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Relativistic motion of a Λ particle in hypernuclei and phenomenological analysis of its binding energy

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A phenomenological analysis of the ground-state binding energy B_Λ of a Λ is made based on the Dirac equation with a rectangular scalar potential and fourth component of a vector potential of the same radius. Analytic expressions are obtained for the small and large component of the radial wave function and the eigenvalue equation for B_Λ is also given. The latter leads to approximate "semiempirical mass formulas" $B_\Lambda = B_\Lambda(A)$ for sufficiently large values of the mass number. Estimates are made of B_Λ and also of the (average) effective mass of the Λ and for the depth of the Schrödinger-equivalent potential by fitting to the experimental values of B_Λ.

I. INTRODUCTION

The states of the Λ particle in hypernuclei have been treated traditionally nonrelativistically.[1,2] One of the usual problems has been the phenomenological analysis of the ground-state binding energy B_Λ by assuming a well of a given shape for the Λ nucleus central potential and determining the depth of the well (and possibly other parameters) by fitting to known experimental data for $B_\Lambda^{\exp}$.[1-15] The value of the well depth D determined empirically in this way has also been compared with the value of the binding energy of the Λ particle in nuclear matter as calculated from many-body calculations.[16]

The simplest approach is to assume a square well[3,4,6,10,14] of depth D and radius $R = r_0 A^{1/3}$ which leads to the well-known eigenvalue equation

$$\cot R[2\mu\hbar^{-2}(D - B_\Lambda)]^{1/2} = -\frac{B_\Lambda^{1/2}}{(D - B_\Lambda)^{1/2}} \tag{1}$$

from which B_Λ may be computed, having determined D and r_0 by means of a fitting procedure. In the above equation the reduced mass between the Λ and the core nucleus of mass number A

$$\mu_{\Lambda A} = \frac{m \cdot m_A}{m + m_A} = \frac{m}{1 + (m/m_N)A^{-1}}$$

may be used or even the Λ mass m (for heavier hypernuclei). The parameter m may be treated, however, as an effective mass of the Λ as it was pointed out in Ref. 12 (in connection with the asymptotic expression, cited below [Eq. (2)].

For very large A Eq. (1) may be solved for B_Λ:[3,6,12,1,2]

$$B_\Lambda \simeq D - \frac{\hbar^2}{2m}\frac{\pi^2}{r_0^2 A^{2/3}} . \tag{2}$$

This expression plays the role of an (elementary) "semiempirical mass formula for B_Λ."[12] For less heavy hypernuclei other approximate expressions giving explicitly the binding energy of the Λ in terms of the mass number A may be derived.[3,4,17]

This study attempts to make a phenomenological analysis of the Λ binding energy which is analogous to the above-mentioned one but in which the motion of the Λ is treated relativistically by means of the Dirac equation with a potential containing both an attractive (generated from scalar boson exchange) part $U_S(r)$ and a repulsive one $U_V(r)$ (resulting from the time component of vector boson exchange).[18]

Relativistic approaches to nuclear many-body systems have been considered long ago,[19-23] but there has been a revival of interest in the last 15 years (see, for example, Refs. 24–46). There is a proliferation of papers and the appearance in nuclear physics of novel expressions such as "Dirac phenomenology," "quantum hadrodynamics" (QHD), etc.[35] Problems of both nuclear structure and nuclear collisions have been treated relativistically.

Relativistic approaches have also attracted interest in the domain of hypernuclear physics.[47-50] On the basis of relativistic Hartree calculations the central Hartree potential as well as the spin-orbit interaction for a Λ in a hypernucleus are considerably reduced in comparison with the corresponding nucleon-nucleus ones.[47] This conclusion is in agreement with the phenomenological analysis of Ref. 54. Further, relativistic treatments of Λ as well as of Σ (and Ξ) hypernuclei have been pursued (e.g., Refs. 51–53).

It should be clear that relativistic effects are not important in all cases. Furthermore, the developed relativistic approaches, in spite of their success, are not free from open questions and also progress is necessary concerning their theoretical foundations.[35] They have provided us, however, with useful and very promising means in investigating a variety of nuclear and hypernuclear problems.

In the following section the formalism adopted for the simplified phenomenological analysis described earlier in the Introduction is summarized. In Sec. III, analytic expressions are given for the large and small components of the radial ground-state wave function, and the energy eigenvalue equation is discussed under the assumption that the potentials $U_S(r)$ and $U_V(r)$ are of rectangular shape and have the same radius. In Sec. IV the eigenvalue equation is solved approximately in the case of "sufficiently large" values of A. This leads, as in the nonrelativistic treatment, to approximate "semiempirical mass formulas" for the ground-state binding energy of the Λ. The last section is devoted to the presentation of the results of numerical calculations obtained by fitting the B_Λ obtained from the numerical solution of the eigenvalue equation to known experimental values. A comparison is also made with the B_Λ values obtained with the various "semiempirical mass formulas." In addition the B_Λ values resulting from the relativistic eigenvalue equation are compared with the corresponding nonrelativistic ones. Estimates of the values of other quantities as, for example, of the (average) effective mass for the Λ particle, etc., are given and a summary is made.

II. THE FORMALISM

For the relativistic treatment of the motion of a Λ in a hypernucleus we use the Dirac equation[47,31]

$$[c\boldsymbol{\alpha}\cdot\mathbf{p}+\beta\mu c^2+\beta U_S(r)+U_V(r)]\psi=E\psi \ , \tag{3}$$

where $\boldsymbol{\alpha}$ and β are the usual Dirac matrices, ψ the Dirac spinor for the Λ, and E the total energy $E=\epsilon+\mu c^2=-B'_\Lambda+\mu c^2$. We follow mostly the notation of Refs. 29, 31, 47, and 49.

The average local Λ-nucleus potential is constructed by means of an attractive scalar relativistic single particle potential $U_S(r)$ and of a repulsive relativistic single particle potential $U_V(r)$ which is the fourth component of a vector potential.

In this section we give, following previous work on Dirac theory, some basic equations which we shall use in the present treatment (see Sec. III and the Appendix) which is a simplified shell model Dirac phenomenology for the Λ particle in hypernuclei

The Dirac spinors[29] may be written as

$$\psi=\psi_{nljm}=\begin{bmatrix} iG_{nlj}(r)/r \\ F_{nlj}(r)\boldsymbol{\sigma}\cdot\mathbf{r}/r \end{bmatrix}\varphi_{ljm} \ , \tag{4}$$

where $\varphi_{ljm}=(\Upsilon_l^{m_l}\otimes\chi_{1/2}^{m_s})_{jm}$ and $\chi_{1/2}^{m_s}$ are the Pauli spinors. One then has the coupled radial Dirac equations for the large G and the small components F:[29,49]

$$\frac{dG}{dr}=D(r)F(r)-\frac{K}{r}G(r) \ , \tag{5}$$

$$\frac{dF}{dr}=H(r)G(r)+\frac{K}{r}F(r) \tag{6}$$

(where we have suppressed the quantum numbers nlj, etc.) $K=\pm(j+\frac{1}{2})$ for $j=l\mp\frac{1}{2}$; $D(r)$ and $H(r)$ are given by

$$D(r)=\frac{1}{\hbar c}[\mu c^2+E+U_-(r)]$$
$$=\frac{1}{\hbar c}[2\mu c^2-B'_\Lambda+U_-(r)] \ , \tag{7}$$

$$H(r)=\frac{1}{\hbar c}[\mu c^2-E+U_+(r)]$$
$$=\frac{1}{\hbar c}[B'_\Lambda+U_+(r)] \ , \tag{8}$$

with

$$U_\pm(r)=U_S(r)\pm U_V(r) \ . \tag{9}$$

$D(r)$ is closely related to $m^*(r)$ (Refs. 37 and 52) which is an r-dependent effective mass (different from the usual effective mass in mean-field theory).

Instead of solving Eqs. (5) and (6) one could solve a second-order equation for the function $G(r)$ and then obtain the function $F(r)$ from Eq. (5), that is

$$F(r)=\left[G'(r)+\frac{K}{r}G(r)\right]D^{-1}(r) \ . \tag{10}$$

This second-order equation for $G(r)$ is[29]

$$G''(r)-\frac{D'(r)}{D(r)}G'(r)+\left[-\frac{D'(r)}{D(r)}\frac{K}{r}-D(r)\cdot H(r)-\frac{l(l+1)}{r^2}\right]G(r)=0 \ . \tag{11}$$

The first derivative of G in this equation may be eliminated by the transformation

$$g(r)=D^{-1/2}(r)\cdot G(r) \ . \tag{12}$$

The equation for $g(r)$ is then

$$g''(r)+\left[-\tfrac{3}{4}D^{-2}(r)[D'(r)]^2+\tfrac{1}{2}D^{-1}(r)D''(r)-D^{-1}(r)D'(r)\frac{K}{r}-D(r)H(r)-\frac{l(l+1)}{r^2}\right]g(r)=0 \ . \tag{13}$$

From this one then obtains the following Schrödinger-type equation for $g(r)$:[29]

$$g''(r)-\left[\frac{l(l+1)}{r^2}+\frac{2\mu}{\hbar^2}(V_{\text{centr}}+V_{\text{s.o.}}+B'_\Lambda)\right]g(r)=0 \ , \tag{14}$$

where

$$V_{\text{centr}}(r,B'_\Lambda)=U_+(r)+\frac{\hbar^2}{2\mu}\left[\frac{1}{\hbar^2c^2}[U_+(r)+B'_\Lambda][U_-(r)-B'_\Lambda]-D^{-1}(r)\cdot D'(r)r^{-1}-[2D(r)]^{-1}D''(r)+\tfrac{3}{4}D^{-2}(r)[D'(r)]^2\right], \quad (15)$$

$$V_{\text{s.o.}}(r,B'_\Lambda)=-\frac{\hbar}{2\mu}\frac{1}{[2\mu c^2-B'_\Lambda+U_-(r)]}\times\frac{1}{r}\frac{dU_-(r)}{dr}\mathbf{l}\cdot\boldsymbol{\sigma}\ . \quad (16)$$

Equation (14) has the same eigenvalues as those of Eq. (11), and also the asymptotic behavior of $g(r)$ for large r is the same as that of $G(r)$.[31,33] In Eq. (14) both central and spin-orbit components of the single particle potential appear. The latter depends on dU_-/dr; V_{centr} and $V_{\text{s.o.}}$ are energy dependent. A slightly different definition of the central Schrödinger potential has also been used in the literature (e.g., Refs. 30 and 31). This differs from that of Eq. (15) in that $-B'^2_\Lambda/2\mu c^2$ does not appear. This term has been combined instead with B'_Λ in the Schrödunger equation (14). The (modified) central potential defined in this way is $\tilde{V}_{\text{centr}}=V_{\text{centr}}+B'^2_\Lambda/2\mu c^2$ and approaches zero for $r\to\infty$.

III. THE GROUND-STATE RADIAL WAVE FUNCTION AND THE ENERGY EIGENVALUE EQUATION FOR THE Λ IN THE RELATIVISTIC SQUARE WELL MODEL

We assume that U_+ and U_- are square wells with the same radius R and with depths D_+ and D_-, respectively,

$$U_\pm(r)=-D_\pm[1-\theta(r-R)]\ . \quad (17)$$

U_S and U_V and therefore U_+ and U_- are, in general, not expected to have the same radius. The use of a common radius R however simplifies the calculations considerably. The assumption that $U_S(r)$ and $U_V(r)$ differ only in strength has also been made for the simplified model in Ref. 31 which deals with the nucleon-nucleus optical potential. Also in Ref. 30 this simplification was considered in the nuclear case using Woods-Saxon potentials.

The expressions for G, F, and for the energy will be derived for the ground state ($l=0$, $K=-1$, $B'_\Lambda=B_\Lambda$) following the analogous treatment of the Dirac equation for a square well potential.[55,56]

For $U_\pm(r)$ of the form (17) we write $g(r)$ as

$$g(r)=[1-\theta(r-R)]g_{\text{in}}(r)+\theta(r-R)g_{\text{ex}}(r)\ . \quad (18)$$

The internal $g_{\text{in}}(r)$ and the external $g_{\text{ex}}(r)$ parts satisfy

$$g''_{\text{in}}(r)+\eta^2g_{\text{in}}(r)=0,\quad 0\le r<R\ , \quad (19)$$

$$g''_{\text{ex}}(r)-\eta_0^2g_{\text{ex}}(r)=0,\quad R<r<\infty\ , \quad (20)$$

where

$$\eta=\left[\frac{2\mu}{\hbar^2}(D_+-B_\Lambda)[1-(B_\Lambda+D_-)(2\mu c^2)^{-1}]\right]^{1/2}, \quad (21)$$

$$\eta_0=\left[\frac{2\mu}{\hbar^2}B_\Lambda[1-B_\Lambda(2\mu c^2)^{-1}]\right]^{1/2}. \quad (22)$$

The solutions of Eqs. (19) and (20) which satisfy

$$g_{\text{in}}(0)=0 \text{ and } g_{\text{ex}}(\infty)=0 \quad (23)$$

are easily found, as well as the corresponding functions $G_{\text{in}}(r)$ and $G_{\text{ex}}(r)$ [see Eq. (12)] which satisfy in addition the continuity condition

$$G_{\text{in}}(R-0)=G_{\text{ex}}(R+0)=G(R)\ . \quad (24)$$

The final result is

$$G(r)=N_0\{[1-\theta(r-R)]\sin\eta r+\theta(r-R)(\sin\eta R)e^{-\eta_0(r-R)}\},\quad 0\le r<\infty\ , \quad (25)$$

$$F(r)=N_0c\hbar\left[[1-\theta(r-R)]\frac{1}{(2\mu c^2-B_\Lambda-D_-)}\left[\eta\cos\eta r-\frac{1}{r}\sin\eta r\right]+\theta(r-R)\frac{-\sin\eta R}{(2\mu c^2-B_\Lambda)}\left[\eta_0+\frac{1}{r}\right]e^{-\eta_0(r-R)}\right],\quad 0\le r<\infty\ , \quad (26)$$

where N_0 is determined from

$$\int_0^\infty[G^2(r)+F^2(r)]dr=1\ . \quad (27)$$

The continuity of F at R

$$F_{\text{in}}(R-0)=F_{\text{ex}}(R+0)=F(R) \quad (28)$$

gives the following energy eigenvalue equation:

$$\cot\eta R = -\frac{\eta_0}{\eta}\left[\frac{1-\{B_\Lambda+D_-[1+(\eta_0 R)^{-1}]\}(2\mu c^{2})^{-1}}{1-B_\Lambda(2\mu c^2)^{-1}}\right] . \tag{29}$$

Equation (29) may be written in two slightly different forms which may make the comparison with the corresponding nonrelativistic equation more transparent. The first one is

$$\cot R\left[\frac{2\mu}{\hbar^2}(D_{\text{rel}}-B_\Lambda)\right]^{1/2} = -\frac{B_\Lambda^{1/2}}{(D_{\text{rel}}-B_\Lambda)^{1/2}}f , \tag{30}$$

where

$$\begin{aligned} D_{\text{rel}} &= \left[D_+\left[1-\frac{D_-}{2\mu c^2}\right]+\frac{B_\Lambda}{2\mu c^2}(B_\Lambda-D_++D_-)\right] \\ &= [(D_S+D_V)-(D_S^2-D_V^2)(2\mu c^2)^{-1}-B_\Lambda D_V(\mu c^2)^{-1}+B_\Lambda^2(2\mu c^2)^{-1}] \end{aligned} \tag{31}$$

and

$$f=(1-\{B_\Lambda+D_-[1+(\eta_0 R)^{-1}]\}(2\mu c^2)^{-1})[1-B_\Lambda(2\mu c^2)^{-1}]^{-1/2} . \tag{32}$$

The quantity D_{rel} corresponds to the "depth" of the Schrödinger-equivalent potential"[31,33] in Eq. (14).

The second form is

$$\cos R\left[\frac{2\mu}{\hbar^2}(D_{\text{rel}}-B_\Lambda)\right]^{1/2} = -\frac{B_\Lambda^{1/2}}{(D_{\text{rel}}-B_\Lambda)^{1/2}}+C \tag{33}$$

with

$$C=\frac{B_\Lambda^{1/2}}{(D_{\text{rel}}-B_\Lambda)^{1/2}}\left[1-\left[1-\frac{B_\Lambda}{2\mu c^2}\right]^{1/2}+\frac{D_-}{2\mu c^2\left[1-\frac{B_\Lambda}{2\mu c^2}\right]}\left[\frac{1}{R\left[\frac{2\mu}{\hbar^2}B_\Lambda\right]^{1/2}}+\left[1-\frac{B_\Lambda}{2\mu c^2}\right]^{1/2}\right]\right] . \tag{34}$$

The relativistic eigenvalue equation has a similar structure as the nonrelativistic one with, however, two differences. First the left-hand side of the relativistic eigenvalue equation is the same as the corresponding nonrelativistic one but instead of the usual potential depth the energy dependent depth D_{rel} appears. It also depends on both $D_+=D_S+D_V$ and $D_-=D_S-D_V$ and the magnitude of D_{rel} is mainly determined by D_+. The energy independent part of D_{rel} is

$$D_{\text{rel}}^{(0)}=D_+\left[1-\frac{D_-}{2\mu c^2}\right] . \tag{35}$$

Second, the right-hand side of Eq. (1) is modified, either by a correction factor f [Eq. (30)] or equivalently by an additive correction C [Eq. (33)] which depend on the energy, on the radius R, and on D_-; f is, however, independent of D_+. That $f\neq 1$ and $C\neq 0$ is mainly due to the fact that D_- turns out to be rather large and that therefore terms such as $D_-\cdot(2\mu c^2)^{-1}$ are not negligible.

We end this section by pointing out that the energy eigenvalue equation may be derived in a different way by integrating a second-order equation which follows from Eq. (11). The details of this derivation, which is analogous to the procedure of finding the eigenvalue equation in Schrödinger problems with δ-function potentials are given in the Appendix. This alternative derivation provides check of the validity of the eigenvalue equation (29) and gives also immediately an expression for the size of the discontinuity of $G'(r)$ at R in terms of $F(R)$.

IV. APPROXIMATE EXPLICIT EXPRESSIONS FOR B_Λ, ELEMENTARY "SEMIEMPIRICAL MASS FORMULAS"

As one can see from Eq. (29), it is not possible to obtain an explicit expression for B_Λ in terms of $R=r_0A^{1/3}$. This can only be done in an approximate way as in the nonrelativistic case,[2–4,6b,17] though the eigenvalue equation is now more complicated. It is useful to keep in mind that B_Λ varies from a few MeV for small A to around 25 MeV for large A. Also we may expect that D_+ and D_- should not differ very much from the central values of the corresponding Λ-nucleus potentials known from earlier studies.[49,52] Therefore D_+ should be around 30 MeV while D_- is several hundred MeV. D_+, D_- and r_0 are assumed to be independent of A and B_Λ.

To derive our approximate expressions we write the eigenvalue equation as

$$\pi-\eta R=\arctan x , \tag{36}$$

where

$$x=\frac{\eta[1-B_\Lambda(2\mu c^2)^{-1}]}{\eta_0\{1-[B_\Lambda+D_-+D_-(\eta_0 R)^{-1}](2\mu c^2)^{-1}\}} . \quad (37)$$

We assume that $x \ll 1$ so that

$$\pi-\eta R \simeq 0 . \quad (38)$$

This approximation corresponds to large R, i.e., large A, and gives

$$B_\Lambda^2-\frac{2\mu c^2}{\lambda}[1+\lambda D_+(2\mu c^2)^{-1}]B_\Lambda - \left[\frac{\hbar^2\pi^2}{2\mu R^2}-\frac{D_+}{\lambda}\right]2\mu c^2 \simeq 0 , \quad (39)$$

where

$$\lambda=[1-D_-(2\mu c^2)^{-1}]^{-1} . \quad (40)$$

Solving Eq. (39) we obtain the approximation

$$B_\Lambda^{(0)} \simeq \frac{\mu c^2}{\lambda}[1+\lambda D_+(2\mu c^2)^{-1}]\left\{1-\left[1+2\lambda(\mu c^2)^{-1}\cdot\left[\frac{\hbar^2\pi^2\lambda}{2\mu R^2}-D_+\right][1+\lambda D_+(2\mu c^2)^{-1}]^{-2}\right]^{1/2}\right\} . \quad (41)$$

This is the relativistic version of the simple nonrelativistic semiempirical mass formula (2). It may be simplified considerably if slightly less accuracy is tolerated by expanding the square root and keeping only the two first terms. This gives (by further retaining the most significant terms)

$$B_\Lambda^{as} \simeq D_+ - \frac{\hbar^2\pi^2}{2m^* R^2} , \qquad D_+=D_S+D_V, \quad R=r_0 A^{1/3} , \quad (42)$$

where

$$m^*=m\left[1-\frac{(D_++D_-)}{2}(mc^2)^{-1}\right] . \quad (43)$$

Thus, for sufficiently large A the variation of B_Λ with A is the same, with the above approximations, as in the nonrelativistic case for a square well of depth $D_+=D_S+D_V$ and effective mass m^*. m^* is the "average effective mass" in the terminology of Ref. 31. The accuracy of (42) is investigated in Sec. V together with that of the other approximate expressions. Equation (42) (an elementary "semiempirical mass formula") may be derived directly from (38) observing that $x \ll 1$ implies $B_\Lambda \simeq D_+$ and then using this A independent value to estimate η.

Another mass formula similar to (42) is obtained if the two terms in the expansion of the square root in expression (41) are kept, without making further expansions and retaining the leading terms. This formula, denoted by $\tilde{B}_\Lambda^{as}$ is

$$\tilde{B}_\Lambda^{as} \simeq \frac{D_+}{(1+\lambda D_+(2\mu c^2)^{-1})} - \frac{\hbar^2\pi^2\lambda}{2\mu(1+\lambda D_+(2\mu c^2)^{-1})R^2} . \quad (44)$$

Improved mass formulas may be derived as in the nonrelativistic case.[3,17] One simple way is the following. Instead of putting $\arctan x \simeq 0$ we keep the first term in the expansion of the $\arctan x$ in powers of x. This leads to

$$B_\Lambda^{(i)}=D_+-\frac{\hbar^2\pi^2}{2\mu g[1+(\tilde{f}\eta_0 R)^{-1}]^2 R^2} , \quad (45)$$

where

$$g=1-(B_\Lambda+D_-)(2\mu c^2)^{-1} \quad (46)$$

and

$$\tilde{f}=\frac{(1-\{B_\Lambda+D_-[1+(\eta_0 R)^{-1}]\}(2\mu c^2)^{-1})}{[1-B_\Lambda(2\mu c^2)^{-1}]} \quad (47)$$

with η_0 given by (22). The quantities g, $\tilde{f}$, and η_0 depend on B_Λ but their value in (45) may be estimated by using an approximate expression $B_{\rm appr}$ for B_Λ, as, for example,

$$B_{\rm appr}=D_+, \quad B_{\rm appr}=B_\Lambda^{as}, \quad B_{\rm appr}=\tilde{B}_\Lambda^{as} .$$

The corresponding mass formulas will be denoted by $B_\Lambda^{(1)}$, $B_\Lambda^{(2)}$, $B_\Lambda^{(3)}$, respectively. If the first choice for B_Λ is made then the μg in the denominator of (45) is just the effective mass $\mu^*=\mu[1-D_S(\mu c^2)^{-1}]$. This is the "average effective mass of the Λ particle" m^* [expression (43) if $\mu=m$].

Expressions (45) unlike the asymptotic one (42) contain higher negative powers of R. If an expansion of $B_\Lambda^{(1)}$ is made, in analogy with the nonrelativistic case, an expression similar to that of Walecka[4] is derived which contains relativistic effects.

Further improved semiempirical mass formulas for B_Λ may be derived,[57] but they are more complicated.

Finally it should be clear that all the "semiempirical mass formulas for B_Λ" discussed in this section, as well as the corresponding well-known ones in the nonrelativistic case, even those which may be suitable for the comparatively lighter hypernuclei, are "elementary" in the sense that they contain only a part, though the most significant one, of a proper hypernuclear mass formula.[2] They are only meant to reproduce approximately the average trend of B_Λ with A (excluding smaller A). A proper hypernuclear mass formula should take into account the diffuseness of the surface and contain in addition a symmetry term, a charge symmetry breaking term, etc.[2]

TABLE I. The parameters D_-, D_+, r_0, m^*, and $D_{\rm rel}^{(0)}$, together with the values of B_Λ and $D_{\rm rel}$ for various A, obtained with the relativistic eigenvalue equation (29) with $\mu=\mu_{\Lambda A}$ in fitting to experimental B_Λ (see text). The corresponding nonrelativistic results are also shown.

	First set of parameters $D_-=300$ MeV (fixed) $D_+=30.55$ MeV, $r_0=1.010$ fm $m^*=0.852$ m, $D_{\rm rel}^{(0)}=26.44$ MeV		Second set of parameters $D_-=443$ MeV (fixed), $D_+=30.77$ MeV, $r_0=1.022$ fm $m^*=0.788$ m, $D_{\rm rel}^{(0)}=24.66$ MeV		Third set of parameters $D_-=462.92$ MeV $D_+=31$ MeV (fixed) $r_0=1.009$ fm $m^*=0.779$ m, $D_{\rm rel}^{(0)}=24.56$		Nonrelativistic $D=29.55$ MeV, $r_0=1.033$ fm
A	B_Λ (MeV)	$D_{\rm rel}$ (MeV)	B_Λ (MeV)	$D_{\rm rel}$ (MeV)	B_Λ (MeV)	$D_{\rm rel}$ (MeV)	B_Λ (nonrelativistic) (MeV)
12	11.23	27.57	11.18	26.39	10.99	26.37	12.02
16	13.68	28.00	13.62	27.00	13.46	27.01	14.38
20	15.42	28.27	15.35	27.41	15.22	27.44	16.04
32	18.62	28.77	18.56	28.15	18.48	28.21	19.03
40	19.93	28.97	19.88	28.44	19.83	28.51	20.23
90	23.69	29.53	23.70	29.24	23.73	29.36	23.64
140	25.20	29.76	25.24	29.59	25.30	29.72	24.98
208	26.28	29.92	26.35	29.83	26.44	29.96	25.93

V. NUMERICAL RESULTS AND SUMMARY

In this section we report the results of calculations using the expressions in previous sections.

Equation (29) for B_Λ was solved using for μ the reduced mass of the Λ-core system. An unweighted least-square fit of the calculated B_Λ to the experimental $B_\Lambda^{\rm exp}$ was made. The $B_\Lambda^{\rm exp}$ were those of ${}^{13}_{\Lambda}C$ and of heavier hypernuclei as in Ref. 14. For the latter the upper limits of $B_\Lambda^{\rm exp}$ given in Ref. 9 which correspond to the higher mass numbers were used. Unfortunately it has not become possible to obtain with this set of data satisfactory results by treating all three parameters D_-, D_+, and r_0 as adjustable. If, however, a guess of D_- or D_+ is made on the basis of other treatments[49,52] the values of the remaining parameters determined by the least-square fit are fairly close to the values deduced form other studies. If D_- is chosen to be 300 MeV then the best fit values for D_+ and r_0 are $D_+=30.55$ MeV and $r_0=1.01$ fm. If the choice $D_-=443$ MeV is made, the corresponding best-fit values are $D_+=30.77$ MeV and $r_0=1.022$ fm. [Note that our preliminary results (Ref. 58) have been obtained with slightly different values.] The best-fit values obtained by using a fixed value for $D_+=31$ MeV are $D_-=462.9$ MeV $r_0=1.009$ fm. It should be clear that the decimal figures included in the results quoted in this paper are not meant to indicate the expected accuracy which should be mostly, considerably lower. They serve, simply, to make the comparison between the results in the various cases, more transparent.

Using the above values of D_- and D_+ the corresponding values of m^* [Eq. (43)] and of $D_{\rm rel}^{(0)}$ [Eq. (35) with $\mu=m$] are easily calculated. These are given in Table I for various A together with the B_Λ obtained from Eq. (29) and also of $D_{\rm rel}$ obtained from (31). The nonrelativistic B_Λ values obtained with the corresponding best-fit values ($D=29.55$ MeV, $r_0=1.033$ fm) are also given. It is seen from Table I that the values of B_Λ and $D_{\rm rel}$ calculated with different sets of values are, in most cases, quite close. Also the relativistic B_Λ are usually quite close to the nonrelativistic ones.

Calculations have also been made for $\mu=m$. The results are very similar to the corresponding ones of Table I though there are some small differences for light hypernuclei (the relativistic B_Λ values and $D_{\rm rel}$ are a little larger in this case).

In Table II the values C and f are given for various hypernuclei, for $D_-=443$ MeV, $D_+=30.77$ MeV, $r_0=1.022$ fm obtained with $\mu_{\Lambda A}$. It is seen that the variation of C and f with A is stronger for the lighter hypernuclei.

In Table III the B_Λ obtained with various "mass formulas" are compared with the "exact values" obtained with Eq. (29) for $D_-=443$ MeV, $D_+=30.77$ MeV, and $r_0=1.022$ fm (see Table I). Some of the B_Λ are either too small (B_Λ^{as}, $\tilde{B}_\Lambda^{as}$, $B_\Lambda^{(0)}$) or too large ($B_\Lambda^{(2)}$, $B_\Lambda^{(3)}$) for small A and in certain cases it was not even possible to obtain a value for B_Λ (because of the square root of a negative quantity). Also the asymptotic expression B_Λ^{as} is fairly accurate only for very large A ($A \gtrsim 150$). However $B_\Lambda^{(1)}$ is quite accurate even for rather small A ($A \gtrsim 16$)

It should finally be noted that the agreement of the approximate expression for B_Λ with the "exact" one is

TABLE II. The values of C [Eq. (34)] and f [Eq. (32)] for various A, obtained with $\mu_{\Lambda A}$, $D_-=443$ MeV, $D_+=30.77$ MeV, and $r_0=1.022$ fm.

A	C	f
12	0.295	0.656
16	0.318	0.685
20	0.337	0.701
32	0.382	0.726
40	0.406	0.734
90	0.508	0.754
140	0.575	0.761
208	0.644	0.766

TABLE III. B_Λ vs A, calculated with approximate expressions B_Λ^{as} [Eq. (42)],[a] $\tilde{B}_\Lambda^{as}$ [Eq. (44)], $B_\Lambda^{(0)}$ [Eq. (41)], $B_\Lambda^{(i)}$, $i=1,2,3$ [Eq. (45)] and with the "exact expression" [Eq. (29)] ($\mu_{\Lambda A}$, $D_- = 443$ MeV, $D_+ = 30.77$ MeV, $r_0 = 1.022$ fm).

A	B_Λ^{as} (MeV)[a]	$\tilde{B}_\Lambda^{as}$ (MeV)	$B_\Lambda^{(0)}$ (MeV)	$B_\Lambda^{(1)}$ (MeV)	$B_\Lambda^{(2)}$ (MeV)	$B_\Lambda^{(3)}$ (MeV)	B_Λ ("exact") (MeV)
12	−14.35	−13.22	−13.12	10.27		—	11.18
16	−5.39	−4.62	−4.61	12.98		—	13.62
20	0.16	0.72	0.72	14.87	30.77	29.79	15.35
32	9.00	9.23	9.27	18.30	22.62	22.53	18.56
40	12.18	12.29	12.38	19.68	22.40	22.37	19.88
90	20.16	20.00	20.23	23.63	24.26	24.27	23.70
140	22.92	22.66	22.95	25.20	25.49	25.51	25.24
208	24.76	24.44	24.78	26.32	26.48	26.49	26.35

[a]With $\mu_{\Lambda A}$ instead of m in the expression for m^*.

much improved, in general, if the values of the parameters are chosen to be the corresponding best-fit values (see Table IV). The best-fit values of r_0 obtained with B_Λ^{as} and $B_\Lambda^{(0)}$ differ, however, considerably from those resulting from the fitting of B_Λ ("exact") and $B_\Lambda^{(1)}$.

Further calculations were made in an effort to investigate the possibility to achieve a more satisfactory fitting procedure. For this purpose a set of experimental B_Λ values used in a recent investigation[59] in connection with the nonrelativistic case, was employed. This consists of the $B_\Lambda^{\exp}$ of the light hypernuclei: ${}^{10}_{\Lambda}B$, ${}^{13}_{\Lambda}C$, ${}^{15}_{\Lambda}N$, ${}^{15}_{\Lambda}O$, and ${}^{32}_{\Lambda}S$ together with a selection of the (upper limits of the) binding energies corresponding to heavier hypernuclei, namely, those with $A=63$, 72, 80, 93, and 103 for the core nuclei, following a suggestion by Goyal. We have used here the higher values of A, since we consider them preferable in view of the better quality of the fit obtained with them in comparison with the one obtained with the lower values of A (35, 44, 52, 65, 75, respectively). It is interesting to note that with the above-mentioned set of experimental data it became possible to obtain meaningful results by fitting either the B_Λ ("exact") or the $B_\Lambda^{(1)}$ and taking all three parameters D_+, D_-, and r_0 as adjustable. The value of D_- which is the parameter less accurately determined, is (mainly when $B_\Lambda^{(1)}$ is used) larger than the one which is to be expected (Refs. 49 and 52). In addition, with the above set of experimental data a weighted least-square fit leads to satisfactory results by not including D_- among the fitting parameters but taking $D_- = 443$ MeV. These results almost coincide with those obtained with the corresponding unweighted fit. The best-fit values (with $\mu_{\Lambda A}$) and the corresponding values of m^*, $D_{\rm rel}^{(0)}$, B_Λ, and $D_{\rm rel}$ for various A are displayed in Table V in the case of two adjustable parameters, mentioned previously, and in the case of three adjustable parameters, in Table VI (where the nonrelativistic results are also given). It is seen that these values are usually quite close to the corresponding ones of Table I.

In conclusion we summarize our main results.

In the framework of a simplified model, namely, that of Dirac phenomenology with rectangular attractive U_S and repulsive U_V potentials of the same radius, analytic solutions are given for the Λ ground-state radial wave functions and B_Λ. Approximate "semiempirical mass formulas for B_Λ" are derived which are valid for "sufficiently large A" (e.g., $A \gtrsim 16$ for $B_\Lambda^{(1)}$). An interesting analogy between the simplest, the "asymptotic mass-formula" [Eq. (42)] valid for very large A and the "standard semiempirical mass formula" (2) was obtained. This might have been expected on the basis of the general formalism used.[52] Finally the results of fitting the calculated B_Λ obtained from Eq. (29) to the experimental B_Λ, treating D_+ and r_0 or D_- and r_0 as adjustable parameters, are compared with the B_Λ obtained with the approximate mass formulas. Additional calculations were made for another set of experimental data. In this case the values of all

TABLE IV. B_Λ for approximate expressions with $\mu_{\Lambda A}$ and $D_- = 443$ MeV and the best-fit values of D_+ and r_0 obtained from the fit of each expression B_Λ^{as} [Eq. (42)],[a] $B_\Lambda^{(0)}$ [Eq. (41)], $B_\Lambda^{(1)}$ [Eq. (45)] to the experimental B_Λ. The values of B_Λ calculated with the "exact" expression [Eq. (29)] are also given.

A	$D_+ = 27.24$ (MeV), $r_0 = 1.743$ fm B_Λ^{as} (MeV)[a]	$D_+ = 27.28$ (MeV), $r_0 = 1.733$ fm $B_\Lambda^{(0)}$ (MeV)	$D_+ = 29.59$ (MeV), $r_0 = 1.136$ fm $B_\Lambda^{(1)}$ (MeV)	$D_+ = 30.77$ (MeV), $r_0 = 1.022$ fm B_Λ ("exact") (MeV)
12	11.78	11.78	11.99	11.18
16	14.85	14.83	14.41	13.62
20	16.75	16.73	16.08	15.35
32	19.78	19.76	19.07	18.56
40	20.87	20.85	20.26	19.88
90	23.61	23.61	23.64	23.70
140	24.55	24.56	24.97	25.24
208	25.18	25.20	25.92	26.35

[a]With $\mu_{\Lambda A}$ instead of m in the expression for m^*.

TABLE V. The best-fit values of the parameters D_+ and r_0 obtained with $\mu_{\Lambda A}$, $D_- = 443$ MeV, using either the "exact" eigenvalue equation or the approximate expression $B_\Lambda^{(1)}$ [Eq. (45)] in a weighted least square fitting, using a set of ten experimental B_Λ (see text). The corresponding values of the effective mass m^* [Eq. (43)] and of $D_{rel}^{(0)}$ [Eq. (35) with $\mu = m$] together with the binding energies and D_{rel} [Eq. (31)] for various A are also given.

	$D_- = 443$ (fixed) MeV, $D_+ = 30.63$ MeV, $r_0 = 1.047$ fm, $m^* = 0.787$ m, $D_{rel}^{(0)} = 24.55$ MeV		$D_- = 443$ (fixed) MeV, $D_+ = 29.03$ MeV, $r_0 = 1.147$ fm, $m^* = 0.788$ m, $D_{rel}^{(0)} = 23.27$ MeV	
A	B_Λ ("exact") (MeV)	D_{rel} (MeV)	$B_\Lambda^{(1)}$ (MeV)	D_{rel} (MeV)
12	11.62	26.37	11.75	25.16
16	14.02	26.97	14.12	25.75
20	15.72	27.38	15.76	26.14
32	18.85	28.09	18.69	26.81
40	20.13	28.38	19.87	27.07
90	23.83	29.19	23.18	27.79
140	25.31	29.50	24.48	28.07
208	26.39	29.73	25.41	28.26

three parameters D_-, D_+, and r_0 were determined from the fit.

Typical values, consistent with the present simplified approach, of the "average effective mass" m^* and of the energy independent part of the depth D_{rel}: $D_{rel}^{(0)}$ [Eq. (35) with $\mu = m$] are $m^* \simeq 0.8$ m and $D_{rel}^{(0)} \simeq 24.0$ MeV. These are in reasonable agreement with the corresponding values deduced from other studies (e.g., Refs. 47 and 52). The relativistic values of B_Λ have also been compared with those obtained with the numerical solution of the nonrelativistic eigenvalue equation (1) by treating the depth D of the square well and the radius parameter r_0 as adjustable parameters in fitting to experimental data. The difference between the relativistic and the "corresponding nonrelativistic" B_Λ values is usually very small as expected.

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APPENDIX

In this appendix we give an alternative derivation of the eigenvalue equation (29). Consider

$$G''(r) - D'(r)F(r) - \left[D(r)H(r) + \frac{l(l+1)}{r^2}\right]G(r) = 0 ,$$

$$0 \le r < \infty \qquad \text{(A1)}$$

which follows from the second-order equation (11) by taking into account (10). $D(r)$ and $H(r)$ which appear in

TABLE VI. The best-fit values of D_-, D_+, and r_0 obtained with $\mu_{\Lambda A}$ and using either the exact eigenvalue equation or the approximate expression $B_\Lambda^{(1)}$ [Eq. (45)] in an unweighted least square fitting using a set of ten experimental data (see text). The corresponding values of the effective mass m^* [Eq. (43)] and of $D_{rel}^{(0)}$ [Eq. (35) with $\mu = m$] together with the binding energies and D_{rel} [Eq. (31)] for various A, as well as the corresponding nonrelativistic results are also given.

	$D_- = 489.20$ MeV, $D_+ = 30.57$ MeV, $r_0 = 1.054$ fm, $m^* = 0.767$ m, $D_{rel}^{(0)} = 23.86$ MeV		$D_- = 590.15$ MeV, $D_+ = 29.50$ MeV, $r_0 = 1.123$ fm, $m^* = 0.722$ m, $D_{rel}^{(0)} = 21.70$ MeV		$D = 28.97$ MeV, $r_0 = 1.034$ fm
A	B_Λ ("exact") (MeV)	D_{rel} (MeV)	$B_\Lambda^{(1)}$ (MeV)	D_{rel} (MeV)	B_Λ (Nonrelativistic) (MeV)
12	11.59	25.89	9.92	23.71	11.57
16	13.97	26.55	12.71	24.62	13.90
20	15.66	26.99	14.61	25.22	15.53
32	18.78	27.78	17.95	26.23	18.50
40	20.05	28.10	19.27	26.62	19.70
90	23.75	28.98	22.98	27.68	23.08
140	25.23	29.33	24.42	28.09	24.40
208	26.30	29.58	25.46	28.37	25.35

Eq. (A1) depend on $U_{\pm}(r)$, given by expression (17). We integrate (A1) from $R-\epsilon_1$ to $R-\epsilon_2$, $\epsilon_1>0$, $\epsilon_2>0$ and then we take the limit for $\epsilon_1\to 0$ and $\epsilon_2\to 0$:

$$\lim_{\substack{\epsilon_1\to 0\\ \epsilon_2\to 0}}\left[\int_{R-\epsilon_1}^{R+\epsilon_2}\frac{d^2G(r)}{dr^2}dr-\int_{R-\epsilon_1}^{R+\epsilon_2}D'(r)F(r)dr-\int_{R-\epsilon_1}^{R+\epsilon_2}\left[D(r)H(r)+\frac{l(l+1)}{r^2}\right]G(r)\right]=0\ . \quad \text{(A2)}$$

The last integral does not contribute in the above limit. Since $D'(r)=(D_-/\hbar c)\delta(r-R)$ and $F(r)$ must be continuous at R, we obtain

$$G'(R+0)-G'(R-0)=\frac{D_-}{\hbar c}F(R)\ . \quad \text{(A3)}$$

Substituting the expressions for $G'(R+0)$,$G'(R-0)$ derived from the expressions for $G(r)$ (this must be continuous) and of $F(R)$ $[F(R+0)]$ by means of relation (10), we obtain

$$\eta_0+\eta\cot\eta R=\frac{D_-}{2\mu c^2-B_\Lambda}\left[\eta_0+\frac{1}{R}\right] \quad \text{(A4)}$$

from which Eq. (29) follows.

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