). The formal expression of collective modes is presented in terms of spectral functions. In the calculations, we use the Dirac nucleon propagator obtained by the relativistic Hartree approximation which includes the effects of vacuum fluctuations (VF).[5,12] Then the results are compared with those obtained by using the free propagator in order to illustrate the role of the effective nucleon mass. We study the dispersion relations of the longitudinal and transverse collective modes which approach the mode of meson propagation in the limit of low temperature and density. The temperature and density dependence of such modes is reported. Next, we examine the effects of temperature and density on the meson masses in extreme matter. In experiments of energetic heavy-ion collisions, the detection of a change of the invariant meson mass may be a key to finding exotic matter. We define an effective meson mass as the energy needed to create one meson at rest. In the framework of the RPA, we have found a considerable change of meson masses. The effects of VF corrections are also examined. They fairly contribute to the collective modes through the effective mass of the Dirac nucleon given by the relativistic Hartree calculation.

In Sec. V, we summarize our results and give some remarks on the present model.

II. REVIEW OF TFD

The main purpose of TFD is to express the thermal average of an arbitrary operator A as the expectation value with respect to the temperature-dependent vacuum $|0(\beta)\rangle$:[8]

$$\langle A \rangle \equiv \mathrm{Tr}(Ae^{-\beta(H-\mu B)})/\mathrm{Tr}(e^{-\beta(H-\mu B)}) \tag{2.1}$$

$$= \langle 0(\beta)|A|0(\beta)\rangle , \tag{2.2}$$

where $\beta \equiv 1/k_B T$, H is the total Hamiltonian of the system, μ is the chemical potential, and B is the Baryon number operator. For this purpose, the Hilbert space in which the system is described is doubled, namely, a fictitious operator set $\tilde{\mathcal{U}}=\{\tilde{A}\}$ is introduced in addition to the physical set $\mathcal{U}=\{A\}$. The Fock space is also doubled. There is the one-to-one correspondence called the tilde conjugation between the two sets. In TFD, the operators and vacuum are governed by the following axioms.[10] (i) The equal time (anti) commutation relations

$$[A(t),\tilde{B}(t)]_{\pm}=0 \quad \text{for } \forall A(t)\in\mathcal{U}, \quad \forall\tilde{B}(t)\in\tilde{\mathcal{U}} . \tag{2.3}$$

The upper sign is relevant when both operators are fermionic, while the lower sign is relevant otherwise. (ii) The tilde-conjugation rules

$$(AB)^{\sim}=\tilde{A}\tilde{B} , \tag{2.4}$$

$$(c_1A+c_2B)^{\sim}=c_1^*\tilde{A}+c_2^*\tilde{B} , \tag{2.5}$$

$$(A^\dagger)^{\sim}=\tilde{A}^\dagger , \tag{2.6}$$

for $\forall A,B\in\mathcal{U}$, $\forall\tilde{A},\tilde{B}\in\tilde{\mathcal{U}}$, and $c_1,c_2\in\mathbf{C}$. (iii) The invariance of the vacuum under the tilde-conjugation

$$[|0(\beta)\rangle]^{\sim}=|0(\beta)\rangle . \tag{2.7}$$

(iv) The space-time translation generated by the energy-momentum operator $P^\mu=(H,\mathbf{P})$:

$$A(x)=e^{iP\cdot x}A(0)e^{-iP\cdot x} , \tag{2.8}$$

$$\tilde{A}(x)=e^{-i\tilde{P}\cdot x}\tilde{A}(0)e^{i\tilde{P}\cdot x} , \tag{2.9}$$

for $\forall A,P^\mu\in\mathcal{U}$, and $\forall\tilde{A},\tilde{P}^\mu\in\tilde{\mathcal{U}}$. Note that the above two equations are unified when we introduce the operator $\hat{P}^\mu\equiv P^\mu-\tilde{P}^\mu$:

$$\begin{bmatrix} A(x) \\ \tilde{A}(x) \end{bmatrix} = e^{i\hat{P}\cdot x}\begin{bmatrix} A(0) \\ \tilde{A}(0) \end{bmatrix} e^{-i\hat{P}\cdot x} . \tag{2.10}$$

This implies that the total Lagrangian density should be defined by $\hat{\mathcal{L}}=\mathcal{L}-\tilde{\mathcal{L}}$. (v) The thermal state condition[13,14]

$$A(t,\mathbf{x})|0(\beta)\rangle=\sigma_A e^{-\beta\mu N_A/2}\tilde{A}^\dagger(t-i\beta/2,\mathbf{x})|0(\beta)\rangle , \tag{2.11}$$

for $\forall A\in\mathcal{U}$ and $\forall\tilde{A}\in\tilde{\mathcal{U}}$, where N_A is the fermion number of A,[15] and

$$\sigma_A=\begin{cases} 1 & \text{for bosonic } A , \\ i & \text{for fermionic } A . \end{cases} \tag{2.12}$$

The well-known Kubo-Martin-Schwinger (KMS) condition[16] is led from Eq. (2.11). (vi) The double tilde-conjugation rule[17]

$$\tilde{\tilde{A}}=A \quad \text{for } \forall A\in\mathcal{U} . \tag{2.13}$$

Now let us see heuristically why the thermal average of A, Eq. (2.1), is equivalent to the vacuum expectation value of A, Eq. (2.2). For definiteness and simplicity, we consider the real scalar field $\phi(x)$ with mass m. The field $\phi(x)$ and the tilde-conjugated one $\tilde{\phi}(x)$ are assumed to become their asymptotic fields $\phi_{\rm in}(x)$ and $\tilde{\phi}_{\rm in}(x)$ when the interaction is switched off in the infinite past:

$$\begin{bmatrix} \phi(x) \\ \tilde{\phi}(x) \end{bmatrix} \to \begin{bmatrix} \phi_{\rm in}(x) \\ \tilde{\phi}_{\rm in}(x) \end{bmatrix} . \tag{2.14}$$

The fields $\phi_{\rm in}(x)$ and $\tilde{\phi}_{\rm in}(x)$, which obey the free-field equation, can be expanded by plane waves

$$\phi_{\rm in}(x)=\frac{1}{\sqrt{\Omega}}\sum_{\mathbf{p}}\frac{1}{\sqrt{2\omega_p}}(a_{\mathbf{p}}e^{-ip\cdot x}+a_{\mathbf{p}}^\dagger e^{ip\cdot x})_{p_0=\omega_p} , \tag{2.15}$$

$$\tilde{\phi}_{\rm in}(x)=\frac{1}{\sqrt{\Omega}}\sum_{\mathbf{p}}\frac{1}{\sqrt{2\omega_p}}(\tilde{a}_{\mathbf{p}}e^{ip\cdot x}+\tilde{a}_{\mathbf{p}}^\dagger e^{-ip\cdot x})_{p_0=\omega_p} , \tag{2.16}$$

where $\omega_p=(\mathbf{p}^2+m^2)^{1/2}$, and Ω is the volume of the system. The creation and annihilation operators satisfy the following commutation relations:

$$[a_{\mathbf{p}},a_{\mathbf{q}}^\dagger]=[\tilde{a}_{\mathbf{p}},\tilde{a}_{\mathbf{q}}^\dagger]=\delta_{\mathbf{pq}} , \tag{2.17}$$

and all other commutators vanish. The vacuum $|0\rangle$ for these operators is defined as

$$a_{\mathbf{p}}|0\rangle=\tilde{a}_{\mathbf{p}}|0\rangle=0 . \tag{2.18}$$

Next, we perform the following Bogoliubov transformation, and introduce the temperature-dependent operator and vacuum along the line of Takahashi and Umezawa:[8]

$$a_{\mathbf{p}}(\beta)\equiv e^{-iG}a_{\mathbf{p}}e^{iG} , \tag{2.19}$$

$$\tilde{a}_{\mathbf{p}}(\beta)\equiv e^{-iG}a_{\mathbf{p}}e^{iG} , \tag{2.20}$$

$$|0(\beta)\rangle\equiv e^{-iG}|0\rangle , \tag{2.21}$$

where the generator G of the transformation is given by

$$G \equiv i \sum_{p} \theta_p (a_p^\dagger \tilde{a}_p^\dagger - \tilde{a}_p a_p) . \tag{2.22}$$

Here θ_p is the angle of the transformation. The Fock space is spanned by the set of operators $[a_p^\dagger(\beta), \tilde{a}_p^\dagger(\beta)]$ and the vacuum $|0(\beta)\rangle$. Note that Eqs. (2.19) and (2.20) can be rewritten as

$$\begin{bmatrix} a_p \\ \tilde{a}_p^\dagger \end{bmatrix} = \begin{bmatrix} \cosh\theta_p & \sinh\theta_p \\ \sinh\theta_p & \cosh\theta_p \end{bmatrix} \begin{bmatrix} a_p(\beta) \\ \tilde{a}_p^\dagger(\beta) \end{bmatrix} . \tag{2.23}$$

The pair of operators in Eq. (2.23) is called the thermal doublet. The new transformed operators satisfy the following commutation relations:

$$[a_p(\beta), a_q^\dagger(\beta)] = [\tilde{a}_p(\beta), \tilde{a}_q^\dagger(\beta)] = \delta_{pq} , \tag{2.24}$$

and all other commutators vanish. The new vacuum has the property of

$$a_p(\beta)|0(\beta)\rangle = \tilde{a}_p(\beta)|0(\beta)\rangle = 0 . \tag{2.25}$$

To determine the angle parameter θ_p, we evaluate the thermal average of the number operator $a_p^\dagger a_p$ with respect to $|0(\beta)\rangle$ as

$$\langle 0(\beta)|a_p^\dagger a_p|0(\beta)\rangle = \sinh^2\theta_p , \tag{2.26}$$

where Eq. (2.23) has been used. Since the left-hand side of Eq. (2.26) should give the Bose distribution function, we choose the angle as

$$\sinh^2\theta_p = \frac{1}{e^{\beta\omega_p} - 1} . \tag{2.27}$$

Using Eq. (2.21), we can write $|0(\beta)\rangle$ as

$$|0(\beta)\rangle = \prod_p (1/\cosh\theta_p) \exp(a_p^\dagger \tilde{a}_p^\dagger \tanh\theta_p)|0\rangle \tag{2.28}$$

$$= \prod_p (1 - e^{-\beta\omega_p})^{1/2} \exp(a_p^\dagger \tilde{a}_p^\dagger e^{-\beta\omega_p/2})|0\rangle \tag{2.29}$$

$$= \prod_p (1 - e^{-\beta\omega_p})^{1/2} \prod_p \sum_n \frac{1}{n!} e^{-\beta n\omega_p/2} (a_p^\dagger)^n (\tilde{a}_p^\dagger)^n |0\rangle \tag{2.30}$$

$$= \prod_i (1 - e^{-\beta\omega_i})^{1/2} \sum_{n_1, n_2, \cdots} \exp[-\beta(n_1\omega_1 + n_2\omega_2 + \cdots)/2] |n_1 n_2 \cdots\rangle \otimes |\tilde{n}_1 \tilde{n}_2 \cdots\rangle , \tag{2.31}$$

where, in the last line, we have numbered the states and defined

$$|n_1 n_2 \cdots\rangle \otimes |\tilde{n}_1 \tilde{n}_2 \cdots\rangle = \frac{1}{n_1! n_2! \cdots} (a_1^\dagger)^{n_1} (a_2^\dagger)^{n_2} \cdots (\tilde{a}_1^\dagger)^{n_1} (\tilde{a}_2^\dagger)^{n_2} \cdots |0\rangle . \tag{2.32}$$

We can now calculate the thermal average of A as

$$\langle A \rangle = \prod_i (1 - e^{-\beta\omega_i}) \sum_{n_1, n_2 \cdots} \sum_{n_1', n_2' \cdots} \exp[-\beta(n_1\omega_1 + n_2\omega_2 + \cdots)/2] \exp[-\beta(n_1'\omega_1 + n_2'\omega_2 + \cdots)/2] \times \langle n_1 n_2 \cdots | A | n_1' n_2' \cdots\rangle \langle n_1 n_2 \cdots | n_1' n_2' \cdots\rangle \tag{2.33}$$

$$= \prod_i (1 - e^{-\beta\omega_i}) \sum_{n_1, n_2 \cdots} \exp[-\beta(n_1\omega_1 + n_2\omega_2 + \cdots)] \langle n_1 n_2 \cdots | A | n_1 n_2 \cdots\rangle \tag{2.34}$$

$$= \mathrm{Tr}(A e^{-\beta H}) / \mathrm{Tr}(e^{-\beta H}) . \tag{2.35}$$

Note that when H is written in terms of asymptotic fields, it takes the form of a free Hamiltonian. Equation (2.35) is the expected result for the real scalar field [cf. Eq. (2.1)]. The case of fermion or vector boson can also be considered in a similar way.

As illustrated above, the thermal average in TFD is given by the vacuum expectation value by virtue of the doubled Hilbert space. As we shall see later, the perturbation theory using the Feynman diagram technique is available in TFD,[18,19] and free propagators at finite temperature are separated into two parts: the temperature-independent part and the temperature-dependent part. The renormalization at finite temperature is hence possible by using appropriate counterterms set up at zero temperature.[10,18] In the imaginary-time formalism of the finite temperature field theory,[20] one finds that the frequency sum and the analytic continuation of frequency variables are very complicated, and that the use of many properties of operator formalism such as the dynamical map, the Ward-Takahashi identities, etc. is quite difficult. Moreover, one sometimes encounters the ill-defined singularity such as $[\delta(p^2 - m^2)]^n$ in the real-time formalism of Ref. 11. We can avoid all of these difficulties when the TFD formalism is used.[21] The equivalence of TFD to the path ordered formalism, the C^*-algebra approach, etc. are studied in Refs. 10, 17, and 22. For further discussions on TFD, see Ref. 9.

III. THE σ-ω MODEL IN TFD

In this section, we develop the σ-ω model in TFD, and present the Feynman rules and some useful relations for perturbative calculations. We consider that the system is static and uniform.

A. Free fields

The free fields for the Dirac nucleon (ψ), σ meson (ϕ), and ω meson (V^μ) are expanded in terms of plane waves as

$$\psi(x)=\frac{1}{\sqrt{\Omega}}\sum_{\mathbf{p},s}[c_{\mathbf{p},s}u(\mathbf{p},s)e^{-ip\cdot x}+d^\dagger_{\mathbf{p},s}v(\mathbf{p},s)e^{ip\cdot x}]_{p_0=E_p}\,, \tag{3.1}$$

$$\phi(x)=\frac{1}{\sqrt{\Omega}}\sum_{\mathbf{p}}\frac{1}{\sqrt{2\omega_s(p)}}(a_{\mathbf{p}}e^{-ip\cdot x}+a^\dagger_{\mathbf{p}}e^{ip\cdot x})_{p_0=\omega_s(p)}\,, \tag{3.2}$$

$$V^\mu(x)=\frac{1}{\sqrt{\Omega}}\sum_{\mathbf{p},\lambda}\frac{e^\mu(\mathbf{p},\lambda)}{\sqrt{2\omega_v(p)}}\times(b_{\mathbf{p},\lambda}e^{-ip\cdot x}+b^\dagger_{\mathbf{p},\lambda}e^{-ip\cdot x})_{p_0=\omega_v(p)}\,, \tag{3.3}$$

where $\sum_{\mathbf{p}}$ stands for $\Omega\int d^3p/(2\pi)^{3/2}$, s specifies a spin state of the Dirac nucleon, and λ denotes a component of the polarization vector e^μ for the ω meson.[23] The energies of the nucleon and σ and ω mesons are expressed by $E_p=(M^2+\mathbf{p}^2)^{1/2}$, $\omega_s(p)=(m_s^2+\mathbf{p}^2)^{1/2}$, and $\omega_v(p)=(m_v^2+\mathbf{p}^2)^{1/2}$ with their masses M, m_s, and m_v. The creation (annihilation) operators of the nucleon, antinucleon, and σ and ω mesons are $c^\dagger(c)$, $d^\dagger(d)$, $a^\dagger(a)$ and $b^\dagger(b)$, respectively. Here we follow the notation of Serot and Walecka.[5] The tilde-fields are similarly expanded as follows:

$$\tilde\psi(x)=\frac{1}{\sqrt{\Omega}}\sum_{\mathbf{p},s}[\tilde c_{\mathbf{p},s}u^*(\mathbf{p},s)e^{ip\cdot x}+\tilde d^\dagger_{\mathbf{p},s}v^*(\mathbf{p},s)e^{-ip\cdot x}]_{p_0=E_p}\,, \tag{3.4}$$

$$\tilde\phi(x)=\frac{1}{\sqrt{\Omega}}\sum_{\mathbf{p}}\frac{1}{\sqrt{2\omega_s(p)}}(\tilde a_{\mathbf{p}}e^{ip\cdot x}+\tilde a^\dagger_{\mathbf{p}}e^{-ip\cdot x})_{p_0=\omega_s(p)}\,, \tag{3.5}$$

$$\tilde V^\mu(x)=\frac{1}{\sqrt{\Omega}}\sum_{\mathbf{p},\lambda}\frac{e^\mu(\mathbf{p},\lambda)}{\sqrt{2\omega_v(p)}}(\tilde b_{\mathbf{p},\lambda}e^{ip\cdot x}+\tilde b^\dagger_{\mathbf{p},\lambda}e^{-ip\cdot x})_{p_0=\omega_v(p)}\,. \tag{3.6}$$

Here the creation and annihilation operators follow the usual commutation relations:

$$[\alpha_{\mathbf{p},\nu},\alpha^\dagger_{\mathbf{q}\nu'}]_\pm=\delta_{\nu\nu'}\delta_{\mathbf{pq}}\,, \tag{3.7}$$

$$[\tilde\alpha_{\mathbf{p}\nu},\tilde\alpha^\dagger_{\mathbf{q}\nu'}]_\pm=\delta_{\nu\nu'}\delta_{\mathbf{pq}}\,, \tag{3.8}$$

and other commutators vanish, where $\alpha^\dagger(\alpha)$ stands for the creation (annihilation) operator for each particle and ν denotes quantum states except for momenta. Then we define the thermal doublet for each field as

$$\psi^{(a)}(x)\equiv\begin{bmatrix}\psi^{(1)}(x)\\ \psi^{(2)}(x)\end{bmatrix}\equiv\begin{bmatrix}\psi(x)\\ i^t\tilde\psi^\dagger(x)\end{bmatrix}\,, \tag{3.9}$$

$$\phi^{(a)}(x)\equiv\begin{bmatrix}\phi^{(1)}(x)\\ \phi^{(2)}(x)\end{bmatrix}\equiv\begin{bmatrix}\phi(x)\\ \tilde\phi(x)\end{bmatrix}\,, \tag{3.10}$$

$$V^{(a)}_\mu(x)\equiv\begin{bmatrix}V^{(1)}_\mu(x)\\ V^{(2)}_\mu(x)\end{bmatrix}\equiv\begin{bmatrix}V_\mu(x)\\ \tilde V_\mu(x)\end{bmatrix}\,, \tag{3.11}$$

where t means the transposition with respect to the spinor index, and a $(=1$ or $2)$ specifies a component of the thermal doublet. The first component $(a=1)$ is physical, and the other is fictitious.

Now the temperature can be taken into account by the Bogoliubov transformation which introduces the temperature-dependent operators $[\alpha^\dagger_{\mathbf{p}\nu}(\beta),\tilde\alpha^\dagger_{\mathbf{p}\nu}(\beta)]$ and the temperature-dependent unperturbed vacuum $|0,\beta\rangle$:

$$\begin{bmatrix}c_{\mathbf{p},s}\\ i\tilde c^\dagger_{\mathbf{p},s}\end{bmatrix}=\begin{bmatrix}\cos\theta_+ & \sin\theta_+\\ -\sin\theta_+ & \cos\theta_+\end{bmatrix}\begin{bmatrix}c_{\mathbf{p},s}(\beta)\\ i\tilde c^\dagger_{\mathbf{p},s}(\beta)\end{bmatrix}\,, \tag{3.12}$$

$$\begin{bmatrix}d_{\mathbf{p},s}\\ i\tilde d^\dagger_{\mathbf{p},s}\end{bmatrix}=\begin{bmatrix}\cos\theta_- & \sin\theta_-\\ -\sin\theta_- & \cos\theta_-\end{bmatrix}\begin{bmatrix}d_{\mathbf{p},s}(\beta)\\ i\tilde d^\dagger_{\mathbf{p},s}(\beta)\end{bmatrix}\,, \tag{3.13}$$

$$\begin{bmatrix}a_{\mathbf{p}}\\ \tilde a^\dagger_{\mathbf{p}}\end{bmatrix}=\begin{bmatrix}\cosh\theta_s & \sinh\theta_s\\ \sinh\theta_s & \cosh\theta_s\end{bmatrix}\begin{bmatrix}a_{\mathbf{p}}(\beta)\\ \tilde a^\dagger_{\mathbf{p}}(\beta)\end{bmatrix}\,, \tag{3.14}$$

$$\begin{bmatrix}b_{\mathbf{p},\lambda}\\ \tilde b^\dagger_{\mathbf{p},\lambda}\end{bmatrix}=\begin{bmatrix}\cosh\theta_v & \sinh\theta_v\\ \sinh\theta_v & \cosh\theta_v\end{bmatrix}\begin{bmatrix}b_{\mathbf{p},\lambda}(\beta)\\ \tilde b^\dagger_{\mathbf{p},\lambda}(\beta)\end{bmatrix}\,, \tag{3.15}$$

and

$$\alpha_{\mathbf{p},\nu}(\beta)|0,\beta\rangle=\tilde\alpha_{\mathbf{p},\nu}(\beta)|0,\beta\rangle=0\,. \tag{3.16}$$

The angles of θ's are determined so as to reproduce the thermal distribution of each particle. They are given by

$$f_\pm(E_p,\beta)\equiv\frac{(2\pi)^3}{\Omega}\langle 0,\beta|\begin{bmatrix}c^\dagger_{\mathbf{p},s}c_{\mathbf{p},s}\\ d^\dagger_{\mathbf{p},s}d_{\mathbf{p},s}\end{bmatrix}|0,\beta\rangle =\sin^2\theta_\pm=\frac{1}{1+e^{\beta(E_p\mp\mu)}}\quad\text{for}\begin{bmatrix}\text{nucleon}\\ \text{antinucleon}\end{bmatrix} \tag{3.17}$$

and

$$g_{\binom{s}{v}}(p,\beta)\equiv\frac{(2\pi)^3}{\Omega}\langle 0,\beta|\begin{bmatrix} a^\dagger_{\mathbf{p}}a_{\mathbf{p}} \\ b^\dagger_{\mathbf{p},\lambda}b_{\mathbf{p},\lambda}\end{bmatrix}|0,\beta\rangle = \sinh^2\theta_{\binom{s}{v}} = \frac{1}{e^{\beta\omega_{\binom{s}{v}}(p)}-1} \quad \text{for} \begin{bmatrix}\text{scalar}\\ \text{vector}\end{bmatrix} \text{meson} . \tag{3.18}$$

The temperature-dependent creation and annihilation operators obey similar commutation relations to Eqs. (3.7) and (3.8), i.e.,

$$[\alpha_{\mathbf{p}\nu}(\beta),\alpha^\dagger_{\mathbf{q}\nu'}(\beta)]=\delta_{\nu\nu'}\delta_{\mathbf{pq}} , \tag{3.19}$$

$$[\tilde{\alpha}_{\mathbf{p}\nu}(\beta),\tilde{\alpha}^\dagger_{\mathbf{q}\nu'}(\beta)]=\delta_{\nu\nu'}\delta_{\mathbf{pq}} . \tag{3.20}$$

Now the free propagator for each field can be written in the form of the thermal 2×2 matrix.[14,24]

1. Dirac nucleon propagator $G^{0(ab)}_{\alpha\beta}$

The Dirac nucleon propagator is defined as

$$iG^{0(ab)}_{\alpha\beta}(x_1-x_2)\equiv\langle 0,\beta|T\psi^{(a)}_\alpha(x_1)\bar{\psi}^{(b)}_\beta(x_2)|0,\beta\rangle \tag{3.21}$$

$$\equiv i\int d\!\!{}^-{}^4p\, e^{-ip\cdot(x_1-x_2)}G^{0(ab)}_{\alpha\beta}(p) , \tag{3.22}$$

where $d\!\!{}^-{}^np\equiv d^np/(2\pi)^n$. The propagator can be separated into two parts, i.e., the usual Feynman part $G^{0(ab)}_{F\alpha\beta}(p)$ and the density-dependent part $G^{0(ab)}_{D\alpha\beta}(p)$:

$$G^{0(ab)}_{\alpha\beta}(p)=G^{0(ab)}_{F\alpha\beta}(p)+G^{0(ab)}_{D\alpha\beta}(p) , \tag{3.23}$$

where

$$G^{0(ab)}_{F\alpha\beta}(p)=(\not p+M)_{\alpha\beta}\times\begin{bmatrix}\dfrac{1}{p^2-M^2+i\varepsilon} & 0 \\ 0 & \dfrac{1}{p^2-M^2-i\varepsilon}\end{bmatrix}^{(ab)} , \tag{3.24}$$

$$G^{0(ab)}_{D\alpha\beta}(p)=2\pi i\delta(p^2-M^2)(\not p+M)_{\alpha\beta}\times\begin{bmatrix}\sin^2\theta_{p_0} & \frac{1}{2}\sin2\theta_{p_0} \\ \frac{1}{2}\sin2\theta_{p_0} & -\sin^2\theta_{p_0}\end{bmatrix}^{(ab)} , \tag{3.25}$$

with

$$\cos\theta_{p_0}=\frac{\theta(p_0)}{(1+e^{-x})^{1/2}}+\frac{\theta(-p_0)}{(1+e^{x})^{1/2}} , \tag{3.26}$$

$$\sin\theta_{p_0}=\frac{e^{-x/2}\theta(p_0)}{(1+e^{-x})^{1/2}}-\frac{e^{x/2}\theta(-p_0)}{(1+e^{x})^{1/2}} , \tag{3.27}$$

where $x=\beta(p_0-\mu)$. The $\theta(p_0)$ is the usual step function. Moreover, the propagator can be rewritten in a very simple form when we use the matrix $U_F(p_0)$ which characterizes the Bogoliubov transformation for fermion:[9]

$$G^{0(ab)}_{\alpha\beta}(p)=\left[U_F(p_0)\begin{bmatrix}\bar{G}^0(p) & 0 \\ 0 & \bar{G}^{0\star}(p)\end{bmatrix}_{\alpha\beta}U_F^{-1}(p_0)\right]^{(ab)} , \tag{3.28}$$

where

$$U_F(p_0)=\begin{bmatrix}\cos\theta_{p_0} & \sin\theta_{p_0} \\ -\sin\theta_{p_0} & \cos\theta_{p_0}\end{bmatrix} , \tag{3.29}$$

$$\bar{G}^0(p)=(\not p+M)/(p^2-M^2+i\varepsilon) . \tag{3.30}$$

In Eq. (3.28), the ☆ means that one must take the complex conjugation of $\bar{G}^0$ except for γ matrices in it. We can find relations among the components of the thermal 2×2 matrix: $G^{0(12)}_{\alpha\beta}=G^{0(21)}_{\alpha\beta}$ and $G^{0(22)}_{\alpha\beta}=G^{0(11)\star}_{\alpha\beta}$.

2. σ meson propagator $\Delta^{0(ab)}$

The free propagator

$$i\Delta^{0(ab)}(x_1-x_2)\equiv\langle 0,\beta|T\phi^{(a)}(x_1)\phi^{(b)}(x_2)|0,\beta\rangle \tag{3.31}$$

$$\equiv i\int d\!\!{}^-{}^4p\, e^{-ip\cdot(x_1-x_2)}\Delta^{0(ab)}(p) \tag{3.32}$$

is given by

$$\Delta^{0(ab)}(p)=\Delta^{0(ab)}_F(p)+\Delta^{0(ab)}_D(p) , \tag{3.33}$$

where

$$\Delta^{0(ab)}_F(p)=\begin{bmatrix}\dfrac{1}{p^2-m_s^2+i\varepsilon} & 0 \\ 0 & \dfrac{-1}{p^2-m_s^2-i\varepsilon}\end{bmatrix}^{(ab)} , \tag{3.34}$$

$$\Delta^{0(ab)}_D(p)=-2\pi i\delta(p^2-m_s^2)\times\begin{bmatrix}\sinh^2\varphi_{p_0} & \frac{1}{2}\sinh2\varphi_{p_0} \\ \frac{1}{2}\sinh2\varphi_{p_0} & \sinh^2\varphi_{p_0}\end{bmatrix}^{(ab)} , \tag{3.35}$$

with

$$\cosh\varphi_{p_0}=\frac{1}{(1-e^{-|y|})^{1/2}} , \tag{3.36}$$

$$\sinh\varphi_{p_0}=\frac{e^{-|y|/2}}{(1-e^{-|y|})^{1/2}} , \tag{3.37}$$

where $y=\beta p_0$. If we use the matrix for boson field

$$U_B(p_0)=\begin{bmatrix}\cosh\varphi_{p_0} & \sinh\varphi_{p_0} \\ \sinh\varphi_{p_0} & \cosh\varphi_{p_0}\end{bmatrix} , \tag{3.38}$$

we can rewrite the propagator in a simple form as

$$\Delta^{0(ab)}(p)=\left[U_B(p_0)\begin{bmatrix}\bar{\Delta}^0(p) & 0\\ 0 & -\bar{\Delta}^{0*}(p)\end{bmatrix}U_B(p_0)\right]^{(ab)}, \tag{3.39}$$

where

$$\bar{\Delta}^0(p)=1/(p^2-m_s^2+i\varepsilon)\ . \tag{3.40}$$

Note that $\Delta^{0(12)}=\Delta^{0(21)}$ and $\Delta^{0(22)}=-\Delta^{0(11)*}$.

3. ω meson propagator $D_{\mu\nu}^{0(ab)}$

The free propagator

$$iD_{\mu\nu}^{0(ab)}(x_1-x_2)\equiv\langle 0,\beta|TV_\mu^{(a)}(x_1)V_\nu^{(b)}(x_2)|0,\beta\rangle \tag{3.41}$$

$$\equiv i\int d^4p e^{-ip\cdot(x_1-x_2)}D_{\mu\nu}^{0(ab)}(p) \tag{3.42}$$

is given by

$$D_{\mu\nu}^{0(ab)}(p)=D_{F\mu\nu}^{0(ab)}(p)+D_{D\mu\nu}^{0(ab)}(p)\ , \tag{3.43}$$

where

$$D_{F\mu\nu}^{0(ab)}(p)=\xi_{\mu\nu}(p)\times\begin{bmatrix}\dfrac{1}{p^2-m_v^2+i\varepsilon} & 0\\ 0 & \dfrac{-1}{p^2-m_v^2-i\varepsilon}\end{bmatrix}^{(ab)}, \tag{3.44}$$

$$D_{D\mu\nu}^{0(ab)}(p)=-2\pi i\delta(p^2-m_v^2)\xi_{\mu\nu}(p)\times\begin{bmatrix}\sinh^2\varphi_{p_0} & \frac{1}{2}\sinh 2\varphi_{p_0}\\ \frac{1}{2}\sinh 2\varphi_{p_0} & \sinh^2\varphi_{p_0}\end{bmatrix}^{(ab)}, \tag{3.45}$$

with $\xi_{\mu\nu}=-g_{\mu\nu}+p_\mu p_\nu/m_v^2$. The $p_\mu p_\nu$ term will not contribute to physical quantities, since the vector meson couples to the conserved Baryon current.[5] The propagator can also be expressed as

$$D_{\mu\nu}^{0(ab)}(p)=\xi_{\mu\nu}(p)\left[U_B(p_0)\begin{bmatrix}\bar{D}^0(p) & 0\\ 0 & -\bar{D}^{0*}(p)\end{bmatrix}\times U_B(p_0)\right]^{(ab)}, \tag{3.46}$$

where

$$\bar{D}^0(p)=1/(p^2-m_v^2+i\varepsilon)\ . \tag{3.47}$$

Note that $D^{0(12)}=D^{0(21)}$ and $D^{0(22)}=-D^{0(11)*}$.

B. Perturbation theory in TFD

In TFD, the n-point Green's function for arbitrary operators $O_i(x)$ ($i=1-n$) can be calculated by using the Gell-Mann-Low formula[19]

$$\langle 0(\beta)|TO_1(x_1)\cdots O_n(x_n)|0(\beta)\rangle=\frac{\langle 0,\beta|TO_1(x_1)\cdots O_n(x_n)\exp\left[-i\int_{-\infty}^{+\infty}d\tau\hat{H}_{\rm int}(\tau)\right]|0,\beta\rangle}{\langle 0,\beta|\exp\left[-i\int_{-\infty}^{+\infty}d\tau\hat{H}_{\rm int}(\tau)\right]|0,\beta\rangle}, \tag{3.48}$$

when the operators obey the commutation relations of the harmonic oscillator type. Here the left-hand side of Eq. (3.48) is in the Heisenberg picture, while the right-hand side is in the interaction picture. The interaction Hamiltonian has been denoted by $H_{\rm int}$, and $\hat{H}_{\rm int}\equiv H_{\rm int}-\tilde{H}_{\rm int}$.

The total Lagrangian density of the present σ-ω model is given in terms of the thermal doublets as follows:

$$\hat{\mathcal{L}}=\mathcal{L}-\tilde{\mathcal{L}}$$

$$=\sum_{a=1,2}\varepsilon_a P_a\left[\bar{\psi}^{(a)}\left[\frac{i}{2}\overleftrightarrow{\partial\!\!\!/}-g_v V\!\!\!/^{(a)}\right]\psi^{(a)}+g_s\bar{\psi}^{(a)}\phi^{(a)}\psi^{(a)}-M\bar{\psi}^{(a)}\psi^{(a)}+\tfrac{1}{2}(\partial_\mu\phi^{(a)})^2-\tfrac{1}{2}m_s\phi^{(a)2}-\tfrac{1}{4}F_{\mu\nu}^{(a)}F^{(a)\mu\nu}+\tfrac{1}{2}m_v^2(V_\mu^{(a)})^2+\delta\mathcal{L}^{(a)}\right] \tag{3.49}$$

$$=\hat{\mathcal{L}}_0+\hat{\mathcal{L}}_{\rm int}\ , \tag{3.50}$$

where g_s and g_v are the coupling constants, $F_{\mu\nu}^{(a)}$ is the kinetic term of the ω meson, $\delta\mathcal{L}^{(a)}$ is the counterterm for the renormalization, and the interaction Lagrangian density $\hat{\mathcal{L}}_{\rm int}$ is

$$\hat{\mathcal{L}}_{\rm int}=\sum_{a=1,2}\varepsilon_a P_a[\bar{\psi}^{(a)}(-g_v V\!\!\!/^{(a)}+g_s\phi^{(a)})\psi^{(a)}+\delta\mathcal{L}^{(a)}]\ , \tag{3.51}$$

with

$$\varepsilon_a=\begin{cases}1 & \text{for } a=1\ ,\\ -1 & \text{for } a=2\ .\end{cases} \tag{3.52}$$

The P_a is the ordering operator defined as[9]

$$P_a(A^{(a)}B^{(a)}\cdots C^{(a)})=\begin{cases}A^{(1)}B^{(1)}\cdots C^{(1)} & \text{for } a=1\ ,\\ C^{(2)}\cdots B^{(2)}A^{(2)} & \text{for } a=2\ ,\end{cases} \tag{3.53}$$

for arbitrary operators A, B, C, etc.

The Wick's theorem is available in TFD.[19] Then one can use the perturbation theory and construct the Feynman rules, which are very similar to the usual ones at zero temperature. The rules will be summarized in a later subsection.

C. Spectral representation for full propagators

We construct the full propagators of the Dirac nucleon, σ and ω mesons in the Heisenberg picture. For the detailed derivation, see Appendix A. (Also, see Refs. 9, 14, 25, and 26.)

1. Dirac nucleon propagator $G_{\alpha\beta}^{(ab)}$

The full propagator is defined as

$$iG_{\alpha\beta}^{(ab)}(x_1-x_2)\equiv\langle 0(\beta)|T\psi_{\alpha}^{(a)}(x_1)\bar{\psi}_{\beta}^{(b)}(x_2)|0(\beta)\rangle \tag{3.54}$$

$$\equiv i\int d^4p\, e^{-ip\cdot(x_1-x_2)}G_{\alpha\beta}^{(ab)}(p)\ . \tag{3.55}$$

By using the complete set of the Dirac operators and the thermal state condition for fermion

$$\psi^{\dagger}(0)|0(\beta)\rangle=ie^{-\beta\mu/2}e^{\beta\hat{H}/2}\tilde{\psi}(0)|0(\beta)\rangle\ , \tag{3.56}$$

we can obtain a simple representation for the propagator in momentum space

$$G_{\alpha\beta}^{(ab)}(p)=\int_{-\infty}^{+\infty}dw\,\rho_{\alpha\beta}(w,|\mathbf{p}|)\times\left[\frac{A_F^{(ab)}(p_0)}{p_0-w+i\varepsilon}+\frac{B_F^{(ab)}(p_0)}{p_0-w-i\varepsilon}\right]\ , \tag{3.57}$$

where

$$A_F^{(ab)}(p_0)=\frac{1}{1+e^{-x}}\begin{bmatrix}1 & -e^{-x/2}\\ -e^{-x/2} & e^{-x}\end{bmatrix}^{(ab)}\ , \tag{3.58}$$

$$B_F^{(ab)}(p_0)=\frac{1}{1+e^{-x}}\begin{bmatrix}e^{-x} & e^{-x/2}\\ e^{-x/2} & 1\end{bmatrix}^{(ab)}\ . \tag{3.59}$$

Here the spectral function $\rho_{\alpha\beta}(p)$ is given by[14]

$$\rho_{\alpha\beta}(p)=\rho_{1t}(p_0,|\mathbf{p}|)(\gamma^0)_{\alpha\beta}+\rho_{1s}(p_0,|\mathbf{p}|)(\mathbf{p}\cdot\boldsymbol{\gamma})_{\alpha\beta}+\rho_2(p_0,|\mathbf{p}|)\delta_{\alpha\beta}\ . \tag{3.60}$$

The translational invariance, the parity conservation and the time-reversal invariance give constraint on the form of the spectral function. The functions ρ_{1t}, ρ_{1s}, and ρ_2 are real. Furthermore, when we introduce

$$\bar{G}(p)=\int_{-\infty}^{+\infty}dw\,\rho(w,|\mathbf{p}|)\chi(p_0,w)\ , \tag{3.61}$$

with

$$\chi(p_0,w)=\frac{\theta(w)}{p_0-w+i\varepsilon}+\frac{\theta(-w)}{p_0-w-i\varepsilon}=\frac{\theta(p_0)}{p_0-w+i\varepsilon}+\frac{\theta(-p_0)}{p_0-w-i\varepsilon}\ , \tag{3.62}$$

the propagator can be rewritten in a convenient and simple form as

$$G_{\alpha\beta}^{(ab)}(p)=\left[U_F(p_0)\begin{bmatrix}\bar{G}(p) & 0\\ 0 & \bar{G}^{\star}(p)\end{bmatrix}U_F^{-1}(p_0)\right]^{(ab)}\ . \tag{3.63}$$

If we take

$$\rho(p)=\frac{\not{p}+M}{2E_p}[\delta(p_0-E_p)-\delta(p_0+E_p)]\ , \tag{3.64}$$

we can obtain the free propagator from Eqs. (3.61) and (3.63).

2. σ meson propagator $\Delta^{(ab)}$

The full propagator is defined by

$$i\Delta^{(ab)}(x_1-x_2)\equiv\langle 0(\beta)|T\phi^{(a)}(x_1)\phi^{(b)}(x_2)|0(\beta)\rangle \tag{3.65}$$

$$\equiv i\int d^4p e^{-ip\cdot(x_1-x_2)}\Delta^{(ab)}(p)\ . \tag{3.66}$$

The complete set and the thermal state condition for the boson field give us a simple form as

$$\Delta^{(ab)}(p)=\int_{-\infty}^{+\infty}dw\,\rho_s(w,|\mathbf{p}|)\left[\frac{A_B^{(ab)}(p_0)}{p_0-w+i\varepsilon}-\frac{B_B^{(ab)}(p_0)}{p_0-w-i\varepsilon}\right]\ , \tag{3.67}$$

where

$$A_B^{(ab)}(p_0)=\frac{1}{1-e^{-y}}\begin{bmatrix}1 & e^{-y/2}\\ e^{-y/2} & e^{-y}\end{bmatrix}^{(ab)}\ , \tag{3.68}$$

$$B_B^{(ab)}(p_0)=\frac{1}{1-e^{-y}}\begin{bmatrix}e^{-y} & e^{-y/2}\\ e^{-y/2} & 1\end{bmatrix}^{(ab)}\ , \tag{3.69}$$

and $\rho_s(p)$ is the real spectral function for the σ field. When we introduce the function $\bar{\Delta}(p)$ as

$$\bar{\Delta}(p)=\int_{-\infty}^{+\infty}dw\,\rho_s(w,|\mathbf{p}|)\chi(p_0,w)\ , \tag{3.70}$$

then $\Delta^{(ab)}(p)$ is given by

$$\Delta^{(ab)}(p)=\left[U_B(p_0)\begin{bmatrix}\bar{\Delta}(p) & 0\\ 0 & -\bar{\Delta}^{*}(p)\end{bmatrix}U_B(p_0)\right]^{(ab)}\ . \tag{3.71}$$

If we set ρ_s in Eq. (3.70) as

$$\rho_s(p)=\frac{1}{2\omega_s(p)}[\delta(p_0-\omega_s(p))-\delta(p_0+\omega_s(p))]\ , \tag{3.72}$$

we can obtain the free propagator from Eq. (3.71).

3. ω meson propagator $D_{\mu\nu}^{(ab)}$

For the full ω meson propagator, we can find a similar form to that of the σ meson. The propagator is defined by

$$iD_{\mu\nu}^{(ab)}(x_1-x_2)\equiv\langle 0(\beta)|TV_\mu^{(a)}(x_1)V_\nu^{(b)}(x_2)|0(\beta)\rangle \tag{3.73}$$

$$\equiv i\int d^4p\, e^{-ip\cdot(x_1-x_2)}D_{\mu\nu}^{(ab)}(p)\ . \tag{3.74}$$

The $D_{\mu\nu}^{(ab)}(p)$ is written as

$$D_{\mu\nu}^{(ab)}(p)=\int_{-\infty}^{+\infty}dw\,\rho_{v\,\mu\nu}(w,|\mathbf{p}|)\times\left[\frac{A_B^{(ab)}(p_0)}{p_0-w+i\varepsilon}-\frac{B_B^{(ab)}(p_0)}{p_0-w-i\varepsilon}\right]\ , \tag{3.75}$$

where $\rho_{v\,\mu\nu}$ is the real spectral function. Furthermore, if $\overline{D}_{\mu\nu}(p)$ is introduced as

$$\overline{D}_{\mu\nu}(p)=\int_{-\infty}^{+\infty}dw\,\rho_{v\,\mu\nu}(w,|\mathbf{p}|)\chi(p_0,w)\ , \tag{3.76}$$

we can rewrite

$$D_{\mu\nu}^{(ab)}(p)=\left[U_B(p_0)\begin{bmatrix}\overline{D}(p) & 0\\ 0 & -\overline{D}^*(p)\end{bmatrix}_{\mu\nu}U_B(p_0)\right]^{(ab)} \tag{3.77}$$

If we set ρ_v in Eq. (3.76) as

$$\rho_{v\,\mu\nu}(p)=-\frac{g_{\mu\nu}}{2\omega_v(p)}[\delta(p_0-\omega_v(p))-\delta(p_0+\omega_v(p))]\ , \tag{3.78}$$

we can find the free propagator from Eq. (3.77) except for the $p_\mu p_\nu$ term.

D. Dyson's equations

The Dyson's equation for the Dirac field is

$$G^{(ab)}(p)=G^{0(ab)}(p)+\sum_{cd}G^{0(ac)}(p)\Sigma_F^{(cd)}(p)G^{(db)}(p)\ , \tag{3.79}$$

where $\Sigma_F^{(ab)}(p)$ is the proper nucleon self-energy. We have omitted the spinor index for simplicity. From this equation, we can limit the general form of the self-energy to

$$\Sigma_F^{(ab)}(p)=[G^{0(ab)}(p)]^{-1}-[G^{(ab)}(p)]^{-1}=\left[U_F(p_0)\begin{bmatrix}\overline{\Sigma}_F(p) & 0\\ 0 & \overline{\Sigma}_F^{☆}(p)\end{bmatrix}U_F^{-1}(p_0)\right]^{(ab)}\ , \tag{3.80}$$

where Eqs. (3.28) and (3.63) have been used. The $\overline{\Sigma}_F(p)$ is a complex function. We can therefore obtain the inverse of $G^{(ab)}(p)$ as

$$[G^{(ab)}(p)]^{-1}=[G^{0(ab)}(p)]^{-1}-\Sigma_F^{(ab)}(p)=\left[U_F(p_0)\begin{bmatrix}\not p-M-\overline{\Sigma}_F(p) & 0\\ 0 & \not p-M-\overline{\Sigma}_F^{☆}(p)\end{bmatrix}U_F^{-1}(p_0)\right]^{(ab)}\ . \tag{3.81}$$

Then the following relations among four components in the thermal 2×2 matrix of self-energy can be found by using the explicit form of U_F:[26,27]

$$\Sigma_F^{(12)}(p)=\Sigma_F^{(21)}(p)=-i\tan 2\theta_{p_0}\cdot\text{Im}\Sigma_F^{(11)}(p)\ , \tag{3.82}$$

$$\Sigma_F^{(22)}(p)=\Sigma_F^{(11)☆}(p) \tag{3.83}$$

and

$$\text{Re}\overline{\Sigma}_F(p)=\text{Re}\Sigma_F^{(11)}(p)\ , \tag{3.84}$$

$$\text{Im}\overline{\Sigma}_F(p)=\text{Im}\Sigma_F^{(11)}(p)/\cos 2\theta_{p_0}\ . \tag{3.85}$$

For the σ meson, the Dyson's equation is

$$\Delta^{(ab)}(p)=\Delta^{0(ab)}(p)+\sum_{cd}\Delta^{0(ac)}(p)\Sigma_s^{(cd)}(p)\Delta^{(db)}(p)\ , \tag{3.86}$$

where $\Sigma_s^{(ab)}(p)$ is the proper self-energy. This is also represented with a complex function $\overline{\Sigma}_s(p)$ as

$$\Sigma_s^{(ab)}(p)=\left[U_B^{-1}(p_0)\begin{bmatrix}\overline{\Sigma}_s(p) & 0\\ 0 & -\overline{\Sigma}_s^*(p)\end{bmatrix}U_B^{-1}(p_0)\right]^{(ab)}\ . \tag{3.87}$$

Then the inverse of the propagator is expressed as

$$[\Delta^{(ab)}(p)]^{-1}=\left[U_B^{-1}(p_0)\begin{bmatrix}p^2-m_s^2-\overline{\Sigma}_s(p) & 0\\ 0 & -p^2+m_s^2+\overline{\Sigma}_s^*(p)\end{bmatrix}U_B^{-1}(p_0)\right]^{(ab)}\ . \tag{3.88}$$

Here, $U_B^{-1}=\tau U_B\tau$ with

$$\tau=\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}.$$

The relations among four components in the self-energy matrix are

$$\Sigma_s^{(12)}(p)=\Sigma_s^{(21)}(p)=-i\tanh2\varphi_{p_0}\cdot\mathrm{Im}\Sigma_s^{(11)}(p)\ , \tag{3.89}$$

$$\Sigma_s^{(22)}(p)=-\Sigma_s^{(11)*}(p) \tag{3.90}$$

and

$$\mathrm{Re}\overline{\Sigma}_s(p)=\mathrm{Re}\Sigma_s^{(11)}(p)\ , \tag{3.91}$$

$$\mathrm{Im}\overline{\Sigma}_s(p)=\mathrm{Im}\Sigma_s^{(11)}(p)/\cosh2\varphi_{p_0}\ . \tag{3.92}$$

For the ω meson, the Dyson's equation is

$$D^{(ab)}(p)=D^{0(ab)}(p)+\sum_{cd}D^{0(ac)}(p)\Sigma_v^{(cd)}(p)D^{(db)}(p)\ , \tag{3.93}$$

where $\Sigma_v^{(ab)}(p)$ is the proper self-energy. We here abbreviate the Lorentz index for simplicity. The self-energy is given with a complex function $\overline{\Sigma}_v(p)$ as

$$\Sigma_v^{(ab)}(p)=\left[U_B^{-1}(p_0)\begin{bmatrix}\overline{\Sigma}_v(p) & 0\\ 0 & -\overline{\Sigma}_v^*(p)\end{bmatrix}U_B^{-1}(p_0)\right]^{(ab)}, \tag{3.94}$$

and the inverse of the propagator is

$$[D_{\mu\nu}^{(ab)}(p)]^{-1}=\left[U_B^{-1}(p_0)\begin{bmatrix}\zeta(p)-\overline{\Sigma}_v(p) & 0\\ 0 & -\zeta(p)+\overline{\Sigma}_v^*(p)\end{bmatrix}_{\mu\nu}U_B^{-1}(p_0)\right]^{(ab)}, \tag{3.95}$$

where

$$\zeta_{\mu\nu}(p)=-(p^2-m_v^2)g_{\mu\nu}+p_\mu p_\nu\ .$$

The relations among four components in the self-energy matrix are

$$\Sigma_{v\,\mu\nu}^{(12)}(p)=\Sigma_{v\,\mu\nu}^{(21)}(p)$$

$$=-i\tanh2\varphi_{p_0}\cdot\mathrm{Im}\Sigma_{v\,\mu\nu}^{(11)}(p)\ , \tag{3.96}$$

$$\Sigma_{v\,\mu\nu}^{(22)}(p)=-\Sigma_{v\,\mu\nu}^{(11)*}(p) \tag{3.97}$$

and

$$\mathrm{Re}\overline{\Sigma}_{v\,\mu\nu}(p)=\mathrm{Re}\Sigma_{v\,\mu\nu}^{(11)}(p)\ , \tag{3.98}$$

$$\mathrm{Im}\overline{\Sigma}_{v\,\mu\nu}(p)=\mathrm{Im}\Sigma_{v\,\mu\nu}^{(11)}(p)/\cosh2\varphi_{p_0}\ . \tag{3.99}$$

Once one knows the real and imaginary parts of the (11) component in the self-energy matrix for each particle, the other components can be immediately evaluated by using the above relations.

E. Feynman rules

In this subsection, we summarize the Feynman rules for the present model. The rules for the nth order contribution are as follows (see Fig. 1 and cf. Ref. 5): (1) Draw all topologically distinct diagrams with two external propagator lines and n vertices connected by internal propagator lines. Do not include diagrams with com-

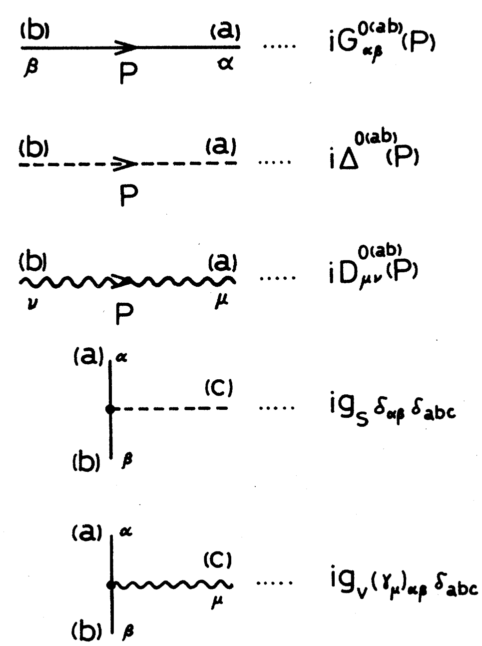

FIG. 1. Feynman rules. The nucleon propagator is the solid line, the σ meson propagator is the dashed line, and the ω meson propagator is the wiggly line.

$$\delta_{abc}=\begin{cases}1 & \text{for } a=b=c\ ,\\ 0 & \text{otherwise}\ .\end{cases}$$

pletely disconnected pieces. (2) Assign a direction to each line. Associate a directed four-momentum with each line and conserve energy and momentum at each vertex. (3) Each directed line gets a factor of $(i)\times$one of the propagators $\Delta^{0(ab)}$, $G^{0(ab)}_{\alpha\beta}$, or $D^{0(ab)}_{\mu\nu}$. (4) each vertex and propagator are assigned a factor and indices of thermal doublet as shown in Fig. 1. (5) Sum over all repeated indices. (6) Integrate $\int d^4q$ over all independent four-momenta. (7) Include a factor of (-1) for each closed fermion loop. (8) Any single fermion line forming a tadpole loop is interpreted as $i\times\exp(i\varepsilon_a \eta p_0)G^{0(ab)}_{\alpha\beta}(p)$ where $\eta\to+0$ at the end of calculation.

These rules are very similar to the usual ones at zero temperature.

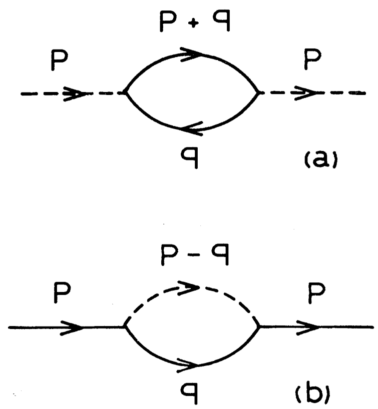

FIG. 2. Product rules: (a) Fermion-fermion bubble. (b) Fermion-boson bubble.

F. Product rules

In the RPA or Hartree-Fock calculation, one encounters the Feynman diagrams shown in Fig. 2. We therefore summarize the following formulas which are very useful in the calculations of these diagrams.[28] For the detailed derivation, see Appendix B.

1. Fermion-fermion product: Fig. 2(a)

The following quantity [the left-hand side of Eq. (3.100)] emerges in the diagram of the fermion bubble, and it can be rewritten as the right-hand side:

$$\int_{-\infty}^{+\infty} \bar{d}q_0 \left[U_F(p_0+q_0) \begin{bmatrix} \chi(w,p_0+q_0) & 0 \\ 0 & \chi^*(w,p_0+q_0) \end{bmatrix} U_F^{-1}(p_0+q_0) \right]^{(ab)} \times \left[U_F(q_0) \begin{bmatrix} \chi(w',q_0) & 0 \\ 0 & \chi^*(w',q_0) \end{bmatrix} U_F^{-1}(q_0) \right]^{(ab)} = i \left[U_B^{-1}(p_0) \begin{bmatrix} \sigma_{ff}(w,w',p_0) & 0 \\ 0 & -\sigma^*_{ff}(w,w',p_0) \end{bmatrix} U_B^{-1}(p_0) \right]^{(ab)} , \quad (3.100)$$

where

$$\sigma_{ff}(w,w',p_0) = \left[\frac{1}{1+e^{\beta(w'-\mu)}} - \frac{1}{1+e^{\beta(w-\mu)}} \right] \chi(w-w',p_0) \; . \quad (3.101)$$

2. Fermion-boson product: Fig. 2(b)

In the Hartree-Fock calculation, one encounters the following quantity [the left-hand side of Eq. (3.102)], and it can be expressed as the right-hand side:

$$\int_{-\infty}^{+\infty} \bar{d}q_0 \left[U_B(p_0-q_0) \begin{bmatrix} \chi(w,p_0-q_0) & 0 \\ 0 & -\chi^*(w,p_0-q_0) \end{bmatrix} U_B(p_0-q_0) \right]^{(ab)} \times \left[U_F(q_0) \begin{bmatrix} \chi(w',q_0) & 0 \\ 0 & \chi^*(w',q_0) \end{bmatrix} U_F^{-1}(q_0) \right]^{(ab)} = i \left[U_F(p_0) \begin{bmatrix} \sigma_{fb}(w,w',p_0) & 0 \\ 0 & \sigma^*_{fb}(w,w',p_0) \end{bmatrix} U_F^{-1}(p_0) \right]^{(ab)} , \quad (3.102)$$

where

$$\sigma_{fb}(w,w',p_0) = \left[\frac{1}{1+e^{\beta(w'-\mu)}} - \frac{1}{1-e^{-\beta w}} \right] \chi(w+w',p_0) \; . \quad (3.103)$$

IV. COLLECTIVE MODES IN EXTREME MATTER

In this section, we study collective modes[29,30] in the σ-ω model at finite temperature. The spectral representation for each propagator and the product rules are used to describe the dielectric function which characterizes collective modes arising from density fluctuations of nuclear or neutron matter. To study the propagation of the σ and ω mesons including their mixing, we here consider a set of ring diagrams (the RPA).

In our calculation, we use the relativistic Hartree propagator of the Dirac nucleon.[5,12] Hence, corresponding to M and E_p in the previous expressions, the effective mass M^* of the nucleon and its energy $E_p^*=(M^{*2}+\mathbf{p}^2)^{1/2}$ are introduced. In Figs. 3(a) and 3(b), we show the effective nucleon mass in the relativistic Hartree approximation with VF corrections. Here the Fermi momentum p_F is defined by $\rho_B=\gamma p_F^3/6\pi^2$, where ρ_B is the Baryon density, and γ is the degeneracy factor for spin and isospin of the Dirac nucleon, with $\gamma=2$ for neutron matter and $\gamma=4$ for nuclear matter. A monotonous decrease of the ratio M^*/M is found in Fig. 3(a), while a slight increase at low temperatures and a decrease at high temperatures are observed in Fig. 3(b). One can see that the ratio rapidly reduces in the region of $T=200-300$ MeV.

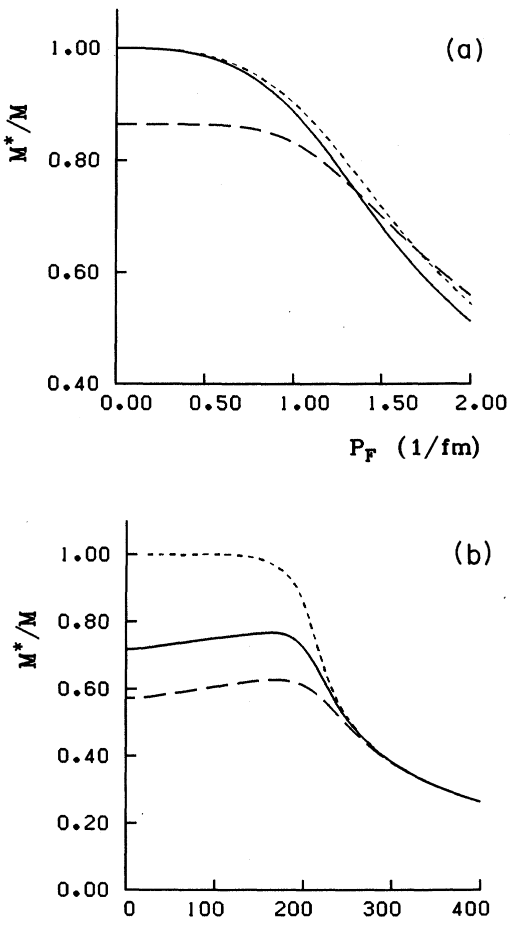

FIG. 3. (a) Density dependence of M^*. $T=0$ MeV is the solid line, $T=100$ MeV is the dotted line, $T=200$ MeV is the dashed line. (b) Temperature dependence of M^*. $p_F=0\ \mathrm{fm}^{-1}$ is the dotted line, $p_F=1.42\ \mathrm{fm}^{-1}$ is the solid line, $p_F=1.8\ \mathrm{fm}^{-1}$ is the dashed line.

In the following subsections (IV A–IV D), we study the dielectric function, the density-dependent parts of polarization functions, and the effects of VF corrections. Our numerical results are found in subsections IV E and IV F.

A. Dielectric function and collective modes

It is convenient to rewrite the interaction Lagrangian density (3.51) in the five-dimensional representation as[30]

$$\hat{\mathcal{L}}_{\rm int}=\sum_{a=1,2}\varepsilon_a P_a(\bar{\psi}^{(a)}\Lambda_\tau F^{(a)\tau}\psi^{(a)}+\delta\mathcal{L}^{(a)}) , \qquad (4.1)$$

where τ runs from 0 to 4,

$$\Lambda^\tau=\begin{cases}\gamma^\tau & \text{for } \tau=0\sim 3 ,\\ \mathbb{1} & \text{for } \tau=4 ,\end{cases} \qquad (4.2)$$

and

$$F_\tau^{(a)}=\begin{cases}-g_v V_\tau^{(a)} & \text{for } \tau=0\sim 3 ,\\ g_s\phi^{(a)} & \text{for } \tau=4 .\end{cases} \qquad (4.3)$$

The full meson propagator in the five-dimensional representation $\mathbf{C}_{\tau\lambda}^{(ab)}(q)$ consists of $D_{\mu\nu}^{(ab)}(q),\Delta^{(ab)}(q)$ and the mixing part $M_\mu^{(ab)}(q)$, where μ and ν run from 0 to 3:

$$\mathbf{C}_{\tau\lambda}^{(ab)}(q)=\begin{bmatrix}D^{(ab)}(q) & M^{(ab)}(q)\\ M^{(ab)}(q) & \Delta^{(ab)}(q)\end{bmatrix}_{\tau\lambda} . \qquad (4.4)$$

Here the mixing part $M_\mu^{(ab)}(q)$ can be written in the form of the thermal 2×2 matrix as

$$M_\mu^{(ab)}(q)=\left[U_B(q_0)\begin{bmatrix}\bar{M}(q) & 0\\ 0 & -\bar{M}^*(q)\end{bmatrix}_\mu U_B(q_0)\right]^{(ab)} , \qquad (4.5)$$

where

$$\bar{M}_\mu(q)=\int_{-\infty}^{+\infty}dw\,\rho_{M\mu}(w,|\mathbf{q}|)\chi(q_0,w) , \qquad (4.6)$$

with the real spectral function $\rho_{M\mu}(q)$. The free propagator in the five-dimensional representation $\mathbf{C}_{\tau\lambda}^{0(ab)}(q)$ is then defined by

$$\mathbf{C}_{\tau\lambda}^{0(ab)}(q)=\begin{bmatrix}D^{0(ab)}(q) & 0\\ 0 & \Delta^{0(ab)}(q)\end{bmatrix}_{\tau\lambda} . \qquad (4.7)$$

In both expressions of free and full propagators, we can factorize out the common matrix $U_B(q_0)$ and rewrite them in the form of the thermal 2×2 matrix. The full propagator becomes

$$\mathbf{C}_{\tau\lambda}^{(ab)}(q)=\left[U_B(q_0)\begin{bmatrix}\bar{\mathbf{C}}(q) & 0\\ 0 & -\bar{\mathbf{C}}^*(q)\end{bmatrix}_{\tau\lambda} U_B(q_0)\right]^{(ab)} , \qquad (4.8)$$

where

$$\bar{\mathbf{C}}_{\tau\lambda}(q)=\begin{bmatrix}\bar{D}(q) & \bar{M}(q)\\ \bar{M}(q) & \bar{\Delta}(q)\end{bmatrix}_{\tau\lambda}, \tag{4.9}$$

and the free propagator is

$$\mathbf{C}^{0(ab)}_{\tau\lambda}(q)=\left[U_B(q_0)\begin{bmatrix}\bar{\mathbf{C}}^0(q) & 0\\ 0 & -\bar{\mathbf{C}}^{0*}(q)\end{bmatrix}_{\tau\lambda}U_B(q_0)\right]^{(ab)}, \tag{4.10}$$

with

$$\bar{\mathbf{C}}^0_{\tau\lambda}(q)=\begin{bmatrix}-g_{\mu\nu}\bar{D}^0(q) & 0\\ 0 & \bar{\Delta}^0(q)\end{bmatrix}. \tag{4.11}$$

Further, we can also write the proper polarization function $\mathbf{\Pi}^{(ab)}_{\tau\lambda}(q)$ in the five-dimensional form by using the product rules [Eqs. (3.100) and (3.101)]:

$$\mathbf{\Pi}^{(ab)}_{\tau\lambda}(q)=\begin{bmatrix}\Pi^{(ab)}_{v}(q) & \Pi^{(ab)}_{M}(q)\\ \Pi^{(ab)}_{M}(q) & \Pi^{(ab)}_{s}(q)\end{bmatrix}_{\tau\lambda}, \tag{4.12}$$

where $\Pi^{(ab)}_{v\,\mu\nu}(q)$ and $\Pi^{(ab)}_{s}(q)$ are the ω and σ meson polarization functions, respectively, and $\Pi^{(ab)}_{M\mu}(q)$ represents their mixing process. These polarization functions are written in the form of the thermal 2×2 matrix with complex functions $\bar{\Pi}_{v\,\mu\nu}(q)$, $\bar{\Pi}_s(q)$, and $\bar{\Pi}_{M\mu}(q)$ as follows:

$$\Pi^{(ab)}_{v\,\mu\nu}(q)=\left[U_B^{-1}(q_0)\begin{bmatrix}\bar{\Pi}_v(q) & 0\\ 0 & -\bar{\Pi}^*_v(q)\end{bmatrix}_{\mu\nu}U_B^{-1}(q_0)\right]^{(ab)}, \tag{4.13}$$

$$\Pi^{(ab)}_{s}(q)=\left[U_B^{-1}(q_0)\begin{bmatrix}\bar{\Pi}_s(q) & 0\\ 0 & -\bar{\Pi}^*_s(q)\end{bmatrix}U_B^{-1}(q_0)\right]^{(ab)}, \tag{4.14}$$

$$\Pi^{(ab)}_{M\mu}(q)=\left[U_B^{-1}(q_0)\begin{bmatrix}\bar{\Pi}_M(q) & 0\\ 0 & -\bar{\Pi}^*_M(q)\end{bmatrix}_{\mu}U_B^{-1}(q_0)\right]^{(ab)}, \tag{4.15}$$

or

$$\mathbf{\Pi}^{(ab)}_{\tau\lambda}(q)=\left[U_B^{-1}(q_0)\begin{bmatrix}\bar{\mathbf{\Pi}}(q) & 0\\ 0 & -\bar{\mathbf{\Pi}}^*(q)\end{bmatrix}_{\tau\lambda}U_B^{-1}(q_0)\right]^{(ab)}, \tag{4.16}$$

where

$$\bar{\mathbf{\Pi}}_{\tau\lambda}(q)=\begin{bmatrix}\bar{\Pi}_v(q) & \bar{\Pi}_M(q)\\ \bar{\Pi}_M(q) & \bar{\Pi}_s(q)\end{bmatrix}_{\tau\lambda}. \tag{4.17}$$

The components of Eq. (4.12) are

$$\Pi^{(ab)}_{v\,\mu\nu}(q)=-ig_v^2\int d^4p\ \mathrm{Tr}[\gamma_\mu G^{(ba)}(p)\gamma_\nu G^{(ab)}(p+q)], \tag{4.18}$$

$$\Pi^{(ab)}_{s}(q)=-ig_s^2\int d^4p\ \mathrm{Tr}[G^{(ba)}(p)G^{(ab)}(p+q)], \tag{4.19}$$

$$\Pi^{(ab)}_{M\mu}(q)=ig_sg_v\int d^4p\ \mathrm{Tr}[\gamma_\mu G^{(ba)}(p)G^{(ab)}(p+q)], \tag{4.20}$$

or the components of Eq. (4.17) are

$$\bar{\Pi}_{v\,\mu\nu}(q)=g_v^2\int d^3p\int dw\int dw'\,\mathrm{Tr}[\gamma_\mu\rho(w',\mathbf{p})\gamma_\nu\rho(w,\mathbf{p}+\mathbf{q})]\sigma_{ff}(w,w',q_0), \tag{4.21}$$

$$\bar{\Pi}_{s}(q)=g_s^2\int d^3p\int dw\int dw'\,\mathrm{Tr}[\rho(w',\mathbf{p})\rho(w,\mathbf{p}+\mathbf{q})]\sigma_{ff}(w,w',q_0), \tag{4.22}$$

$$\bar{\Pi}_{M\mu}(q)=-g_vg_s\int d^3p\int dw\int dw'\,\mathrm{Tr}[\gamma_\mu\rho(w',\mathbf{p})\rho(w,\mathbf{p}+\mathbf{q})]\sigma_{ff}(w,w',q_0), \tag{4.23}$$

where ρ is the spectral function of the Dirac propagator Eq. (3.60). The expressions (4.21), (4.22), and (4.23) give us the formal representation of the polarization functions in the RPA.

The Dyson's equation for the full propagator $\mathbf{C}^{(ab)}_{\tau\lambda}(q)$ is given by

$$\mathbf{C}^{(ab)}(q)=\mathbf{C}^{0(ab)}(q)+\sum_{cd}\mathbf{C}^{0(ac)}(q)\mathbf{\Pi}^{(cd)}(q)\mathbf{C}^{(db)}(q). \tag{4.24}$$

In order to study the meson propagation in matter, we define the following dielectric function $\varepsilon(q)$ from Eq. (4.24):[20,29]

$$\varepsilon(q)=\det[\varepsilon^\lambda_\tau(q)], \tag{4.25}$$

with

$$\varepsilon^\lambda_\tau(q)=\delta^\lambda_\tau-\bar{\mathbf{C}}^0_{\tau\rho}(q)\bar{\mathbf{\Pi}}^{\rho\lambda}(q). \tag{4.26}$$

Poles of the full propagator or zero-points of the dielectric function indicate collective modes of the system. In general, as pointed out by Chin,[30] one finds the meson branch which consists of two modes, i.e., the longitudinal and transverse modes in the timelike region and the longitudinal collective mode as zero sound in the spacelike region. The former two modes become the free meson propagation when the density approaches zero. The latter mode, i.e., the zero sound, in general, may not be observed in the system where attractive force dominates.[20,29,30]

B. Real parts of density-dependent polarization functions

We give the real parts of density-dependent polarization functions by using the relativistic Hartree nucleon propagator in Eqs. (4.18), (4.19), and (4.20). The real parts in which at least one factor of G^0_D in Eq. (3.23) is included become finite:

$$\mathrm{Re}\Pi_{DL}^{(11)}(q)\equiv\mathrm{Re}[\Pi_D^{(11)33}(q)-\Pi_D^{(11)00}(q)] \tag{4.27}$$

$$=-16\gamma g_v^2\int d^3p\frac{\mathcal{F}_+(E_p^*)}{2E_p^*}\frac{q^2(E_p^{*2}-|\mathbf{p}|^2\cos^2\theta)}{q^4-4(p\cdot q)^2}\bigg|_{p_0=E_p^*} \tag{4.28}$$

$$=-\frac{g_v^2}{2\pi^2}\frac{q^2}{|\mathbf{q}|^3}\gamma\int_0^\infty dp\,p\frac{\mathcal{F}_+(E_p^*)}{E_p^*}[|\mathbf{p}||\mathbf{q}|-(E_p^{*2}-q_0E_p^*+q^2/4)L_1(p,q)/2$$

$$-(E_p^{*2}+q_0E_p^*+q^2/4)L_2(p,q)/2]\;, \tag{4.29}$$

$$\mathrm{Re}\Pi_{DT}^{(11)}(q)\equiv\mathrm{Re}\Pi_D^{(11)11}(q)=\mathrm{Re}\Pi_D^{(11)22}(q) \tag{4.30}$$

$$=-16\gamma g_v^2\int d^3p\frac{\mathcal{F}_+(E_p^*)}{2E_p^*}\frac{(p\cdot q)^2-q^2|\mathbf{p}|^2(1-\cos^2\theta)/2}{q^4-4(p\cdot q)^2}\bigg|_{p_0=E_p^*} \tag{4.31}$$

$$=\frac{g_v^2}{2\pi^2}\frac{\gamma}{|\mathbf{q}|^3}\int_0^\infty dp\,p\frac{\mathcal{F}_+(E_p^*)}{E_p^*}\{|\mathbf{q}|^3|\mathbf{p}|+|\mathbf{q}||\mathbf{p}|q^2/2-[q^2(E_p^*-q_0/2)^2+M^{*2}|\mathbf{q}|^2+q^2|\mathbf{q}|^2/4]L_1(p;q)/4$$

$$-[q^2(E_p^*+q_0/2)^2+M^{*2}|\mathbf{q}|^2+q^2|\mathbf{q}|^2/4]L_2(p,q)/4\}\;, \tag{4.32}$$

$$\mathrm{Re}\Pi_{DM}^{(11)0}(q)=-16\gamma M^*g_sg_v\int d^3p\frac{\mathcal{F}_-(E_p^*)}{2E_p^*}\frac{q^2E_p^*-q_0(p\cdot q)}{q^4-4(p\cdot q)^2}\bigg|_{p_0=E_p^*} \tag{4.33}$$

$$=-\frac{M^*g_sg_v}{4\pi^2}\frac{\gamma}{|\mathbf{q}|}\int_0^\infty dp\,p\frac{\mathcal{F}_-(E_p^*)}{E_p^*}[(E_p^*-q_0/2)L_1(p,q)+(E_p^*+q_0/2)L_2(p,q)]\;, \tag{4.34}$$

$$\mathrm{Re}\Pi_{DS}^{(11)}(q)=16\gamma g_s^2\int d^3p\frac{\mathcal{F}_+(E_p^*)}{2E_p^*}\frac{M^{*2}q^2-(p\cdot q)^2}{q^4-4(p\cdot q)^2}\bigg|_{p_0=E_p^*} \tag{4.35}$$

$$=\frac{g_s^2}{2\pi^2}\frac{\gamma}{|\mathbf{q}|}\int_0^\infty dp\,p\frac{\mathcal{F}_+(E_p^*)}{E_p^*}\{|\mathbf{p}||\mathbf{q}|-(q^2-4M^{*2})[L_1(p,q)+L_2(p,q)]/8\}\;, \tag{4.36}$$

where

$$\mathcal{F}_\pm(E)=f_+(E,\beta)\pm f_-(E,\beta)\;, \tag{4.37}$$

$$L_{\binom{1}{2}}(p,q)=\ln|(2q_0E_p^*\mp q^2\mp 2|\mathbf{p}||\mathbf{q}|)/(2q_0E_p^*\mp q^2\pm 2|\mathbf{p}||\mathbf{q}|)|\;. \tag{4.38}$$

Here we only show the (11) component, which is physical, of each polarization function. The subscript D means the density-dependent part. The current conservation gives us two independent modes, i.e., the longitudinal L and transverse T modes in the ω meson propagation. One finds the relation $q^0\Pi_M^{(11)0}=|\mathbf{q}|\Pi_M^{(11)3}$ in the mixing part when the direction of $\mathbf{q}$ is parallel with the z axis. In Eqs. (4.29), (4.32), (4.34), and (4.36), the integrals with respect to angular variables have been performed.

C. Imaginary parts of density-dependent polarization functions

The density-dependent imaginary parts of polarization functions are

$$\mathrm{Im}\Pi_{DL}^{(11)}(q)=\pi g_v^2\gamma\int d^3p\frac{E_p^{*2}-|\mathbf{p}|^2\cos^2\theta}{E_p^*E_{p+q}^*}\{F(E_p^*,E_{p+q}^*)[\delta(q_0+E_p^*-E_{p+q}^*)+\delta(q_0-E_p^*+E_{p+q}^*)]$$

$$+F(E_p^*,-E_{p+q}^*)[\delta(q_0+E_p^*+E_{p+q}^*)+\delta(q_0-E_p^*-E_{p+q}^*)]\}_{s=1}\;, \tag{4.39}$$

$$\mathrm{Im}\Pi_{DT}^{(11)}(q)=\frac{\pi}{2}g_v^2\gamma\int d^3p\frac{q^2/2-|\mathbf{p}|^2(1-\cos^2\theta)}{E_p^*E_{p+q}^*}\{F(E_p^*,E_{p+q}^*)[\delta(q_0+E_p^*-E_{p+q}^*)+\delta(q_0-E_p^*+E_{p+q}^*)]$$

$$+F(E_p^*,-E_{p+q}^*)[\delta(q_0+E_p^*+E_{p+q}^*)+\delta(q_0-E_p^*-E_{p+q}^*)]\}_{s=1}\;, \tag{4.40}$$

$$\mathrm{Im}\Pi_{DS}^{(11)}(q)=-\pi g_s^2(M^{*2}-q^2/4)\gamma\int d^3p\frac{1}{E_p^*E_{p+q}^*}\{F(E_p^*,E_{p+q}^*)[\delta(q_0+E_p^*-E_{p+q}^*)+\delta(q_0-E_p^*+E_{p+q}^*)]$$
$$+F(E_p^*,-E_{p+q}^*)[\delta(q_0+E_p^*+E_{p+q}^*)+\delta(q_0-E_p^*-E_{p+q}^*)]\}_{s=1}\ , \tag{4.41}$$

$$\mathrm{Im}\Pi_{DM}^{(11)0}(q)=\frac{\pi}{2}M^*g_sg_v\gamma\int d^3p\frac{1}{E_p^*E_{p+q}^*}\{F(E_p^*,E_{p+q}^*)(E_p^*+E_{p+q}^*)[\delta(q_0+E_p^*-E_{p+q}^*)+\delta(q_0-E_p^*+E_{p+q}^*)]$$
$$+F(E_p^*,-E_{p+q}^*)(E_p^*-E_{p+q}^*)[\delta(q_0+E_p^*+E_{p+q}^*)$$
$$+\delta(q_0-E_p^0-E_{p+q}^*)]\}_{s=-1}\ , \tag{4.42}$$

where

$$F(E,E')|_{s=\pm1}=\begin{cases}f_+(E,\beta)[1-f_+(E',\beta)]+sf_-(E,\beta)[1-f_-(E',\beta)] & \text{for } E'>0\ ,\\ f_+(E,\beta)[1-f_-(-E',\beta)]+sf_-(E,\beta)[1-f_+(-E',\beta)] & \text{for } E'<0\ .\end{cases} \tag{4.43}$$

For the meson branch, we are interested in the timelike region. Hence we consider only one δ function $\delta(q_0-E_p^*-E_{q+p}^*)$ when $q_0\geq 0$. On the other hand, for acoustic sound, the collective mode propagates in the spacelike region. In this region, we may take account of either $\delta(q_0-E_p^*+E_{q+p}^*)$ or $\delta(q_0+E_p^*-E_{q+p}^*)$. Then we obtain the imaginary parts in the timelike region:

$$\mathrm{Im}\Pi_{DL}^{(11)}(q)=-\frac{q^2\gamma}{4\pi|\mathbf{q}|^3}g_v^2\int_{z_-}^{z_+}dE_p^*[(E_p^*-q_0/2)^2-|\mathbf{q}|^2/4]F(E_p^*,E_p^*-q_0)|_{s=1}\ , \tag{4.44}$$

$$\mathrm{Im}\Pi_{DT}^{(11)}(q)=\frac{\gamma g_v^2}{8\pi|\mathbf{q}|^3}\int_{z_-}^{z_+}dE_p^*[q^2(E_p^*-q_0/2)^2+|\mathbf{q}|^2(M^{*2}+q^2/4)]F(E_p^*,E_p^*-q_0)|_{s=1}\ , \tag{4.45}$$

$$\mathrm{Im}\Pi_{DS}^{(11)}(q)=-\frac{\gamma g_s^2}{4\pi|\mathbf{q}|}(M^{*2}-q^2/4)\int_{z_-}^{z_+}dE_p^*F(E_p^*,E_p^*-q_0)|_{s=1}\ , \tag{4.46}$$

$$\mathrm{Im}\Pi_{DM}^{(11)0}(q)=\frac{M^*\gamma g_sg_v}{8\pi|\mathbf{q}|}\int_{z_-}^{z_+}dE_p^*(2E_p^*-q_0)F(E_p^*,E_p^*-q_0)|_{s=-1}\ , \tag{4.47}$$

and the imaginary parts in the spacelike region:

$$\mathrm{Im}\Pi_{DL}^{(11)}(q)=-\frac{\gamma g_v^2q^2}{4\pi|\mathbf{q}|^3}\int_{|z_-|}^{\infty}dE_p^*[(E_p^*+q_0/2)^2-|\mathbf{q}|^2/4][F(E_p^*,E_p^*+q_0)+F(E_p^*+q_0,E_p^*)]|_{s=1}\ , \tag{4.48}$$

$$\mathrm{Im}\Pi_{DT}^{(11)}(q)=\frac{\gamma g_v^2}{8\pi|\mathbf{q}|}\int_{|z_-|}^{\infty}dE_p^*[q^2(E_p^*+q_0/2)^2/|\mathbf{q}|^2+q^2/4+M^{*2}][F(E_p^*,E_p^*+q_0)+F(E_p^*+q_0,E_p^*)]|_{s=1}\ , \tag{4.49}$$

$$\mathrm{Im}\Pi_{DS}^{(11)}(q)=-\frac{\gamma g_s^2}{4\pi|\mathbf{q}|}(M^{*2}-q^2/4)\int_{|z_-|}^{\infty}dE_p^*[F(E_p^*,E_p^*+q_0)+F(E_p^*+q_0,E_p^*)]|_{s=1}\ , \tag{4.50}$$

$$\mathrm{Im}\Pi_{DM}^{(11)0}(q)=\frac{M^*\gamma g_sg_v}{8\pi|\mathbf{q}|}\int_{|z_-|}^{\infty}dE_p^*(2E_p^*+q_0)[F(E_p^*,E_p^*+q_0)+F(E_p^*+q_0,E_p^*)]|_{s=-1}\ , \tag{4.51}$$

where

$$z_\pm=\tfrac{1}{2}(q_0\pm|\mathbf{q}|\sqrt{1-4M^{*2}/q^2})\quad\text{for } q^2\geq 4M^{*2}\ . \tag{4.52}$$

The imaginary parts vanish in the region where the condition $q^2\geq 4M^{*2}$ is not satisfied.

D. Vacuum fluctuation effects

The effects of VF corrections in the RPA can be calculated by the method of the dimensional regularization.[31] To evaluate the effects, we add the following counterterms to the Lagrangian density:

$$\delta\mathcal{L}^{(a)}=\tfrac{1}{2}\xi_s(\partial_\mu\phi^{(a)})^2+\sum_{n=1}^{4}\frac{\alpha_n}{n!}\phi^{(a)n}+\tfrac{1}{4}\xi_v(F_{\mu\nu}^{(a)})^2\ , \tag{4.53}$$

where the first two terms denote the renormalizations of the wave function and the fermion loop for the σ meson, and the last term represents the renormalization of the wave function for the ω meson. Here the coefficients α in Eq. (4.53) are the same as those of Chin.[30]

The Feynman parts of polarization functions are given by

$$\Pi_{FS}^{(11)}(q)=-ig_s^2\int d^n p\frac{\mathrm{Tr}[(\not p+M^*)(\not p+\not q+M^*)]}{(p^2-M^{*2}+i\varepsilon)[(p+q)^2-M^{*2}+i\varepsilon]}\ , \tag{4.54}$$

for the σ meson and

$$\Pi_{Fv\,\mu\nu}^{(11)}(q)=-ig_v^2\int d^n p\frac{\mathrm{Tr}[(\not p+M^*)\gamma_\mu(\not p+\not q+M^*)\gamma_\nu]}{(p^2-M^{*2}+i\varepsilon)[(p+q)^2-M^{*2}+i\varepsilon]}\ , \tag{4.55}$$

for the ω meson. Thus we obtain

$$\xi_s=-\frac{g_s^2}{12\pi^2}[3\Gamma(-1+\varepsilon/2)+5+3\ln M^2]\ , \tag{4.56}$$

$$\xi_v=\frac{g_v^2}{6\pi^2}[\Gamma(-1+\varepsilon/2)+1+\ln M^2]\ , \tag{4.57}$$

where $\varepsilon\to+0$. Consequently, we obtain the renormalized Feynman parts as

$$\Pi_{FS}^{R(11)}(q)=-\frac{3g_s^2}{2\pi^2}\left[\int_0^1 dx[M^{*2}-x(1-x)q^2]\ln[(M^{*2}-x(1-x)q^2)/M^2]-M^2+4MM^*-3M^{*2}+q^2/6\right]\ , \tag{4.58}$$

$$\Pi_{Fv\,\mu\nu}^{R(11)}(q)=-\frac{g_v^2}{\pi^2}(g_{\mu\nu}q^2-q_\mu q_\nu)\int_0^1 dx\,x(1-x)\ln[(M^{*2}-x(1-x)q^2)/M^2]\ . \tag{4.59}$$

E. Dielectric function and dispersion relations

From Eq. (4.26), the dielectric tensor $\varepsilon_\tau^\lambda(q)$ is written as follows:

$$\varepsilon_\tau^\lambda(q)=\begin{bmatrix} 1+\bar D^0(q)\bar\Pi_v^{00}(q) & 0 & 0 & \bar D^0(q)\bar\Pi_v^{03}(q) & \bar D^0(q)\bar\Pi_M^0(q) \\ 0 & 1-\bar D^0(q)\bar\Pi_v^{11}(q) & 0 & 0 & 0 \\ 0 & 0 & 1-\bar D^0(q)\bar\Pi_v^{22}(q) & 0 & 0 \\ -\bar D^0(q)\bar\Pi_v^{30}(q) & 0 & 0 & 1-\bar D^0(q)\bar\Pi_v^{33}(q) & -\bar D^0(q)\bar\Pi_M^3(q) \\ -\bar\Delta^0(q)\bar\Pi_M^0(q) & 0 & 0 & -\bar\Delta^0(q)\bar\Pi_M^3(q) & 1-\bar\Delta^0(q)\bar\Pi_s(q) \end{bmatrix}_\tau^\lambda\ , \tag{4.60}$$

Then the dielectric function $\varepsilon(q)$ given by Eq. (4.25) is

$$\varepsilon(q)=\varepsilon_L(q)\varepsilon_T^2(q)\ , \tag{4.61}$$

where

$$\varepsilon_L(q)=[1-\bar D^0(q)\bar\Pi_L(q)][1-\bar\Delta^0(q)\bar\Pi_s(q)]-\frac{q^2}{|\mathbf{q}|^2}\bar D^0(q)\bar\Delta^0(q)[\bar\Pi_M^0(q)]^2\ , \tag{4.62}$$

$$\varepsilon_T(q)=1-\bar D^0(q)\bar\Pi_T(q)\ . \tag{4.63}$$

One can study the collective modes by searching zero-points of the above dielectric function.

In our calculation, we used the same coupling constants and meson masses as those in the relativistic Hartree calculation of Ref. 30: $g_s^2=62.89$, $g_v^2=79.78$, $m_s=550$ MeV, and $m_v=783$ MeV. In Figs. 4(a)–4(d), we show the dispersion relation of meson branch in nuclear matter in which relation the VF corrections are taken into account. In the momentum transfer regions of these figures, the polarization functions in Eq. (4.60) can be replaced by their real parts because q^2 is below the threshold for decay of collective modes, i.e., Eq. (4.52). Since results for neutron matter are very similar to those for nuclear matter, we do not show them here.

The temperature and density dependence of the dispersion relation is shown in Figs. 4(a)–4(c). We find that the difference between the longitudinal and transverse modes of the ω meson is very small, which in fact is quite different from the case of photon propagation in solids.[32] This is due to the fact that the bare ω meson mass is considerably large. Higher densities yield the larger difference between the two modes. The comparison between the results in which M^* and M are used is illustrated in Fig. 4(c), where $T=0$ MeV and $p_F=1.42$ fm^{-1} (normal nuclear density). The M^* largely reduces the collective modes. In Fig. 4(d), we present the result in which the effects of VF corrections, i.e., Eqs. (4.58) and (4.59), are excluded. The collective modes are very largely softened, namely, values of q_0 in each mode are pushed down, by taking account of the VF effects.

F. Masses of the σ and ω mesons at finite temperature

It is very interesting to study the behavior of the mass of mesons embedded in hot and dense matter because measurements of the invariant meson mass will give us important information on extreme states produced by energetic heavy-ion collisions.[1] Using the present model, we can discuss the σ and ω meson masses in hadronic matter at finite temperature and density.

Now let us define the effective meson mass m_i^* ($i = s$ or v) as the energy of the meson at rest. In other words, the m_i^* is the energy that is needed to create one corresponding meson at rest in the medium. In the rest frame, the longitudinal and transverse modes degenerate, and the branches of the σ and ω mesons completely separate each other because the mixing parts of polarization functions vanish. Then, the effective meson masses are obtained from

$$D_v(q_0)|_{q_0=m_v^*} \equiv q_0^2 - m_v^2 - \overline{\Pi}_L(q)|_{|\mathbf{q}|=0,q_0=m_v^*} = 0 , \quad (4.64)$$

$$D_s(q_0)|_{q_0=m_s^*} \equiv q_0^2 - m_s^2 - \overline{\Pi}_s(q)|_{|\mathbf{q}|=0,q_0=m_s^*} = 0 , \quad (4.65)$$

where the real parts of polarization functions are

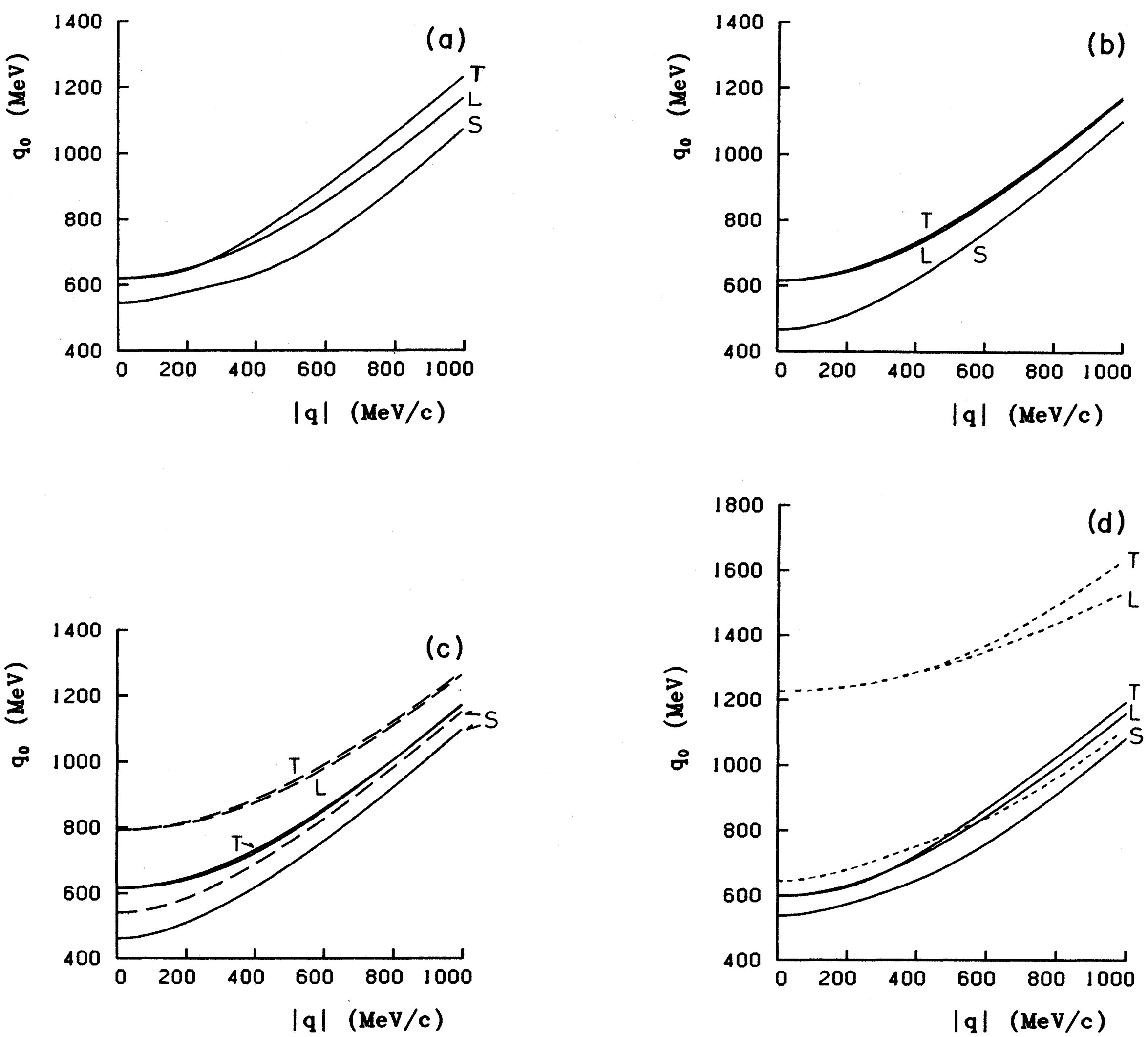

FIG. 4. (a) Dispersion relations, with $T=0$ MeV and $p_F=1.8$ fm^{-1}. L is the longitudinal mode of the ω meson, T is the transverse mode of the ω meson, and S is the σ meson mode. (b) Same as (a): $T=100$ MeV and $p_F=1.42$ fm^{-1}. (c) Effects of M^* on dispersion relations with $T=0$ MeV and $p_F=1.42$ fm^{-1}. The results of M and M^* are denoted by dashed and solid lines respectively. (d) Effects of vacuum fluctuation on dispersion relations with $T=100$ MeV and $p_F=1.8$ fm^{-1}. The dotted lines show the results where the VF corrections [Eqs. (4.58) and (4.59)] are not taken into account. The solid lines represent the results with the VF corrections.

$$\mathrm{Re}\overline{\Pi}_L(q_0) \equiv \mathrm{Re}[\Pi^{(11)}_{DL}(q_0)+\Pi^{(11)}_{FL}(q_0)] = \mathrm{Re}[\Pi^{(11)}_{DT}(q_0)+\Pi^{(11)}_{FT}(q_0)]$$
$$= \frac{g_v^2}{6\pi^2}\gamma\int_{M^*}^{\infty} dE_p^* \mathcal{F}_+(E_p^*)\frac{(E_p^{*2}-M^{*2})^{1/2}}{E_p^*}(2E_p^{*2}+M^{*2})\left[\frac{1}{2E_p^*+q_0}+\frac{1}{2E_p^*-q_0}\right]$$
$$+\frac{q_0^2}{\pi^2}\int_{-1/2}^{1/2} dx\,(\tfrac{1}{4}-x^2)\ln(|M^{*2}-q_0^2/4+q_0^2x^2/4|/M^2)\ , \tag{4.66}$$

$$\mathrm{Re}\overline{\Pi}_s(q_0) \equiv \mathrm{Re}[\Pi^{(11)}_{DS}(q_0)+\Pi^{(11)}_{FS}(q_0)]$$
$$= \frac{g_s^2}{2\pi^2}\gamma\int_{M^*}^{\infty} dE_p^* \mathcal{F}_+(E_p^*)\frac{(E_p^{*2}-M^{*2})^{3/2}}{E_p^*}\left[\frac{1}{2E_p^*+q_0}+\frac{1}{2E_p^*-q_0}\right]$$
$$-\frac{3}{2\pi^2}\left[\int_{-1/2}^{1/2} dx\,(M^{*2}-q_0^2/4+q_0^2x^2/4)\ln(|M^{*2}-q_0^2/4+q_0^2x^2/4|/M^2)\right.$$
$$\left. -(M-M^*)(M-3M^*)+q_0^2/6\right] \tag{4.67}$$

and their imaginary parts are

$$\mathrm{Im}\overline{\Pi}_L(q_0) = \mathrm{Im}\overline{\Pi}_T(q_0)$$
$$= \begin{cases} \dfrac{g_v^2}{12\pi}\dfrac{(q_0^2/4-M^{*2})^{1/2}}{q_0\cosh 2\varphi_{p_0}}(q_0^2+2M^{*2})[\gamma F(q_0/2,-q_0/2)-4]|_{s=1} & \text{for } q_0 \ge 2M^*\ , \\ 0 \quad \text{for } 0 \le q_0 \le 2M^*\ , \end{cases} \tag{4.68}$$

$$\mathrm{Im}\overline{\Pi}_s(q_0) = \begin{cases} \dfrac{g_s^2}{2\pi}\dfrac{(q_0^2/4-M^{*2})^{3/2}}{q_0\cosh 2\varphi_{p_0}}[\gamma F(q_0/2,-q_0/2)-4]|_{s=1} & \text{for } q_0 \ge 2M^*\ , \\ 0 \quad \text{for } 0 \le q_0 \le 2M^*\ . \end{cases} \tag{4.69}$$

In Figs. 5(a)–5(j), we show the meson masses and widths. The VF effects are taken into account. The results for nuclear matter are presented in Figs. 5(a)–5(i), and Fig. 5(j) shows the meson masses in neutron matter.

The temperature and density dependence of the meson masses is presented in Figs. 5(a)–5(d). Figures 5(a) and 5(c) show the density dependence: the behavior of mass spectra at $T=200$ MeV in both figures differs considerably from that at temperatures below 100 MeV in the low density region. Figure 5(b) and 5(d) show the temperature dependence: we can see that both meson masses are almost degenerate at temperatures above 200 MeV and that some difference depending on densities exists at low temperatures. Furthermore, one can observe the cusp structure for the ω meson at $T=330$ MeV and the shoulder for the σ meson at around $T=300$ MeV. These characteristic structures are attributed to the threshold effect for the decay of mesons into a nucleon and antinucleon pair, namely, the imaginary parts of Eqs. (4.68) and (4.69) emerge at these temperatures. On the other hand, in Figs. 5(a) and 5(c), the kinematical conditions are below the threshold: $q_0 \le 2M^*$.

In Fig. 5(e), we show the width Γ_i ($i=s$ or v) of each meson. Here we do not consider the native widths of the σ and ω mesons. When $\Gamma_i \ll m_i^*$, Eqs. (4.64) and (4.65) can be expanded in the Taylor series about the real part of q_0, and hence one obtains, in the first order,[29]

$$\Gamma_i \simeq -\mathrm{Im}D_i(q_0)/(\partial D_i(q_0)/\partial q_0)|_{q_0=m_i^*}\ . \tag{4.70}$$

Here m_i^* is the real part of the solution of $D_i=0$, the real part of which may be approximated by the solution of

$$\mathrm{Re}D_i(q_0)|_{q_0=m_i^*}=0\ . \tag{4.71}$$

The width of the ω meson suddenly appears at $T=330$ MeV and rapidly increases, while that of the σ meson smoothly grows at temperatures above 300 MeV. These temperatures correspond to those at which the outstanding structures have been observed in the mass spectra. The difference between the two widths near the threshold points is due to that of the q_0 dependence between the imaginary parts of polarization functions given by Eqs. (4.68) and (4.69).

In Figs. 5(f) and 5(g), we show the comparison between the results where the effective nucleon mass and the free one have been used. Figure 5(f) gives the density dependence: the meson masses vary slowly. Figure 5(g) shows the temperature dependence: the relatively rich structures are observed. These are due to the characteristic behavior of the effective nucleon mass presented in Figs. 3(a) and 3(b): the shallow dip for the ω meson and the rapid increase for the σ meson at temperatures around 220 MeV are attributed to the steep decrease of the effective nucleon mass at those temperatures.

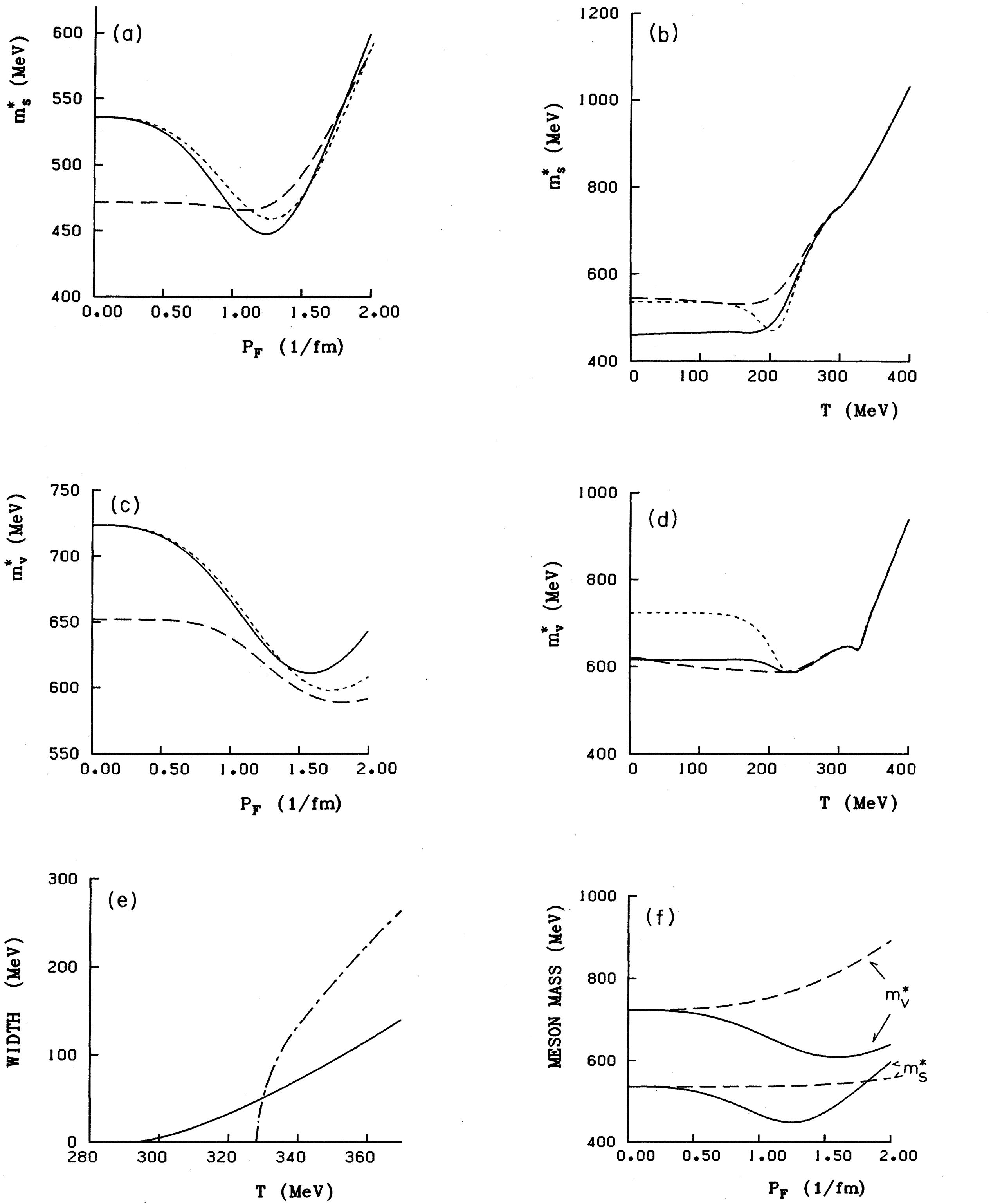

FIG. 5. (a) Density dependence of m_s^*. The curves are denoted as in Fig. 3(a). (b) Temperature dependence of m_s^*. The curves are denoted as in Fig. 3(b). (c) Same as (a) for m_v^*. (d) Same as (b) for m_v^*. (e) Widths of σ (solid) and ω (dot-dashed) mesons vs temperature. $p_F = 1.42$ fm^{-1}. (f) Effects of M^* on the meson masses (density dependence) with $T = 0$ MeV. The curves are denoted as in Fig. 4(c). (g) Same as (f) (temperature dependence) with $p_F = 1.42$ fm^{-1}. (h) Effects of vacuum fluctuation on the meson masses (density dependence) with $T = 0$ MeV. The curves are denoted as in Fig. 4(d). (i) Same as (h) (temperature dependence) with $p_F = 1.42$ fm^{-1}. (j) Meson masses in neutron matter as a function of density with $T = 0$ MeV. The σ meson is the solid line and the ω meson is the dot-dashed line.

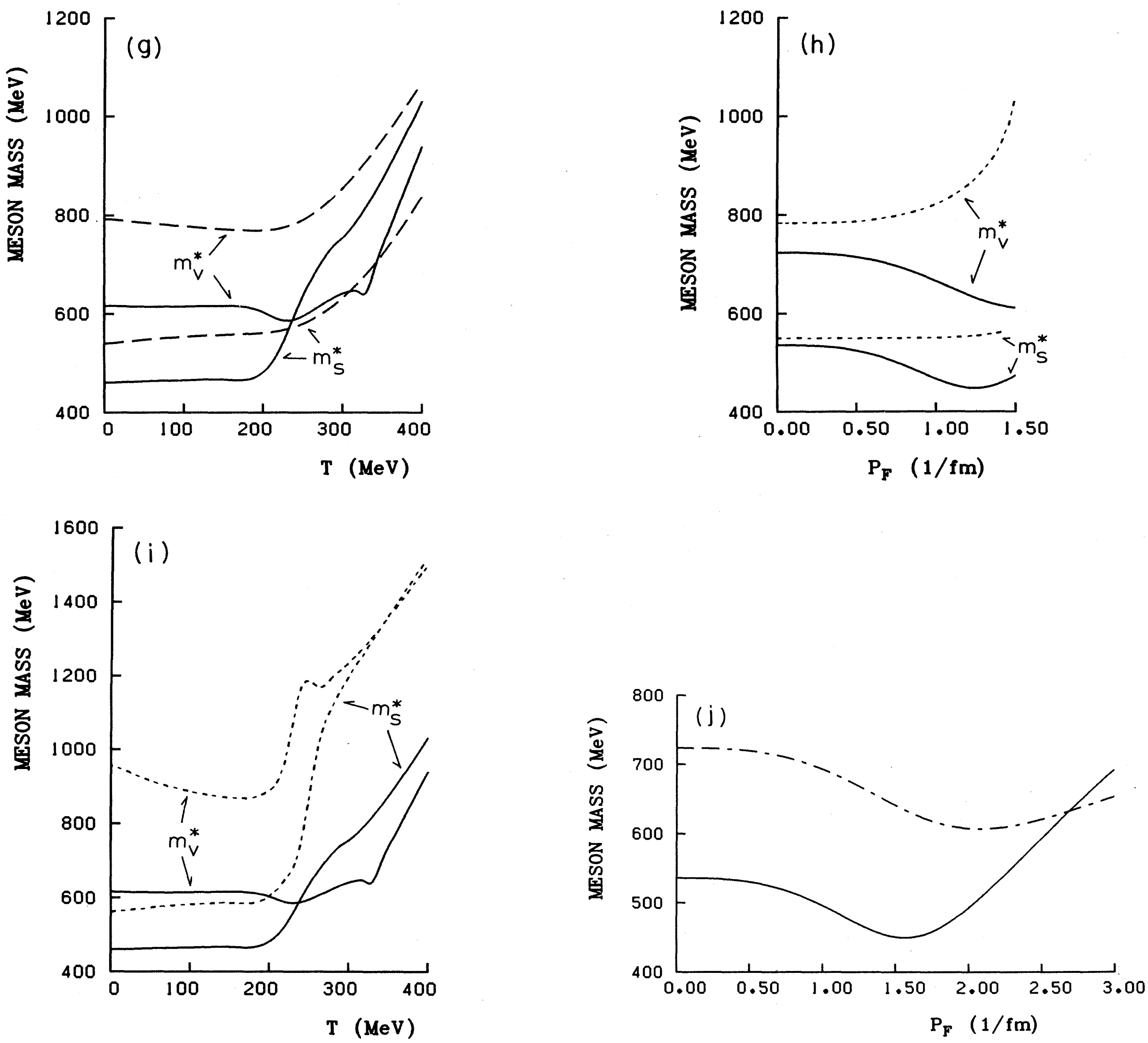

FIG. 5. (*Continued*).

The effects of VF corrections calculated from Eqs. (4.58) and (4.59) are illustrated in Figs. 5(h) and 5(i). They largely contribute to the spectra of meson masses. We can find from the figures that the real parts of density-dependent polarization functions enhance the meson masses while those of VF corrections reduce them. We show the density dependence in Fig. 5(h) up to $p_F = 1.5\ \text{fm}^{-1}$. This is because the numerical calculation is difficult at high densities.

Finally, in Fig. 5(j), we show the spectra of meson masses in dense neutron matter at zero temperature. The spectra change very little within the region where temperatures are less than some tens of MeV. The shapes of the mass spectra are very similar to those of Figs. 5(a) and 5(c), but the dips in them move toward higher densities.

V. SUMMARY AND REMARKS

In the first half of this paper, we have constructed the σ-ω model at finite temperature and density in terms of thermo field dynamics; the expressions for the Green's functions, the Feynman rules, and some useful formulae have been presented. In the latter half, we have studied the dispersion relations of collective modes and the mass spectra of the σ and ω mesons at rest in hot and dense matter within the RPA. Note that the Feynman diagram technique we used here is also applicable to calculations of various quantities for hadronic matter at any temperature and density.

Concerning the collective modes, we found the following for nuclear matter: (1) The dispersion relations of collective modes are largely affected by the effective nucleon mass. The vacuum fluctuation also contributes to them considerably. They soften the propagation of collective modes in a medium. (2) The longitudinal and transverse modes of the ω meson in the meson branch almost degenerate at low densities because the bare ω mass is large. Higher densities make the difference between the two modes larger. (3) We examined the mass spectra of the σ and ω mesons at rest in hot and dense matter. The effective nucleon mass and the vacuum fluctuation again play important roles in the mass spectra. At low temperatures and densities, the meson masses fairly depend on them. On the other hand, the meson masses increase monotonously at temperatures above 250 MeV. The density-dependent parts of polarization functions make the meson masses heavy, while the Feynman parts, i.e., the VF corrections, largely lighten them. From our results, we can conclude that meson mass in a hot and dense medium sensitively reflects its properties. (4) The behavior of the σ meson width near the threshold point is quite different from that of the ω meson width: the σ meson width grows smoothly, while the ω meson one suddenly appears and increases rapidly. Correspondingly, the characteristic threshold effect for each meson is clearly seen in the mass spectra.

For neutron matter we found that the dispersion relations of collective modes are very similar to those of nuclear matter. There exists some difference between the meson mass spectra in nuclear and neutron matter at zero temperature, a difference which depends on density. Both mass spectra will coincide as temperatures become higher.

Next, we give the following remarks: (1) The present method is applicable to the system where the degrees of freedom of quarks and gluons are explicitly included. We can study various properties of the quark-gluon plasma produced in ultrarelativistic heavy-ion collisions or in the early universe. At very high temperatures and densities, the present results should be compared with such studies. (2) We will be able to study the longitudinal acoustic sound mode, i.e., zero sound, in hot and dense matter. Such a mode, in general, cannot be observed in the system where attractive force dominates.[20] Hence one does not find zero sound in the normal nuclear matter at zero temperature. However, at finite temperatures and/or higher densities, there is a possibility to find zero sound.[29] At high temperatures, the polarization functions for the σ and ω mesons decouple each other, because the effective nucleon mass becomes very small and hence the mixing polarization function Π_M correspondingly vanishes. Then, zero sound will appear in the sector of the ω meson. At higher densities, one will also find zero sound because the ω meson field becomes dominant as compared with the σ meson field. In order to obtain a conclusive result of zero sound, we must do more careful studies of the polarization functions, in which the imaginary parts of polarization functions should be fully taken into account. (3) It is necessary to extend the present model to the chiral invariant one because the chiral symmetry becomes more significant as the temperature becomes higher. For this purpose, we must consider the most important ingredient: the π meson. It is interesting to study the propagation and mass spectrum of the π meson in hot and dense matter by taking account of the chiral symmetry.

APPENDIX A: SPECTRAL REPRESENTATION FOR PROPAGATORS

In this appendix, we derive the full propagators of the Dirac nucleon, the σ and ω mesons in the spectral representation.

1. Dirac nucleon propagator

From the definition of Eq. (3.54), the full propagator is written as

$$iG^{(ab)}_{\alpha\beta}(x_1-x_2)=\langle 0(\beta)|T\psi^{(a)}_\alpha(x_1)\bar{\psi}^{(b)}_\beta(x_2)|0(\beta)\rangle \tag{A1}$$

$$=\theta(x^0_1-x^0_2)\langle 0(\beta)|e^{i\hat{P}\cdot x_1}\psi^{(a)}_\alpha(0)e^{-i\hat{P}\cdot(x_1-x_2)}\bar{\psi}^{(b)}_\beta(0)e^{-i\hat{P}\cdot x_2}|0(\beta)\rangle$$
$$-\theta(x^0_2-x^0_1)\langle 0(\beta)|e^{i\hat{P}\cdot x_2}\bar{\psi}^{(b)}_\beta(0)e^{-i\hat{P}\cdot(x_2-x_1)}\psi^{(a)}_\alpha(0)e^{-i\hat{P}\cdot x_1}|0(\beta)\rangle\ , \tag{A2}$$

where $\hat{P}_\mu$ is the generator of the space-time translation [see Eq. (2.10)], and

$$\hat{P}|0(\beta)\rangle=0\ . \tag{A3}$$

We use the following completeness relation:

$$\sum_{nn'mm'}|nn',\tilde{m}\,\tilde{m}'\rangle\langle\tilde{m}'\tilde{m},n'n|=1\ , \tag{A4}$$

where $|nn',\tilde{m}\,\tilde{m}'\rangle$ means the state vector given by the operator of $[c^\dagger_{ps}(\beta)]^n$, $[d^\dagger_{ps}(\beta)]^{n'}$, $[\tilde{c}^\dagger_{ps}(\beta)]^m$, and $[\tilde{d}^\dagger_{ps}(\beta)]^{m'}$ on the physical vacuum $|0(\beta)\rangle$. Then Eq. (A2) is rewritten as

$$iG^{(ab)}_{\alpha\beta}(x_1-x_2)=\theta(x_1^0-x_2^0)\sum_{\substack{nn'mm'\\kk'll'}}\langle 0(\beta)|\psi^{(a)}_\alpha(0)|nn',\tilde{m}\,\tilde{m}'\rangle\langle\tilde{m}'\tilde{m},n'n|e^{-i\hat{P}\cdot(x_1-x_2)}|kk',\tilde{l}\,\tilde{l}'\rangle$$
$$\times\langle\tilde{l}'\tilde{l},k'k|\bar{\psi}^{(b)}_\beta(0)|0(\beta)\rangle$$
$$-\theta(x_2^0-x_1^0)\sum_{\substack{nn',mm'\\kk',ll'}}\langle 0(\beta)|\bar{\psi}^{(b)}_\beta(0)|nn',\tilde{m}\,\tilde{m}'\rangle\langle\tilde{m}'\tilde{m},n'n|e^{-i\hat{P}\cdot(x_2-x_1)}|kk',\tilde{l}\,\tilde{l}'\rangle$$
$$\times\langle\tilde{l}'\tilde{l},k'k|\psi^{(a)}_\alpha(0)|0(\beta)\rangle \tag{A5}$$
$$=\theta(x_1^0-x_2^0)\sum_{NM}e^{-i(p_N-p_M)\cdot(x_1-x_2)}\langle 0(\beta)|\psi^{(a)}_\alpha(0)|N,\tilde{M}\rangle\langle\tilde{M},N|\bar{\psi}^{(b)}_\beta(0)|0(\beta)\rangle$$
$$-\theta(x_2^0-x_1^0)\sum_{NM}e^{-i(p_N-p_M)\cdot(x_2-x_1)}\langle 0(\beta)|\bar{\psi}^{(b)}_\beta(0)|N,\tilde{M}\rangle\langle\tilde{M},N|\psi^{(a)}_\alpha(0)|0(\beta)\rangle\ , \tag{A6}$$

where we shortened the indices of (nn') and (mm') to N and M respectively, and $P^\mu|N,\tilde{M}\rangle=p_N^\mu|N,\tilde{M}\rangle$ and $\tilde{P}^\mu|N,\tilde{M}\rangle=-p_M^\mu|N,\tilde{M}\rangle$. By using the integral representation of the step function in Eq. (A6), we obtain the propagator in momentum space

$$G^{(ab)}_{\alpha\beta}(p)=\int_{-\infty}^{+\infty}dw\left[\frac{\sigma^{(ab)}_{1\alpha\beta}(w,\mathbf{p})}{p_0-w+i\varepsilon}+\frac{\sigma^{(ab)}_{2\alpha\beta}(w,\mathbf{p})}{p_0-w-i\varepsilon}\right]\ , \tag{A7}$$

where

$$\sigma^{(ab)}_{1\alpha\beta}(p)=(2\pi)^3\sum_{NM}\langle 0(\beta)|\psi^{(a)}_\alpha(0)|N,\tilde{M}\rangle\langle\tilde{M},N|\bar{\psi}^{(b)}_\beta(0)|0(\beta)\rangle\delta^4(p-p_N+p_M)\ , \tag{A8}$$
$$\sigma^{(ab)}_{2\alpha\beta}(p)=(2\pi)^3\sum_{NM}\langle 0(\beta)|\bar{\psi}^{(b)}_\beta(0)|N,\tilde{M}\rangle\langle\tilde{M},N|\psi^{(a)}_\alpha(0)|0(\beta)\rangle\delta^4(-p-p_N+p_M)\ . \tag{A9}$$

From the thermal state condition Eq. (2.11), we find the following relations:

$$\langle 0(\beta)|\psi^{(2)}(0)|N,\tilde{M}\rangle=-e^{\beta\mu/2}e^{-\beta(E_N-E_M)/2}\langle 0(\beta)|\psi^{(1)}(0)|N,\tilde{M}\rangle\ , \tag{A10}$$
$$\langle 0(\beta)|\bar{\psi}^{(1)}(0)|N,\tilde{M}\rangle=-ie^{\beta\mu/2}e^{\beta(E_N-E_M)/2}\langle M,\tilde{N}|\bar{\psi}^{(1)}(0)|0(\beta)\rangle\ , \tag{A11}$$
$$\langle 0(\beta)|\bar{\psi}^{(2)}(0)|N,\tilde{M}\rangle=-i\langle M,\tilde{N}|\bar{\psi}^{(1)}(0)|0(\beta)\rangle\ , \tag{A12}$$
$$\langle\tilde{M},N|\bar{\psi}^{(2)}(0)|0(\beta)\rangle=-e^{\beta\mu/2}e^{-\beta(E_N-E_M)/2}\langle\tilde{M},N|\bar{\psi}^{(1)}(0)|0(\beta)\rangle\ , \tag{A13}$$
$$\langle\tilde{M},N|\psi^{(1)}(0)|0(\beta)\rangle=ie^{\beta\mu/2}e^{\beta(E_N-E_M)/2}\langle 0(\beta)|\psi^{(1)}(0)|\tilde{N},M\rangle\ , \tag{A14}$$
$$\langle\tilde{M},N|\psi^{(2)}(0)|0(\beta)\rangle=i\langle 0(\beta)|\psi^{(1)}(0)|\tilde{N},M\rangle\ . \tag{A15}$$

When we define the spectral function as

$$\rho_{\alpha\beta}(p)\equiv(2\pi)^3(1+e^{-\beta(p_0-\mu)})\sum_{NM}\delta^4(p-p_N+p_M)\langle 0(\beta)|\psi^{(1)}_\alpha(0)|N\tilde{M}\rangle\langle\tilde{M}N|\bar{\psi}^{(1)}_\beta(0)|0(\beta)\rangle\ , \tag{A16}$$

we can write

$$\sigma^{(ab)}_{1\alpha\beta}(p)=\rho_{\alpha\beta}(p)A_F^{(ab)}(p_0)\ , \tag{A17}$$
$$\sigma^{(ab)}_{2\alpha\beta}(p)=\rho_{\alpha\beta}(p)B_F^{(ab)}(p_0)\ . \tag{A18}$$

Here the matrices A_F and B_F are defined by Eqs. (3.58) and (3.59). We can hence obtain Eq. (3.57) from Eqs. (A7), (A17), and (A18).

Furthermore, if we define $\bar{G}(p)$ by Eq. (3.61) and use the identities

$$\theta(\pm p_0)\frac{1}{p_0-w+i\varepsilon}=\theta(\pm p_0)\left[\frac{\theta(w)}{p_0-w\pm i\varepsilon}+\frac{\theta(-w)}{p_0-w\mp i\varepsilon}\right]\ , \tag{A19}$$

$$\theta(\pm p_0)\frac{1}{p_0-w-i\varepsilon}=\theta(\pm p_0)\left[\frac{\theta(w)}{p_0-w\mp i\varepsilon}+\frac{\theta(-w)}{p_0-w\pm i\varepsilon}\right]\ , \tag{A20}$$

we find

$$G_{\alpha\beta}^{(ab)}(p)=\bar{G}_{\alpha\beta}(p)\begin{bmatrix} \cos^2\theta_{p_0} & -\frac{1}{2}\sin 2\theta_{p_0} \\ -\frac{1}{2}\sin 2\theta_{p_0} & \sin^2\theta_{p_0} \end{bmatrix}+\bar{G}_{\alpha\beta}^{\star}(p)\begin{bmatrix} \sin^2\theta_{p_0} & \frac{1}{2}\sin 2\theta_{p_0} \\ \frac{1}{2}\sin 2\theta_{p_0} & \cos^2\theta_{p_0} \end{bmatrix}, \tag{A21}$$

where θ_{p_0} is defined by Eqs. (3.26) and (3.27), and the realness of ρ, i.e., $\rho=\rho^{\star}$, has been used (see also Ref. 14). Then, from this expression, we can obtain Eq. (3.28).

2. σ meson propagator

The definition of the σ meson propagator is given by Eq. (3.65). When we use the completeness relation for the σ meson, we can find the propagator in momentum space

$$\Delta^{(ab)}(p)=\int_{-\infty}^{+\infty}dw\left[\frac{\sigma_{s1}^{(ab)}(w,\mathbf{p})}{p_0-w+i\varepsilon}-\frac{\sigma_{s2}^{(ab)}(w,\mathbf{p})}{p_0-w-i\varepsilon}\right], \tag{A22}$$

where

$$\sigma_{s1}^{(ab)}(p)\equiv(2\pi)^3\sum_{nm}\langle 0(\beta)|\phi^{(a)}(0)|n,\tilde{m}\rangle\langle\tilde{m},n|\phi^{(b)}(0)|0(\beta)\rangle\delta^4(p-p_n+p_m), \tag{A23}$$

$$\sigma_{s2}^{(ab)}(p)\equiv(2\pi)^3\sum_{nm}\langle 0(\beta)|\phi^{(b)}(0)|n,\tilde{m}\rangle\langle\tilde{m},n|\phi^{(a)}(0)|0(\beta)\rangle\delta^4(-p-p_n+p_m). \tag{A24}$$

Next, from the thermal state condition for the σ meson, the following relations are obtained:

$$\langle 0(\beta)|\phi^{(2)}(0)|n,\tilde{m}\rangle=e^{-\beta(E_n-E_m)/2}\langle 0(\beta)|\phi^{(1)}(0)|n,\tilde{m}\rangle, \tag{A25}$$

$$\langle 0(\beta)|\phi^{(1)}(0)|n,\tilde{m}\rangle=e^{\beta(E_n-E_m)/2}\langle m,\tilde{n}|\phi^{(1)}(0)|0(\beta)\rangle, \tag{A26}$$

$$\langle 0(\beta)|\phi^{(2)}(0)|n,\tilde{m}\rangle=\langle m,\tilde{n}|\phi^{(1)}(0)|0(\beta)\rangle, \tag{A27}$$

$$\langle\tilde{m},n|\phi^{(2)}(0)|0(\beta)\rangle=e^{-\beta(E_n-E_m)/2}\langle\tilde{m},n|\phi^{(1)}(0)|0(\beta)\rangle, \tag{A28}$$

$$\langle\tilde{m},n|\phi^{(1)}(0)|0(\beta)\rangle=e^{\beta(E_n-E_m)/2}\langle 0(\beta)|\phi^{(1)}(0)|\tilde{n},m\rangle, \tag{A29}$$

$$\langle\tilde{m},n|\phi^{(2)}(0)|0(\beta)\rangle=\langle 0(\beta)|\phi^{(1)}(0)|\tilde{n},m\rangle. \tag{A30}$$

By using these relations, we can write

$$\sigma_{s1}^{(ab)}(p)=\rho_s(p)A_B^{(ab)}(p_0), \tag{A31}$$

$$\sigma_{s2}^{(ab)}(p)=\rho_s(p)B_B^{(ab)}(p_0), \tag{A32}$$

where the matrices A_B and B_B are defined by Eqs. (3.68) and (3.69). The spectral function is

$$\rho_s(p)=(2\pi)^3(1-e^{-\beta p_0})\sum_{nm}\delta^4(p-p_n+p_m)|\langle 0(\beta)|\phi^{(1)}(0)|n,\tilde{m}\rangle|^2. \tag{A33}$$

Then, Eq. (3.67) is found. When we define $\bar{\Delta}(p)$ by Eq. (3.70) and φ_{p_0} by Eqs. (3.36) and (3.37), we obtain

$$\Delta^{(ab)}(p)=\bar{\Delta}(p)\begin{bmatrix} \cosh^2\varphi_{p_0} & \frac{1}{2}\sinh 2\varphi_{p_0} \\ \frac{1}{2}\sinh 2\varphi_{p_0} & \sinh^2\varphi_{p_0} \end{bmatrix}-\bar{\Delta}^{*}(p)\begin{bmatrix} \sinh^2\varphi_{p_0} & \frac{1}{2}\sinh 2\varphi_{p_0} \\ \frac{1}{2}\sinh 2\varphi_{p_0} & \cosh^2\varphi_{p_0} \end{bmatrix}. \tag{A34}$$

Then, we can find the final result Eq. (3.71).

3. ω meson propagator

We can also obtain the full propagator of the ω meson defined by Eq. (3.73) in a similar manner. By using the completeness relation for the ω meson, the propagator in momentum space is written as

$$D_{\mu\nu}^{(ab)}(p)=\int_{-\infty}^{+\infty}dw\left[\frac{\sigma_{v1\mu\nu}^{(ab)}(w,\mathbf{p})}{p_0-w+i\varepsilon}-\frac{\sigma_{v2\mu\nu}^{(ab)}(w,\mathbf{p})}{p_0-w-i\varepsilon}\right], \tag{A35}$$

where

$$\sigma^{(ab)}_{v1\,\mu\nu}(p)\equiv(2\pi)^3\sum_{nm}\langle 0(\beta)|V^{(a)}_\mu(0)|n,\tilde m\rangle\langle\tilde m,n|V^{(b)}_\nu(0)|0(\beta)\rangle\delta^4(p-p_n+p_m)\ , \tag{A36}$$

$$\sigma^{(ab)}_{v2\,\mu\nu}(p)\equiv(2\pi)^3\sum_{nm}\langle 0(\beta)|V^{(b)}_\nu(0)|n,\tilde m\rangle\langle\tilde m,n|V^{(a)}_\mu(0)|0(\beta)\rangle\delta^4(-p-p_n+p_m)\ . \tag{A37}$$

From the thermal state condition, we find

$$\langle 0(\beta)|V^{(2)}_\mu(0)|n,\tilde m\rangle=e^{-\beta(E_n-E_m)/2}\langle 0(\beta)|V^{(1)}_\mu(0)|n,\tilde m\rangle\ , \tag{A38}$$

$$\langle 0(\beta)|V^{(1)}_\mu(0)|n,\tilde m\rangle=e^{\beta(E_n-E_m)/2}\langle m,\tilde n|V^{(1)}_\mu(0)|0(\beta)\rangle\ , \tag{A39}$$

$$\langle 0(\beta)|V^{(2)}_\mu(0)|n,\tilde m\rangle=\langle m,\tilde n|V^{(1)}_\mu(0)|0(\beta)\rangle\ , \tag{A40}$$

$$\langle\tilde m,n|V^{(2)}_\mu(0)|0(\beta)\rangle=e^{-\beta(E_n-E_m)/2}\langle\tilde m,n|V^{(1)}_\mu(0)|0(\beta)\rangle\ , \tag{A41}$$

$$\langle\tilde m,n|V^{(1)}_\mu(0)|0(\beta)\rangle=e^{\beta(E_n-E_m)/2}\langle 0(\beta)|V^{(1)}_\mu(0)|\tilde n,m\rangle\ , \tag{A42}$$

$$\langle\tilde m,n|V^{(2)}_\mu(0)|0(\beta)\rangle=\langle 0(\beta)|V^{(1)}_\mu(0)|\tilde n,m\rangle\ . \tag{A43}$$

Then, we can rewrite Eqs. (A36) and (A37) as

$$\sigma^{(ab)}_{v1\,\mu\nu}(p)=\rho_{v\,\mu\nu}(p)A^{(ab)}_B(p_0)\ , \tag{A44}$$

$$\sigma^{(ab)}_{v2\,\mu\nu}(p)=\rho_{v\,\mu\nu}(p)B^{(ab)}_B(p_0)\ , \tag{A45}$$

where the spectral function has been defined by

$$\rho_{v\,\mu\nu}(p)=(2\pi)^3(1-e^{-\beta p_0})\sum_{nm}\delta^4(p-p_n+p_m)\langle 0(\beta)|V^{(1)}_\mu(0)|n,\tilde m\rangle\langle\tilde m,n|V^{(1)}_\nu(0)|0(\beta)\rangle\ , \tag{A46}$$

and the relation $\rho_{v\,\mu\nu}=\rho_{v\nu\mu}=\rho^*_{v\mu\,\nu}$ has been used. These equations lead to Eq. (3.77).

APPENDIX B: PRODUCT RULES

Here we prove the product rules for the fermion-fermion and fermion-boson cases.

1. Fermion-fermion product

First, we note that the arguments of U_F and U_F^{-1} in the left-hand side (l.h.s.) of Eq. (3.100) can be replaced by w and w' as follows:

$$[\text{l.h.s. of }(3.100)]^{(ab)}=\int_{-\infty}^{+\infty}đq_0\left[U_F(w)\begin{bmatrix}\chi(w,p_0+q_0) & 0\\ 0 & \chi^*(w,p_0+q_0)\end{bmatrix}U_F^{-1}(w)\right]^{(ab)}\times\left[U_F(w')\begin{bmatrix}\chi(w',q_0) & 0\\ 0 & \chi^*(w',q_0)\end{bmatrix}U_F^{-1}(w')\right]^{(ab)}\ . \tag{B1}$$

Using Eq. (3.62) and defining

$$u_\pm(w)\equiv\frac{1}{(1+e^{\pm\beta(w-\mu)})^{1/2}}\ , \tag{B2}$$

we obtain

$$[\text{l.h.s. of }(3.100)]^{(ab)}=\int_{-\infty}^{+\infty}đq_0\left[\frac{1}{p_0+q_0-w+i\varepsilon}\begin{bmatrix}u_-^2(w) & -u_+(w)u_-(w)\\ -u_+(w)u_-(w) & u_+^2(w)\end{bmatrix}+\frac{1}{p_0+q_0-w-i\varepsilon}\begin{bmatrix}u_+^2(w) & u_+(w)u_-(w)\\ u_+(w)u_-(w) & u_-^2(w)\end{bmatrix}\right]^{(ab)}$$
$$\times\left[\frac{1}{q_0-w'+i\varepsilon}\begin{bmatrix}u_-^2(w') & -u_+(w')u_-(w')\\ -u_+(w')u_-(w') & u_+^2(w')\end{bmatrix}+\frac{1}{q_0-w'-i\varepsilon}\begin{bmatrix}u_+^2(w') & u_+(w')u_-(w')\\ u_+(w')u_-(w') & u_-^2(w')\end{bmatrix}\right]^{(ab)}\ . \tag{B3}$$

Then, the identities

$$\int_{-\infty}^{+\infty} dx \frac{1}{x-\alpha+i\varepsilon}\frac{1}{x-\beta+i\varepsilon}=0 , \tag{B4}$$

$$\int_{-\infty}^{+\infty} dx \frac{1}{x-\alpha\pm i\varepsilon}\frac{1}{x-\beta\mp i\varepsilon}=\frac{\mp i}{\alpha-\beta\mp i\varepsilon} \tag{B5}$$

lead to

$$\begin{aligned}[\text{l.h.s. of } (3.100)]^{(ab)}=&\frac{i}{-p_0+w-w'+i\varepsilon}\begin{bmatrix} u_+^2(w)u_-^2(w') & -u_+(w)u_-(w)u_+(w')u_-(w') \\ -u_+(w)u_-(w)u_+(w')u_-(w') & u_-^2(w)u_+^2(w')\end{bmatrix}^{(ab)} \\ &-\frac{i}{-p_0+w-w'+i\varepsilon}\begin{bmatrix} u_-^2(w)u_+^2(w') & -u_-(w)u_-(w)u_+(w')u_-(w') \\ -u_+(w)u_-(w)u_+(w')u_-(w') & u_+^2(w)u_-^2(w')\end{bmatrix}^{(ab)} .\end{aligned} \tag{B6}$$

Next, we use the identity

$$\frac{1}{1+e^A}\frac{1}{1+e^B}=\left[\frac{1}{1+e^A}-\frac{1}{1+e^{-B}}\right]\frac{1}{1-e^{A+B}} \tag{B7}$$

to rewrite the (11) component of Eq. (B6) as follows:

$$\begin{aligned}[\text{l.h.s. of } (3.100)]^{(11)}=&\,i\left[\frac{1}{1+e^{\beta(w-\mu)}}-\frac{1}{1+e^{\beta(w'-\mu)}}\right] \\ &\times\left[\frac{1}{-p_0+w-w'+i\varepsilon}\frac{1}{1-e^{\beta p_0}}+\frac{1}{-p_0+w-w'-i\varepsilon}\frac{1}{1-e^{-\beta p_0}}\right]\end{aligned} \tag{B8}$$

$$=i\left[\cosh^2\varphi_{p_0}\sigma_{ff}(w,w',p_0)-\sinh^2\varphi_{p_0}\sigma_{ff}^*(w,w',p_0)\right] , \tag{B9}$$

where we have defined φ_{p_0} by Eqs. (3.36) and (3.37), and $\sigma_{ff}(w,w',p_0)$ by Eq. (3.101). Other components can be calculated in a similar manner. These expressions lead to the simple matrix form shown in the right-hand side of Eq. (3.100).

2. Fermion-boson product

In the same way, we can prove Eq. (3.102). The arguments of U_B and U_F in the l.h.s. of Eq. (3.102) can be replaced by w and w'. Then, defining

$$h_\pm(w)=\frac{1}{1-e^{\pm\beta w}} \quad \text{and} \quad h_0(w)=\frac{e^{\beta w/2}}{1-e^{\beta w}} , \tag{B10}$$

we can find

$$\begin{aligned}[\text{l.h.s. of } (3.102)]^{(ab)}=&\frac{-i}{p_0-w-w'+i\varepsilon}\begin{bmatrix} h_-(w)u_-^2(w') & h_0(w)u_+(w')u_-(w') \\ h_0(w)u_+(w')u_-(w') & -h_+(w)u_+^2(w')\end{bmatrix}^{(ab)} \\ &+\frac{i}{p_0-w-w'-i\varepsilon}\begin{bmatrix} h_+(w)u_+^2(w') & h_0(w)u_+(w')u_-(w') \\ h_0(w)u_+(w')u_-(w') & -h_-(w)u_-^2(w')\end{bmatrix}^{(ab)} .\end{aligned} \tag{B11}$$

The identity

$$\frac{1}{1-e^A}\frac{1}{1+e^B}=\left[\frac{1}{1-e^A}-\frac{1}{1+e^{-B}}\right]\frac{1}{1+e^{A+B}} \tag{B12}$$

is used to rewrite Eq. (B11), for example,

$$[\text{l.h.s. of } (3.102)]^{(11)} = -i\left[\frac{1}{1-e^{-\beta w}} - \frac{1}{1+e^{\beta(w'-\mu)}}\right] \times \left[\frac{1}{p_0 - w - w' + i\varepsilon}\frac{1}{1+e^{-\beta(p_0-\mu)}} + \frac{1}{p_0 - w - w' - i\varepsilon}\frac{1}{1+e^{\beta(p_0-\mu)}}\right] \tag{B13}$$

$$= i[\cos^2\theta_{p_0}\sigma_{fb}(w,w',p_0) + \sin^2\theta_{p_0}\sigma^*_{fb}(w,w',p_0)] , \tag{B14}$$

where we have defined θ_{p_0} by Eqs. (3.26) and (3.27) and $\sigma_{fb}(w,w',p_0)$ by Eq. (3.103). Then we obtain the final expression, i.e., the right-hand side of Eq. (3.102).

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