Polarization response functions and the $(\vec{e},e'\vec{p})$ reaction

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The first comprehensive study of the full set of 18 response functions relevant to the $(\vec{e},e'\vec{p})$ reaction is presented. Benchmark analytical features and limiting cases of the response functions are described. Numerical predictions contrasting nonrelativistic and relativistic (Dirac) dynamics and on- and off-shell final-state interaction effects are presented. Basic physical characteristics and dependences of the response functions are identified. The outlook for future experimental studies of the $(\vec{e},e'\vec{p})$ polarization response functions is discussed.

I. INTRODUCTION

Experimental studies of both inclusive and exclusive quasielastic electron scattering have yielded results which have, so far, defied explanation within the context of traditional nuclear theory. In the case of the inclusive reaction,[1–8] the difficulties lie in the experimentally observed suppression of the longitudinal response and in the region of the "dip" between the quasielastic and resonance peaks in the transverse response. In the case of the exclusive reactions,[9–13] which have so far been limited to the $(e,e'p)$ reaction using certain restricted choices of kinematics the difficulties are the observation of insufficient spectral response[9–11] and an excess transverse response at large missing energies.[11] These difficulties have lead to considerable theoretical activity in improving the application of traditional many-body methods to the description of these reactions,[14–26] and in the application of more exotic models involving relativistic dynamics[27–42] or quark degrees of freedom.[43–45] So far these efforts have resulted in varying degrees of success. The only thing that is clear, at this point, is that neither the dominant physical mechanisms nor the ambiguities in analyses are understood. Continued work in these areas will be of interest for the foreseeable future.

The difficulty in using the available experimental results to discriminate between various models is related to the intrinsic properties of the quasielastic response. Since the quasielastic response is in large part determined by the nuclear momentum distribution and the phase space available to the reactions, the quasielastic response functions tend to be smooth broad peaks. As a result, given the range of assumptions which can reasonably be made in constructing models of these processes, it is quite easy to fit any limited set of quasielastic data by the adjustment of a small number of parameters. At present, the data are not capable of distinguishing unambiguously among the various classes of models. On the other hand, some of the characteristics of present data seem to defy explanation within the context of any of the current models. It is therefore necessary to determine classes of experiments which have the potential for discriminating among the various models and for providing additional insight into the physical mechanisms responsible for the inadequacies of current models.

For the inclusive reaction the possibilities are to continue the process of obtaining Rosenbluth separations of longitudinal and transverse response functions for a variety of nuclear targets under a wide range of momentum transfers. The inherent complexity of the inclusive reaction, due to the many open reaction channels which contribute to the cross sections, makes it difficult to test any of the detailed properties of any model. Inclusive processes may be more useful in providing global constraints on proposed resolutions of defects encountered in the exclusive context. On the other hand, the exclusive reactions have the potential of introducing a greater simplicity into the analysis of the reaction by allowing attention to be focused on a particular reaction channel while also providing a greater richness of structure due to the greater number of observables available. For the $(e,e'N)$ reactions a considerable amount of freedom is still available in simply choosing kinematics which vary from the widely used parallel and perpendicular kinematics, in using the known symmetries and the known functional form of the cross section to extract longitudinal, transverse, and interference cross sections, and in studying the $(e,e'n)$ reaction. In addition, it is possible to increase the richness of available information in this reaction by use of polarization of the electron beam,[46] the target[47–51] and the ejected nucleon.[50–52] It is also, of course, possible to extend the study of exclusive reactions to processes resulting in more complicated final states such as the $(e,e'2N)$ reaction. However, the complexity of these final states will obviously result in a greater experimental and analytical complexity due to the presence of at least three additional continuous kinematical variables. For example, the theoretical situation is complicated by the elimination of simplifying techniques, such as the distorted-wave-impulse approximation, which are available in the case of $(e,e'N)$. The ultimate goal of any theory of quasielastic electron scattering must be to construct a consistent treatment of all significant reaction channels which contribute to the inclusive cross section.

In this paper we focus on the $(\vec{e},e'\vec{N})$ reaction where polarized electrons are used to eject polarized nucleons from an unpolarized nucleus.[50–52] This reaction has several advantages as a means for increasing the available information necessary to constrain theory. The additional measurable quantities are discrete spin degrees of freedom which can be accessed by providing a polarized electron beam and/or using a polarimeter for the ejected nucleons. Both of these elements exist and the advent of the coming generation of high duty factor electron accelerators should make possible their simultaneous use in coincidence experiments. The discreteness of the spin degrees of freedom can also be used to minimize systematic experimental errors by allowing all of the continuous kinematical variables to be fixed while the spin of the beam is flipped. While this is also true of coincidence experiments using polarized targets, the measurement of ejectile spin circumvents the difficulties of producing polarized targets which can be used in a high current electron beam. From the theoretical standpoint, the $(\vec{e},e'\vec{N})$ reaction provides direct access to the spin response of the nuclear system. This is, of course, of considerable importance since the strong interactions of the nuclear system are explicitly spin dependent as is the electromagnetic interaction of the electrons with the hadrons of the nucleus. There is, by inference from recent developments in elastic proton scattering,[53–55] from the electrodisintegration of the deuteron,[47,51] and from the unexpected results of longitudinal/transverse separations in inclusive quasielastic electron scattering,[1–8] every reason to believe that the addition of these spin observables will considerably constrain the various elements of models of quasielastic electron scattering.

In a previous paper[52] we present a formal framework for the description of the $(\vec{e},e'\vec{N})$ reaction which provides a direct generalization of the usual description of the unpolarized reaction and which treats the spin of the ejected nucleon in a manner consistent with that used in elastic proton scattering. This framework was constructed to explicitly display the dependence of the differential cross section on the polarization of the ejected nucleon. In that paper, we presented a discussion of the constraints placed on the 18 response functions (13 of which depend on ejectile spin) by various symmetries. We also provided limited preliminary results of the first distorted-wave-impulse approximation (DWIA) calculations of these response functions for a many-nucleon system, primarily to give some indication of the sizes of the new response functions. In the present work we provide a more complete presentation of these results showing their characteristic dependence on final-state interactions, relativistic dynamics, and nuclear off-shell effects. In order to more clearly explicate some of the characteristics of the polarization response functions, we provide a detailed discussion of these response functions in the simplest, nontrivial model of the reaction, the plane-wave-impulse approximation (PWIA). This is done for both the Schrödinger and the Dirac dynamical approaches, with a semirelativistic approximation to the latter used to show some of the problems inherent in certain approximations to the $(e,e'p)$ reaction which are currently in use. We also present some discussion of the consequences which many-body corrections to the DWIA can be expected to have on the response functions.

The next section of this paper consists of a review of the cross section for the $(\vec{e},e'\vec{N})$ reaction and of the properties and limiting values of the response functions which can be determined by use of various symmetries. This is followed by a discussion of the response functions in the various realizations of the PWIA in the succeeding section. The following section contains a presentation and discussion of the results of our DWIA calculations. Finally, we present conclusions which can be inferred from the work presented in this paper, and discuss some of the extensions necessary to provide a firm theoretical foundation for analysis of the physics contained in the $(\vec{e},e'\vec{p})$ reaction.

II. REVIEW OF FORMALISM

By virtue of general symmetry principles, the differential cross section for the $(\vec{e},e'\vec{N})$ reaction, when the residual nucleon is left in its ground state or some discrete excited state, can be written as[52]

$$\left[\frac{d^3\sigma}{d\epsilon_{k'}d\Omega_{k'}d\Omega_{\mathbf{p}'}}\right]_{h,s'}=\frac{m|\mathbf{p}'|}{2(2\pi)^3}\left[\frac{d\sigma}{d\Omega_{k'}}\right]_{\text{Mott}}\{V_L(R_L+R_L^n\mathcal{S}_n)+V_T(R_T+R_T^n\mathcal{S}_n)$$
$$+V_{TT}[(R_{TT}+R_{TT}^n\mathcal{S}_n)\cos2\beta+(R_{TT}^l\mathcal{S}_l+R_{TT}^t\mathcal{S}_t)\sin2\beta]$$
$$+V_{LT}[(R_{LT}+R_{LT}^n\mathcal{S}_n)\cos\beta+(R_{LT}^l\mathcal{S}_l+R_{LT}^t\mathcal{S}_t)\sin\beta]$$
$$+hV_{LT'}[(R_{LT'}+R_{LT'}^n\mathcal{S}_n)\sin\beta+(R_{LT'}^l\mathcal{S}_l+R_{LT'}^t\mathcal{S}_t)\cos\beta]$$
$$+hV_{TT'}(R_{TT'}^l\mathcal{S}_l+R_{TT'}^t\mathcal{S}_t)\}\ , \tag{2.1}$$

where $\mathbf{k}$ and ϵ_k ($\mathbf{k}'$ and $\epsilon_{k'}$) are the momentum and energy of the incident (scattered) electron, h is the incident electron helicity, θ is the electron scattering angle, and m is the nucleon mass. Defining $\mathbf{q}$ and ω as the momentum and energy transfer from the electron, and $Q^2=-q^2=\mathbf{q}^2-\omega^2$, the Mott cross section is

$$\left[\frac{d\sigma}{d\Omega_{k'}}\right]_{\text{Mott}}=\left[\frac{\alpha\cos\theta/2}{2\epsilon_k\sin^2\theta/2}\right]^2 =\left[\frac{2\alpha\epsilon_{k'}\cos\theta/2}{Q^2}\right]^2 . \tag{2.2}$$

The kinematic factors are defined as

$$\begin{aligned} V_L&=\frac{Q^4}{\mathbf{q}^4}\,,\\ V_T&=\left[\frac{Q^2}{2\mathbf{q}^2}+\tan\tfrac{1}{2}\theta\right],\\ V_{TT}&=\frac{Q^2}{2\mathbf{q}^2}\,,\\ V_{LT}&=\frac{Q^2}{\mathbf{q}^2}\left[\frac{Q^2}{\mathbf{q}^2}+\tan^2\tfrac{1}{2}\theta\right]^{1/2},\\ V_{LT'}&=\frac{Q^2}{\mathbf{q}^2}\tan\tfrac{1}{2}\theta\,,\\ V_{TT'}&=\tan\tfrac{1}{2}\theta\left[\frac{Q^2}{\mathbf{q}^2}+\tan^2\tfrac{1}{2}\theta\right]^{1/2}. \end{aligned} \tag{2.3}$$

The ejected nucleon has momentum $\vec{p}\,'$ and energy

$$E'=E(\mathbf{p}')=(\mathbf{p}'^2+m^2)^{1/2}\,.$$

The direction of the ejectile rest frame spin is given by the unit vector $\hat{\mathbf{s}}'_R$. The direction of $\mathbf{p}'$ is specified by the polar and azimuthal angles α and β relative to the coordinate system shown in Fig. 1. The unit vectors $\hat{\mathbf{n}}$, $\hat{\mathbf{l}}$, and $\hat{\mathbf{t}}$, shown in Fig. 1, form a right-handed coordinate system defined such that $\hat{\mathbf{l}}$ lies along $\mathbf{p}'$, $\hat{\mathbf{n}}=(\mathbf{q}\times\hat{\mathbf{l}})/|\mathbf{q}\times\hat{\mathbf{l}}|$, and $\hat{\mathbf{t}}=\hat{\mathbf{n}}\times\hat{\mathbf{l}}$. Projections of the spin unit vector on to these basis vectors are defined as

$$\mathcal{S}_n=\hat{\mathbf{n}}\cdot\hat{\mathbf{s}}'_R,\quad \mathcal{S}_l=\hat{\mathbf{l}}\cdot\hat{\mathbf{s}}'_R,\quad\text{and}\quad \mathcal{S}_t=\hat{\mathbf{t}}\cdot\hat{\mathbf{s}}'_R\,. \tag{2.4}$$

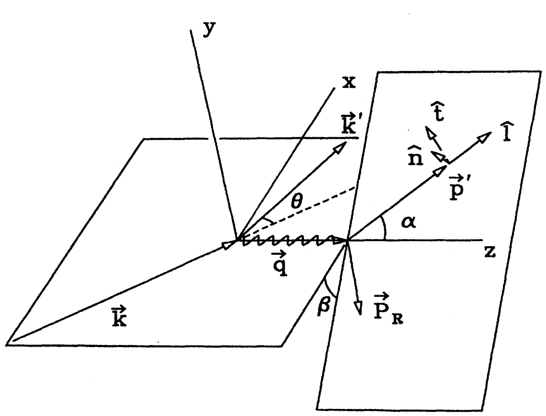

FIG. 1. Coordinate system used in describing the $(\vec{e},e'\vec{N})$ reaction.

The plane containing $\mathbf{p}'$ and $\mathbf{q}$ (or, alternatively, $\hat{\mathbf{l}}$, and $\hat{\mathbf{t}}$) contains the momentum of the ejectile $\mathbf{p}'$ and the recoil momentum of the residual nuclear system $\mathbf{P}_R=\mathbf{q}-\mathbf{p}'$, and plays the same role in this case as does the scattering plane in elastic proton scattering. For convenience we will refer to this as the hadronic or photonuclear scattering plane. The nuclear response functions R_j^i are determined from the electromagnetic four-vector current density of the nucleus by (A3) and (A4) in the Appendix. The expressions and conventions of (2.1) differ from that found in Ref. 2. The relationships between the two conventions are described in detail in the Appendix.

Table I contains an extensive summary of the properties of the 18 response functions. The first of these is the behavior of the response functions under time reversal (T) and parity (P) transformations. As described in Ref. 52, the TP operation can be used to show that the nuclear response tensor $W^{\mu\nu}(\hat{\mathbf{s}}'_R)$ (see the Appendix) has the property

$$W^{\mu\nu}(\hat{\mathbf{s}}'_R,(-))=W^{\nu\mu}(-\hat{\mathbf{s}}'_R,(+))\,, \tag{2.5}$$

where for the purposes of the discussion we have also displayed the dependence on the final state boundary condition, incoming $(-)$ and outgoing $(+)$ scattered waves. For the $(\vec{e},e'\vec{p})$ reaction, the $(-)$ condition is appropriate. In the general case, (2.5) does not imply an immediately useful result. However, in cases where the boundary conditions can be ignored, (2.5) states that the symmetric part of $W^{\mu\nu}(\hat{\mathbf{s}}'_R)$ is independent of $\hat{\mathbf{s}}'_R$ and that the antisymmetric part of $W^{\mu\nu}(\hat{\mathbf{s}}'_R)$ is proportional to $\hat{\mathbf{s}}'_R$. This is because the dependence of $W^{\mu\nu}(\hat{\mathbf{s}}'_R)$ on $\hat{\mathbf{s}}'_R$ is at most linear. These results really apply only in idealized models where there is no scattered wave at infinity, such as the PWIA. However, the PWIA is a useful conceptual limit so that the behavior of the response functions under such conditions is of interest. In the column of Table I headed "TP," response functions which survive the conditions (2.5) when the boundary conditions are ignored are labeled "even" while those which do not are labeled "odd."

Additional properties of the response functions are of interest from the experimental viewpoint. Since it is more difficult to measure ejected nucleons which have momenta lying out of the electron scattering plane (because it is necessary to move either the electron beam or the hadron spectrometer out of the plane of the floor of the experimental hall), it is useful to know which of the response functions can be detected in the electron scattering plane ($\beta=0$ or $\beta=\pi$) and which require going out of plane. The column of Table I labeled "survives in-plane" indicates which response functions contribute in the electron scattering plane.

Coincidence experiments are often performed with the

TABLE I. Properties of response functions.

Response function	TP	Survives in-plane	Survives in parallel kinematics	Electron polarization required	Reflection symmetry
R_L	even	yes	yes	no	even
R_L^n	odd	yes	no	no	even
R_T	even	yes	yes	no	even
R_T^n	odd	yes	no	no	even
R_{TT}	even	yes	no	no	even
R_{TT}^n	odd	yes	no	no	even
R_{TT}^l	odd	no	no	no	odd
R_{TT}^t	odd	no	no	no	odd
R_{LT}	even	yes	no	no	odd
R_{LT}^n	odd	yes	yes	no	odd
R_{LT}^l	odd	no	no	no	even
R_{LT}^t	odd	no	yes	no	even
$R_{LT'}$	odd	no	no	yes	even
$R_{LT'}^n$	even	no	yes	yes	even
$R_{LT'}^l$	even	yes	no	yes	odd
$R_{LT'}^t$	even	yes	yes	yes	odd
$R_{TT'}^l$	even	yes	yes	yes	even
$R_{TT'}^t$	even	yes	no	yes	even

ejectile momentum parallel ($\alpha=0$) or antiparallel ($\alpha=\pi$) to the momentum transfer. This is the so-called parallel-antiparallel kinematics, where the parallel/antiparallel distinction is conventionally made according to whether[9] the recoil momentum $\mathbf{P}_R$ is parallel or antiparallel to $\mathbf{q}$ or, alternately, whether[12] the "missing" momentum $\mathbf{p}_m=-\mathbf{P}_R$ is parallel or antiparallel to $\mathbf{q}$ (in PWIA, only, $\mathbf{p}_m$ is the initial nucleon momentum). The column of Table I labeled "survives in parallel kinematics" indicates which of the response functions contribute to the cross section under these kinematical conditions. The general definitions of the response functions depend upon the angle β between the electron and photonuclear scattering planes. Because β is not well defined in parallel/antiparallel kinematics, some care is needed in considering this limit. The simplest way to obtain the appropriate limit is to take the spin unit vector $\hat{\mathbf{s}}'_R$ to point in an arbitrary direction relative to the coordinate system of Fig. 1 and then to consider the cross section (2.1) [or $W^{\mu\nu}(\hat{\mathbf{s}}'_R)$] as the limit $\alpha\to 0(\pi)$ is approached. Assuming that the cross section is a well-behaved function in this limit, the cross section must then become independent of the angle β, which is no longer well defined. Since the response functions are independent of β, it is only necessary to examine the explicit β dependence in (2.1) and the β dependence implicit in the scalar products involving $\hat{\mathbf{s}}'_R$. The cross section is then put in the form of a Fourier series in β and by requiring that the coefficients of all β-dependent terms vanish, constraints can be placed on some of the response functions in the limit $\alpha\to 0(\pi)$. Those response functions denoted by "no" in the "survives in parallel kinematics" column of Table I must vanish in the limit $\alpha\to 0(\pi)$. The response functions R_L, R_T, and $R_{TT'}^l$, are not constrained, whereas the remaining response functions must satisfy the constraints

$$R_{LT}^n \mp R_{LT}^t \to 0 \quad \text{and} \quad R_{LT'}^n \pm R_{LT'}^t \to 0 \ , \tag{2.6}$$

where the upper (lower) sign refers to the limit $\alpha\to 0$ ($\alpha\to\pi$). Thus the four response functions of (2.6) effectively collapse into two independent response functions in parallel kinematics. The above limits also provide useful checks on numerical calculations of the response functions. Similar results can also be obtained directly from the definitions of the response functions by recognizing that finite experiments effectively integrate over β as $\alpha\to(\pi)$.

Once these constraints have been imposed, an independent set of surviving spin-dependent response functions can be taken to be just R_{LT}^n, $R_{LT'}^t$, and $R_{TT'}^l$. The surviving contributions of these terms in the limit $\alpha\to 0(\pi)$ are such that the LT' response contribution to the cross section is proportional to $\hat{\mathbf{x}}\cdot\hat{\mathbf{s}}'_R$, the LT contribution is proportional to $\hat{\mathbf{y}}\cdot\hat{\mathbf{s}}'_R$, and the TT' contribution is proportional to $\hat{\mathbf{z}}\cdot\hat{\mathbf{s}}'_R$. This implies that there is a natural choice for the spin coordinate system in this limit. The unit vector $\hat{\mathbf{l}}$ is chosen to point along $\mathbf{p}'$,$\hat{\mathbf{n}}$ points in the positive y direction and $\hat{\mathbf{t}}=\hat{\mathbf{n}}\times\hat{\mathbf{l}}$. This corresponds to the limiting process of first taking the limit $\beta\to 0$ and then $\alpha\to 0(\pi)$, that is, the natural spin coordinate system in parallel kinematics is defined by simply allowing $\beta=0$ in Fig. 1. Thus the spin-dependent response functions R_{LT}^n, $R_{LT'}^t$, and $R_{TT'}^l$ determine the transverse component normal to the electron scattering plane, transverse component in the electron scattering plane, and the longitudinal component of the ejectile polarization vector. Note from (2.1) that the detection of the in-plane components of the polarization vector require a polarized electron beam whereas the normal component does not. Because the response functions corresponding to the in-plane polarizations are predicted to be large (see later discussion) and because those corresponding to the normal polarization are predicted to be small, but dynamically sensitive, ejectile polarization measurements in the case of parallel/antiparallel kinematics appear to be a promising means of extracting dynamical information about the $(\vec{e},e'\vec{p})$ reaction process.

Since the polarization of existing electron beams is limited to about 40% and such beams have limited current, coincidence experiments which do not require beam polarization can be performed more rapidly. The column labeled "electron polarization required" in Table I indicates which of the response functions contribute only when the beam is polarized.

Finally, terms contributing to the cross section in the electron scattering plane with a contribution to the cross section which changes sign under the changes in kinematics $\beta=0\rightarrow\beta=\pi$, can be most easily separated from the total cross section, at least in principle, by making the change in azimuthal angle and subtracting cross sections. The last column of Table I, labeled "reflection symmetry," gives the symmetry of the contribution to the cross section associated with each of the response functions under the reflection of the ejectile momentum $\mathbf{p}'$ through the yz plane (irrespective of any associated direction changes of $\hat{\mathbf{n}}$, $\hat{\mathbf{l}}$, and $\hat{\mathbf{t}}$). The terms of special interest are those with odd symmetry which contribute in the electron scattering plane.

III. THE PLANE-WAVE-IMPULSE APPROXIMATION (PWIA)

The various response functions are to a large extent the result of a polarization transfer from the virtual photon to the ejected nucleon. For transversely polarized photons this results in a flipping of the component of the nucleon spin parallel to $\mathbf{q}$ when consistent with angular momentum conservation. Some feeling for the relative sizes of the response functions can be obtained by examining the simplest nontrivial calculation of the $(\vec{e},e'\vec{N})$ reaction, that is the plane-wave-impulse approximation (PWIA). This is also useful in obtaining analytic expressions which allow a simple comparison of the contrasting characteristics of those response functions which are nonvanishing in this limit. In addition, deviations from the plane wave results will show the importance of coupling to other reaction channels in more sophisticated models of this process. Finally, this approximation serves as a useful example of possible problems arising from violation of current conservation, ambiguities in the off-shell form of the current, the effects of relativistic kinematics, and of potential sensitivities of the response functions to medium modifications of free-nucleon properties such as electromagnetic form factors.

In the traditional nonrelativistic Schrödinger version of this approximation, the nuclear response tensor can be written as

$$W^{\mu\nu}(\hat{\mathbf{s}}'_R)=\sum_{m,s'}\chi^\dagger_{s'}\tfrac{1}{2}(1+\boldsymbol{\sigma}\cdot\hat{\mathbf{s}}'_R)J^\nu(p',q)\Psi_{nljm}(\mathbf{p}'-\mathbf{q})\Psi^\dagger_{nljm}(\mathbf{p}'-\mathbf{q})J^{\mu\dagger}(p',q)\chi_{s'}\ , \tag{3.1}$$

where the nonrelativistic current operators in momentum space can be written as

$$\begin{aligned} J^0(p',q)&=F_1(Q^2)\ ,\\ \mathbf{J}(p',q)&=F_1(Q^2)\frac{2\mathbf{p}'-\mathbf{q}}{2m}+G_M(Q^2)\frac{i\boldsymbol{\sigma}\times\mathbf{q}}{2m}\ ,\end{aligned} \tag{3.2}$$

with the free-nucleon electromagnetic form factors defined such that

$$G_E(Q^2)=F_1(Q^2)-\tau F_2(Q^2)\ , \tag{3.3}$$

and

$$G_M(Q^2)=F_1(Q^2)+F_2(Q^2)\ , \tag{3.4}$$

with $\tau=Q^2(4m^2)$. Note that (3.3) can be derived by a nonrelativistic expansion of the free Dirac current matrix elements to order $(1/m)$. As long as the lowest order is sufficient, there are no ambiguities in (3.2) associated with the use of the Gordon identity. There is, however, some ambiguity associated with the choice of form factors in (3.2). From (3.3) it is clear that that form factor $F_1(Q^2)$ differs from the Sachs form factor $G_E(Q^2)$ by a term proportional to τ. Since τ is manifestly of order $(1/m)^2$, the difference between the two form factors is of higher order than is retained in the nonrelativistic expansion. Under circumstances where the difference between $F_1(Q^2)$ and $G_E(Q^2)$ becomes quantitatively significant, the use of this lowest-order expansion clearly becomes invalid, and higher-order corrections must be carefully treated.

The expression for the nuclear response tensor can be simplified by noting that

$$\begin{aligned}\sum_m\Psi_{nljm}(\mathbf{p}'-\mathbf{q})\Psi^\dagger_{nljm}(\mathbf{p}'-\mathbf{q})&=\tfrac{1}{2}n_{nlj}(|\mathbf{p}'-\mathbf{q}|)\\ &=\frac{(2j+1)}{8\pi}R^2_{nlj}(|\mathbf{p}'-\mathbf{q}|)\ ,\end{aligned} \tag{3.5}$$

where $n_{nlj}(|\mathbf{p}|)$ is the momentum density distribution for a proton (neutron) in the nlj subshell and R_{nlj} is the single-particle radial wave function. The nuclear response tensor therefore reduces to

$$W^{\mu\nu}(\hat{\mathbf{s}}'_R)=\mathrm{Tr}[\tfrac{1}{4}(1+\boldsymbol{\sigma}\cdot\hat{\mathbf{s}}'_R)J^\nu(p',q)J^{\mu\dagger}(p',q)]n_{nlj}(|\mathbf{p}'-\mathbf{q}|)\ . \tag{3.6}$$

Performing the traces for various combinations of the currents and comparing to (A4), the response functions are

$$
\begin{aligned}
&R_L=F_1^2(Q^2)n_{nlj}(|\mathbf{p}'-\mathbf{q}|)\ ,\\
&R_L^n=0\ ,\\
&R_T=\left[F_1^2(Q^2)\frac{\mathbf{p}'^2}{m^2}\sin^2\alpha+G_M^2(Q^2)\frac{\mathbf{q}^2}{2m^2}\right]n_{nlj}(|\mathbf{p}'-\mathbf{q}|)\ ,\\
&R_T^n=0\ ,\\
&R_{TT}=F_1^2(Q^2)\frac{\mathbf{p}'^2}{m^2}\sin^2\alpha n_{nlj}(|\mathbf{p}'-\mathbf{q}|)\ ,\\
&R_{TT}^n=R_{TT}^l=R_{TT}^t=0\ ,\\
&R_{LT}=-2F_1^2(Q^2)\frac{|\mathbf{p}'|}{m}\sin\alpha n_{nlj}(|\mathbf{p}'-\mathbf{q}|)\ ,\\
&R_{LT}^n=R_{LT}^l=R_{LT}^t=0\ ,\\
&R_{LT'}=0\ ,\\
&R_{LT'}^n=F_1(Q^2)G_M(Q^2)\frac{|\mathbf{q}|}{m}n_{nlj}(|\mathbf{p}'-\mathbf{q}|)\ ,\\
&R_{LT'}^l=-F_1(Q^2)G_M(Q^2)\frac{|\mathbf{q}|}{m}\sin\alpha n_{nlj}(|\mathbf{p}'-\mathbf{q}|)\ ,\\
&R_{LT'}^t=-F_1(Q^2)G_M(Q^2)\frac{|\mathbf{q}|}{m}\sin\alpha n_{nlj}(|\mathbf{p}'-\mathbf{q}|)\ ,\\
&R_{TT'}^l=\left[G_M^2(Q^2)\frac{\mathbf{q}^2}{2m^2}\cos\alpha+F_1(Q^2)G_M(Q^2)\frac{|\mathbf{p}'||\mathbf{q}|}{m^2}\sin^2\alpha\right]n_{nlj}(|\mathbf{p}'-\mathbf{q}|)\ ,\\
&R_{TT'}^t=\left[-G_M^2(Q^2)\frac{\mathbf{q}^2}{2m^2}\sin\alpha+F_1(Q^2)G_M(Q^2)\frac{|\mathbf{p}'||\mathbf{q}|}{2m^2}\sin2\alpha\right]n_{nlj}(|\mathbf{p}'-\mathbf{q}|)\ .
\end{aligned}
\tag{3.7}
$$

We note that even at this rudimentary level, given the oversimplified nuclear structure, current operator, and final state interaction (FSI) models assumed, (3.7) illustrates an important point despite the fact that many of the response functions vanish in the PWIA limit. The point is that while the response functions must be constructed from $|\mathbf{p}'|,|\mathbf{q}|$, the angle α between $\mathbf{p}'$ and $\mathbf{q}$, and the form factors $F_1(Q^2)$ and $G_M(Q^2)$, which are the only available ingredients, the set of response functions displays considerable diversity in dependence upon these ingredients. Of particular interest in regard to nuclear medium modifications to free-nucleon form factors is the differing dependences and interference effects involving the nucleon form factors which are displayed by the various response functions above. This behavior carries over to the relativistic case considered next. Also, because this nonrelativistic limit is unique, it provides a convenient benchmark for considering relativistic corrections such as those which arise from relativistic forms of the current operator.

It is straightforward to generalize to a Dirac plane wave approximation. In this approximation, the nuclear response tensor can be written as

$$W^{\mu\nu}(\hat{\mathbf{s}}'_R)\sum_{m,s''}\bar{u}(\mathbf{p}',s'')\tfrac{1}{2}(1+\gamma_5\not{s}')\Gamma^\nu(q)\Psi_{nljm}(\mathbf{p}'-\mathbf{q})\overline{\Psi}_{nljm}(\mathbf{p}'-\mathbf{q})\overline{\Gamma}^\mu(q)u(\mathbf{p}',s'')\ ,\tag{3.8}$$

where

$$s'=\left[\frac{\mathbf{p}'\cdot\hat{\mathbf{s}}'_R}{m},\hat{\mathbf{s}}'_R+\frac{(\mathbf{p}'\cdot\hat{\mathbf{s}}'_R)\mathbf{p}'}{m(E'+m)}\right]\ ,\tag{3.9}$$

$$\Gamma^\mu(q)=F_1(Q^2)\gamma^\mu+\frac{F_2(Q^2)}{2m}i\sigma^{\mu\alpha}q_\alpha\ ,\tag{3.10}$$

and $\overline{\Gamma}^\mu=\gamma^0\Gamma^{\mu\dagger}\gamma^0$ denotes the Dirac adjoint. Again, the choice of the current operator as the usual form of the free Dirac current is somewhat arbitrary. The general form of the fully off-shell current operator can be constructed using general symmetry arguments and the properties of the Dirac γ matrices. From four-momentum conservation there are only two independent four-momenta at the vertex. Using any two four-momenta along with the γ matrices and their commutation relations, it can be shown that there are 12 independent four-vector forms which can be constructed in the Dirac space and which transform properly under parity.[56] In constructing these 12 forms, all scalars involving four-momenta contracted with γ matrices are incorporated, leaving three remaining momentum-space scalars which can be constructed using only the two independent four-momenta. Each of the 12 four-vectors in the Dirac space is therefore multiplied in general by a form factor which is an arbitrary function of these three scalars. For example, the form factors can be chosen to be functions of the invariant masses of the photon and the two nucleons which join at the vertex. The commutation relations for the γ matrices can be used to construct generalized Gordon identities which allow for the rearrangement of the various contributions to the vertex functions. In order to uniquely determine the complete off-shell behavior of the vertex function, it is necessary to have a dynamical theory for the nucleon. It may be possible, however, to place some constraints on the vertex function using reactions such as Compton scattering or meson electroproduction, or by theoretical constraints such as the Ward-Takahashi identity.[57]

In the absence of this additional information about the off-shell vertex function, it is usual to keep only a subset of the possible terms in the current operator. This subset is chosen to be both linearly independent and nonvanishing on shell and it is usual to ignore the dependence of form factors on the invariant masses of the nucleons. Two obvious choices for the operator are then (3.10) and the Gordon form of the operator. While these give identical results on shell, they produce different off-shell contributions. Clearly then, the choice of current operator is only poorly constrained and reflects a lack of knowledge concerning off-shell contributions to the vertex function. In all of the calculations presented in Sec. IV, the Dirac current operator given by (3.10) is used exclusively.

Using Dirac independent-particle bound state wave functions, the nuclear response tensor can be simplified by noting that

$$\sum_m \Psi_{nljm}(\mathbf{p}'-\mathbf{q})\overline{\Psi}_{nljm}(\mathbf{p}'-\mathbf{q}) = \tfrac{1}{4}[\not{n}^V_{nlj}(\mathbf{p}'-\mathbf{q})+n^S_{nlj}(|\mathbf{p}'-\mathbf{q}|)] , \quad (3.11)$$

where

$$\begin{aligned} n^{V\mu}_{nlj}(\mathbf{p}) &= \sum_m \overline{\Psi}_{nljm}(\mathbf{p})\gamma^\mu\Psi_{nljm}(\mathbf{p}) , \\ n^S_{nlj}(|\mathbf{p}|) &= \sum_m \overline{\Psi}_{nljm}(\mathbf{p})\Psi_{nljm}(\mathbf{p}) . \end{aligned} \quad (3.12)$$

The possible contributions to the right-hand side of (3.11) from pseudovector and antisymmetric tensor terms vanish as a result of *TP* symmetry. The pseudoscalar term vanishes because the upper to lower component spin density vanishes, leaving just the vector and scalar terms shown.

Using standard trace techniques, the nuclear response tensor can therefore be written as

$$W^{\mu\nu}(\hat{s}'_R) = \mathrm{Tr}\left[\tfrac{1}{2}(1+\gamma_5\not{s}')\Gamma^\nu(q)\tfrac{1}{4}[\not{n}^V_{nlj}(\mathbf{p}'-\mathbf{q})+n^S_{nlj}(|\mathbf{p}'-\mathbf{q}|)]\overline{\Gamma}^\mu(q)\frac{\not{p}'+m}{2m}\right] . \quad (3.13)$$

The calculation of the various response functions from (3.13) is straightforward, although very tedious, and the resulting expressions are exceedingly complicated. There is, however, a particular case in which the results are both simpler and interesting from a pedagogical standpoint. If the bound-state Dirac equation is projected onto the positive energy (plane-wave basis) space, eliminating all coupling to the negative-energy space, the solution of the Dirac equation can be written as the spinor wave function

$$\Psi_{nljm}(\mathbf{p}) = \left[\frac{E(\mathbf{p})+m}{2E(\mathbf{p})}\right]^{1/2}\left[\begin{array}{c} 1 \\ \dfrac{\sigma\cdot\mathbf{p}}{E(\mathbf{p})+m}\end{array}\right]\Phi_{nljm}(\mathbf{p}) , \quad (3.14)$$

where the wave function is normalized such that

$$\int d^3p\,\overline{\Psi}_{nljm}(\mathbf{p})\gamma^0\Psi_{nljm}(\mathbf{p}) = \int d^3p\,\Phi^\dagger_{nljm}(\mathbf{p})\Phi_{nljm}(\mathbf{p}) = 1 . \quad (3.15)$$

This simplification eliminates the dynamical aspects of relativity inherent in the Dirac equation. That is, the effects of coupling to virtual negative-energy states have been eliminated while the relativistic kinematics of the Dirac equation have been retained. It is in this sense that we refer to equivalent nonrelativistic calculations, as we did in Ref. 33.

In this case the momentum distributions are given by

$$\begin{aligned} n^{V0}_{nlj}(\mathbf{p}) &= n_{nlj}(|\mathbf{p}|) = \frac{2j+1}{4\pi}|R_{nlj}(|\mathbf{p}|)|^2 , \\ \mathbf{n}^V_{nlj}(\mathbf{p}) &= \frac{\mathbf{p}}{E(\mathbf{p})}n_{nlj}(|\mathbf{p}|) , \\ n^S_{nlj}(|\mathbf{p}|) &= \frac{m}{E(\mathbf{p})}n_{nlj}(|\mathbf{p}|) , \end{aligned} \quad (3.16)$$

where $R_{nlj}(|\mathbf{p}|)$ is the radial part of $\Phi_{nljm}(\mathbf{p})$. With these definitions of the momentum densities, the nuclear response tensor can be written as

$$W^{\mu\nu}(\hat{\mathbf{s}}'_R) = \mathrm{Tr}\left[\tfrac{1}{2}(1+\gamma_5\not{s}')\Gamma^\nu(q)\frac{1}{2}\frac{\not{p}'-\bar{\not{q}}+m}{2m} \times\overline{\Gamma}^\mu(q)\frac{\not{p}'+m}{2m}\right]n^S_{nlj}(|\mathbf{p}'-\mathbf{q}|) , \quad (3.17)$$

where

$$\bar{q} = (E(\mathbf{p}')-E(\mathbf{p}'-\mathbf{q}),\mathbf{q}) = (E'-E,\mathbf{q}) = (\bar{\omega},\mathbf{q})$$

so that the four-vector $p'-\bar{q}$ is on-mass-shell. This expression leads to a factorizable expression for the response functions. It is, however, not current conserving since the term in the bold parentheses depends on both the true four-momentum transfer q and the "on-shell" four-momentum transfer $\bar{q}$. An infinite number of possibilities exist for imposing a current conservation property on this expression by either adding terms or in changing q to $\bar{q}$ or $\bar{q}$ to q in such a way that $W^{\mu\nu}(\hat{\mathbf{s}}'_R)$ can be written in terms of manifestly current conserving forms. One popular method is the de Forrest "CC1" prescription[59] in which q is replaced by $\bar{q}$ everywhere in (3.13) except in the arguments of the form factors. Note that this "off-shell prescription" actually corresponds to placing the struck nucleon on mass-shell and is current conserving with respect to the corresponding mass-shell momentum transfer four-vector $\bar{q}$ rather than the true four-momentum transfer q which is really appropriate to the kinematics of the $(e,e'N)$ reaction. However, since this prescription leads to relatively simple expressions for the response functions and is also widely used, we present explicit forms for the response functions in this approximation

$$
\begin{aligned}
R_L &= \left[[F_1^2(Q^2)+\bar{\tau}F_2^2(Q^2)] \left[\frac{E'+E}{2m} \right]^2 - \frac{\mathbf{q}^2}{4m^2} G_M^2(Q^2) \right] n_{nlj}^S(|\mathbf{p}'-\mathbf{q}|) , \\
R_L^n &= 0 , \\
R_T &= \left[[F_1^2(Q^2)+\bar{\tau}F_2^2(Q^2)] \frac{\mathbf{p}'^2}{m^2} \sin^2\alpha + 2\bar{\tau} G_M^2(Q^2) \right] n_{nlj}^S(|\mathbf{p}'-\mathbf{q}|) , \\
R_T^n &= 0 , \\
R_{TT} &= [F_1^2(Q^2)+\bar{\tau}F_2^2(Q^2)] \frac{\mathbf{p}'^2}{m^2} \sin^2\alpha\, n_{nlj}^S(|\mathbf{p}'-\mathbf{q}|) , \\
R_{TT}^n &= R_{TT}^l = R_{TT}^t = 0 , \\
R_{LT} &= -2[F_1^2(Q^2)+\bar{\tau}F_2^2(Q^2)] \left[\frac{E'+E}{2m} \right] \frac{|\mathbf{p}'|}{m} \sin\alpha\, n_{nlj}^S(|\mathbf{p}'-\mathbf{q}|) , \\
R_{LT}^n &= R_{LT}^l = R_{LT}^t = 0 , \\
R_{LT'} &= 0 , \\
R_{LT'}^n &= \left[F_1(Q^2) + F_2(Q^2) \frac{E'\bar{\omega} - |\mathbf{q}||\mathbf{p}'|\cos\alpha}{2m^2} \right] G_M(Q^2) \frac{|\mathbf{q}|}{m} n_{nlj}^S(|\mathbf{p}'-\mathbf{q}|) , \\
R_{LT'}^l &= -\left[F_1(Q^2) \frac{E'}{m} + F_2(Q^2) \frac{\bar{\omega}}{2m} \right] G_M(Q^2) \frac{|\mathbf{q}|}{m} \sin\alpha\, n_{nlj}^S(|\mathbf{p}'-\mathbf{q}|) , \\
R_{LT'}^t &= -\left[F_1(Q^2)\cos\alpha + F_2(Q^2) \frac{E'\bar{\omega}\cos\alpha - |\mathbf{q}||\mathbf{p}'|}{2m^2} \right] G_M(Q^2) \frac{|\mathbf{q}|}{m} n_{nlj}^S(|\mathbf{p}'-\mathbf{q}|) , \\
R_{TT'}^l &= \left[F_1(Q^2) \frac{|\mathbf{p}'||\mathbf{q}| - \bar{\omega}E'\cos\alpha}{m^2} - F_2(Q^2) \frac{\bar{q}^2}{2m^2} \cos\alpha \right] G_M(Q^2) n_{nlj}^S(|\mathbf{p}'-\mathbf{q}|) , \\
R_{TT'}^t &= \left[F_1(Q^2) \frac{\bar{\omega}}{m} + F_2(Q^2) \frac{E'\bar{q}^2}{2m^3} \right] G_M(Q^2) \sin\alpha\, n_{nlj}^S(|\mathbf{p}'-\mathbf{q}|) ,
\end{aligned}
\tag{3.18}
$$

where $\bar{\tau}=\bar{Q}^2/(4m^2)=-\bar{q}^2/(4m^2)$. Since these results do not contain the physics of coupling ot the negative-energy Dirac space in the bound state, we refer to this as the semirelativistic plane-wave-impulse approximation (SRPWIA). First we note that, given the ambiguity in using $G_E(Q^2)$ or $F_1(Q^2)$, Eqs. (3.18) reduce to the comparable expressions in (3.7) in the limit where the momenta are small compared to the nucleon mass. Equations (3.18) exhibit a diversity of dependence on the ingredients from which the response functions are formed that goes a bit beyond that found in the nonrelativistic limit (3.7). The additional structure in (3.18) due to relativistic effects arises exclusively from higher-order terms in the $(1/m)$ expansion of the current operator. Furthermore, purely off-shell terms appear for other choices of the relativistic current operator, yielding different and much more complex forms for the response functions. Thus, the relativistic corrections exhibited in (3.18) must be regarded as representative, only. Of course, model dependence introduced by ambiguities in the off-shell contributions bears directly on the degree of precision with which predictions can be reliably made.

IV. DWIA CALCULATIONS OF $(\vec{e},e'\vec{p})$

The logical extension of the plane-wave results discussed above is the distorted-wave-impulse approximation (DWIA). In this section we present the first results of DWIA calculations of the spin-dependent response functions for a many-body nucleus. In examining these results, however, certain limitations of the DWIA should be kept in mind. Inherent in the DWIA approximation to the $(e,e'p)$ reaction is an inconsistent treatment of more complicated additional reaction channels open at a given energy transfer ω. While such channels are included in the optical potential, in principle, resulting in the absorptive part of the potential, and may also be included as virtual contributions to the bound-state mean-field potential by means, for example, of a Brueckner-Hartree-Fock approximation, the electromagnetic current operator is taken to be a strictly one-body operator which cannot couple the bound state directly to any of the more complex reaction channels. The most immediate consequence of the inadequacy of this approximation is a violation of current conservation. This defect may be remedied by either a solution of a coupled-channel model containing all of the significant channels at each energy, or equivalently, effective one-body current operators can be constructed which contain information about the many-body channels which is consistent with that contained in the one-body potential employed. The latter approach illustrates the possible effects of many-body corrections to the DWIA.

Consider the contribution of two-body currents such as the nonrelativistic meson exchange current. Such a two-body current must be present at the Hamiltonian level to provide consistency with the one-pion exchange potential. Unlike the free one-body current operator, such current operators do not determine a unique value of $\mathbf{p}$ for given values of $\mathbf{p}'$ and $\mathbf{q}$ even in PWIA, but sample a range of values centered about $|\mathbf{p}'-\mathbf{q}|$. This is due to the interaction between the ejected nucleon and the rest of the nucleus which allows the momentum transfer $\mathbf{q}$ to be shared by the valence nucleon and the residual nucleus. As a result, the cross section is no longer as closely constrained by the one-body momentum distribution. The effect of such exchange currents can be mostly clearly identified where the impulse approximation gives a small result, such as at large recoil momentum where the impulse approximation is suppressed by the one-body momentum distribution, whereas momentum sharing allows the effective current operator to make a larger relative contribution. This feature will also be characteristic of other many-body corrections to the effective current operator such as ground-state correlations and inelastic rescattering effects.

In addition, the dependence of the effective current operator on the external momenta $\mathbf{q}$ and $\mathbf{p}'$ is more complicated than that of the free current operator. The functional dependence on the asymptotic momenta and spin of the impulse and many-body contributions can, therefore, be expected to be distinctive. This raises the possibility that kinematical regions may be identified for the various response functions which will tend to emphasize one or more dynamical contributions to the reaction. The additional freedom provided by the measurement of the recoil polarization can be expected to facilitate such attempts to isolate individual physical processes. This has been shown to be the case in existing calculations of electrodisintegration of the deuteron.[47,51] Thus, although the DWIA results which follow exhibit a number of physically interesting characteristics, this study by no means exhausts the physically interesting issues associated with the $(\vec{e},e'\vec{p})$ reaction. Considerably more analytical sophistication will be required to fully circumscribe the dynamics relevant to this reaction. DWIA results represent a first step in this direction.

Before discussing our results in detail, it is useful to make some comments on the likely extraction of the information contained in the $(\vec{e},e'\vec{p})$ cross section. It is clearly a formidable task to undertake the separation of all 18 response functions. For simple systems such as the deuteron, it may be possible for a compressive program of measurements, including target and ejected nucleon polarization, to completely determine the transition current densities up to an overall phase. In this case, the greater degree of certainty with which dynamical models may be applied to the two-nucleon system may justify the effort inherent in such a comprehensive separation of response functions. However, the greater degree of uncertainty in our understanding of reactions in many-body systems where many reaction channels are open, seems to militate against such an ambitious approach. A more modest and realistic approach seems to be to select response functions which show a high degree of sensitivity to the special characteristics of a given model of the reaction, or for which different dynamical models give disparate predictions. In order to focus on specific physical issues of special interest, it is also important to incorporate response functions which closely constrain other aspects of the reaction process. For example, an accurate description of response functions which are very sensitive to FSI effects allows the analysis of other response functions sensitive to medium-modified nucleon properties to proceed with greater confidence. Thus, from the theoretical standpoint, it is important to study the variations in predictions of the response functions caused by various dynamical ingredients or models. The choice of response functions will of course also be greatly influenced by the degree of difficulty required to separate them from the cross section.

Figures 2–4 display the first comprehensive results of DWIA predictions for the full set of 18 $(\vec{e},e'\vec{p})$ response functions. For fixed $|\mathbf{p}'|$, the response functions are functions only of $|\mathbf{q}|$ and the angle α between $\mathbf{p}'$ and $\mathbf{q}$, or equivalently $|\mathbf{q}|$ and $|\mathbf{p}'-\mathbf{q}|$. This is due to the explicit extraction of all β and spin dependence in defining the response functions in terms of the nuclear response tensor $W^{\mu\nu}(\hat{\mathbf{s}}'_R)$ and is illustrated by the PWIA expressions in Sec. III. The physically attainable values of $|\mathbf{q}|$ and $|\mathbf{p}'-\mathbf{q}|$ are represented by the shaded region in Fig. 5. The boundary of this region corresponds to the so-called parallel-antiparallel kinematics. The results displayed in Figs. 2–4 correspond to a choice of variables such that $|\mathbf{q}|=|\mathbf{p}'|$, which is represented by the horizontal dashed

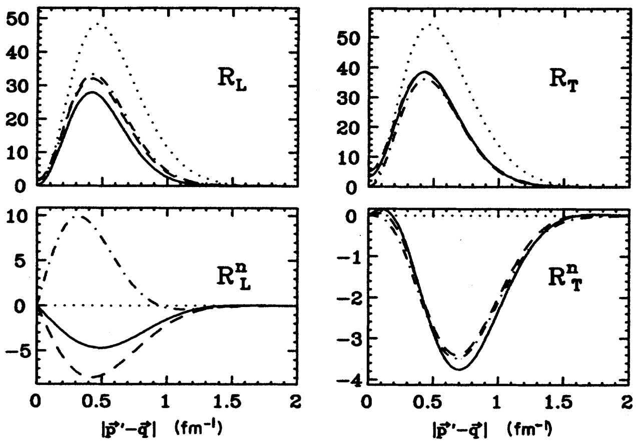

FIG. 2. Response functions for the ejection of a $T_{p'}=135$ MeV proton from the $1p_{1/2}$ shell of ^{16}O. The response functions are shown for fixed momentum transfer $|\mathbf{q}|=2.641$ fm^{-1} as a function of the magnitude of the recoil momentum. The solid and dashed lines represent the relativistic and nonrelativistic DWIA calculations, while the dotted line represents the relativistic PWIA and the dot-dashed line represents the relativistic "on-shell" calculation, as described in the text.

line in Fig. 5. However, the results displayed in Figs. 2–4 are representative of the general character of the response functions over the full region of Fig. 5. The results depicted in Figs. 2–4 represent an extension of the calculations of Ref. 33 to include the 13 response functions which depend on the spin of the ejected proton. Results are presented for the specific case of ejection of a 135 MeV proton from the $1p_{1/2}$ shell of ^{16}O at a constant momentum transfer of 2.641 fm^{-1}.

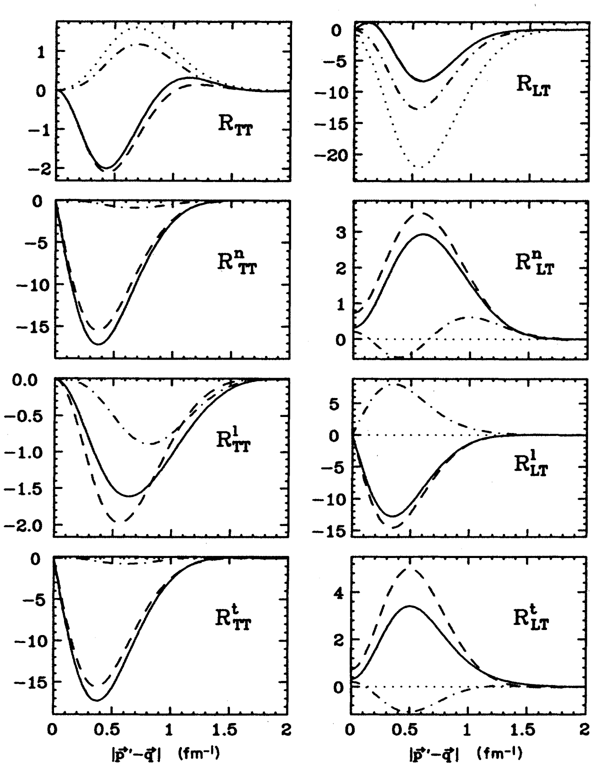

FIG. 3. Same as Fig. 2.

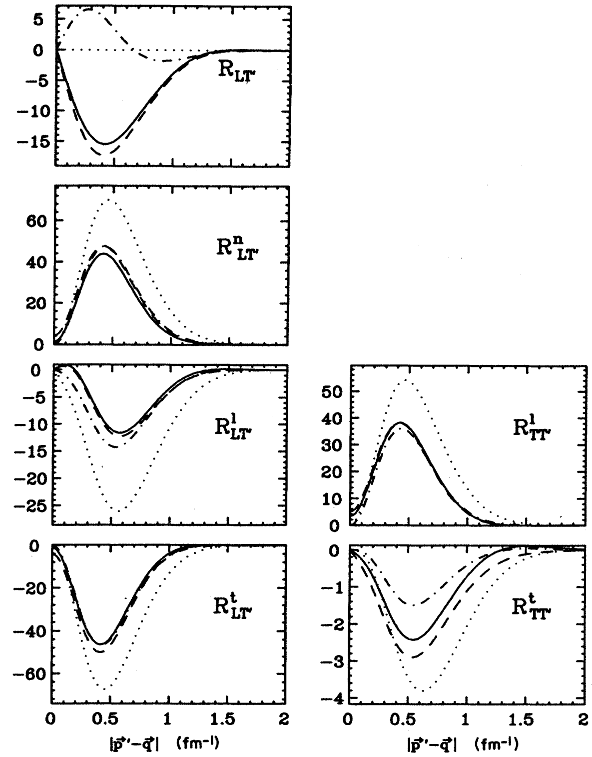

FIG. 4. Same as Fig. 2.

Four different dynamical calculations are presented for each of the 18 response functions in Figs. 2–4. The solid lines represent the unfactorized Dirac DWIA calcula-

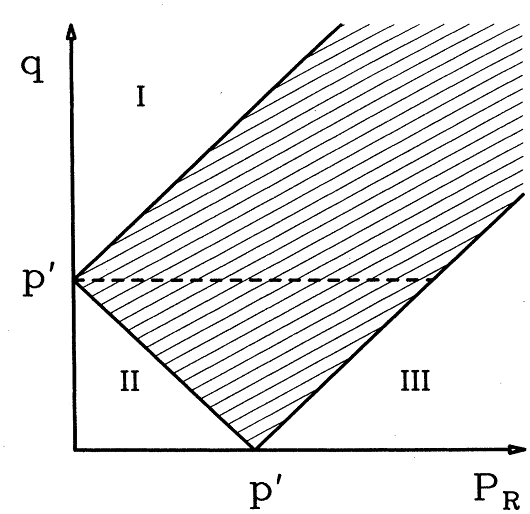

FIG. 5. The hatched region represents the physically available values of momentum transfer and recoil momentum for the $(\vec{e},e'\vec{N})$ reaction. Parallel/antiparallel kinematics correspond to the borders of this region, while the kinematics used in Figs. 2–4 is represented by the dashed line.

tions as described in Ref. 33. These use Dirac optical model scattering wave functions for the ejected nucleon using the Dirac optical potential of Ref. 55 and Dirac-Hartree independent particle bound-state wave functions.[38] The free Dirac current operator as given by (3.10) using the Höhler 8.2 parametrization[60] of the nucleon form factors is used in all of the calculations presented in this section. The dotted lines represent the Dirac PWIA calculation with the nuclear response tensor described by (3.13). The equivalent nonrelativistic DWIA calculations, as described by (3.17), are represented by the dashed lines. A calculation, which for convenience we refer to as "on shell," is represented by the dot-dashed lines. In this calculation, only the pole part of the propagator which appears in the Møller operator for the scattering wave function is kept. This forces the nucleon-nucleus scattering t matrix, which appears in this Møller operator, to be on shell. By comparison with the full Dirac and nonrelativistic calculations, this calculation can be used as a rough measure of the sensitivity of the DWIA calculations to the off-shell components of the t matrix which are not so highly constrained by experimental elastic proton scattering.

A careful examination of the 18 response functions shown in Figs. 2–4 show that there is no consistent relationship among the various calculations which holds for all of the response functions. This is not surprising since 7 of the 18 response functions cannot even contribute in the PWIA, but do so in the various distorted wave calculations. This diversity alone suggests that it is indeed likely that a selective separation of response may be useful in assessing the merits of various models applicable to this reaction. Although there seems to be no global relationship between the four calculations, some interesting patterns do appears in Figs. 2–4.

First we note from the figures that a number of the polarization response functions are very large, many being comparable in size to the familiar R_L and R_T. Of these $R_{LT'}^n$, $R_{LT'}^l$, $R_{LT'}^t$, and $R_{TT'}^l$ tend to be largest because they are large even in the PWIA limit.[61] This is due to the fact that these response functions arise from the antisymmetric part of $W^{\mu\nu}(\hat{\mathbf{s}}_R')$, which also means that a polarized electron beam is necessary to resolve them. However, we recall from Sec. II and Table I that $R_{LT'}^n$, $R_{LT'}^t$, and $R_{TT'}^l$ are among the few response functions which survive in parallel kinematics and are the only ones whose contributions to the cross section change sign with the electron helicity in this case. Thus these response functions can be readily accessed with in-plane measurements in parallel kinematics by flipping the electron helicity and detecting the ejectile polarization in the scattering plane. The other large polarization response functions, which do not survive in parallel kinematics, are R_{TT}^n, R_{TT}^t, and R_{LT}^l. These response functions vanish in PWIA yet are very large in DWIA predictions due to final-state interactions. Because of this they are very sensitive to both on- and off-shell final-state interaction effects, as is evident from the figures. The additional polarization response functions which do survive in parallel kinematics, R_{LT}^n and R_{LT}^t, are similarly sensitive but are predicted to be small. These results for the polarization response functions are encouraging for further theoretical and experimental work: a number of the response functions are predicted to be large while others are small, some contribute in the electron plane and for parallel kinematics, and the response functions show considerable sensitivity to the subset of realistic dynamical effects treated here.

More specifically, we see from the figures that, with the exception of R_T^n, all of the TP even response functions (those which vanish in PWIA) are very sensitive to the off-shell components of the scattering wave function. For example $R_{LT'}$, which is predicted to be relatively large, is extremely sensitive to off-shell effects but displays little sensitivity to the differences between Dirac and nonrelativistic dynamics. In contrast R_L^n, although predicted to be somewhat smaller, is very sensitive to both off-shell and relativistic effects. As can be seen from Fig. 2 and in Ref. 33, the transverse-transverse response function R_{TT} is also sensitive to off-shell components by virtue of the sensitive cancellation between the squares of the two transverse components of the transition current density. Much like $R_{LT'}$, the response function R_{LT}, which is predicted to be appreciable, also shows considerable sensitivity to off-shell effects while being totally insensitive to the difference between Dirac and nonrelativistic dynamics. Unlike $R_{LT'}$ however, R_{LT} is very large in the PWIA limit, so that its response to final-state effects is very different from and complementary to that of $R_{LT'}$.

For the large response functions the dynamical differences between the relativistic and nonrelativistic DWIA calculations result in differences in size of on the order of 5% to 10%, with the longitudinal response function R_L showing an enhanced effect of 10% to 20%. This apparent relativistic suppression of R_L relative to R_T is especially interesting in view of an analogous suppression which has been observed in inclusive electron scattering. For the smaller response functions the dynamical effects of relativity are on the order of 5% to 10% with the exceptions of R_{LT}^n, R_{LT}^t, and R_{TT}^l where the effects vary from 20% to 35%, and R_L^n where the effect is 75%.

The response functions are bilinear combinations of the various spin-dependent transition amplitudes [see (A4) and (6.1)–(6.15) of Ref. 52]. For a specific class of response functions, those labeled by the same subscript L, T, TT, etc., the response functions within the class are constructed from the same set of amplitudes. In connection with this, the calculations of Figs. 2–4 show an interesting pattern: Half of the response functions are relatively large while the other half are relatively small, the grouping of response functions is the same for all classes. That is, the unpolarized response function along with the response function for spin aligned along $\hat{\mathbf{l}}$ form one group, while the response functions for the perpendicular spin directions $\hat{\mathbf{n}}$ and $\hat{\mathbf{t}}$ form the other. For each class response functions in one group are relatively large while those in the other are relatively small. For example, R_{TT} and R_{TT}^l are relatively small, while R_{TT}^n and R_{TT}^t are relatively large. Given this pattern along with the fact that the response functions for a given class are constructed from combinations of the same transition amplitudes, it is clear that the small response functions involve destructive interference, while the large response functions involve constructive interference. Response functions which are very small due to the delicate cancellation of large amplitudes interfering destructively are most likely to show sensitivity to variations in the details of computational models which may tend to modify this cancellation. Indeed, the calculations presented in Figs. 2–4 are consistent with this expectation since all of the response functions which are particularly sensitive to differences between the relativistic and nonrelativistic models are relatively small. The converse, however, is not the case since there are small response functions which display very little sensitivity to this difference. These small response functions, however, appear to show promise as a means of discriminating between models of quasielastic electrons scattering, and should be examined for the effects of more sophisticated descriptions of nuclear structure, off-shell current operator ambiguities, and meson exchange currents.

V. CONCLUSIONS

The first comprehensive predictions for the full set of 18 response functions for the $(\vec{e},e'\vec{p})$ reaction have been presented. The diversity in sensitivity of the response functions to the various dynamical models considered here suggests that the measurement of response functions depending on ejected nucleon and/or electron polarizations may be useful in discriminating between dynamical models of the $(e,e'N)$ reaction. Many of the polarization response functions are comparable in size to the familiar unpolarized response functions, while others are predicted to be small. Relativistic effects are significant for a number of the response functions. For example, the response functions show a sensitivity to relativistic dynamics which vary from a few percent to 75%. Off-shell final-state interaction effects range from being small for some response functions to being physically dominant for others. The explicit PWIA expressions for the response functions given in Sec. III indicate that medium-modifications to free-nucleon properties and off-shell current operator effects will continue to follow this trend of diversity of produced effects on the various response functions. However, the size of such effects is presently unknown.

Because of the complexity involved in dealing with such a large number of response functions, a selective focus on a subset of the response functions appears to be called for. This subset can be chosen to isolate specific physical implications of dynamical models, or for their sensitivity to special features of a given model, which closely constrain other important physical ingredients. The special simplifications of parallel kinematics (only five independent response functions survive) appears promising since it allows ready access to some of the large polarization response functions. This entails only in-plane measurements, the ability to flip the electron helicity, and a final ejectile spin determination.

The flexibility provided by final-state polarization measurements in the $(\vec{e},e'\vec{p})$ reaction is considerable. In our study, every variation we have considered produces distinctive implications for some subset of the response functions. As in the case of medium and off-shell current operator effects, there is every reason to expect this trend to continue as additional realistic physical processes are explored. For example, realistic nuclear structure implications, exchange currents, and further relativistic effects remain to be explored. Also, the four-momentum transfer behavior of the response functions needs to be explored, especially as a function of differing dynamical models. The results of the present study suggest that such investigations will prove interesting. It is also clear from this initial study that potential advantages to be gained by measuring these new response functions merit an investment in studies of the feasibility of separating some or all of these response functions from the cross section, and efforts to develop any new experimental techniques which may be necessary to achieve this goal.

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APPENDIX

In order to be consistent with what is becoming a recognized convention in describing the $(e,e'N)$ reaction, we have defined the azimuthal angle β used to specify the ejected nucleon momentum to be measured relative to the electron scattering plane, rather than to the normal electron scattering plane as we did in Refs. 33 and 53. The relationship of the "old" angles of our previous papers to those used in this work is defined by

$$\alpha=\alpha_{\text{old}} , \tag{A1}$$

$$\beta=\beta_{\text{old}}-\frac{\pi}{2} . \tag{A2}$$

As a result of this change of variables, the associated change in the labels of the coordinate axes, and a desire to present the cross section (2.1) in a manner where all terms enter formally with the same (positive) sign, the definitions of the response functions used in the present work differ from those of the "old" response functions of our previous papers by at most a sign.

Using the definition of the nuclear response tensor as

$$W^{\mu\nu}(\hat{\mathbf{s}}'_R)=\overline{\sum_i}\sum_F \int d^3P'\delta^4(q+P-P'-p')\langle p',\hat{\mathbf{s}}'_R,(-);F,P'|J^{\nu}(q)|I,P\rangle\langle I,P|J^{\mu\dagger}(q)|p',\hat{\mathbf{s}}'_R,(-);F,P'\rangle , \tag{A3}$$

the "new" response functions for $(\vec{e},e'\vec{N})$ are defined by

$$\begin{aligned}
&\tfrac{1}{2}(R_L+R_L^n\mathcal{S}_n)=\int_{\text{line}}dE'\,W^{00}(\hat{\mathbf{s}}'_R)\ ,\\
&\tfrac{1}{2}(R_T+R_T^n\mathcal{S}_n)=\int_{\text{line}}dE'[W^{11}(\hat{\mathbf{s}}'_R)+W^{22}(\hat{\mathbf{s}}'_R)]\ ,\\
&\tfrac{1}{2}[(R_{TT}+R_{TT}^n\mathcal{S}_n)\cos2\beta+(R_{TT}^l\mathcal{S}_l+R_{TT}^t\mathcal{S}_t)\sin2\beta]=\int_{\text{line}}dE'[W^{11}(\hat{\mathbf{s}}'_R)-W^{22}(\hat{\mathbf{s}}'_R)]\ ,\\
&\tfrac{1}{2}[(R_{LT}+R_{LT}^n\mathcal{S}_n)\cos\beta+(R_{LT}^l\mathcal{S}_l+R_{LT}^t\mathcal{S}_t)\sin\beta]=-\int_{\text{line}}dE'[W^{01}(\hat{\mathbf{s}}'_R)+W^{10}(\hat{\mathbf{s}}'_R)]\ ,\\
&\tfrac{1}{2}[(R_{LT'}+R_{LT'}^n\mathcal{S}_n)\sin\beta+(R_{LT'}^l\mathcal{S}_l+R_{LT'}^t\mathcal{S}_t)\cos\beta]=i\int_{\text{line}}dE'[W^{20}(\hat{\mathbf{s}}'_R)-W^{02}(\hat{\mathbf{s}}'_R)]\ ,\\
&\tfrac{1}{2}(R_{TT'}^l\mathcal{S}_l+R_{TT'}^t\mathcal{S}_t)=i\int_{\text{line}}dE'[W^{12}(\hat{\mathbf{s}}'_R)-W^{21}(\hat{\mathbf{s}}'_R)]\ .
\end{aligned}\tag{A4}$$

These definitions yield β-independent response functions consistent with the expression for the cross section (2.1). The relationship between these "new" response functions and the "old" ones can be written as

$$R_j^i=C_j^i(R_j^i)_{\text{old}}\ ,\tag{A5}$$

where no summation of indices is implied. The coefficients C_j^i are given by

$$\begin{aligned}
&C_L=C_L^n=1\ ,\\
&C_T=C_T^n=1\ ,\\
&C_{TT}=C_{TT}^n=C_{TT}^l=C_{TT}^t=-1\ ,\\
&C_{LT}=C_{LT}^n=-1,\quad C_{LT}^l=C_{LT}^t=1\ ,\\
&C_{LT'}=C_{LT'}^n=1,\quad C_{LT'}^l=C_{LT'}^t=-1\ ,\\
&C_{TT'}^l=C_{TT'}^t=1\ .
\end{aligned}\tag{A6}$$

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