Effective masses in relativistic approaches to the nucleon-nucleus mean field

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In relativistic descriptions of the mean field in nuclei or in nuclear matter, the expression "effective mass" has been used to denote different quantities. The relationship between these various quantities is clarified. It is exhibited which one among them is most closely related to the effective mass that is derived from nonrelativistic analyses of scattering and bound-state data. This nonrelativistic-type effective mass has a characteristic energy dependence near the Fermi energy whenever one goes beyond the relativistic Hartree or Hartree-Fock approximations. By making use of dispersion relations that connect the real and imaginary parts of the microscopic mean field, it is shown that the occurrence of this "Fermi surface anomaly" is quite general. It has the same origin as in the nonrelativistic case, namely the frequency dependence of the mean field. Despite this qualitative similarity between the relativistic and nonrelativistic cases, a striking difference exists between the size of the Fermi surface anomaly in the two cases. The physical origins of the effective mass are also shown to be very different in the relativistic and nonrelativistic descriptions.

I. INTRODUCTION

Many recent papers which deal with relativistic descriptions of the nuclear mean field contain the expression "effective mass." However, this expression has been used to denote different quantities, with the result that "there is some confusion about how to define an effective mass."[1] This confusion goes beyond a mere inconvenience to the reader. Indeed, these different relativistic effective masses have occasionally been compared to the same empirical value, namely the one derived from analyses of experimental data in the framework of a nonrelativistic shell or optical model. This straightforward comparison is meaningful only if the expression "effective mass" denotes the same physical quantity in the relativistic as in the nonrelativistic description. We thus feel that a clarification is needed. The main purpose of the present paper is threefold. Firstly, we critically survey the definitions of the various quantities that have been associated with the expression "effective mass" in relativistic approaches to the nuclear mean field. Secondly, we exhibit the fact that in the case of the relativistic Brueckner-Hartree-Fock approximation, the effective mass that is most closely related to its nonrelativistic analog has a characteristic energy dependence near the Fermi energy, as in the nonrelativistic Brueckner-Hartree-Fock approximation. Thirdly, we show that, despite the fact that this qualitative property is common to the nonrelativistic and relativistic effective masses, its magnitude is quite different in the two cases.

In the nonrelativistic theory, the microscopic potential is nonlocal and frequency dependent. This led one to introduce three different effective masses:[2,3] the *k mass* which characterizes the nonlocality of the microscopic potential in the spatial coordinates, the *E mass* which characterizes its frequency dependence (or equivalently its nonlocality in time), and the *effective mass* proper which characterizes the energy dependence of a *local* potential that is equivalent to the microscopic potential. Here, the word "potential" denotes the real part of the mean field and the expression "equivalent" refers to a local mean field that yields the same scattering cross sections and single-particle energies as the microscopic potential. We shall extend these definitions and concepts to the relativistic mean field. In that case too, the microscopic potential is in general nonlocal and frequency dependent. An additional complication appears because the relativistic microscopic potential is a sum of several components which have different Lorentz transformation properties. For simplicity, we shall mainly deal with the simple case of nuclear matter and shall consider only two Lorentz components, namely a scalar and the fourth component of a vector field. The extension to finite nuclei and to other Lorentz components is fairly straightforward.

The present paper is organized as follows. In Sec. II, we survey the definitions and concepts used in the nonrelativistic approach. In Sec. III, we investigate the case of an energy-independent relativistic mean field; this is the situation encountered in many phenomenological descriptions, as well as in the relativistic Hartree approximation[4] or in Walecka's solution of the σ-ω model.[5] Section IV is mainly devoted to the relativistic Brueckner-Hartree-Fock approximation;[6–9] in this approximation, the potential is nonlocal and frequency dependent. Section V contains a summary and our conclusions.

Throughout, we set $\hbar=c=1$ and usually omit any explicit reference to the spin-orbit coupling.

II. EFFECTIVE MASSES IN THE NONRELATIVISTIC DESCRIPTION

A. Phenomenological analyses

In the phenomenological optical and shell models, one considers the Schrödinger equation:

$$\left[\frac{1}{2m}\mathbf{p}^2+U_S(r;\epsilon)\right]\Psi_S(\mathbf{r};\epsilon)=\epsilon\Psi_S(\mathbf{r};\epsilon) . \tag{2.1}$$

Here, the index S refers to Schrödinger and $\mathbf{p}=-i\nabla$ is the momentum operator. In the scattering case, $\epsilon>0$ is the kinetic energy of the scattered nucleon, while for a bound state $\epsilon<0$ is the energy of the single-particle level. As indicated, the phenomenological nonrelativistic mean field is local but energy dependent. It is usually complex:

$$U_S(r;\epsilon)=V_S(r;\epsilon)+iW_S(r;\epsilon) . \tag{2.2}$$

The nonrelativistic *effective mass* $m_S^*(r;\epsilon)$ is defined by

$$m_S^*(r;\epsilon)/m=1-\frac{d}{d\epsilon}V_S(r;\epsilon) . \tag{2.3}$$

It thus characterizes the energy dependence of the real part of the phenomenological nonrelativistic mean field. In practice, $V_S(r;\epsilon)$ is assumed to have a Woods-Saxon shape whose depth is associated with the potential energy of a nucleon in nuclear matter. The corresponding effective mass will be denoted by $m_S^*(\epsilon)$:

$$m_S^*(\epsilon)/m=1-\frac{d}{d\epsilon}V_S(r=0;\epsilon)=1-\frac{d}{d\epsilon}V_S(\epsilon) . \tag{2.4}$$

The Fermi energy ϵ_F is the energy of a nucleon whose momentum is equal to the Fermi momentum k_F:

$$\epsilon_F=\frac{1}{2m}k_F^2+V_S(\epsilon_F) . \tag{2.5a}$$

At saturation, the empirical values of ϵ_F and k_F in symmetric nuclear matter are the following:

$$k_F\approx 1.36\ \text{fm}^{-1};\quad \epsilon_F\approx -16\ \text{MeV} . \tag{2.5b}$$

B. Empirical value of the nonrelativistic effective mass

The effective mass usually depends upon energy. In recent analyses of the neutron-^{208}Pb mean field,[10,11] it has been found that

$$m_S^*(\epsilon=\epsilon_F)/m\approx 0.82 , \tag{2.6a}$$

$$m_S^*(\epsilon\approx 30\ \text{MeV})/m\approx 0.74 . \tag{2.6b}$$

The result quoted in Eq. (2.6a) has been taken as reference for comparison with nuclear matter, see, e.g., Ref. 12. Caution must be exercised when doing so, because of the following two main complications. (a) Finiteness effects are not negligible, even at the nuclear center.[13,14] (b) The n-^{208}Pb mean field is influenced by the existence of a neutron excess in ^{208}Pb. A recent analysis[15] of the proton-^{208}Pb mean field yields

$$m_S^*(\epsilon=\epsilon_F)/m\approx 0.63 . \tag{2.7}$$

Note that the latter value is itself influenced by a third effect, namely, (c) Coulomb energy corrections.[16] If one takes into account complications (a) and (c), Eqs. (2.6a) and (2.7) yield the following rough estimate for the central value of the effective mass associated with the isoscalar component of the mean field at the Fermi energy:[15]

$$m_S^*(\epsilon=\epsilon_F)/m\approx 0.74 . \tag{2.8}$$

C. Nonlocality in space and time

Microscopic calculations of the nonrelativistic mean field usually yield an operator which is nonlocal in the space and time coordinates. This corresponds to using the following Schrödinger equation:

$$\frac{1}{2m}\mathbf{p}^2\Psi_S(\mathbf{r};t)+\int d\mathbf{r}'\int dt'\mathcal{U}_S(\mathbf{r},\mathbf{r}';t-t')\Psi_S(\mathbf{r}';t')=i\frac{\partial}{\partial t}\Psi_S(\mathbf{r};t) . \tag{2.9}$$

For simplicity, we consider nuclear matter and a nucleon with given frequency

$$\Psi_S(\mathbf{r};t)=e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)} . \tag{2.10}$$

Let us introduce the double Fourier transform

$$\mathcal{U}_S(k;\omega)=(2\pi)^{-4}\int d(\mathbf{r}-\mathbf{r}')\times\int d(t-t')\mathcal{U}_S(|\mathbf{r}-\mathbf{r}'|;t-t')\times e^{-i\mathbf{k}\cdot(\mathbf{r}-\mathbf{r}')}e^{i\omega(t-t')} . \tag{2.11}$$

Note that the dependence of $\mathcal{U}_S(k;\omega)$ upon the momentum k reflects the fact that the microscopic mean field is nonlocal in the spatial coordinates, while its dependence upon the frequency ω reflects a nonlocality in time. For ω real, one defines the real and imaginary parts of $\mathcal{U}_S$ by

$$\mathcal{V}_S(k;\omega)+i\mathcal{W}_S(k;\omega)=\lim_{\eta\to+0}\mathcal{U}_S(k;\omega+i\eta) . \tag{2.12}$$

The quantity $\mathcal{U}_S(k;\omega)$ is analytic in the upper half of the complex ω plane,[17] which implies the dispersion relation

$$\mathcal{V}_S(k;\omega)=\mathcal{V}_S^{(\infty)}(k)+\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{\mathcal{W}_S(k;\omega')}{\omega'-\omega}d\omega' , \tag{2.13}$$

if $\mathcal{U}_S(k;\omega)$ is well behaved for large $|\omega|$. In Eq. (2.13), the integral is a principal value, while

$$\mathcal{V}_S^{(\infty)}(k)=\lim_{|\omega|\to\infty}\mathcal{V}_S(k;\omega) . \tag{2.14}$$

Equations (2.9)–(2.11) yield the following frequency-momentum relation:

$$\frac{1}{2m}k^2+\mathcal{V}_S(k;\omega)=\omega , \tag{2.15}$$

where, for simplicity, we have neglected the role of

$\mathcal{W}_S(k;\omega)$. Equation (2.15) defines a function $k(\omega)$. The quantity $V_S(\epsilon)$ defined by

$$V_S(\epsilon)=\mathcal{V}_S(k(\epsilon);\epsilon) \tag{2.16}$$

does not depend explicitly upon k. It is thus local: it is the "*local equivalent*" of $\mathcal{V}_S(k;\omega)$. This quantity $V_S(\epsilon)$ is the one that should be compared with the potential depth derived from phenomenological nonrelativistic analyses, since these use a local potential. This is why we used the notation V_S for the function defined by Eq. (2.16) and the notation ϵ for its energy variable, see Eqs. (2.2)–(2.4).

The origin of the dependence of the phenomenological $V_S(\epsilon)$ upon ϵ is twofold: (a) the dpendence upon k of the microscopic potential $\mathcal{V}_S$, i.e., its nonlocality in the spatial coordinates, (b) the dependence of $\mathcal{V}_S$ upon ω, i.e., its nonlocality in time. These two types of nonlocality have been characterized by two effective masses, respectively called the k *mass* $\tilde{m}$ and the E mass $\overline{m}$, defined as follows:[2]

$$\tilde{m}(\epsilon)/m=\left[1+\frac{m}{k}\frac{\partial}{\partial k}\mathcal{V}_S(k;\epsilon)\right]^{-1}_{k=k(\epsilon)}, \tag{2.17}$$

$$\overline{m}(\epsilon)/m=\left[1-\frac{\partial}{\partial\epsilon}\mathcal{V}_S(k;\epsilon)\right]_{k=k(\epsilon)}. \tag{2.18}$$

The effective mass (2.4) is given by the two *equivalent* expressions

$$m^*(\epsilon)/m=1-\frac{d}{d\epsilon}\mathcal{V}_S(k(\epsilon);\epsilon) \tag{2.19a}$$

$$=\left[1+\frac{m}{k}\frac{d}{dk}\mathcal{V}_S(k;\epsilon(k))\right]^{-1}_{k=k(\epsilon)}, \tag{2.19b}$$

where $\epsilon(k)$ is the function of k defined by the energy-momentum relation (1.15), i.e., by

$$\epsilon=\frac{1}{2m}k^2+V_S(\epsilon). \tag{2.20}$$

The three effective masses are related by the following equation:[2]

$$\frac{m^*(\epsilon)}{m}=\frac{\tilde{m}(\epsilon)}{m}\cdot\frac{\overline{m}(\epsilon)}{m}. \tag{2.21}$$

III. SCALAR-VECTOR MODEL FOR THE RELATIVISTIC MEAN FIELD

A. Dirac equation

Much interest is presently devoted to the use of a Dirac rather than a Schrödinger equation in the optical and shell models. For simplicity, we restrict our discussion to the case of a relativistic mean field that is the sum of a Lorentz scalar (U_σ) and of the fourth component of a Lorentz vector (U_0). The Dirac equation then reads

$$\{\boldsymbol{\alpha}\cdot\mathbf{p}+\gamma_0[m+U_\sigma(r)+\gamma_0U_0(r)]\}\phi(\mathbf{r};E)=E\phi(\mathbf{r};E), \tag{3.1}$$

where ϕ is a four-component Dirac spinor, while

$$E=\epsilon+m \tag{3.2}$$

is the total energy. The notation U_σ has been used, rather than U_S, in order to avoid any confusion with the nonrelativistic potential of Eq. (2.1); it does not imply that the component U_σ is due to the exchange of a σ meson. In the present Sec. III we assume that the complex relativistic potentials U_σ and U_0 are local and independent of E. This simple case is encountered in the theoretical mean field and Hartree approximations.[4,5] It is useful for illustrating the difference between various quantities which have been related to the expression "effective mass."

We consider the limiting case of nuclear matter; most of the resulting relations can easily be adapted to spherical nuclei.[18] In nuclear matter, $\phi(\mathbf{r};E)$ is a plane wave:

$$\phi(\mathbf{r};E)=u(\mathbf{k})\exp[i(\mathbf{k}\cdot\mathbf{r}-Et)]. \tag{3.3}$$

Equations (3.1) and (3.3) imply the following frequency-momentum relation:

$$k^2+(m+U_\sigma)^2=(E-U_0)^2. \tag{3.4}$$

B. Schrödinger-equivalent potential

Equation (3.4) can be written in the following Schrödinger-type form:

$$\frac{1}{2m}k^2+U_e(\epsilon)=\frac{1}{2m}k^2_\infty, \tag{3.5}$$

where

$$U_e(\epsilon)=U_\sigma+U_0+\frac{\epsilon}{m}U_0+\frac{1}{2m}(U_\sigma^2-U_0^2), \tag{3.6}$$

$$k^2_\infty=\epsilon^2+2m\epsilon=E^2-m^2. \tag{3.7}$$

The main difference between Eq. (3.5) and the nonrelativistic relation (2.20) is that in Eq. (3.5) the expression of the asymptotic momentum k_∞ takes relativistic kinematics into account [note that for small ϵ ($\epsilon<100$ MeV), the relativistic correction (ϵ^2) is small compared to $2m\epsilon$].

In finite nuclei, a quantity $U_e(r:\epsilon)$ defined in a similar way as in Eq. (3.6) can also be identified with the central part of a nonrelativistic potential, except for a minor correction.[18] The corresponding wave function $\Psi_e(\mathbf{r};\epsilon)$ is related to the upper components $\phi_u(\mathbf{r};\epsilon)$ of the Dirac spinor ϕ by the following equation:

$$\Psi_e(\mathbf{r};\epsilon)=A(r;\epsilon)\phi_u(\mathbf{r};\epsilon), \tag{3.8a}$$

where

$$A(r;\epsilon)=\left[\frac{E+m+U_\sigma(r)-U_0(r)}{E+m}\right]^{-1/2}. \tag{3.8b}$$

At large distance, $\Psi_e(\mathbf{r};\epsilon)$ and $\phi_u(\mathbf{r};\epsilon)$ are proportional to one another. Hence the Schrödinger equation with $U_e(r;\epsilon)$ yields the same scattering cross sections and single-particle energies as the original relativistic potential. The quantity $U_e(r;\epsilon)$ has thus been called the "Schrödinger-equivalent potential";[18] the lower index e

on $U_e(r;\epsilon)$ refers to that "equivalence" property. We note that it might be more proper to call Eq. (3.5) a Klein-Gordon-type equation since, in finite nuclei, it corresponds to the following time-dependent equation (we recall that we do not consider the spin-orbit coupling):

$$\left[-\mathbf{p}^2+\frac{\partial^2}{\partial t^2}+2mU_e(r;\epsilon)\right]\psi_e(\mathbf{r};\epsilon)=0 . \quad (3.9)$$

We emphasize that U_e is obtained by a mere rewriting of the relativistic equation. Its use does *not* imply that a nonrelativistic limit is taken. One must remain aware, however, that the wave function $\Psi_e(\mathbf{r};\epsilon)$ has no straightforward physical meaning; in particular, it should not be related to a probability density.

In the remainder of the present Sec. III, we omit the contribution to U_e of the imaginary components (W_σ, W_0) of the relativistic mean field; this contribution will be considered in Sec. IV F. Equations (3.4)–(3.6) then become

$$k^2+(m+V_\sigma)^2=(E-V_0)^2 , \quad (3.10)$$

$$\frac{1}{2m}k^2+V_e(\epsilon)=\epsilon+\epsilon^2/2m , \quad (3.11)$$

$$V_e(\epsilon)=V_\sigma+V_0+\frac{\epsilon}{m}V_0+\frac{1}{2m}(V_\sigma^2-V_0^2) . \quad (3.12)$$

As in the nonrelativistic case, the Fermi energy ϵ_F is the energy of a nucleon whose momentum is equal to the Fermi momentum:

$$(E_F-V_0)^2=k_F^2+(m+V_\sigma)^2 , \quad (3.13a)$$

$$E_F=\epsilon_F+m . \quad (3.13b)$$

Equation (3.11) yields

$$\epsilon_F(1+\epsilon_F/2m)=\frac{1}{2m}k_F^2+V_e(\epsilon_F) . \quad (3.13c)$$

C. Dirac mass

Equation (3.9) contains the combination

$$M^*=m+V_\sigma . \quad (3.14)$$

This quantity M^* is the one that is most often called "the effective mass" in the literature on relativistic mean-field approaches, see, e.g., Refs. 9, 19, and 20. This expression had been used by Walecka.[5] It already appeared in the pioneering work of Duerr,[21] who also used the expression "apparent mass." In Ref. 20, the quantity (3.14) is called the "Dirac mass," a term that we shall adopt here. This Dirac mass is *not* closely related to the effective mass $m_s^*(\epsilon)$ defined by Eq. (2.4). Hence M^* should *not* be identified with the effective mass determined from analyses of the experimental data performed in the framework of the nonrelativistic shell and optical models.

D. Nonrelativistic-type effective mass

Since the nonrelativistic-type potential $V_e(\epsilon)$ plays the same role as the Schrödinger potential $V_s(\epsilon)$, a nonrelativistic-type effective mass $m_e^*(\epsilon)$ can be defined in a similar way as in Eq. (2.4), namely,[18]

$$m_e^*(\epsilon)/m=1-\frac{d}{d\epsilon}V_e(\epsilon) . \quad (3.15)$$

Equation (3.12) yields

$$m_e^*/m=1-V_0/m . \quad (3.16)$$

In the present model, the nonrelativistic-type effective mass is thus energy independent. It is this quantity $m_e^*(\epsilon)$ that should be compared with the empirical value (2.6a)–(2.8) extracted from analyses performed in the framework of the nonrelativistic optical and shell models. It is also $m_e^*(\epsilon)$ that determines the spacing between single-particle energies.[3]

We emphasize that the definition (3.15) *does not* imply that a nonrelativistic limit is taken. We also note that the energy dependence of the Schrödinger-type potential $V_e(\epsilon)$ is *not* related to a nonlocality of the *relativistic* potential in space or in time. This is obvious from our present model, since the relativistic potential is local in space and is independent of frequency. The energy dependence of $V_e(\epsilon)$ in the present model has been interpreted as resulting from a modification of the strengths of the Lorentz components of the potential when measured in the rest frame of the nucleon instead of the rest frame of the nucleus.[21,22] For this reason, in Sec. IV we shall call m_e^* the Lorentz mass.

One could have used Eq. (3.11) to define a function $\epsilon(k)$ and, relatedly, a momentum-dependent (nonlocal) potential

$$\tilde{V}_e(k)=V_e(\epsilon(k)) . \quad (3.17)$$

By analogy with the nonrelativistic case [compare with Eq. (2.19b)], one could then have defined a nonrelativistic-type effective mass by

$$\tilde{m}_e(\epsilon)/m=\left[1+\frac{m}{k}\frac{d}{dk}\tilde{V}_e(k)\right]^{-1}_{k=k(\epsilon)} . \quad (3.18a)$$

One easily checks that

$$m_e^*=\tilde{m}_e[1-\epsilon(m-\tilde{m}_e)/(m\tilde{m}_e)] . \quad (3.18b)$$

The quantities m_e^* and $\tilde{m}_e$ would be identical if one would neglect ϵ^2 on the right-hand side of Eq. (3.7), which is justified at low energy. In practice, the nonlocal Schrödinger-type potential (3.17) and the related $\tilde{m}_e$ are not of much interest, because phenomenological nonrelativistic analyses of the experimental data usually use a Schrödinger equation with a *local* energy-dependent mean field.

E. Group velocity

The group velocity is given by

$$v_g=\frac{d}{dk}\epsilon(k) , \quad (3.19)$$

where the function $\epsilon(k)$ is defined by Eqs. (3.11) and (3.12). These yield

$$v_g = k/m_g^*(\epsilon) , \quad (3.20)$$

where the "group mass" m_g^* is given by

$$m_g^*(\epsilon) = \epsilon - V_0 + m = [k^2 + (M^*)^2]^{1/2} . \quad (3.21)$$

In Eq. (3.20), we wrote the group velocity in a form appropriate to nonrelativistic kinematics (the momentum divided by a mass). We emphasize, however, that the relations (3.19)–(3.21) hold exactly in the present model with energy-independent relativistic potentials: they do *not* imply that a nonrelativistic limit is taken. Equations (3.16) and (3.21) give the following relation between m_g^* and m_e^*:

$$m_g^* = m_e^* + \epsilon . \quad (3.22)$$

The two quantities are thus equal at $\epsilon = 0$ and are very nearly equal at low energy.

F. Landau mass

The Landau theory of Fermi liquids involves quantities evaluated at the Fermi energy. It has been extended to the relativistic case in Ref. 23. One of the basic Landau parameters, denoted by F_1, is defined as follows:

$$\left[\frac{dk}{d\epsilon}\right]_{\epsilon=\epsilon_F} = (E_F/k_F)(1 + \tfrac{1}{3}F_1) . \quad (3.23)$$

It is related to the Landau mass m_L^* by the equation

$$m_L^* = E_F(1 + \tfrac{1}{3}F_1) . \quad (3.24)$$

Equations (3.19), (3.21), (3.23), and (3.24) show that

$$m_L^* = [k_F^2 + (M^*)^2]^{1/2} = \epsilon_F - V_0 + m . \quad (3.25)$$

At the Fermi energy, m_L^* is thus equal to the group mass m_g^*. One readily finds that

$$\tfrac{1}{3}F_1 = -V_0/\{V_0 + [k_F^2 + (M^*)^2]^{1/2}\} ; \quad (3.26)$$

this expression of F_1 is identical to that derived by Matsui[24] in the framework of a relativistic microscopic mean-field approximation, see also Refs. 25 and 26. In Ref. 12, the Landau mass m_L^* was identified with the nonrelativistic-type effective mass $m_e^*(\epsilon_F)$ defined by Eq. (3.15); this is very accurate since Eqs. (3.15) and (3.24) yield

$$m_L^* = m_e^*(\epsilon_F) + \epsilon_F \approx m_e^*(\epsilon_F) . \quad (3.27)$$

G. Low-energy approximation to the nonrelativistic-type mass

In Ref. 9, the authors defined a "nonrelativistic mass" $m_{\rm NR}^*$ by the following relation:

$$m_{\rm NR}^*(\epsilon)/m = \left[1 + \frac{m}{k}\frac{d}{dk}V_{\rm NR}(k)\right]_{k=k(\epsilon)}^{-1} , \quad (3.28)$$

where the quantity $V_{\rm NR}(k)$ is obtained by subtracting the free energy from the total energy:

$$V_{\rm NR}(k) = \epsilon(k) + m - (k^2 + m^2)^{1/2} . \quad (3.29)$$

Here, $\epsilon(k)$ is defined by Eq. (3.11); one could as well use the latter equation to express k as a function $k(\epsilon)$, and have defined the following energy-dependent quantity:

$$V_{\rm NR}(\epsilon) = \epsilon + m - \{[k(\epsilon)]^2 + m^2\}^{1/2} , \quad (3.30)$$

where for convenience we used the same symbol $V_{\rm NR}$ as in Eq. (3.29). The quantity $V_{\rm NR}$ has been introduced by Duerr.[21] It has also been considered in Refs. 27–29. It is related to the Schrödinger-equivalent potential V_e of Eq. (3.11) by the following equation ($E = \epsilon + m$):[29]

$$V_{\rm NR}(\epsilon) = E - [E^2 - 2mV_e(\epsilon)]^{1/2} . \quad (3.31)$$

The quantity $V_{\rm NR}$ has a straightforward physical meaning only *at low energy,* where it is legitimate to write the total energy as the sum of a kinematic and a potential contribution (plus the rest mass). In particular, $V_{\rm NR}$ is not the potential that appears when one writes the equation for the upper components of the Dirac spinor in a Schrödinger-type form. At low energy, however, the quantities $V_{\rm NR}(\epsilon)$ and $V_e(\epsilon)$ are very close to each other. This implies that the effective masses m_e^* and $m_{\rm NR}^*(\epsilon)$ are nearly equal, for energies between -50 and 100 MeV. In contrast, these quantities differ for large ϵ; there indeed, m_e^*/m is independent of ϵ, while $m_{\rm NR}^*(\epsilon)/m$ approaches unity.

H. Less reliable nonrelativistic approximations

In Refs. 30 and 31, the expression "effective mass" was used to denote the following quantity:

$$M_*^{\rm NR}/m = 1 + (V_\sigma - V_0)/2m . \quad (3.32)$$

Note that $M_*^{\rm NR}$ is the average between the Dirac mass and the Lorentz mass:

$$M_*^{\rm NR} = \tfrac{1}{2}(M^* + m_e^*) . \quad (3.33)$$

We now briefly describe how $M_*^{\rm NR}$ appears. The Dirac equation (3.1) yields coupled equations for the upper (ϕ_u) and lower (ϕ_l) components of ϕ:

$$\boldsymbol{\sigma}\cdot\mathbf{p}\phi_u = (E + m + V_\sigma - V_0)\phi_l , \quad (3.34a)$$

$$\boldsymbol{\sigma}\cdot\mathbf{p}\phi_l = (E - m - V_\sigma - V_0)\phi_u . \quad (3.34b)$$

If one uses the approximation

$$E + m + V_\sigma - V_0 \approx 2m + V_\sigma - V_0 \quad (3.35)$$

in the right-hand side of Eq. (3.34a), derives $\phi_l(\epsilon)$ from the resulting equation, and introduces the result in Eq. (3.34b), one obtains the following nonrelativistic approximation to the energy-momentum relation (3.11):

$$\frac{1}{2m}k^2 + V^{\rm NR}(\epsilon) \approx \epsilon , \quad (3.36a)$$

with

$$V^{\rm NR}(\epsilon) = \frac{M_*^{\rm NR}}{m}(V_0 + V_\sigma) + \frac{\epsilon}{2m}(V_0 - V_\sigma) . \quad (3.36b)$$

The quantity $M_*^{\rm NR}$ is seen to be given by

$$M_*^{\rm NR}/m = 1 - \frac{d}{d\epsilon}V^{\rm NR}(\epsilon) , \quad (3.36c)$$

compare with Eq. (3.15)

We note that $V^{\rm NR}(\epsilon)$ has not been obtained from a systematic expansion of $V_e(\epsilon)$ in powers of ϵ/m. A more systematic expansion would yield[18]

$$\frac{1}{2m}k^2+V^{\rm LE}(k)\approx\epsilon\ , \qquad (3.37a)$$

where the upper label LE stands for "low energy" and where

$$V^{\rm LE}(k)\approx V_\sigma+V_0-\frac{1}{2mM^*}k^2V_\sigma-\frac{1}{8(M^*)^3}k^4\ . \qquad (3.37b)$$

If one replaces M^* by m on the right-hand side of Eq. (3.37b), one obtains the same result as from two successive Foldy-Wouthuysen transformations.[18] If one omits the last term on the right-hand side of Eq. (3.37b) and defines a low-energy approximation to the effective mass by the relation [see Eq. (3.18a)]

$$M_*^{\rm LE}(\epsilon)/m=\left[1+\frac{m}{k}\frac{d}{dk}V^{\rm LE}(k)\right]^{-1}_{k=k(\epsilon)}\ , \qquad (3.37c)$$

one obtains

$$M_*^{\rm LE}\approx M^*\ , \qquad (3.37d)$$

where M^* is the Dirac mass of Eq. (3.14). The approximation (3.37d) must be considered as inaccurate, in particular because the last term on the right-hand side of Eq. (3.37b) is not negligible.

I. Discussion

Most phenomenological analyses of the experimental data are based on the following Schrödinger-type equation:

$$\left[\frac{1}{2m}\mathbf{p}^2+V_S(r;\epsilon)\right]\phi_S(\mathbf{r};\epsilon)=\frac{1}{2m}k_\infty^2\phi_S(\mathbf{r},\epsilon)\ , \qquad (3.38)$$

see, e.g., Refs. 32–34. Equations (3.11) and (3.38) show that it is the quantity $V_e(\epsilon)$ that should be identified with the depth of the empirical average potential derived from nonrelativistic phenomenological analyses of the experimental data. Correspondingly, it is the nonrelativistic-type effective mass $m_e^*(\epsilon)$, Eq. (3.15), that should be identified with the empirical effective mass derived from these analyses, e.g., from those carried out in Refs. 10 and 11. We also recall that, *at low energy* ($|\epsilon|<50$ MeV), the quantities $m_e^*(\epsilon)$ [Eq. (3.15)], $m_g^*(\epsilon)$ [Eq. (3.21)], and $m_{\rm NR}^*(\epsilon)$ [Eq. (3.28)] are practically equal. They all differ from the Dirac mass M^* defined by Eq. (3.14).

Some typical magnitudes are useful. Since ϵ_F and k_F are known empirically [Eq. (2.5b)], the potential depth $V_e(\epsilon_F)$ at the Fermi energy is also known [see Eq. (3.13c)]:

$$V_e(\epsilon_F)\approx-55\ \text{MeV}\ . \qquad (3.39)$$

The present model for $V_e(\epsilon)$ only involves two unknowns, namely V_0 and V_σ. Therefore only one additional empirical constraint is needed besides the value of ϵ_F. This additional constraint could for instance be the effective mass at some energy. A large body of phenomenological nonrelativistic analyses of experimental scattering data indicate that

$$m_S^*(\epsilon\approx30\ \text{MeV})/m\approx0.70\pm0.05\ ; \qquad (3.40)$$

the quoted error is a rather conservative one, which takes into account uncertainties in the empirical potential radius and in the energy dependence of the symmetry potential. Equation (3.15) then yields

$$V_0\approx(300\pm30)\ \text{MeV}\ . \qquad (3.41a)$$

From Eqs. (3.13c), (3.39), and (3.41a), one obtains the following value for the scalar potential:

$$V_\sigma\approx(-375\pm40)\ \text{MeV}\ . \qquad (3.41b)$$

These estimates are in semiquantitative agreement with the empirical spin-orbit coupling (see, e.g., Refs. 18, 35, and 36), as well as with values extracted from phenomenological relativistic analyses of the experimental cross sections (see, e.g., Ref. 37). At a more quantitative level, however, empirical as well as theoretical evidence indicates that the model considered in the present section is too simple. For instance, the model nonrelativistic-type effective mass m_e^* is independent of energy; above, we chose the parameters in such a way that m_e^* is in agreement with the empirical value of $m_s^*(\epsilon)$ for $\epsilon\approx30$ MeV, but it is then necessarily too small at the Fermi energy ϵ_F: compare Eqs. (2.8) and (3.40). This disagreement is of importance because the value of m_e^* at ϵ_F plays a significant role in the Landau theory of Fermi liquids, see, e.g., Refs. 12 and 26. It indicates that the relativistic potentials V_0 and V_σ should be allowed to depend upon energy. This is confirmed by detailed phenomenological relativistic analyses of scattering data at intermediate energy[37,38] as well as by microscopic calculations,[6,8,9,39,40] that we now discuss.

IV. MICROSCOPIC RELATIVISTIC MODELS

A. Definitions and properties

As in the nonrelativistic case (Sec. II C), microscopic calculations of the relativistic mean field yield Lorentz components that depend upon momentum and frequency if one goes beyond the Hartree approximation. This is considered in the present section. For simplicity, we still limit ourselves to the case where the relativistic potential only contains two Lorentz components: a scalar and the fourth component of a vector. Other components could easily be included, at the expense of somewhat more complicated expressions. The Schrödinger-type potential is derived in Sec. IV B, and the effective masses in Sec. IV C. The relativistic Brueckner-Hartree-Fock approximation is discussed in Sec. IV D, and a correction to it in Sec. IV E. Section IV F deals with the modification of the imaginary part of the Schrödinger-equivalent potential and of the mean free path that is due to the nonlocality of the real part of the relativistic mean field.

We closely follow the notation used in Secs. II C, III A, and III B. The relativistic mean field (self-energy) is written in the form

$$\mathcal{U}(k;\omega)=\mathcal{U}_\sigma(k;\omega)+\gamma_0\mathcal{U}_0(k;\omega) . \quad (4.1)$$

As in the nonrelativistic case, the dependence of $\mathcal{U}_\sigma$ or $\mathcal{U}_0$ upon k reflects a nonlocality in the space coordinates, while the dependence upon ω implies a nonlocality in time. For ω real, we define

$$\mathcal{U}(k;\omega)=\lim_{\eta\to+0}\mathcal{U}(k;\omega+i\eta) \quad (4.2a)$$

$$=\mathcal{V}(k;\omega)+i\mathcal{W}(k;\omega) . \quad (4.2b)$$

The microscopic mean field is an analytic function of ω in the upper half plane;[39] if $\mathcal{U}(k;\omega)$ is well behaved for large $|\omega|$, the following dispersion relation thus holds:[39]

$$\mathcal{V}(k;\omega)=\mathcal{V}^{(\infty)}(k)+\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{\mathcal{W}(k;\omega')}{\omega'-\omega}d\omega' . \quad (4.3)$$

Here, we do not consider possible complications associated with the left-hand cut ($\omega \lesssim -m$); these lie considerably beyond the scope of the present paper and are of no practical importance in the present context. Quite general phase space arguments[41] show that $\mathcal{W}_\sigma(k;\omega)$ and $\mathcal{W}_0(k;\omega)$ vanish like $(\omega-\epsilon_F)^2$ for ω close to ϵ_F. The dependence upon k of these quantities is weak, because it is not influenced by phase space considerations. Hence we make the following approximations for ω close to ϵ_F and for k close to $k(\omega)$:

$$\mathcal{W}_\sigma(k;\omega)\approx W_\sigma(\omega)\sim c_\sigma(\omega-\epsilon_F)^2+O((\omega-\epsilon_F)^3) , \quad (4.4)$$

$$\mathcal{W}_0(k;\omega)\approx W_0(\omega)\sim c_0(\omega-\epsilon_F)^2+O((\omega-\epsilon_F)^3) . \quad (4.5)$$

B. Schrödinger-equivalent potential

We introduce the quantity [see Eq. (3.6)]

$$\mathcal{U}_e(k;\omega)=\mathcal{U}_\sigma(k;\omega)+\mathcal{U}_0(k;\omega)+\frac{\omega}{m}\mathcal{U}_0(k;\omega)+\frac{1}{2m}[\mathcal{U}_\sigma^2(k;\omega)-\mathcal{U}_0^2(k;\omega)] , \quad (4.6)$$

which enters in the following energy-momentum relation:

$$k^2+2m\,\mathcal{U}_e(k;\epsilon)=\epsilon^2+2m\epsilon ; \quad (4.7a)$$

$\mathcal{U}_e(k;\omega)$ is complex:

$$\mathcal{U}_e(k;\omega)=\mathcal{V}_e(k;\omega)+i\mathcal{W}_e(k;\omega) . \quad (4.7b)$$

The role of the imaginary component $\mathcal{W}_e(k;\omega)$ will be discussed in Sec. IV F; here, we approximate Eqs. (4.7a) and (4.7b) by [see Eq. (3.11)]:

$$k^2+2m\,\mathcal{V}_e(k;\epsilon)=\epsilon^2+2m\epsilon . \quad (4.8)$$

This relation defines a real function $k(\epsilon)$; the quantity

$$U_e(\epsilon)=V_e(\epsilon)+iW_e(\epsilon) , \quad (4.9a)$$

with

$$V_e(\epsilon)=\mathcal{V}_e(k(\epsilon);\epsilon), \quad W_e(\epsilon)=\mathcal{W}_e(k(\epsilon);\epsilon) , \quad (4.9b)$$

can be identified with the local complex mean field obtained from nonrelativistic analyses of the experimental data. As in Sec. III B, we call it the Schrödinger-equivalent potential. The extension to finite nuclei is straightforward. In keeping with Eq. (4.9b), we shall use the notation

$$V_\sigma(\epsilon)=\mathcal{V}_\sigma(k(\epsilon);\epsilon); \quad W_\sigma(\epsilon)=\mathcal{W}_\sigma(k(\epsilon);\epsilon) , \quad (4.9c)$$

$$V_0(\epsilon)=\mathcal{V}_0(k(\epsilon);\epsilon); \quad W_0(\epsilon)=\mathcal{W}_0(k(\epsilon);\epsilon) . \quad (4.9d)$$

C. Effective masses

Most authors refer to the following Dirac mass [see Eq. (3.14)]:

$$M^*(\epsilon)=m+V_\sigma(\epsilon) \quad (4.10)$$

as the "effective mass" associated with the relativistic mean field, see, e.g., Refs. 8 and 9. As we discussed in Sec. III D, this Dirac mass $M^*(\epsilon)$ should *not* be identified with the effective mass derived from phenomenological analyses of the experimental data in the framework of the nonrelativistic shell and optical models. Rather, the latter empirical effective mass should be identified with the following nonrelativistic-type effective mass [see Eq. (3.15)]:

$$m^*(\epsilon)/m=1-\frac{d}{d\epsilon}V_e(\epsilon) , \quad (4.11)$$

where $V_e(\epsilon)$ is now defined by Eqs. (4.6) and (4.7b). The quantity $m^*(\epsilon)$ is the one that characterizes the energy dependence of the Schrödinger-equivalent potential; note that it does *not* imply that any nonrelativistic limit has been taken. In contrast to the model discussed in Sec. III, the right-hand side of Eq. (4.11) now depends upon ϵ. We now investigate the origin of this energy dependence.

We introduce the following quantities:

$$\mathcal{V}_e(k;\omega)=\mathcal{Y}_e(k;\omega)+\frac{\omega}{m}\mathcal{V}_0(k;\omega) , \quad (4.12a)$$

$$\mathcal{Y}_e(k;\omega)=\mathcal{V}_\sigma(k;\omega)+\mathcal{V}_0(k;\omega)+\frac{1}{2m}(\mathcal{V}_\sigma^2-\mathcal{V}_0^2-\mathcal{W}_\sigma^2+\mathcal{W}_0^2) , \quad (4.12b)$$

$$\mathcal{W}_e(k;\omega)=Z_e(k;\omega)+\frac{\omega}{m}\mathcal{W}_0(k;\omega) , \quad (4.13a)$$

$$Z_e(k;\omega)=\mathcal{W}_\sigma(k;\omega)+\mathcal{W}_0(k;\omega)+\frac{1}{2m}(\mathcal{V}_\sigma\mathcal{W}_\sigma-\mathcal{V}_0\mathcal{W}_0) ; \quad (4.13b)$$

for simplicity we did not write explicitly the dependence upon k and ω of the quantities contained within the square brackets on the right-hand side of Eqs. (4.12b) and (4.13b). The difference between the nonrelativistic-type effective mass $m^*(\epsilon)$ and the nucleon mass m now has a threefold origin.

(i) The factor ω/m in the second term on the right-hand side of Eq. (4.12a). This contribution arises from the Lorentz transformation properties of $\mathcal{V}_\sigma$ and $\mathcal{V}_0$. It is the only one that was encountered in the model discussed in Sec. III. It is described by the Lorentz mass $m_e^*(\epsilon)$ defined as follows [see Eq. (3.15)]:

$$m_e^*(\epsilon)/m = 1 - V_0(\epsilon)/m \ , \tag{4.14}$$

with $V_0(\epsilon)$ given by Eq. (4.9d).

(ii) The fact that the relativistic potential now depends upon k, i.e., is nonlocal in the *spatial* coordinates. This type of nonlocality will be characterized by the appearance of quantities that involve the *partial* derivatives $\partial\mathcal{V}_\sigma(k;\omega)/\partial k$ and $\partial\mathcal{V}_0(k;\omega)/\partial k$. We define the *k mass* $\tilde{m}(\epsilon)$ by the following relation [compare with Eq. (2.17)]:

$$\tilde{m}(\epsilon)/m = \left[1 + \frac{m}{k}\frac{\partial}{\partial k}\mathcal{V}_e(k;\epsilon)\right]^{-1}_{k=k(\epsilon)} . \tag{4.15}$$

(iii) The fact that the relativistic potential is nonlocal in time. This can be characterized by the appearance of the *partial* derivatives $[\partial\mathcal{V}_\sigma(k;\epsilon)/\partial\epsilon]_{k=k(\epsilon)}$ and $[\partial\mathcal{V}_0(k;\epsilon)/\partial\epsilon]_{k=k(\epsilon)}$ or by their following combination:

$$\overline{m}(\epsilon)/m = 1 - \left[\partial\mathcal{Y}_e(k;\epsilon)/\partial\epsilon + \frac{\epsilon}{m}\partial\mathcal{V}_0(k;\epsilon)/\partial\epsilon\right]_{k=k(\epsilon)} . \tag{4.16}$$

By analogy with the nonrelativistic case, we shall call $\overline{m}(\epsilon)$ the *E mass*.

The existence of these different physical contributions to the nonrelativistic -type effective mass $m^*(\epsilon)$ of Eq. (4.11) is exhibited by the following identity:

$$\frac{m^*(\epsilon)}{m} = \frac{\tilde{m}(\epsilon)}{m}\left[\frac{m_e^*(\epsilon)}{m} + \frac{\overline{m}(\epsilon)}{m} - 1 - \frac{\epsilon}{m}\left[\frac{m}{\tilde{m}(\epsilon)} - 1\right]\right] . \tag{4.17}$$

This identity can be derived from Eqs. (4.8) and (4.11). It is the extension of Eq. (2.21) to the relativistic case. In the model considered in Sec. III, one has $\tilde{m}(\epsilon) = m$ and $\overline{m}(\epsilon)/m = 1$; one then recovers the expression (3.16) as should be the case.

In the next section we pay particular attention to the energy dependence of the nonrelativistic-type effective mass at low energy, i.e., for $|\epsilon|/m \ll 1$. Equations (4.15)–(4.17) can then be approximated as follows:

$$\frac{\tilde{m}(\epsilon)}{m} \approx \left[1 + \frac{m}{k}\frac{\partial}{\partial k}\mathcal{Y}_e(k;\epsilon)\right]^{-1}_{k=k(\epsilon)} , \tag{4.18}$$

$$\frac{\overline{m}(\epsilon)}{m} \approx 1 - [\partial\mathcal{Y}_e(k;\epsilon)/\partial\epsilon]_{k=k(\epsilon)} , \tag{4.19}$$

$$\frac{m^*(\epsilon)}{m} \approx \frac{\tilde{m}(\epsilon)}{m}\left[\frac{m_e^*(\epsilon)}{m} + \frac{\overline{m}(\epsilon)}{m} - 1\right] , \tag{4.20}$$

$$\frac{\tilde{m}(\epsilon)}{m} \cdot \frac{\overline{m}(\epsilon)}{m} = 1 - \frac{d}{d\epsilon}\mathcal{Y}_e(k(\epsilon);\epsilon) . \tag{4.21}$$

In order to simplify the discussion, we shall drop the terms contained within the square brackets on the right-hand side of Eqs. (4.12b) and (4.13b). These terms are not negligible but are small; they would not significantly modify our conclusions. We shall thus use the following approximations:

$$\mathcal{Y}_e(k;\omega) \approx \mathcal{V}_\sigma(k;\omega) + \mathcal{V}_0(k;\omega) . \tag{4.22}$$

Correspondingly, we shall approximate $V_e(\epsilon)$ by

$$V_e(\epsilon) \approx V_\sigma(\epsilon) + V_0(\epsilon) . \tag{4.23}$$

These approximations can easily be dropped.

D. Relativistic Brueckner-Hartree-Fock approximation

In the present section, we discuss the type of energy dependence expected for the nonrelativistic-type effective mass $m^*(\epsilon)$ when one uses the relativistic Brueckner-Hartree-Fock (RBHF) approximation. The latter is the one that has been used in the most recent microscopic calculations of the relativistic mean field (see, e.g., Refs. 8, 9, and 20). We first exhibit the analytical properties of the quantities in the vicinity of the Fermi energy. We then give rough quantitative estimates, derived from numerical results contained in Ref. 9.

Let us first consider the k mass $\tilde{m}(\epsilon)$, as approximated by Eq. (4.18). We shall argue below that for $\epsilon \gg \epsilon_F$ the quantity $\overline{m}(\epsilon)$ is expected to be close to unity; for $\epsilon \gg \epsilon_F$, the quantity $\tilde{m}(\epsilon)$ can be estimated by using Eq. (4.21) and from the calculated energy dependence of $V_0(\epsilon)$ and $V_\sigma(\epsilon)$. The latter could be characterized by the quantities

$$m_0^*(\epsilon)/m = 1 - \frac{d}{d\epsilon}V_0(\epsilon); \quad m_\sigma^*(\epsilon)/m = 1 - \frac{d}{d\epsilon}V_\sigma(\epsilon) . \tag{4.24a}$$

Figures 4.1a, 4.1b, and Table 4.2 of Ref. 9 indicate that, in the range $0 < \epsilon < 150$ MeV (which corresponds to $k/k_F < 2$), the potential strengths $V_0(\epsilon)$ and $V_\sigma(\epsilon)$ are roughly linear functions of energy, with

$$m_0^*(\epsilon)/m \approx 1.34; \quad m_\sigma^*(\epsilon)/m \approx 0.69 . \tag{4.24b}$$

Equations (4.12b), (4.21), and (4.24b) yield the following estimate:

$$\tilde{m}/m \approx 1.03 , \tag{4.24c}$$

for $0 < \epsilon < 150$ MeV. Note that $\tilde{m}/m$ is somewhat larger than unity, while in the nonrelativistic case the corresponding quantity [Eq. (2.17)] is sizably smaller than unity. The origin of this difference is that, in the nonrelativistic case, $\tilde{m}$ describes the spatial nonlocality of the microscopic potential, while in the relativistic case it results from the *combined* effect of the spatial nonlocalities of the scalar and of the vector components of the field.

The Lorentz mass $m_e^*(\epsilon)$ is defined in Eq. (4.14). It smoothly increases with increasing ϵ since $V_0(\epsilon)$ smoothly decreases with increasing ϵ. Figure 4.1b and Table 4.2 of Ref. 9 show that

$$m_e^*(\epsilon)/m \approx 0.69 + 3.6\times10^{-4}\epsilon , \tag{4.25}$$

in the energy domain $|\epsilon| < 150$ MeV.

Finally, we turn to the E mass $\overline{m}(\epsilon)$, as approximated by Eq. (4.20). One of the main characteristics of the RBHF approximation is that

$$W_\sigma(\omega)=W_0(\omega)=0 \quad \text{for } \omega<\epsilon_F \ . \tag{4.26}$$

The dispersion relation (4.3) then reduces to

$$\mathcal{V}(k;\omega)\approx \mathcal{V}^{(\infty)}(k)+\Delta V(\omega) \ , \tag{4.27}$$

$$\Delta V(\omega)=\frac{1}{\pi}\int_{\epsilon_F}^{\infty}\frac{W(\omega')}{\omega'-\omega}d\omega' \ , \tag{4.28}$$

where we have assumed that $\mathcal{W}(k;\omega')$ is independent of k in keeping with the approximation (4.4) and (4.5); this is legitimate for our present purpose. For $\omega<\epsilon_F$, one can calculate the derivative of $\Delta\mathcal{V}(\omega)$ with respect to ω by taking the derivative of the integrand on the right-hand side of Eq. (4.28). For $\epsilon<\epsilon_F$, Eqs. (4.19), (4.22), (4.27), and (4.28) show that

$$\frac{\overline{m}(\epsilon)}{m}=1-\frac{1}{\pi}\int_{\epsilon_F}^{\infty}\frac{W_\sigma(\omega')+W_0(\omega')}{(\omega-\omega')^2}d\omega' \ . \tag{4.29}$$

Since the sum $W_\sigma(\omega')+W_0(\omega')$ is negative for all ω' (see Table 4.2 of Ref. 9) and vanishes like $(\omega'-\epsilon_F)^2$ for ω' close to ϵ_F, the quantity $\overline{m}(\epsilon)/m$ has a positive infinite slope at $\epsilon=\epsilon_F$:

$$\left[\frac{d}{d\epsilon}\overline{m}(\epsilon)/m\right]_{\epsilon=\epsilon_F}=+\infty \ , \tag{4.30a}$$

while the slope is positive for $\epsilon<\epsilon_F$:

$$\left[\frac{d}{d\epsilon}\overline{m}(\epsilon)/m\right]>0 \quad \text{for } \epsilon<\epsilon_F \ . \tag{4.30b}$$

These relations (4.30a) and (4.30b) hold *exactly* within the RBHF approximation. In particular, they are influenced neither by the approximations (4.22) and (4.23) nor by our assumption that $\mathcal{W}(k;\omega)$ is independent of k. They directly derive from the dispersion relation (4.28) and from the behavior [(4.4), (4.5)] of $\mathcal{W}_0(k;\omega)$ and of $\mathcal{W}_\sigma(k;\omega)$ for ω close to ϵ_F. They are typical of the (relativistic as well as nonrelativistic) BHF approximation because they originate from the fact that, in this approximation, the imaginary part of the mean field vanishes for $\omega<\epsilon_F$, see Eq. (4.26); we return to this point in Sec. IV E. In the RBHF approximation, the quantity $\overline{m}(\epsilon)/m$ is thus larger than unity for $\epsilon<\epsilon_F$ and has a vertical slope at $\epsilon=\epsilon_F$; since it is continuous at $\epsilon=\epsilon_F$, it continues to increase for ϵ somewhat larger than ϵ_F. We shall see that it then decreases below unity and finally increases again to reach unity for $\epsilon\gg\epsilon_F$. This behavior is *qualitatively* the same as that encountered in the nonrelativistic BHF approximation. However, we now show that a very large *quantitative* difference exists between the relativistic and nonrelativistic cases.

In order to estimate the various quantities, we use the results of the RBHF approximation calculated by ter Haar and Malfliet.[9] Figure 4.7 of Ref. 9 represents the momentum dependence of the quantity $m^*_{\rm NR}/m$, defined in Eq. (3.28); we recall (Sec. III G) that, at low energy, this quantity is very close to the nonrelativistic-type effective mass $m^*(\epsilon)/m$. It is represented by the short-dashed curve in Fig. 1. It is seen to have a characteristic variation in the vicinity of the Fermi momentum. This is

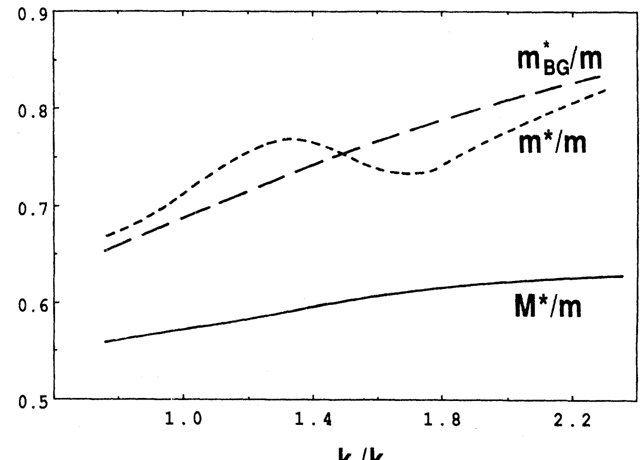


FIG. 1. Dependence upon k/k_F of various quantities related to the effective mass, as evaluated in the framework of the RBHF approximation for $k_F=1.34$ fm^{-1}. The solid curve and the short-dashed line have been adapted from Fig. 4.7 of Ref. 9. The solid curve represents the Dirac mass M^*/m [Eq. (4.10)]. The short-dashed line gives the nonrelativistic-type effective mass $M^*_{\rm NR}/m$ [Eq. (3.28)]. The long-dashed curve is a rough guess of the background contribution $m^*_{\rm BG}/m$, that we identified with the right-hand side of Eq. (4.31a).

in keeping with our discussion on the energy dependence of the E mass $\overline{m}(\epsilon)$. We now give a rough estimate of $\overline{m}(\epsilon)/m$. We expect that, for ϵ close to ϵ_F, the nonmonotonic part of the energy dependence of $m^*(\epsilon)$ is due to $\overline{m}(\epsilon)$, since in Eq. (4.20) the quantities $\tilde{m}(\epsilon)$ and $m_e^*(\epsilon)$ are very smooth functions of energy. The long-dashed curve in Fig. 1 represents a typical value of the "background" contribution

$$\frac{m^*_{\rm BG}(\epsilon)}{m}\approx\frac{\tilde{m}(\epsilon)}{m}\frac{m_e^*(\epsilon)}{m} \ . \tag{4.31a}$$

This long-dashed curve is a guess drawn by eye, using the property that $\overline{m}(\epsilon)/m$ should become close to unity for $\epsilon\gg\epsilon_F$. The background contribution (4.31a) is the value that $m^*(\epsilon)/m$ would take if one would set $\overline{m}(\epsilon)/m$ equal to unity in Eq. (4.20). The latter then yields

$$\overline{m}(\epsilon)/m\approx 1+[m^*(\epsilon)-m^*_{\rm BG}(\epsilon)]/\tilde{m}(\epsilon) \ . \tag{4.31b}$$

We use the latter approximation for estimating $\overline{m}(\epsilon)$, adopting for $m^*(\epsilon)$ and $\tilde{m}(\epsilon)$ the values deduced by combining Eqs. (4.25) and (4.31a); this yields the solid curve in Fig. 2. There, the long-dashed curve represents the value of $\overline{m}(\epsilon)/m$ calculated from the *nonrelativistic* BHF approximation. In the relativistic as well as in the nonrelativistic BHF approximation, $\overline{m}(\epsilon)/m$ has a vertical slope at $k=k_F$. However, this property is hardly visible in the relativistic case. Likewise, the existence of a maximum of $\overline{m}$ for k somewhat larger than k_F is quite pronounced in the nonrelativistic case but is almost invisible in the relativistic case. We now describe the origin of this large difference.

We still omit the dependence of $\mathcal{W}(k;\omega)$ upon k. In

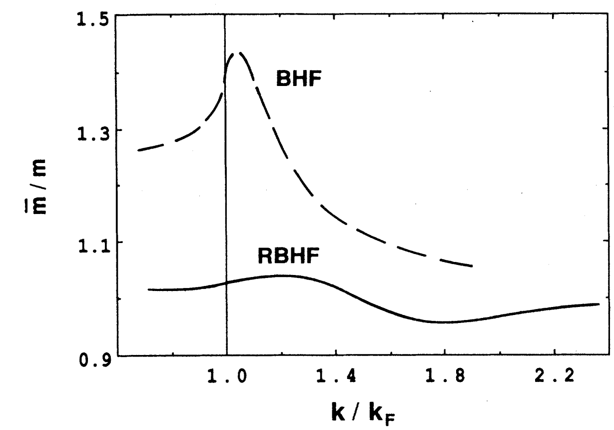


FIG. 2. Dependence of the quantity $\overline{m}/m$ upon k/k_F. The long-dashed curve corresponds to the nonrelativistic BHF approximation for $k_F=1.36\ \mathrm{fm}^{-1}$ (see Fig. 12 of Ref. 2), and the solid curve to the relativistic BHF approximation for $k_F=1.34\ \mathrm{fm}^{-1}$. The solid line has been evaluated from Eq. (4.20), as explained in the text, using as input results given in Ref. 9 (see also Fig. 3 below).

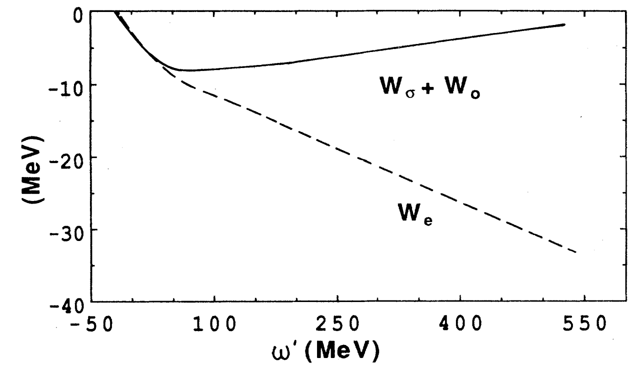


FIG. 3. The solid curve represents the energy dependence of the quantity $W_\sigma(\omega')+W_0(\omega')\approx Z_e(\omega')$ which can be identified with the numerator of the integrand in the dispersion relation (4.32b), for $\omega/m \ll 1$. The dashed line gives the value of $W_e(\omega')=Z_e(\omega')+(\omega'/m)W_0(\omega')$, which approximates the imaginary part of the Schrödinger-equivalent potential. These quantities have been calculated from results of the RBHF approximation given in Ref. 9.

the *nonrelativistic* case, the dispersion integral in Eq. (4.28) then involves the imaginary part of the nonrelativistic mean field:

$$\Delta V_{\mathrm{BHF}}(\omega)=\frac{1}{\pi}\int_{\epsilon_F}^{\infty}\frac{W_{\mathrm{BHF}}(\omega')}{\omega'-\omega}d\omega' \;; \tag{4.32a}$$

the numerator can be approximated by the imaginary part of the nonrelativistic optical-model potential. In the *relativistic* case, Eqs. (4.13a) and (4.28) yield instead, for $\omega/m \ll 1$,

$$\Delta V_{\mathrm{RBHF}}(\omega)=\frac{1}{\pi}\int_{\epsilon_F}^{\infty}\frac{Z_e(\omega')+\frac{\omega}{m}W_0(\omega')}{\omega'-\omega}d\omega' \;. \tag{4.32b}$$

Note that the numerator contains a factor (ω/m), *not* (ω'/m). Therefore this numerator in the integrand in Eq. (4.32b) is *quite different* from $W_e(\omega')$ for large ω'. Indeed, one has

$$W_e(\omega')=Z_e(\omega')+\frac{\omega'}{m}W_0(\omega') \;. \tag{4.33}$$

This difference is exhibited in Fig. 3. Hence, in the relativistic case, the dispersion relation (4.28) does not yield a relation between the phenomenological values of the real and imaginary parts of the Schrödinger-equivalent potential, in sharp contrast with the nonrelativistic case. It has been observed[6,9] that the imaginary part $W_e(\omega')$ of the Schrödinger-equivalent potential is very close to $W_{\mathrm{BHF}}(\omega')$, and that both quantities are in good agreement with the strength of the imaginary part of the phenomenological optical-model potential. This explains why the energy dependence of $\overline{m}(\epsilon)$ for ϵ close to ϵ_F is much less pronounced in the relativistic than in the nonrelativistic case; indeed, the energy numerator in the integrand in Eq. (4.32b) is much smaller than the quantity $W_e(\omega')\approx W_{\mathrm{BHF}}(\omega')$ for large ω', as exhibited by the difference between the solid curve and the dashed curve in Fig. 3.

Finally, few remarks are in order concerning the contribution of $\mathcal{V}_\sigma$ and of $\mathcal{V}_0$ to the quantity $\Delta V(\omega)$. As in the nonrelativistic case, $\mathcal{V}^{(\infty)}(k)$ in Eq. (4.3) can approximately be identified with the Hartree-Fock field, and the quantity $\Delta V(\omega)$ can be called the dispersive contribution.[3] In the relativistic case, $\Delta V(\omega)$ has two Lorentz components, namely (we still omit the dependence upon k),

$$\Delta V_\sigma(\omega)=\frac{1}{\pi}\int_{\epsilon_F}^{\infty}\frac{W_\sigma(\omega')}{\omega'-\omega}d\omega' \;, \tag{4.34a}$$

$$\Delta V_0(\omega)=\frac{1}{\pi}\int_{\epsilon_F}^{\infty}\frac{W_0(\omega')}{\omega'-\omega}d\omega' \;. \tag{4.34b}$$

Since $W_\sigma(\omega')$ is positive and $W_0(\omega')$ negative,[9] one has

$$\Delta V_\sigma(\epsilon)>0,\quad \Delta V_0(\epsilon)<0 \quad \text{for } \epsilon<\epsilon_F \;. \tag{4.35}$$

These inequalities also hold for ϵ somewhat larger than ϵ_F since $\Delta V_\sigma(\epsilon)$ and $\Delta V_0(\epsilon)$ are smooth functions of ϵ. They are in keeping with the numerical results shown in Ref. 8 (Fig. 11) and in Ref. 9 (Figs. 4.1a and 4.1b).

Finally, we discuss the contributions of $\Delta V_\sigma(\epsilon)$ and of $\Delta V_0(\epsilon)$ to the E mass $\overline{m}(\epsilon)$. Equations (4.12b), (4.16), and (4.19) show that, for $\epsilon/m \ll 1$,

$$\overline{m}(\epsilon)/m \approx \overline{m}_\sigma(\epsilon)/m+\overline{m}_0(\epsilon)/m-1 \;, \tag{4.36a}$$

with

$$\overline{m}_\sigma(\epsilon)/m=1-\frac{d}{d\epsilon}\Delta V_\sigma(\epsilon),\quad \overline{m}_0(\epsilon)/m=1-\frac{d}{d\epsilon}\Delta V_0(\epsilon) \;. \tag{4.36b}$$

From a similar reasoning as that carried out previously in connection with relations (4.29)–(4.30b), one can check that, for ϵ close to ϵ_F, the quantity $\overline{m}_\sigma(\epsilon)/m$ is smaller than unity for ϵ close to ϵ_F, with a negative energy derivative that is infinite at $\epsilon=\epsilon_F$. In contrast, $\overline{m}_0(\epsilon)/m$ is larger than unity, with a positive energy derivative that is infinite at $\epsilon=\epsilon_F$. The value of $\overline{m}_0(\epsilon)/m-1$ is larger than that of $1-\overline{m}_\sigma(\epsilon)/m$ because $|W_0(\epsilon)|>|W_\sigma(\epsilon)|$; this explains why $\overline{m}(\epsilon_F)>m$ and why $(d/d\epsilon)\overline{m}(\epsilon)$ is positive infinite at $\epsilon=\epsilon_F$. These results are in keeping with numerical calculations recently carried out by Poschenrieder and Weigel.[39,42] The physical interest of $\overline{m}_0(\epsilon)/m$ and of $\overline{m}_\sigma(\epsilon)/m$ is that they are intimately related to the residues of the one-body Green's functions at the quasiparticle pole.[39]

E. Correlation correction to the RBHF approximation

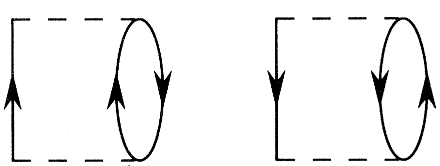
We showed in Sec. IV D that the quantity $\overline{m}(\epsilon)/m$ has a positive infinite energy derivative at $\epsilon=\epsilon_F$, in the RBHF approximation. We now point out that, as in the nonrelativistic case, this singularity of the derivative of $\overline{m}(\epsilon)$ is canceled by the contribution of the "correlation contribution"[2] to the mean field. This correlation contribution is schematically represented by the graph CO in Fig. 4. It corresponds to including a term $\Delta V_{\rm CO}(\omega)$ in the relativistic potential. Equation (4.3) then becomes

$$\mathcal{V}(k;\omega)\approx\mathcal{V}^\infty(k)+\Delta V(\omega)+\Delta V_{\rm CO}(\omega)\ , \tag{4.37a}$$

where the quantity $\Delta V_{\rm CO}(\omega)$ is given by the dispersion relation:

$$\Delta V_{\rm CO}(\omega)=\frac{1}{\pi}\int_{-\infty}^{\epsilon_F}\frac{W(\omega')}{\omega'-\omega}d\omega'\ . \tag{4.37b}$$

In Eqs. (4.37a) and (4.37b), we omitted the dependence of ΔV upon k and we have used the fact that the main characteristic of the correlation contribution is that the corresponding imaginary part $W(\omega)$ vanishes for $\omega>\epsilon_F$.[3] Arguments similar to those developed in Sec. IV D indicate that the contribution $\Delta V_{\rm CO}(\omega)$ alone yields an effective mass $\overline{m}(\epsilon)/m$ which is larger than unity and has a negative derivative for $\epsilon\gtrsim\epsilon_F$. This contribution to $\overline{m}(\epsilon)$ has a vertical slope at $\epsilon=\epsilon_F$; it reaches a maximum for ϵ somewhat smaller than ϵ_F. The energy derivative of the full E mass is finite at $\epsilon=\epsilon_F$. The interest of the existence of $\Delta V_{\rm CO}$ in the present context is that its contribution is expected to increase the value of the nonrelativistic-type effective mass $m^*(\epsilon)/m$ at the Fermi energy, as in the nonrelativistic case.[3] In the nonrelativistic case, the existence of this contribution was pointed out by Brueckner and Goldman,[43] who showed that it plays an important role in the fulfillment of the Hugenholtz–Van Hove theorem;[44] this is also true in the relativistic case.[45]



PO CO

FIG. 4. Second-order contributions to the mean field. Upward pointing arrows are associated with particle states ($k>k_F$) and downward pointing arrows to hole states ($k<k_F$), where particles and holes refer to the Fermi sea (*not* to the Dirac sea). The "polarization" (PO) contribution is included in the RBHF approximation, but not the "correlation" (CO) contribution.

F. Nonlocality corrections to the mean free path of a nucleon

In Eq. (4.8) we have neglected the role of the imaginary component $\mathcal{W}_e(k;\epsilon)$ of the optical potential. Actually, Eq. (4.7a) implies that the momentum $k(\epsilon)$ is a complex quantity $k=k_R+ik_I$; the imaginary part k_I is related to the mean free path λ by $\lambda=(2k_I)^{-1}$. The fact that k_I is complex, associated to the feature that the real part $\mathcal{V}_e(k;\omega)$ of the Schrödinger equivalent depends upon k, modifies the expression of the imaginary component $\mathcal{W}_e$. Arguments similar to those developed in Ref. 46 indeed show that the energy-momentum relation becomes

$$\frac{\epsilon^2}{2m}+\epsilon\approx\frac{k^2}{2m}+V_e(\epsilon)+i\frac{\tilde{m}(\epsilon)}{m}W_e(\epsilon) \tag{4.38a}$$

when k_I is taken into account. In Eq. (4.38a), the quantity $\tilde{m}(\epsilon)/m$ is defined by [see Eq. (4.18)]

$$\frac{\tilde{m}(\epsilon)}{m}=\left[1+\frac{m}{k_R}\frac{\partial\mathcal{V}_e(k_R;\epsilon)}{\partial k_R}\right]_{k_R=k_R(\epsilon)}; \tag{4.38b}$$

here, $k_R(\epsilon)$ is given by Eq. (4.8), with k replaced by k_R. Correspondingly, the quantities $V_e(\epsilon)$ and $W_e(\epsilon)$ are defined by Eqs. (4.9b) and (4.9c), with $k(\epsilon)$ replaced by $k_R(\epsilon)$. Equations (4.38a) and (4.38b) extend an expression used in Ref. 47. The nonlocality correction factor $\tilde{m}/m$ which appears in the last term on the right-hand side of Eq. (4.38a) is formally the same as in the nonrelativistic case. Here, however, its value is expected to be close to unity, while it is sizably smaller than unity in the nonrelativistic case.[46] We recall that the nonrelativistic-type wave function Ψ_e differs from the upper components ϕ_u of the Dirac spinor [Eq. (3.8b)], and that the latter furthermore cannot be associated with a probability density. We return to this point in Sec. V.

G. Overview

The nonrelativistic-type effective mass $m^*(\epsilon)$ defined by Eq. (4.11) is the one that should be compared with the effective mass derived from analyses of the experimental data performed in the framework of the nonrelativistic shell and optical models. The short-dashed line in Fig. 5 represents the value of m^*/m, as evaluated from the RBHF approximation.[9] Equation (4.20) shows that $m^*(\epsilon)$ has a threefold origin.

(i) The k mass $\tilde{m}(\epsilon)$ [Eq. (4.15)], which reflects the fact that the microscopic relativistic potential is nonlocal in the spatial coordinates. The value of $\tilde{m}(\epsilon)$ has been estimated from Eq. (4.31a), where $m^*_{\rm BG}$ is given by the

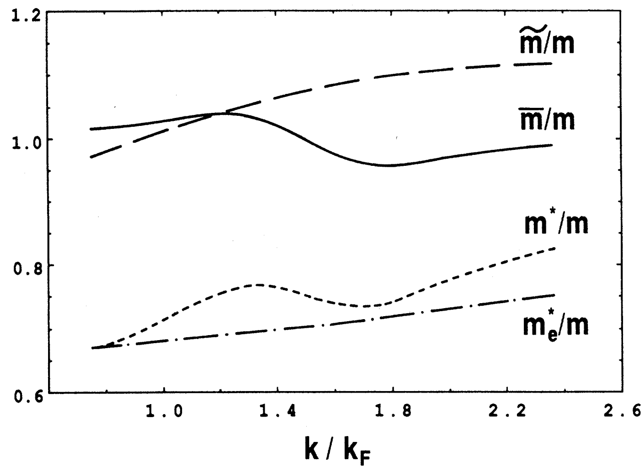


FIG. 5. The short-dashed line gives the nonrelativistic-type effective mass m^*/m. The k mass $\tilde{m}/m$ is represented by the long-dashed line, E mass $\overline{m}/m$ by the solid curve, and the Lorentz mass m_e^*/m by the dash-dotted line. These estimates are derived from results of the RBHF approximation given in Ref. 9.

long-dashed curve in Fig. 2. The result is represented by the long-dashed curve in Fig. 5.

(ii) The E mass $\overline{m}(\epsilon)$ [Eq. (4.16)], which derives from the fact that the microscopic mean field is nonlocal in time. Its energy dependence is represented by the solid curve in Fig. 5.

(iii) The Lorentz mass $m_e^*(\epsilon)$ [Eq. (4.11)], which arises from the Lorentz transformation properties of the vector field. This contribution is the only one that was encountered in Sec. III. It is represented by the dash-dotted line in Fig. 5.

Figure 5 shows that the value of the nonrelativistic-type effective mass $m^*(\epsilon)$ is mainly determined by the Lorentz mass $m_e^*(\epsilon)$, but that the latter is smaller than $m^*(\epsilon)$. This is likely to be reinforced by the correlation contribution (Sec. IV E): the resulting value of $m^*(\epsilon_F)$ should be in good agreement with the empirical value [Eq. (2.8)]. The increase of $m_e^*(\epsilon)$ with increasing ϵ is due to the decrease of $V_0(\epsilon)$. In turn, this decrease mainly reflects the nonlocality of the vector component of the potential in the spatial coordinates. The corresponding range of nonlocality[48] is about 0.6 fm for $k < k_F$ but decreases with increasing k; indeed, Fig. 3 of Ref. 49 suggests that the range of nonlocality becomes close to zero for $k/k_F \gg 1$, where the vector and scalar fields tend to become local. This energy dependence of the range of nonlocality of the vector and scalar fields possibly reflects the property that the exchange of light mesons plays a major role at low energy, while the exchange of heavy mesons becomes dominant at high energy.

V. SUMMARY AND DISCUSSION

The relativistic approaches to the nuclear mean field are characterized by the fact that the single-particle wave equation is the Dirac rather than the Schrödinger equation. By eliminating the lower components of the Dirac spinor, one obtains a Schrödinger-type equation for a function Ψ_e which, at large radial distance, has the same asymptotic behavior as the upper components of the Dirac spinor. Accordingly, the Schrödinger-type equation yields the same scattering cross sections and single-particle energies as the original Dirac equation. This provides a link between the relativistic mean field that appears in the Dirac equation and the nonrelativistic descriptions of the shell- and of the optical-model potentials. Here, this link between the Dirac potential and its "Schrödinger equivalent" [Eq. (3.6)] has been used to identify the quantity $m^*(\epsilon)$ [Eq. (4.11)] that should be compared with the empirical effective mass m_s^* [Eq. (2.4)] derived from nonrelativistic analyses of scattering and bound-state data; this empirical effective mass characterizes the energy dependence of the real part of the nonrelativistic shell- and optical-model potentials. We called $m^*(\epsilon)$ the nonrelativistic-type effective mass; we emphasize that this expression does *not* imply that a nonrelativistic limit has been taken: the word "nonrelativistic" rather refers to a very close analogy with the effective mass used in nonrelativistic descriptions of the shell- and optical-model potentials. For simplicity, we restricted the discussion to nuclear matter and to the case when the Dirac mean field only has two Lorentz components, namely, a scalar (U_σ) and the fourth component of a vector (U_0). The various definitions and relations can easily be extended to finite nuclei and to the case when more Lorentz components exist.

In Sec. III we considered the case where U_0 and U_σ are independent of energy. This simplification is encountered in Walecka's model[5] and in the Hartree approximation.[4] Even though the Dirac potential is independent of energy, its Schrödinger-equivalent potential depends upon energy. This energy dependence is linear, and the corresponding nonrelativistic-type effective mass [Eq. (3.16)] is thus independent of energy. It derives from the difference between the Lorentz transformation properties of U_σ and U_0;[21,22] we therefore call m_e^* the Lorentz mass. In the model of Sec. III, the Lorentz mass and the nonrelativistic-type effective mass are identical, but they differ in more general cases (Sec. IV). The simplicity of the mean-field model considered in Sec. III enabled us to clarify the difference between various quantities which have been related to the expression "effective mass" in relativistic descriptions of the mean field. (i) In most recent papers, e.g., Refs. 9, 19, and 20, this expression is used to denote the Dirac mass M^* [Eq. (3.14)]. The Dirac mass is smaller than the nonrelativistic-type effective mass (Fig. 1). It should therefore not be directly compared with the effective mass m_s^* derived from nonrelativistic shell- and optical-model potentials. (ii) The group velocity involves a quantity m_g^* [Eq. (3.21)] that we called the group mass; in the model of Sec. III, m_g^* is a linear function of energy. (iii) At the Fermi energy, the group mass is identical to the Landau mass [Eq. (3.25)], which is intimately related to the Landau parameter F_1, see Refs. 12, 25, and 26. We emphasize that the definition of the Dirac, group, and Landau masses does not imply that any nonrelativistic limit is taken. At low energy, various approximations have been introduced. The quantity m^*_{NR} [Eq. (3.28)] was used in Ref. 9. At low energy, it is very close to the nonrelativistic-type effective mass m^*, but it differs from m^* at high energy. It is associated with a potential energy $V_{NR}(\epsilon)$ [Eq. (3.29)] that differs from the Schrödinger-equivalent potential at high energy.[29] The quantity M^{NR}_* [Eq. (3.32)] was introduced in Refs. 30 and 31. It is the average of the Dirac mass and the Lorentz mass; it should not be directly compared with the empirical nonrelativistic effective mass, even at low energy.

In Sec. III, the Dirac potential was assumed to be independent of energy. In contrast, microscopic calculations[6,8,9,39] yield Dirac potentials that depend upon the nucleon momentum k and upon the nucleon frequency ω. This is already true in nonrelativistic microscopic calculations of the mean field (Sec. II). The dependence of the mean field upon k reflects its nonlocality in the spatial coordinates; its dependence upon ω reflects its nonlocality in time [Eqs. (2.9)–(2.11)]. In the nonrelativistic case, this led to the introduction of the k mass $\tilde{m}$ [Eq. (2.17)] and of the E mass $\overline{m}$ [Eq. (2.18)].[2] The effective mass proper is equal to the product $\tilde{m}\overline{m}/m$, see Eq. (2.21). In Eqs. (4.15) and (4.16), we have extended the definitions of the k mass and of the E mass to the relativistic approach. In that case, the relation [Eq. (4.17)] between the nonrelativistic-type effective mass m^* [Eq. (4.11)], the k mass, and the E mass involves another quantity, namely the Lorentz mass m^*_e. The latter is the only one that differed from the nucleon mass m in the simple model considered in Sec. III. In the relativistic Brueckner-Hartree-Fock (RBHF) approximation (Sec. IV D), the energy derivative of the E mass is positive for energies smaller or somewhat larger than the Fermi energy and is positive infinite at the Fermi energy [Eqs. (4.30a) and (4.30b)]. The slope of the nonrelativistic-type effective mass m^* is also positive infinite at the Fermi energy. These properties are identical to those encountered in the nonrelativistic Brueckner-Hartree-Fock (BHF) approximation and have the same origin, namely the fact that in the BHF approximation the imaginary part of the mean field vanishes below the Fermi energy. The derivative of m^* at the Fermi energy becomes finite when one takes into account the correlation contribution to the BHF approximation (Sec. IV E). The energy dependence of the E mass near the Fermi energy can most conveniently be discussed in terms of dispersion relations. These can be related to a causality property.[50] Since the Schrödinger-equivalent potential does not fulfill a dispersion relation, it is not "causal." This should not be viewed as being troublesome since the corresponding wave function cannot be interpreted in terms of a probability density (Sec. IV F).

While the behavior of the E mass near the Fermi energy is qualitatively the same in the relativistic as in the nonrelativistic BHF approximation, its energy dependence is quantitatively very different in the two cases (Fig. 2). The origin of this difference is that in the relativistic case the dispersion relation [Eq. (4.3)] that connects the real and imaginary parts of the mean field does *not* yield a dispersion relation between the real and imaginary parts of the Schrödinger-equivalent potential. This is mainly due to the appearance of a factor ω/m (instead of ω'/m) in the numerator of the integrand on the right-hand side of Eq. (4.32b). For large ω', this numerator is thus very different from the imaginary part of the Schrödinger-equivalent potential (Fig. 3). This is why the energy dependence of the quantities $m^*(\epsilon)$ and $\overline{m}(\epsilon)$ near the Fermi energy is much weaker in the relativistic than in the nonrelativistic BHF approximation. It would be of great interest to investigate whether the value of $\overline{m}(\epsilon)$ is related to the occupation probability of the Fermi sea in the case of the RBHF approximation, as it is in the nonrelativistic BHF approximation.[2] Indeed, this would imply that in the relativistic BHF approximation the Fermi sea is almost fully occupied; this is amenable to experimental tests, see, e.g., Ref. 51. It would also be of interest to calculate more accurately the quantities $\overline{m}(\epsilon)$ and $\tilde{m}(\epsilon)$, by performing *partial* derivatives of the mean field. Indeed, the energy dependence of $m^*(\epsilon)$ involves total derivatives, which are not well suited to accurate numerical calculations. The numerical values shown in Figs. 1–3 and 5 should thus be considered as semiquantitative estimates.

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