Limits on the presence of scalar and induced-scalar currents in superallowed β decay

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Limits on the scalar and induced-scalar coupling constants are established from experimental ft values of superallowed Fermi transitions. The role of the uncertainties in the ft values is discussed, leading to minimal coupling strengths that can be extracted from ft values. The implications for the Kobayashi-Maskawa mixing angles are also discussed.

It is a long-standing hypothesis that the vector part of the weak current is a conserved quantity.[1] The principal test of this proposition lies in the study of $J^\pi=0^+$, $T=1\rightarrow 0^+$, $T=1$ nuclear β transitions. Namely, their transition rates should be nucleus independent and given by the relation

$$ft=\frac{K}{G_V^2|M_F|^2}\ , \tag{1}$$

where t is the partial half-life, f is the statistical rate function, $K=8.1201\times 10^{-7}$ is a product of fundamental constants, G_V is the effective vector coupling constant for nucleon β decay [measured in units of $(\hbar c)^3$], and M_F is the Fermi matrix element, $M_F=\langle \psi_f|T_\pm|\psi_i\rangle$.

Two classes of nucleus-dependent corrections, however, must be applied to Eq. (1). The first is radiative corrections[2] to the statistical rate function f, denoted by δ_R, yielding $f_R=f(1+\delta_R)$. The second is corrections to the nuclear matrix element due to the presence of isospin-nonconserving (INC) forces in nuclei,[3] and is denoted by δ_C; that is,

$$|M_F|^2=|M_{F_0}|^2(1-\delta_C),$$

where

$$M_{F_0}=[T(T+1)-T_{Z_i}T_{Z_f}]^{1/2}\delta_{if}\ .$$

With δ_R and δ_C, the "nucleus-independent" ft value for 0^+, $T=1\rightarrow 0^+$, $T=1$ transitions is

$$\mathcal{F}t=ft(1+\delta_R)(1-\delta_C)=\frac{K}{2G_V^2}\ . \tag{2}$$

Using these $\mathcal{F}t$ values, it is possible to test the conserved vector current (CVC) hypothesis. Furthermore, by comparing G_V with the vector coupling constant obtained from muon β decay, the Kobayashi-Maskawa (KM) mixing angle between u and d quarks (v_{ud}) can be determined.[4,5]

Until recently, the $\mathcal{F}t$ values for the most accurately measured transitions (^{14}O, ^{26}Alm, ^{34}Cm, ^{38}Km, ^{42}Sc, ^{46}V, ^{50}Mn, ^{54}Co) failed to yield constant values. Both nucleus-dependent corrections δ_R (Ref. 6) and δ_C (Ref. 7) have been investigated, leading to $\mathcal{F}t$ values that are now constant to the level of $\approx 0.1\%$, with $\mathcal{F}t_{\rm avg}=3077\pm 1.5$ s, with $\chi^2/\nu=0.8$.[7] There are, however, two other "exotic" processes that could, in principle, contribute to these transitions, and it is important to establish upper limits on their presence. These are (i) an induced-scalar current that is present only if the vector current is not conserved, and (ii) the possibility that in addition to the standard V-A components of the electroweak interaction, there is also a scalar coupling. The former is also forbidden in the standard model by the absence of second-class currents, but could arise via a quark doubling mechanism.[8] Likewise, a direct scalar coupling lies outside the conventional Weinberg-Salam-Glashow picture, but could arise from the exchange of a charged Higgs boson if two or more charged Higgs doublets are introduced.[9] Assuming that the lepton current is left handed, the most general weak Hamiltonian that would contribute to 0^+, $T=1\rightarrow 0^+$, $T=1$ transitions is then[10]

$$\begin{aligned} H_{\beta^+}=&\frac{iG_S}{\sqrt{2}}(\bar{u}_n u_p)[\bar{u}_\nu(1+\gamma_5)v_e]\\ &+\frac{iG_V}{\sqrt{2}}\{\bar{u}_n[f_V\gamma_\mu-f_S(\partial/\partial x_\mu-ieA_\mu)]u_p\}\\ &\times[\bar{u}_\nu\gamma_\mu(1+\gamma_5)v_e]\ , \end{aligned} \tag{3}$$

where G_S is the scalar coupling strength, f_V and f_S are the vector and induced-scalar form factors (note that for CVC $f_V=1$ and $G_S=f_S=0$), and A_μ is the potential for the static electric field of the nuclear charge, and is included in order to preserve gauge invariance. Limits on G_S (Ref. 11) and the ratio f_S/f_V (Ref. 12) have been established previously. However, since these studies, the experimental ft values have been measured with greater precision,[13] and the corrections δ_R and δ_C have been reevaluated, and in the case of δ_R, a major error has been corrected.[6] Given these considerations, it is perhaps timely to reestablish the limits at which these "exotic" effects may be present. Further, in this Brief Report, we also investigate the absolute upper limits that can be established from the $\mathcal{F}t$ values given both experimental and theoretical uncertainties. The consequences pertaining to the Kobayashi-Maskawa mixing angles will also be discussed.

In general, the electroweak Hamiltonian may include both the scalar and induced-scalar terms. However, operationally their effect in Fermi transitions is identical, because, writing the induced-scalar interaction in the form

$$H_{\mathrm{IS}} = -\frac{iG_V}{\sqrt{2}}\bar{u}_n f_S(\partial_\mu - ieA_\mu)u_p\bar{u}_\nu\gamma_\mu(1+\gamma_5)\psi_e \ ,$$

where ψ_e is a Coulomb solution to the Dirac equation, we have

$$H_{\mathrm{IS}} = \frac{iG_V}{\sqrt{2}} m_e f_S \bar{u}_n u_p \bar{u}_\nu (1+\gamma_5)\psi_e \ ,$$

which is of the same form as the direct scalar coupling provided one makes the substitution

$$m_e f_S G_V/\sqrt{2} \rightarrow G_S/\sqrt{2} \ .$$

The $\mathcal{F}t$ values are modified in the presence of a scalar current by[14]

$$\mathcal{F}t = (\mathcal{F}t)_0(1-2b\gamma\langle W^{-1}\rangle) \ , \qquad (4)$$

with $(\mathcal{F}t)_0 = K/2G_V^2$, $\gamma = (1-\alpha^2 Z^2)^{1/2}$,

$$b = G_S G_V f_V / [(G_V f_V)^2 + G_S^2] \approx G_S/G_V = m_e f_S / f_V \ ,$$

and $\langle W^{-1}\rangle$ is the average of W^{-1} over the allowed spectrum, i.e.,

$$\langle W^{-1}\rangle = \frac{\int dW\, p(W-W_0)^2 F(Z,W)}{\int dW\, pW(W-W_0)^2 F(Z,W)} \ ,$$

where W is the total β energy in units of $m_e c^2$, W_0 is the β end-point energy, p is the momentum (units of $m_e c$), and $F(Z,W)$ is the Fermi function. Listed in Table I are the experimental ft values,[13] the corrections δ_R (Ref. 6) and δ_C (Ref. 7), the corresponding $\mathcal{F}t$ values, the β end-point energies,[13] and the factors $\gamma\langle W^{-1}\rangle$ for each of the eight superallowed transitions. Performing a least-squares fit to the $\mathcal{F}t$ values, we find $(\mathcal{F}t)_0 = 3078.4 \pm 4.3$ s and $b = (0.6 \pm 2.5)\times 10^{-3}$, $\chi^2/\nu = 0.93$. This is in good agreement with the results obtained previously, $(\mathcal{F}t)_0 = 3077.3 \pm 1.5$ s, in which $G_S = 0.0$ was assumed.[7] Furthermore, we note that the limit given here is only slightly tighter than that established by Hardy and Towner[11] of $b = (0.5 \pm 3.0)\times 10^{-3}$, and is slightly weaker than that found by Szybiszc and Silbergleit[12] of $b = (0.8 \pm 2.1)\times 10^{-3}$. We note, however, that the experimental $\mathcal{F}t$ values used by Szybiscz and Silbergleit had considerably smaller uncertainties than those used here. This is most likely due to the use of a different data set and a somewhat smaller estimate of the uncertainty in the nuclear corrections δ_C. In addition, the older Hardy and Towner survey used $\mathcal{F}t$ values from all known transitions, in particular ^{10}C, although with experimental uncertainties considerably larger than those used here. In this light, however, we note that only a slight improvement on the limits of b could be obtained if the ft value for ^{10}C could be measured with the same accuracy as the eight transitions used here. For example, assuming $\mathcal{F}t = 3077 \pm 4$ and $\gamma\langle W^{-1}\rangle = 0.618$ ($E_0 = 888.2$ keV), we would obtain $b = (0.3 \pm 1.5)\times 10^{-3}$.

From these results we see that the superallowed β-decay data are consistent with $G_S = 0$ and $f_S = 0$. However, the fact that the a limits on the "exotic" effects established by the new data are not significantly tighter than those established previously is somewhat surprising, especially given the fact that the uncertainty in the experimental ft values has improved considerably, and that the $\mathcal{F}t$ values were obtained using the values of δ_R that contained a significant error. It is interesting to speculate at this point what the smallest values of G_S/G_V (and f_S/f_V) are that could be extracted from the $\mathcal{F}t$ values. To do this, it is necessary to examine the sources of uncertainty in $\mathcal{F}t$. Shown in Table II are the contributions to the uncertainty in $\mathcal{F}t$ due to the experimental ft values, the radiative corrections δ_R, and the isospin-mixing corrections δ_C to the nuclear matrix elements. In many cases, it is seen that the contribution due to the theoretical uncertainty in δ_C is of the order of or larger than the experimental error. In this regard, it is interesting to ask what limits on G_S/G_V and f_S/f_V could be established in the event that all $\mathcal{F}t$ values were constant with experimental uncertainties of zero. In this case, we

TABLE I. List of experimental ft values, the corrections δ_R and δ_C, the corresponding $\mathcal{F}t$ values, the β end-point energies E_0, and the quantity $\gamma\langle W^{-1}\rangle$ for the eight superallowed transitions.

Nucleus	ft (s)	δ_R (%)	δ_C (%)	$\mathcal{F}t$ (s)	E_0 (keV)	$\gamma\langle W^{-1}\rangle$
^{14}O	3038.1(23)	1.53(1)	0.19(9)	3078.7(36)	1808.44(27)	0.438
^{26}Al^m	3034.5(14)	1.47(2)	0.24(10)	3071.7(34)	3209.95(25)	0.299
^{34}Cl	3052.0(29)	1.45(3)	0.48(10)	3081.4(44)	4470.27(18)	0.232
^{38}K^m	3045.1(26)	1.44(3)	0.49(14)	3073.7(51)	5020.49(56)	0.211
^{42}Sc	3048.7(63)	1.46(4)	0.39(9)	3081.1(71)	5403.02(28)	0.198
^{46}V	3043.7(22)	1.46(4)	0.21(10)	3081.6(40)	6028.62(69)	0.180
^{50}Mn	3039.9(40)	1.46(5)	0.28(10)	3075.6(53)	6610.01(41)	0.166
^{54}Co	3044.7(23)	1.45(5)	0.35(10)	3077.1(42)	7220.14(32)	0.154

TABLE II. Contributions to the uncertainty in $\mathcal{F}t$ due to the experimental ft values and the corrections δ_R and δ_C.

		$\delta(\mathcal{F}t)$	
Nucleus	ft	δ_R	δ_C
^{14}O	2.3	0.3	2.8
^{26}Al^{m}	1.4	0.6	3.1
^{34}Cl	2.9	0.9	3.1
^{38}K^{m}	2.6	0.9	4.3
^{42}Sc	6.3	1.2	2.8
^{46}V	2.2	1.2	3.1
^{50}Mn	4.0	1.5	3.1
^{54}Co	2.3	1.5	3.1

arrive at the limit $b=\pm 1.9\times 10^{-3}$, with the uncertainty $\delta(\mathcal{F}t)_0$ being ± 3.2 s. From this, we see that the values obtained here are very near the absolute limit that could be established from superallowed $\mathcal{F}t$ values given the uncertainty in the nuclear correction δ_C.

The consequences of these results can be seen when values of the effective vector coupling constant G_V are extracted from $(\mathcal{F}t)_0$. The Kobayashi-Maskawa (KM) mixing matrix element v_{ud} is given by

$$v_{ud}=\frac{G_V}{G_\mu}(1+\Delta_\beta-\Delta_\mu)^{-1/2} ,$$

where

$$G_\mu/(\hbar c)^3=1.166\,37(2)\times 10^{-5}\ \mathrm{GeV}^{-2}$$

(Ref. 15) is the vector coupling constant for muon β decay, and Δ_β and Δ_μ are the "inner" radiative corrections to both nucleon (β) and muon (μ) β decay, with $\Delta_\beta-\Delta_\mu=(2.3\pm 0.2)\times 10^{-2}$.[16] The only condition that is imposed upon the KM matrix is that it should be unitary, i.e.,

$$v^2=v_{ud}^2+v_{us}^2+v_{ub}^2=1 .$$

Deviations from unity would either indicate the possibility of another quark generation ($v^2<1$), or a defect in the standard form of the electroweak interaction. Assuming $G_S=f_S=0$, we obtain $v_{ud}=0.9737\pm 0.0010$.[7] Taking $v_{us}=0.220\pm 0.002$ (Ref. 17) and $v_{ub}<0.0075$,[18] we arrive at $v^2=0.9965\pm 0.0021$. We note that at this point the uncertainty in v^2 is dominated by the uncertainty in $\Delta_\beta-\Delta_\mu$.

Applying the above analysis to the $\mathcal{F}t$ value obtained while including a scalar current, we find $v_{ud}=0.9735\pm 0.0012$ and $v^2=0.9961\pm 0.0025$. We note that this result is not significantly different from that obtained assuming $G_S=f_S=0$. In fact, the somewhat larger uncertainty in $(\mathcal{F}t)_0$ contributes an uncertainty of only 0.0012 to v^2, which is less than that brought about by the "inner" radiative corrections (≈ 0.0019).

Before concluding, we digress with a note about a further generalization of the weak Hamiltonian as defined in Eq. (3). With the inclusion of a scalar current there is no fundamental reason to assume that the lepton currents be necessarily left handed. Including right-handed currents, the scalar part of the Hamiltonian takes the form

$$H_S=\frac{i}{\sqrt{2}}(\bar{u}_n u_p)[\bar{u}_\nu(G_S+G_S'\gamma_5)v_e] ,$$

where in the left-handed limit $G_S=G_S'$. Limits on G_S and G_S' have been established[19] by examining data on the neutron half-life, t_n, the electron neutrino angular correlation, a [in particular for the neutron $a(n)$], and the Fierz interference term [b in Eq. (4)] for pure Fermi and mixed Fermi-Gammow-Teller transitions. They are, however, rather poorly determined: at the 95% confidence level it was found $|G_S/G_V|<0.23$ and $|G_S'/G_V|<0.19$ [mostly from t_n and $a(n)$] with

$$|(G_S+G_S')/G_V|<0.065$$

(mostly from b_{Fermi}). The difference in these limits arises from the fact that both $a(n)$ and t_n are sensitive to the squares of the G_S and G_S', whereas in the Fierz interference term the dependence is linear (cf. Appendix 1 in Ref. 19). Even so, the Fierz interference term for mixed transitions is not particularly useful because of uncertainties in the allowed Gammow-Teller matrix elements and the additional possibility that the weak interaction may contain tensor or second-class currents.[20] Furthermore, the ft values for the superallowed transitions mentioned here are considerably more precise. Because of this precision, in the limit that only left-handed currents are considered, other experiments sensitive to G_S contribute very little to the limits on G_S established by superallowed Fermi transitions.

We conclude that it is unlikely that limits on a possible scalar coupling can be improved in this fashion even if significantly more precise ft data are available. There is one aspect here, however, that bears comment. If the induced scalar mechanism is relevant, or if the scalar coupling is due to Higgs exchange and is proportional to the lepton mass, then even through effects in beta decay are suppressed by $m_e/m_N E_e\sim 10^{-4}$, in a muon capture reaction, scalar effects arise at $\mathcal{O}(m_\mu/m_N)\sim 10\%$—a significant enhancement. Unfortunately, there is, in general, only one parameter in which to deduce the coupling strengths, i.e., the capture rate. By making the following reasonable assumptions:

(i) q^2 dependence of nuclear matrix elements from the impulse approximation and CVC and (ii) an induced pseudoscalar coupling from partial conservation of axial-vector current (PCAC); one can obtain from the capture rates measured for ^{1}H and ^{3}He a limit on a combination of induced-scalar (f_S) and induced-tensor (f_T) strengths[21]

$$m_e\left|f_S-\frac{f_A}{f_V}f_T\right|\le 5\times 10^{-4} ,$$

which is significantly stronger than that found in the $0^+\to 0^+$ analysis. It should be remarked, however, that this result is strongly model dependent.

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