Equation of state of cold nuclear matter extracted from nuclear masses by the droplet model

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The information of the equation of state of cold nuclear matter is extracted from the experimental nuclear masses by the droplet model. The result shows the asymmetry dependence of the equation of state in a more direct way than has been done before. It shows that the spherical or the deformed nuclear region appears alternately as the bulk nucleon density of the core of nuclei increases. An explanation of why the rms fit of the droplet model to the nuclear masses gives a very broad minimum region of the nuclear compressibility is possible based on this work.

The equation of state of cold nuclear matter has been studied by various kinds of approaches for more than a decade,[1-11] but its experimental information is still small. What we have known are the normal nuclear density ~ 0.15 nucleons/fm^3, the normal binding energy per nucleon ~ -16 MeV, and the normal nuclear compressibility $K\sim$ a few hundred MeV. The most important one of these three quantities, for the studies in the macroscopic properties of the nucleus and in the relativistic heavy-ion collisions as well as in the astrophysics, is the nuclear compressibility K, while its experimental information from nuclear and astrophysical evidence is still ambiguous.[12,13] Especially, all of this evidence of the nuclear compressibility K is model dependent: The estimations from astrophysical observations depend on the neutron star models or the supernova models,[12-14] the evaluations from relativistic heavy-ion collisions depend on the models to deal with the measurement data,[15-18] the value of K given by the giant monopole resonance depends on the model to specify the collective motion of the nucleus,[19-20] the normal nuclear compressibility determined from the nuclear masses depends on the model expressed for the nuclear masses,[21-24] and so on. It is worthwhile to note that the above-mentioned case of nuclear masses is that the normal nuclear compressibility K is determined from the parameters appearing in the proposed phenomenological nuclear mass formula while all of the parameters are adjusted to fit the experimental nuclear masses. This is an indirect way to extract the equation of state of cold nuclear matter from nuclear masses; especially, it cannot show the asymmetry dependence of the nuclear equation of state directly. The purpose of this paper is to try to extract the information of the equation of state of cold nuclear matter, especially the asymmetry dependence of the equation of state, from the experimental nuclear masses in a way as direct as possible.

The idea is as follows: First, subtract from the experimental nuclear mass the surface energy, the Coulomb energy, the shell energy, the pair energy, and the other terms to leave the bulk energy of the core of this nucleus; second, calculate the bulk energy density, the bulk nucleon density, and the bulk asymmetry of the core of this nucleus; third, the ensemble of the datum obtained by the foregoing method from all of the experimental nuclear masses gives model-dependent experimental evidence of the equation of state of cold nuclear matter. The droplet model of atomic nuclei[21,25-28] is employed here to calculate the energy terms other than the bulk energy, to calculate the bulk nucleon density, as well as to calculate the bulk asymmetry of the given nucleus.

The bulk energy E_b of a nucleus with a given neutron number N and proton number Z can be extracted from the experimental mass $E_{\rm exp}$ of this nucleus as

$$E_b=E_{\rm exp}-E(N,Z)+E_V ,$$

where the second term $E(N,Z)$ is the total nuclear energy calculated by the droplet model formula in which the volume energy, the surface energy, the Coulomb energy, the shell correction, the even-odd term, the Wigner term, and a phenomenological term are included;[25-28] the third term E_V is the droplet model volume energy and its formula can be found in Refs. 25 and 28. The bulk energy density e of the core of the given nucleus can be calculated from the bulk energy E_V as

$$e=E_b/A ,$$

where $A=N+Z$.

The bulk nucleon density of the core of the given nucleus ρ can be calculated as

$$\rho=(1-3\epsilon)\rho_0 ,$$

where ρ_0 is the normal density of symmetric nuclear matter, and ϵ is the droplet model quantity denoted by $\bar{\epsilon}$ in Refs. 21 and 25–27. The bulk asymmetry δ is another droplet model quantity denoted by $\bar{\delta}$ in Refs. 21 and 25–27. For a given nucleus (N,Z) we can calculate the bulk density ρ and the bulk asymmetry δ by the droplet model formula.[21,25-27]

The experimental nuclear masses $E_{\rm exp}$ are taken from

Ref. 29, and the droplet model parameters which are fitted to these experimental masses and given in Ref. 27 are used in the present work. Some of these parameters are as follows: $a_1=15.96$ MeV, the volume energy coefficient; $a_2=20.69$ MeV, the surface energy coefficient; $J=36.8$ MeV, the symmetry energy coefficient; $r_0=1.18$ fm, the nuclear radius constant; and $K=240$ MeV, the compressibility coefficient. The normal nuclear density corresponding to the nuclear radius constant r_0 is $\rho_0=0.1453$ nucleons/fm^3.

All of 1643 experimental nuclear masses are treated in this way, and the results are shown in Fig. 1 as the energy density $e=E_b/A$ (in unit MeV) vs the relative nucleon density ρ/ρ_0 of the bulk nuclear matter in the core of nuclei. The top plot is for the total of 974 spherical nuclei. The middle plot is for the total of 669 deformed nuclei. It can be seen that there is no overlap region between them in the $e-\rho/\rho_0$ plane. The light nuclei are in the lower right-hand side while the heavy nuclei are in the upper left-hand side of the plot. Roughly speaking, the bulk density ρ decreases while the bulk asymmetry δ increases as the nuclear mass increases. It can be seen also that the spherical or the deformed nuclear region appears alternately as the bulk nucleon density increases, while the alternative interval is roughly $\Delta(\rho/\rho_0)\sim 0.02$. The range of the relative bulk density is about $1.00<\rho/\rho_0<1.18$, while the range of the bulk asymmetry is about $0<\delta<0.16$. In order to show the asymmetry dependence of this plot, the bottom plot gives the points for selected ranges of the bulk asymmetry δ as (0.0185, 0.0215), (0.0385, 0.0415), (0.0585, 0.0615), (0.0785, 0.0815), (0.0985, 0.1015), (0.1185, 0.1215), and (0.1385, 0.1415) from the bottom to the top of the plot, respectively.

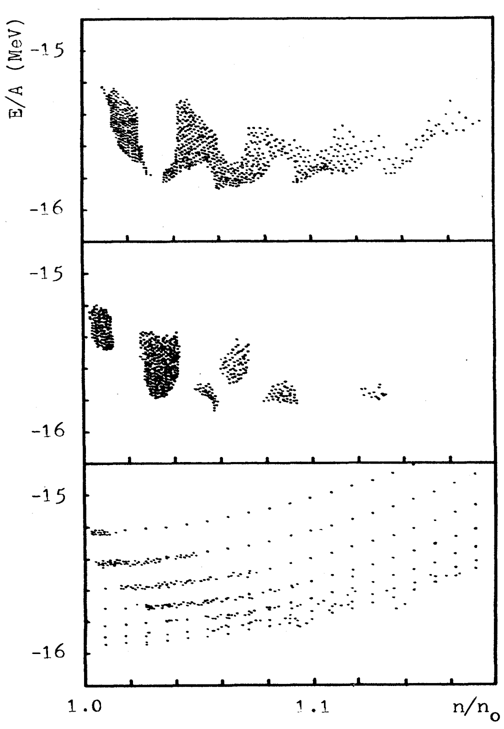

FIG. 1. The energy density (in MeV/nucleon) vs the nucleon density (in unit of normal nuclear density) of the bulk nuclear matter of the core of nuclei. The top plot is for the 974 spherical nuclei. The middle plot is for the 669 deformed nuclei. The bottom plot is for the points with selected ranges of the bulk nuclear asymmetry δ as (0.0185, 0.0215), (0.0385, 0.0415), (0.0585, 0.0615), (0.0785, 0.0815), (0.0985, 0.1015), (0.1185, 0.1215), and (0.1385, 0.1415) from the bottom to the top of the plot, respectively. The dilute dot curves in the bottom plot are calculated by an asymmetry-dependent equation of state of cold nuclear matter (Ref. 11).

The dilute dot curves shown in the bottom plot of Fig. 1 are calculated by an asymmetry-dependent equation of state of cold nuclear matter, and the corresponding asymmetry δ is 0.02, 0.04, 0.06, 0.08, 0.10, 0.12, and 0.14 from the bottom to the top of the plot, respectively. This asymmetry-dependent equation of state of cold nuclear matter is given by the Thomas-Fermi statistical model with the Seyler-Blanchard phenomenological momentum-dependent nuclear interaction, and can be expressed analytically as follows:[11]

$$e(\rho,\delta)=T[\tfrac{3}{4}D(\rho/k)^{2/3}-\tfrac{1}{2}C(\rho/k)^{3/3}+\tfrac{3}{10}B(\rho/k)^{5/3}]\ ,$$

$$T=b^2/2m\ ,$$

$$k=(16\pi/3)(b/2\pi\hbar)^3\ ,$$

$$D=\tfrac{2}{5}[(1+\delta)^{5/3}+(1-\delta)^{5/3}]\ ,$$

$$C=(\alpha+\beta)+(\alpha-\beta)\delta^2\ ,$$

$$B=\alpha[(1+\delta)^{8/3}+(1-\delta)^{8/3}]$$
$$+\beta(1-\delta^2)[(1+\delta)^{2/3}+(1-\delta)^{2/3}]\ ,$$

where α and β are the nuclear interaction strength parameters while b is the critical momentum parameter appearing in the Seyler-Blanchard interaction, m is the nucleon mass, and $\hbar$ is the reduced Planck constant. It has been shown that four parameters α, β, b, and the Yukawa range a in the Seyler-Blanchard interaction are related to the droplet model parameters a_1, a_2, J, and r_0.[21] The parameters appearing in the preceding equation of state of cold nuclear matter thus can be determined by these droplet model parameters chosen earlier as follows:

$$\alpha=1.78005\ ,$$

$$\beta=4.69607\ ,$$

$$b=401.42465\ \text{MeV}/c\ ,$$

$$T=85.81171\ \text{MeV}\ ,$$

$$k=0.56866\ \text{nucleons/fm}^3\ .$$

It can be seen that this asymmetry-dependent equation of state of cold nuclear matter fits to the nuclear mass points treated previously quite well. It is not surprising to find this good agreement, because the droplet model is used in our treatment to deal with the experimental nuclear masses, while the droplet model has been proven to be an approximation to the Thomas-Fermi statistical model with the Seyler-Blanchard phenomenological interaction;[21] therefore, this kind of good agreement is expected as our equation of state of cold nuclear matter is given by the same Thomas-Fermi model.[11]

Another point which is worthwhile to note is that the calculation of normal nuclear compressibility based on the foregoing equation of state gives

$$K = 9\rho^2 \frac{\partial^2 e}{\partial \rho^2}\bigg|_0$$
$$= T[6(\alpha+\beta)(\rho_0/k)^{5/3} - \tfrac{6}{5}(\rho_0/k)^{2/3}]$$
$$= 301.60 \text{ MeV} .$$

This is much larger than the compressibility coefficient used in the present droplet model ($K = 240$ MeV). On the other hand, a least-squares fit of a quadratic equation of state[1] fitted to the above-mentioned points for selected ranges of bulk asymmetry δ gives the results presented in Table I. The large scatter of the values in the last row of Table I can be understood as the compressibility being a quantity which is a measure of the curvature of the curve of the equation of state, so it cannot be determined exactly in a very short interval of density. At the same time, it is possible to give an explanation of why the rms fit of the droplet model to the nuclear masses gives a very broad minimum region of the compressibility K:[12,13] because the nuclear masses distribute on the e-ρ plane in a band instead of a curve. It gives also a suggestion that an asymmetry-dependent compressibility $K(\delta)$ should be used to give a better rms fit of the mass formula to the nuclear masses.

Obviously, all of the results we have given are model dependent, as the droplet model is used to extract the bulk energy density as well as to calculate the bulk nucleon density and the bulk asymmetry. The circumstance is the same as the other evidence from the experiments or the observations mentioned at the beginning of this paper. As we have not had a direct way to measure the energy density and the nucleon density experimentally up to now, we have no choice but to use a model-dependent way to extract the information of the equation of state of nuclear matter from the experiments or the observations.

TABLE I. Results obtained from a least-squares fit of a quadratic equation of state (Ref. 1) fitted to points for selected ranges of bulk asymmetry δ.

Range of δ	Number of points	Compressibility (MeV)
0.0185–0.0215	8	163.99
0.0385–0.0415	20	34.77
0.0585–0.0615	31	140.47
0.0785–0.0815	48	162.83
0.0985–0.1015	53	12.84
0.1185–0.1215	64	77.04
0.1385–0.1415	19	48.57
0.0000–0.1800	1643	369.90

In summary, the information of the equation of state of cold nuclear matter is extracted from the nuclear masses by the droplet model. The result shows the asymmetry dependence of the equation of state in a more direct way than has been done before. As the result is model dependent, it will be important to compare the results obtained by different nuclear models. On the other hand, it will be very interesting to study this asymmetry dependence experimentally. Especially, it may be possible to explain the A dependence of the compressibility K in the giant monopole resonance based on this asymmetry dependence of the equation of state.

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