Laser applications to fundamental weak interaction studies

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The possibility of performing weak interaction experiments using spectroscopic techniques is discussed. In such an experiment, a laser is tuned to an electronic transition of the daughter, and a signal whose strength is proportional to the number of recoil atoms is monitored, such as fluorescence, absorption, or anisotropy induced in radiation emitted in a subsequent nuclear decay. As a specific example of this technique, we consider measuring the angular correlation between the momenta of the electron and the antineutrino emitted in the decay $^{85}\mathrm{Kr}\rightarrow{}^{85}\mathrm{Rb}^{m}+\beta^{-}+\bar{\nu}$ using the laser-induced nuclear orientation technique. Such an experiment is currently under way.

I. INTRODUCTION

Kinematic studies, in which the recoil energy or momentum of the daughters are observed, have been of fundamental importance to our understanding of β decay and weak interactions in general.[1] Such studies yield information about the angular correlation between the momenta of the β and $\bar{\nu}$ emitted in the decay,[2-7] and have provided experimental proof that weak interactions are due to the "vector" and "axial-vector" terms in the Hamiltonian, rather than the "scalar" and "tensor" terms. Kinematic studies have also been used to place an upper bound upon the rest mass of the neutrino[8,9] (although the sensitivity achieved in these experiments is not nearly as good as has been obtained for distortions characteristic of nonzero ν mass near the end points of the β spectra[10,11]). Measurements of the angular correlation are also of interest since they can be used to determine the Fermi and Gamow-Teller matrix elements. This information can be used, for example, to calculate the mixing angle between the u and d quarks as a high-precision test of the three-generation standard model.[12]

Since the correlation coefficient implicitly contains helicity information, angular correlation measurements may aid in the search for right-handed currents in nuclear β decay.[13] Usual searches for this current involve measuring the helicity of the emitted electron (positron). Recently Quin and Girard have pointed out that, for selected transitions involving polarized nuclei, the effects of such currents can be enhanced by an order of magnitude over the unpolarized case.[14] Thus, measuring the angular correlation for polarized β decay could yield important information concerning the possible existence of right-handed currents.

Experiments to measure this angular correlation typically collect the recoil ions and measure their velocities directly. They are quite difficult to perform and, consequently, only a relatively few nuclei have been studied. In this paper, we discuss the possibility of measuring the velocity spectra of recoiling atoms by spectroscopic techniques. In such an experiment, a laser is tuned to an electronic transition of the daughter, and a signal is monitored whose strength is proportional to the number of recoil atoms, such as fluorescence, absorption, or, as we discuss below, anisotropy in a subsequent nuclear decay. If the bandwidth of the laser is narrow enough, the beam will only interact with the small group of atoms whose velocities are such that they are Doppler shifted into resonance. The monitored signal will thus be proportional to the number of atoms in a particular velocity group, and scanning the laser through the Doppler profile of the atoms will allow the velocity spectra to be determined.

Such a technique may offer several advantages over those discussed above. The use of spectroscopic methods eliminates the need to physically sweep out the ions in order to measure their velocities, thus greatly simplifying the experimental apparatus. In addition, as pointed out by Cohen *et al.*,[15] the sensitivity inherent in the spectroscopic approach should result in measurements of much greater precision than are possible using direct detection of the ions. Lastly, this technique has the advantage of being selective of the daughter species, so that the recoil products of decays which proceed either to a variety of nuclear isomeric states, or which occur as a chain of β decays, may be studied individually by selectively tuning the laser to resonant transitions of each species in turn.

Weak interaction decays occur either as two-body processes [electron capture (EC) and bound-state β decay ($B\beta$) (Refs. 16-19)] in which only a neutrino is emitted from the atom, or as three-body processes (β^{+} and β^{-} emission) in which an electron or positron is emitted as well. In EC and $B\beta$ decays, the neutrino is emitted with a well-defined energy, $Q-E_k$, where Q is the energy available in the decay and E_k is the binding energy of the participating atomic electron. The atom recoils with an energy $(Q-E_k)^2/2Mc^2$. In the absence of any polarization of the parent, these decays occur isotropically, resulting in a constant distribution for the z component of the velocity of the daughters, with sharp cutoffs at $\pm v_0=(Q-E_k)/Mc$, provided the parent atoms are initially at rest. (In practice, the parents will have a thermal distribution of velocities, which must be convoluted with this recoil velocity distribution. This has the effect of smoothing out the sharp cutoff at $\pm v_0$.) In such two-

body decays, spectroscopic techniques could be used to accurately determine the end-point energy of the decay, Q, by observing this characteristic cutoff in the recoil velocity distribution. Cohen *et al.*[15] have proposed such an experiment to observe $B\beta$ decay in $^3H \rightarrow {}^3He$. They also discuss the feasibility of using this technique for neutrino mass measurements.

In three-body decays, both an electron (or positron) and a neutrino are emitted, and such a decay will be characterized by daughters with a broad range of recoil energies. The daughters recoil antiparallel to the sum of the electron and neutrino momenta, and their velocity distribution depends strongly upon any angular correlation between these two particles. Careful measurement of the velocity spectra of the recoil atoms will allow the determination of this angular correlation.

If thermalizing processes (such as collisions) occur, the width of the velocity distribution of the daughters will gradually be reduced to its thermal value and the information contained in the initial velocity spectra will be lost. Under such conditions, it is necessary to measure the velocities of the daughters immediately after the decay occurs, in a time of the order of the mean time between collisions. If the daughter is a short-lived radioactive species, its subsequent decay will remove it from resonance with the laser, effectively limiting the time over which its velocity is measured. The velocity distribution may then be measured spectroscopically either by observing the fluorescence yield or absorption (although, in practice, it may be quite difficult to obtain the densities necessary for such measurements) or by observing anisotropy in the subsequent decay, induced by the optical pumping of the daughter (see below).

In EC and $B\beta$ decays, the daughter emerges in a neutral state and may interact with the laser immediately. In β^- decays, however, the daughter is created as an ion, whose electronic transitions are typically in the far ultraviolet. In the absence of tunable high-power lasers in this region, these ions must be neutralized before they can be optically pumped. Collisions occurring during the time required for neutralization will tend to wash out the information contained in the initial velocity distribution of the daughters. As we show below, however, under certain conditions it should still be possible to extract this information.

The advent of tunable high-power ultraviolet lasers would allow the optical pumping of the ion directly. In such a case, neutralization would constitute an "effective" decay, since it would remove the atom from resonance with the laser, and this technique could then be extended to β decays to stable daughters (provided that the densities needed for fluorescence studies could be achieved). Pumping the ion would have the added advantage that, since the laser interacts with the atom at an earlier time than would be the case if neutralization were required, the effects of thermalizing collisions would be reduced.

An example will best serve to illustrate this general technique, and in the remainder of this paper, we consider an experiment using the laser-induced nuclear orientation (LINO) technique to measure the angular correlation between the momenta of the electron and antineutrino emitted in the decay $^{85}Kr \rightarrow {}^{85}Rb^m$. Such an experiment is currently underway at Massachusetts Institute of Technology (MIT). The experimental results and their analysis will appear later. Section II of this paper contains a brief description of the LINO technique in general and this experiment in particular. Section III presents a short review of those aspects of β decay which are of interest to this problem, while Sec. IV contains a treatment of the thermalization and neutralization of the recoil ions in the cell, and includes predictions of the width of the $^{85}Rb^m$ velocity distribution as a function of buffer gas pressure for both the case of pure "Fermi," and pure "Gamow-Teller" decays.

II. LASER-INDUCED NUCLEAR ORIENTATION

The decay scheme of ^{85}Kr is shown in Fig. 1. This nucleus decays by β^- emission, generally to ^{85}Rb. A small percentage of the time (0.4%), however, ^{85}Kr decays to $^{85}Rb^m$, an isomeric state of ^{85}Rb, which subsequently decays by the emission of a 514 keV, $M2$ γ ray, with a half-life of 1.0 μs. The initial velocity distribution of the isomers produced in the β decay is several times wider than that of the thermalized atoms in the gas cell at room temperature, due to the recoil momentum transferred to the daughters during the decay. As shown below, there will be a range of gas-cell pressures over which the isomers will not have time to thermalize before their subsequent decay and their velocity distribution will thus retain information about their initial velocity. By optically pumping the electron levels of the isomers, it is possible to align their spins, relying on the hyperfine coupling between the electrons and the nuclei, and to thus induce anisotropy in the angular distribution of the radiation emitted in their subsequent decay. This technique, known as laser-induced nuclear orientation, has been used to study several short-lived nuclear systems[20–22] [including $^{85}Rb^m$, at high buffer gas pressures (4 Torr)],[20] and can yield very precise measurements of nuclear dipole and quadrupole moments as well as isomer shifts. As we indicate below, it may also be used to measure the Doppler width of the distribution of isomers in the cell. Such a measurement, under proper conditions of cell pressure and temperature, should allow the determination of the initial velocity distribution of the daughters, and hence the angular correlation between the momenta of the emitted β^- and the $\bar{\nu}$.

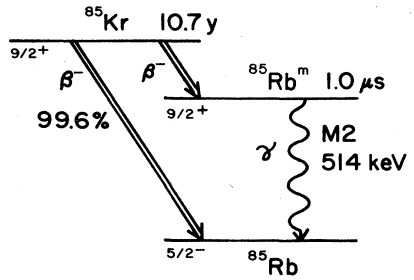

FIG. 1. Decay scheme for ^{85}Kr.

The $^{85}Rb^{m}$'s are produced predominantly in the singly ionized state. (A small percentage are initially doubly ionized due to Auger processes which result in the ejection of an additional electron. This is a small refinement, however, which we will neglect in this analysis.) The optical transitions of $^{85}Rb^{m+}$ lie far in the ultraviolet where tunable dye lasers are not available, so that in order to optically pump these atoms, it is necessary that they be neutralized. This may be accomplished through resonant charge exchange (CE) with natural Rb. Since the natural Rb is in its ground state and the CE process interchanges the electronic levels of the two atoms, the neutral $^{85}Rb^{m}$'s which result are produced in their ground states and are immediately available for optical pumping.

The sample for this experiment would thus consist of a small cell containing Kr gas with a large isotopic fraction of ^{85}Kr. In addition, the cell would contain a small amount of Rb metal. Heating the cell would then drive some of the Rb into the vapor phase, where it would be available for charge exchange. In the previous $^{85}Rb^{m}$ work,[20] it was found that a temperature of 160–180 °C represented a good compromise between efficient charge exchange on the one hand, and effects such as spin depolarization, which tends to destroy the anisotropy signal, as well as excessive absorption of the laser radiation by the natural Rb, on the other.

The neutralized $^{85}Rb^{m}$'s may be optically pumped with either linearly or circularly polarized light. The pumping scheme for circular polarization is shown in Fig. 2. The ground state of Rb consists of two hyperfine manifolds, with total angular momenta $F=4$ and 5, respectively. Two circularly polarized laser fields are used to pump these two manifolds, at rates R_1 and R_2, respectively. Each time an atom absorbs a photon it picks up one unit of angular momentum. In the subsequent decay down to the ground state, the atom either retains the same angular momentum, gains one unit, or loses one unit. Thus, during each such optical pumping cycle, there is a tendency for the atom to gain angular momentum. Since the lifetime of the $^2P_{1/2}$ excited electronic state is 27 ns, several such optical pumping cycles are possible during the lifetime of the isomer, with a resultant accumulation of population in the unpumped ground state level $F=5$, $M_F=5$. This corresponds to aligning the nuclear spins and results in an anisotropy in the angular distribution of the γ rays emitted in the subsequent decay. The electronic states may be appropriately grouped into pumped and unpumped levels, and a simple rate equation approach may be used to describe the optical pumping and accurately predict the anisotropy produced.[23] Thorough discussions of the LINO technique are contained in Refs. 20 and 24.

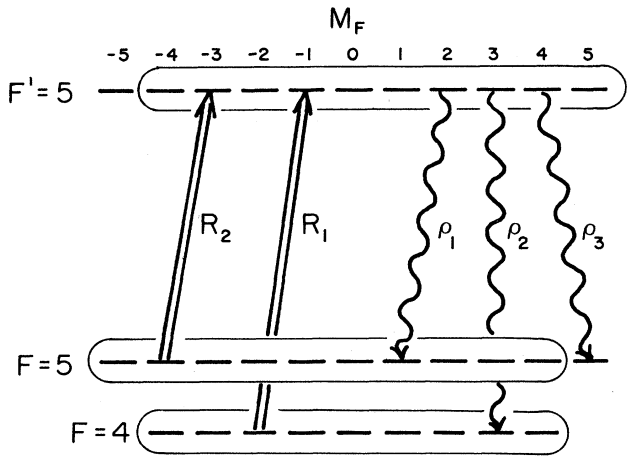

FIG. 2. Scheme for orienting the $^{85}Rb^{m}$ isomers by optically pumping their D_1 transition with two circularly polarized laser fields.

The frequency bandwidth of tunable dye lasers ($\sim$1 MHz) is much narrower than the Doppler width of the atoms in the cell, and thus the radiation would only interact with atoms in the narrow velocity bin, of width γ_S (where γ_S is the power-broadened homogeneous width of the transition, typically 50–100 MHz), which is Doppler shifted into resonance. If both lasers are tuned to be in resonance with the same velocity bin, the frequency dependence of the anisotropy signal will be due solely to $n(v_z)$, the z-component velocity distribution of the isomers. Measuring the anisotropy signal should thus allow the average velocity of the neutral Rb isomers in the cell to be determined.

III. ANGULAR CORRELATION IN β DECAY

The angular correlation between the electron and neutrino emitted during β decay is given by

$$W(\theta)=1+a\frac{v}{c}\cos\theta\ , \tag{1}$$

where v is the electron velocity, c the speed of light, and θ the angle between the momenta of the electron and the neutrino in the rest frame of the parent. The value of the parameter a depends upon the type of interaction which governs the β decay. Experimentally, these interactions have been shown to be the Fermi (or vector) interaction, for which $a=+1$, and the Gamow-Teller (or axial-vector) interaction, for which $a=-\frac{1}{3}$.[1] In the Fermi interaction, the electron and antineutrino are emitted in the singlet state (that is, with their spins antiparallel) and thus carry away no angular momentum. Since the antineutrino has helicity $+1$ and the electron is generated with helicity -1, they tend to be emitted in the same direction. The selection rule for this process is $\Delta I=0$, with no change in parity. In the Gamow-Teller interaction, the electron and antineutrino are emitted in the triplet state (with spins parallel), and thus tend to be emitted in opposite directions. The selection rules for this process are $\Delta I=0,1$ (no $0\rightarrow 0$), again with no change in parity.

The recoil energy transmitted to the daughter will be correspondingly larger in the case of a Fermi interaction than in the Gamow-Teller case, since, for the Fermi interaction, the momenta of the electron and neutrino tend to add, rather then cancel. The system we are studying, $^{85}Kr\rightarrow{}^{85}Rb^{m}+\beta^{-}+\bar{\nu}$, corresponds to a transition from $I=(\frac{9}{2})^{+}$ $I'=(\frac{9}{2})^{+}$, and thus satisfies both Fermi and

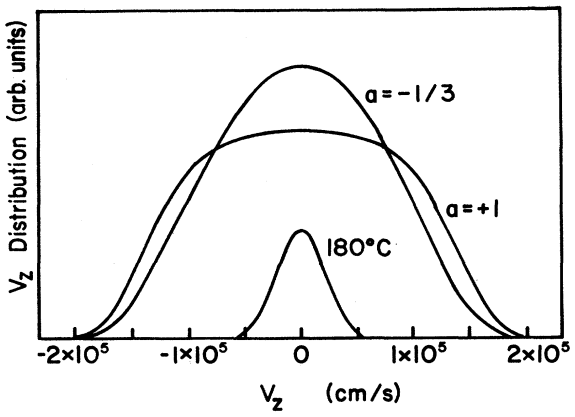

FIG. 3. Calculated initial z-component velocity distributions for $^{85}Rb^m$ ions, for the cases of pure Fermi decay ($a=+1$) and pure Gamow-Teller decay ($a=-\frac{1}{3}$). Also shown (not to the same vertical scale) is the thermal Rb distribution for $T=180\,^{\circ}C$.

Gamow-Teller selection rules, so that, for this case all that can be said is that $-\frac{1}{3}\leq a\leq+1$. Initial z-component velocity distributions of the $^{85}Rb^m$ recoil ions, for the two cases, $a=+1$ and $a=-\frac{1}{3}$, are shown in Fig. 3. It can be seen that these distributions are roughly Gaussian, with a half-width at $1/e$, v_z, of 1.17×10^5 cm/s for $a=-\frac{1}{3}$, and 1.41×10^5 cm/s for $a=+1$. Since the average thermal velocity of Rb at 180 °C is $u=(2kT/M)^{1/2}=2.97\times10^4$ cm/s, the initial velocity distribution of the recoil ions is between 3.9 and 4.8 times wider than that of the thermal atoms in the cell.

IV. THERMALIZATION AND NEUTRALIZATION OF THE RECOIL IONS

After their initial recoil, the $^{85}Rb^m$ ions will undergo velocity changing collisions (vcc's) with the Kr atoms in the cell and will eventually thermalize. In addition, they will undergo resonant charge exchange (CE) with the natural Rb vapor which is also present. The β decay occurs at a steady rate, and the system may be described by the following set of rate equations:

$$\frac{dn^+(v)}{dt}=0=\Lambda N_{Kr}n^0(v)-\Gamma_{CE}n^+(v)-\frac{n^+(v)}{T}-\Gamma^+_{vcc}(v)n^+(v)+\int dv'W^+(v'\rightarrow v)n^+(v')\,, \tag{2a}$$

$$\frac{dn(v)}{dt}=0=\Gamma_{CE}n^+(v)-\frac{n(v)}{T}-\Gamma_{vcc}(v)n(v)+\int dv'W(v'\rightarrow v)n(v)\,, \tag{2b}$$

where $n^+(v)$ and $n(v)$ are the speed distribution densities of the $^{85}Rb^m$ ions and neutrals, respectively. (The system remains isotropic under the effects of both the decay and the collisions, so that these equations depend only upon the magnitude of the velocity, $v=|v|$.) N_{Kr} is the total number density of ^{85}Kr (integrated over velocity), Λ is their decay rate, and $n^0(v)$ is the initial speed distribution of the $^{85}Rb^m$ ions, discussed above. $\Gamma_{CE}=n_{Rb}\sigma_{CE}v_{rel}$ is the charge-exchange rate, where n_{Rb} is the density of natural Rb in the cell, σ_{CE} is the CE cross section, and v_{rel} is the average relative velocity between the $^{85}Rb^m$ atoms and the natural Rb. T is the average lifetime of the isomer (1.4 μs), while $\Gamma^+_{vcc}=n_{Kr}\sigma^+_{vcc}v_{rel}$ and $\Gamma_{vcc}=n_{Kr}\sigma_{vcc}v_{rel}$ are the rates at which $^{85}Rb^m$ ions and neutrals undergo velocity changing collisions, respectively, σ^+_{vcc} and σ_{vcc} being the $^{85}Rb^{m+}\rightarrow Kr$ and $^{85}Rb^m\rightarrow Kr$ vcc cross sections. $W^+(v'\rightarrow v)$ and $W(v'\rightarrow v)$ are collision kernals (for ions and neutrals, respectively) which give the probability per unit time per unit velocity interval that a vcc involving an atom of initial speed v' will scatter that atom so that its final speed is v.

The interpretation of Eqs. (2a) and (2b) is straightforward: The first term in Eq. (2a) is the rate at which $^{85}Rb^m$ ions of speed v are created by the β decay of Kr, the second term is the rate at which they are neutralized by charge exchange, while the third term is the rate at which they undergo nuclear decay. The fourth term is the rate at which collisions scatter ions of speed v into other parts of the speed distribution, while the last term describes the rate at which ions with speeds other than v are scattered so that their final speed is v. The interpretation of Eq. (2b) is similar.

While these equations are conceptually quite simple, their solution is not. Both the source terms and the collision rates are functions of velocity. v_{rel}, in particular, involves the convolution of the steady-state velocity distribution of the recoil ions, which is not known, with a thermal Gaussian. While the differential vcc cross sections $d\sigma^+_{vcc}/d\Omega$ and $d\sigma_{vcc}/d\Omega$, and hence the collision kernals, may be estimated, the dependence of $W(v'\rightarrow v)$ on v and v' is quite complicated. For instance, since this kernal describes collisions between a nonthermal initial distribution and a thermal one, it will *not* be true that $W(v'\rightarrow v)=W(v\rightarrow v')$. Finally, while these rate equations depend only upon the speed, v, the rate equations which describe optical pumping depend upon the component of v along the laser beam v_z,[24] and the coupled system of equations must be strictly treated in three dimensions. While, in principle, this system of equations could be solved on a computer, we may, for the purposes of this example, treat this system in a much simpler way by making several approximations.

For the purposes of determining the β-decay angular correlation, we need only extract v_z^0, the initial velocity width of the $^{85}Rb^m$ ions. Therefore, let us consider a Gaussian distribution of ions, of width $v_z^0>u$ at time $t=0$, which gradually thermalizes as a result of collisions with Kr atoms in the cell, and calculate the width of this distribution as a function of time, $v_z^+(t)$. If we include charge exchange, we may then calculate the width of the distribution of neutral Rb isomers as a function of time, $v_z(t)$. Averaging $v_z(t)$ over time will then yield the average velocity of the neutral Rb isomers in the cell, which may be measured directly in our LINO experiment. Note that this time-dependent approach is equivalent to the steady-state equations discussed above [Eqs. (2a) and (2b)]. In the steady-state case viewpoint, we observe, at

an arbitrary instant of time, the distribution of isomers which were created at all possible times in the past; in the time-dependent view, we start all the isomers at $t=0$ with their initial velocity distribution and average the observations over time.

In order to calculate the average velocity of this ensemble of ions as a function of time, let us consider the rate equation

$$\frac{dv_z^+(t)}{dt} = -\alpha[v_z^+(t)-u]\Gamma_{\rm vcc}^+(t) = -\alpha[v_z^+(t)-u]n_{\rm Kr}\sigma_{\rm vcc}^+ v_{\rm rel}^+(t) , \quad (3)$$

where we have assumed that if the initial velocity distribution is Gaussian, it remains so throughout the thermalization process, and that the rate at which the width of the distribution changes is directly proportional to the collision rate. In Eq. (3), α is a fitting parameter, chosen to give the correct width at a time t_1, when

$$\int_0^{t_1}\Gamma_{\rm vcc}^+(t)dt = 1 , \quad (4)$$

as discussed below. $v_{\rm rel}^+$ is the average relative velocity between the Kr atoms and the $^{85}Rb^m$ ions, given by

$$v_{\rm rel}^+(t) = \int_0^\infty\int_{-\infty}^\infty\int_{-\infty}^\infty dv_x dv_y dv_z \int_0^\infty\int_{-\infty}^\infty\int_{-\infty}^\infty dv_x' dv_y' dv_z' e^{-(|\mathbf{v}|/v_z^+)^2} e^{-(|\mathbf{v}'|/u)^2}|\mathbf{v}-\mathbf{v}'| . \quad (5)$$

Using this expression for $v_{\rm rel}^+$, Eq. (3) again yields a complicated integral-differential equation. It may be considerably simplified, however, by using an approximate expression for $v_{\rm rel}^+$. Noting that when both distributions are thermal, $v_z^+=u$, and $v_{\rm rel}^+=(8/\pi)^{1/2}u\approx 1.6u$, we may write $v_{\rm rel}^+\approx v_z^+ + 0.6u$. Then Eq. (3) becomes

$$\frac{dv_z^+}{dt} = -\alpha(v_z^+-u)n_{\rm Kr}\sigma_{\rm vcc}^+(v_z^+ + 0.6u) . \quad (6)$$

This equation may be integrated directly to yield

$$v_z^+(t) = \frac{1+\xi e^{-\alpha\Gamma_{\rm Th}^+ t}}{1-\xi e^{-\alpha\Gamma_{\rm Th}^+ t}}(0.8)u + 0.2u , \quad (7)$$

where $\Gamma_{\rm Th}^+$ is the thermal vcc rate for the ions, $\Gamma_{\rm Th}^+ = 1.6 n_{\rm Kr}\sigma_{\rm vcc}^+ u$, and

$$\xi = \frac{v_z^0-u}{v_z^0+0.6u} . \quad (8)$$

Checking the limiting behavior of this expression shows that

$$v_z^+(t\to\infty) = 0.8u + 0.2u = u , \quad (9)$$

as expected, while

$$v_z^+(t=0) = \frac{1+\xi}{1-\xi}(0.8)u + 0.2u = v_z^0 \quad (10)$$

again, as expected.

Now, it remains to evaluate α. If the differential vcc cross section $d\sigma_{\rm vcc}^+/d\Omega$ is known, the velocity distribution at the time t_1, at which each atom has undergone one collision on average, may be determined by calculating, for each initial velocity, the final velocity associated with the scattering of the Rb ion through an angle θ in the center-of-mass frame, and weighting that contribution to the scattered distribution by the differential cross section. For example, for a classical billiard-ball collision (hard spheres of radius r) of total cross section $\sigma=\pi b^2$, where $b=2r$ is the maximum impact parameter, the differential cross section is

$$\frac{d\sigma}{d\theta} = \frac{\pi b^2}{2}\sin\theta \quad (11)$$

in the center-of-mass frame. The most probable scattering angle is thus $\theta=\pi/2$. If we consider the Rb to be traveling at speed v in the lab frame and take the Kr to be initially at rest, as is shown in Fig. 4(a), then in the center-of-mass frame, each atom travels with speed $v/2$. After an elastic collision through $\theta=\pi/2$, transforming back to the lab frame reveals that the Rb is now traveling at $v/\sqrt{2}\approx 0.7v$, as is shown in Fig. 4(b). A more detailed calculation, including the effects of motion of the Kr atoms and integrating over all possible initial velocities and scattering angles shows that, for billiard-ball collisions,

$$v_z^+(t_1) = 0.77 v_z^0 . \quad (12)$$

Numerical evaluation of Eq. (7) then yields, again for billiard-ball collisions, $\alpha=0.4$. If the differential cross section is more strongly forward peaked than Eq. (11), the value of α will be correspondingly smaller; if it is peaked more strongly backward, the value of α will be

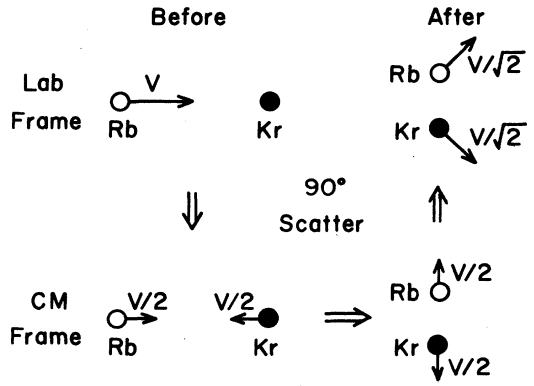

FIG. 4. Kinematics of the scattering of an $^{85}Rb^m$ atom, traveling at speed v, by a Kr atom, initially at rest, through an angle of $\pi/2$ in the center-of-mass frame.

larger. As we discuss below, LINO measurements at several Kr pressures can serve to determine α as well as v_z^0.

Given $v_z^+(t)$, we wish to calculate both the fraction of $^{85}Rb^m$ ions which undergo charge exchange before they decay, the average time it takes for charge exchange to occur, and in particular, the average velocity of the neutral $^{85}Rb^m$'s in the cell. The probability that an ion undergoes a charge-exchange collision in the time interval t to $t+dt$ is

$$P_{CE}(t)dt = (\text{fraction left at } t)(\text{probability of CE in } dt \text{ at } t)$$
$$= \exp\left\{-\int_0^t [\Gamma_{CE}(t')+1/T]dt'\right\}\Gamma_{CE}(t)dt \ , \tag{13}$$

where $\Gamma_{CE}(t)=n_{Rb}\sigma_{CE}[v_z^+(t)+0.6u]$. The fraction of ions which charge exchange before they decay is then

$$F_{CE}=\int_0^\infty P_{CE}(t)dt \ . \tag{14}$$

Note that if Γ_{CE} were constant,

$$F_{CE}=\int_0^\infty e^{-(\Gamma_{CE}+1/T)t}\Gamma_{CE}dt=\frac{\Gamma_{CE}T}{\Gamma_{CE}T+1} \ . \tag{15}$$

This is the charge-exchange fraction for $^{85}Rb^m$ ions in a high-pressure (above a few Torr) cell, where vcc's thermalize the distribution essentially instantaneously, provided we take $\Gamma_{CE}=n_{Rb}\sigma_{CE}(1.6u)$. In the low-pressure regime, $v_z^+(t)$ (and hence $v_{rel})>u$ for all $t<T$, and charge exchange will occur at a higher rate, resulting in a larger fraction of $^{85}Rb^m$ ions undergoing charge exchange before they decay than would be the case if the ions were thermalized immediately.

The average time it takes for charge exchange to occur is

$$\tau_{CE}=\frac{1}{F_{CE}}\int_0^\infty P_{CE}(t)t\,dt \ , \tag{16}$$

where the normalization factor $1/F_{CE}$ must be included to give the conditional probability of charge exchange at time t for those atoms which undergo charge exchange before they decay. Again, if charge exchange is constant, as in the high-pressure regime,

$$\tau_{CE}=\frac{T}{\Gamma_{CE}T+1} \ . \tag{17}$$

Since charge exchange occurs at a higher rate in the low-pressure regime than in the high-pressure regime, τ_{CE} will generally be shorter than the above value.

In order to calculate the average velocity of the $^{85}Rb^m$ neutrals, consider those $^{85}Rb^m$ ions which undergo charge exchange in the time interval t_{CE} to $t_{CE}+dt_{CE}$. These ions have undergone vcc's at a rate Γ_{vcc}^+, so that their average velocity at $t=t_{CE}$ is thus $v_z^+(t_{CE})$. The resulting neutral isomers will subsequently undergo vcc's at a rate Γ_{vcc}, so that their average velocity at a later time t will be

$$v_z[v_z^+(t_{CE}),t]=\frac{1+\xi(t_{CE})e^{-\alpha\Gamma_{Th}(t-t_{CE})}}{1-\xi(t_{CE})e^{-\alpha\Gamma_{Th}(t-t_{CE})}}(0.8)u+0.2u \ , \quad t>t_{CE} \ , \tag{18}$$

where $\Gamma_{Th}=n_{Kr}\sigma_{vcc}(1.6)u$ and

$$\xi(t_{CE})=\frac{v_z^+(t_{CE})-u}{v_z^+(t_{CE})+0.6u} \ . \tag{19}$$

While, for simplicity, we have assumed $\alpha^+=\alpha^{neutral}=\alpha$, in general this need not be true.

The average velocity of the neutral isomers, $\bar{v}_z$, may then be calculated by averaging Eq. (18) over time and then integrating over all possible charge-exchange and decay times, t_{CE} and t_D, respectively, weighting each such time by the appropriate probability factor, so that

$$\bar{v}_z=\frac{1}{F_{CE}}\int_0^\infty dt_D\int_0^{t_D}dt_{CE}P_{CE}(t_{CE})P_D(t_D)\frac{1}{t_D-t_{CE}} \times\int_{t_{CE}}^{t_D}v_z[v_z^+(t_{CE}),t]dt \ . \tag{20}$$

In this equation, P_{CE} is given by Eq. (13), while

$$P_D(t_D)=\frac{1}{T}e^{-t_D/T} \ . \tag{21}$$

The resonant charge exchange and neutral vcc cross sections which appear above have been estimated at $\sigma_{CE}\approx 550$ Å^2 and $\sigma_{vcc}\approx 50$ Å^2, respectively.[20,24] A simple estimate of σ_{vcc}^+ based on Rb ion mobility data[25] shows it to be no greater than about 350 Å^2. Given these numbers, Eq. (20) may be easily evaluated on a computer. The results, for $T=180$ °C and assuming billiard-ball col-

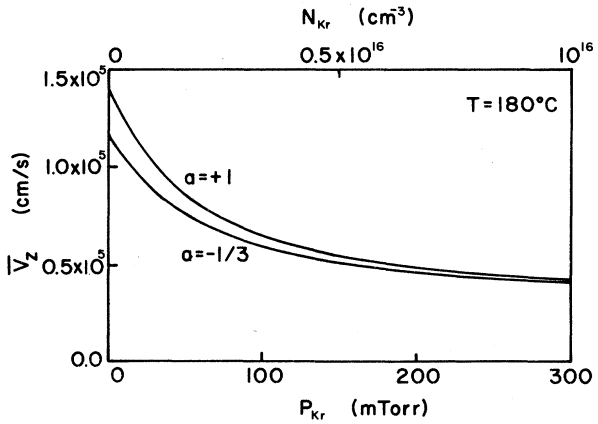

FIG. 5. Calculated average velocities of the neutral $^{85}Rb^m$ isomers in the cell vs Kr density and pressure, for the two cases $a=+1$ and $a=-\frac{1}{3}$. The values of P_{Kr} given correspond to the pressure in the cell at room temperature, not at 180 °C.

lisions for both the ions and the neutrals, are shown in Fig. 5. These curves represent a "worst case." If, as is likely, $d\sigma^{+}_{vcc}/d\Omega$ is more strongly forward peaked than it is for the hard-sphere case, or if $\sigma^{+}_{vcc} < 350$ Å^2, the distribution will thermalize more slowly. From Fig. 5, the average velocities of the neutral isomers in the cell in the two cases, $a=+1$ and $a=-\frac{1}{3}$, for a Kr pressure of 100 mTorr, are 6.4×10^4 and 5.9×10^4 cm/s, respectively. There is thus a difference in the full width of the distribution for these two cases of $\Delta v \approx 10^4$ cm/s, which corresponds to a frequency difference of $\Delta\nu \approx 140$ MHz for $\lambda = 7947$ Å, the wavelength of the ^{85}Rb$^m D_1$ transition. In principle, the use of arbitrarily long counting times should allow the Doppler width to be determined to arbitrary precision. In reality, temperature, laser power, and laser frequency drifts limit the resolution which may be obtained. Nevertheless, it should easily be possible to determine the Doppler width to within 10 MHz.

In actual practice, measurements of v_z would be made at several Kr pressures. The anisotropy signal detected would be a convolution of a Doppler profile of width v_z with a Lorentzian of width $\gamma_S/2$. After deconvolution, the resulting curve, v_z vs n_{Kr}, would be extrapolated back to $n_{Kr}=0$, yielding v_z^0. Fitting this curve with Eq. (20) would then yield values for the parameters σ^{+}_{vcc}, α^{+}, σ_{vcc}, α, and σ_{CE}, as well.

This calculation demonstrates that it should be possible to extract angular correlation information from LINO anisotropy data, provided the buffer gas pressure is sufficiently low that the radioactive daughters do not have time to thermalize before they decay. Determining the angular correlation constant a requires only a knowledge of the width of the recoil velocity distribution. It should be possible, however, to measure the end points and detailed shape of this distribution, in addition to its width. This information could, in principle, be used for high-precision neutrino mass studies. While, at present, the LINO technique is applicable only to parents which decay to short-lived daughters, systems which decay to stable daughters might be treated as well, given the development of tunable high-power ultraviolet lasers.

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