Modified statistical model for the disassembly of hot nuclei formed in intermediate-energy heavy-ion collisions

Sa Ben-Hao and Zheng Yu-Ming
Chinese Center for Advanced Science and Technology (World Laboratory), Beijing, China and Institute of Atomic Energy, Beijing, China

Zhang Xiao-Ze
Institute of Atomic Energy, Beijing, China
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We propose a modified statistical model describing the disassembly of hot nuclei created in intermediate energy heavy-ion reactions. The calculated mass-yield distribution of fragments arising in ^{12}C($E/A=35$ MeV)+^{63}Cu reactions agrees with experimental data quite well.

I. INTRODUCTION

Recently the study of intermediate energy heavy-ion reactions have attracted considerable interest because of the change in the dynamics of the interaction might occur at this energy region. Mean-field dynamics dominates the low-energy reactions ($E/A \leq 10$ MeV) while in the high-energy reactions ($E/A \geq 200$ MeV) two-body dynamics (nucleon-nucleon collisions) plays dominant roles. However, both the one-body and two-body dynamics could not be neglected in the intermediate energy region.

On the contrary, the disassembly of composite nuclei (hot nuclei) created in heavy-ion reactions might not be so sensitive to the incident energy. As the initial interaction (the formation of hot nuclei) is described properly, a well-defined statistical model (for example, the equilibrium statistical model[1-3]) might predict the experimental data of mass-yield distribution of fragments quite well, no matter whether the fragments were produced in high- or intermediate-energy heavy-ion reactions.

The experimental data of mass-yield distribution of fragments arising in ^{12}C ($E/A=35$ MeV)+^{63}Cu reactions have been reported by Cho *et al.*[4] and compared with the cascade evaporation,[5] and the hybrid preequilibrium[6] model calculations. Both calculations were based on the following assumptions: (1) the initial interaction involves incomplete fusion, and (2) the composite nucleus consists of the target plus a transferred piece from projectile and the transferred piece may be alpha particle or ^{6}Li or ^{9}Be or ^{12}C. Neither cascade evaporation model nor hybrid preequilibrium model could satisfactorily describe the experimental facts especially the considerable large yield at medium mass $A_f \simeq 40$ and the rising again of mass-yield distribution at $A_f \simeq 30$.

In order to solve the above puzzle the statistical model[2,7,8] devised for high-energy heavy-ion reaction originally is examined in this paper. The essences of this modification are as follows.

(i) The composite nucleus, formed in intermediate-energy heavy-ion reactions as a product of initial interaction (incomplete fusion), consists of target and a transferred piece from projectile. This piece is composed of the nucleons of projectile located in the overlap region between target and projectile under a given impact parameter.

(ii) After heating and compression the composite nuclear system expands homogeneously towards the freeze-out state (with radius parameter R_{T0} greater than nucleon radius $r_0=1.18$ fm). The excitation energy of the composite nucleus at freeze out is a fraction of available reaction energy and this fractional factor is regarded as a model parameter instead of excitation energy in the previous publication.[8]

(iii) The available reaction energy is calculated due to the reaction kinetics and mass balance[9] assuming that the missing mass in initial interaction escapes as a whole cluster with beam velocity.[4]

(iv) The code of MMCSF (microcanonical ensemble Monte Carlo simulation of the fragmentation[8]) based on the statistical model[2] is used to calculate various distributions of fragments from the disassembly of composite nuclei.

(v) Average over impact parameters sampled upon the area law is taken and the results are then compared with the corresponding experimental data. The agreement between the calculated mass-yield distribution of fragments for the reaction of ^{12}C+^{63}Cu and the data is quite good.

This paper is organized as follows: In Sec. II some details about the model are given. The results of mass-yield distribution of fragments are displayed in Sec. III. A brief discussion and conclusion are given in Sec. IV.

II. MODEL

The systematics of longitudinal momentum-transfer measurements shows that the complete fusion gives way to incomplete fusion when the projectile energy exceeds 8–10 MeV/nucleon.[4] Thus, one might picture the initial interaction in intermediate-energy heavy-ion reactions as involving incomplete fusion. The experimental trend[4] of the fragments close to target are formed in peripheral interaction and those of low mass are formed in more central collisions, is suggestive of a geometric origin. Therefore one might assume further that the composite nucleus created in initial interaction of incomplete fusion is composed of a target and a part of projectile located in the overlap region between the target and projectile under a given impact parameter. The number of nucleons in that part of projectile can be calculated as follows:

$$N_P(b)=\rho_0\int dV\,\theta\{R_P-[x^2+(b-y)^2+z^2]^{1/2}\}$$
$$\times\theta\{R_T-[x^2+y^2]^{1/2}\}\ , \tag{1}$$

where $\rho_0=0.16\ \mathrm{fm}^{-3}$ is the normal nuclear density, θ refers to the step function, R_P and R_T stand for the radii of projectile and target, respectively, and b notes the impact parameter. The results of Eq. (1) is close to but a little bit larger than that of Swiatacki's empirical formula.[10]

Impact parameter b should be sampled in $(0,b_{\max}=R_P+R_T)$ due to square law. For simplicity we select first a certain number of impact parameters $(b_1,b_3,\ldots,b_{2N-1})$ due to equal probability principle

$$\int_0^{b_1} b\,db=\int_{b_1}^{b_2} b\,db=\cdots=\int_{b_{2N-1}}^{b_{2N}} b\,db$$
$$=\frac{1}{2N}\int_0^{b_{\max}} b\,db\ . \tag{2}$$

For a given impact parameter the number $N_P(b)$ is then taken to be an integer by counting the fraction over $\frac{1}{2}$ as one otherwise as zero. That integer is shared between proton and neutron due to the principle of the number of protons is less than but as close as possible to the number of neutrons for the case of the projectile lighter than ^{40}Ar. The mass and charge number of composite nuclei A_{RT} and Z_{RT} are then determined.

By the assumption of missing mass in initial interaction that escapes as a whole cluster with beam velocity, the reaction energy Q can be calculated in virtue of mass balance. From nonrelativistic kinetic energy and momentum conservations, the kinetic energy deposited in the composite nucleus can be expressed[9] as

$$E_k=\frac{N_P}{A_P}\frac{A_T}{A_T+N_P}E\ , \tag{3}$$

where A_P and A_T refer to the mass number of the projectile and target, respectively. The available reaction energy is then given by

$$E_{\mathrm{avai}}=E_k+Q\ . \tag{4}$$

As the size and shape of hot nuclei are very hard to determine and are still an open problem at the moment, we picture the freeze-out volume of hot nuclei as an expanded sphere with radius R_{T0} greater than $r_0=1.18$ fm. We also assume that excitation energy E^* of hot nuclei before breakup is a fraction of the available reaction energy E_{avai} since a certain portion of the energy should be consumed by preequilibrium emission and expansion of composite nuclei themselves. The fractional factors C_f and R_{T0} are regarded as parameters. The values of impact parameter and the corresponding values of N_p, E_k, Q and E^* $(C_f=0.73)$ are given in Table I.

The statistical model of Ref. 2 and the corresponding Metropolis sampling technique of Ref. 8 are then used to describe the disassembly of hot nuclei. In this model it was assumed that the hot nuclei disassemble promptly into a configuration described by a set of variables $\{N_c, N_n, \{A_i,Z_i\}_{i=1}^{N_c}, \{\mathbf{r}_i\}_1^{N_c}, \{\mathbf{p}_i\}_1^{N_c}, \{\epsilon_i\}_1^{N_c}, \{\mathbf{r}_j\}_1^{N_n}, \{\mathbf{p}_j\}_1^{N_n}\}$. Here N_c refers to number of charged fragments including prompt protons. N_n stands for number of prompt and evaporated neutrons. The evaporation of charged particles are neglected in comparison with neutrons and the evaporation of neutrons is accounted concentratingly at the moment of breakup (freeze out). $\{A_i,Z_i\}_i^{N_c}$, $\{\mathbf{r}_i\}_1^{N_c}$, $\{\mathbf{p}_i\}_1^{N_c}$, and $\{\epsilon_i\}_1^{N_c}$ are the set of mass and charge numbers, position, momentum, and internal excitation energy of charged fragments. $\{\mathbf{r}_j\}_1^{N_n}$ and $\{\mathbf{p}_j\}_1^{N_n}$ are the set of position and momentum of neutrons. That means the hot nuclei breakup into an assembly of charged fragments and neutrons simultaneously and promptly. The total energy of hot nucleus at a freeze-out configuration reads

$$E=E_B+E_C+E_{in}^*+E_{kf}+E_{kn}\ , \tag{5}$$

where

$$E_C=\sum_{i<j}^{N_c}\frac{Z_iZ_je^2}{|\mathbf{r}_i-\mathbf{r}_j|} \tag{6}$$

is the Coulomb energy of the fragments, and

TABLE I. The values of impact parameter and corresponding values of N_p, E_k, Q, and E^* (cf. context for their definitions).

b (fm)	1.598	2.768	3.573	4.228	4.794	5.300
Hot	^{75}Br	^{74}Se	^{72}As	^{69}Ge	^{68}Ga	^{66}Zn
nuclei	35	34	33	32	31	30
N_p	12	11	9	6	5	3
	(6,6)[a]	(6,5)	(5,4)	(3,3)	(3,2)	(2,1)
$E_{k\,(\mathrm{MeV})}$	352.0	327.8	275.6	191.7	162.1	100.2
$Q_{(\mathrm{MeV})}$	3.583	−0.652	−12.3	−1.26	−14.3	−9.06
$E^*_{(\mathrm{MeV})}$ [b]	260.0	239.0	192.0	131.0	108.0	66.00

[a]Digits in parentheses are the number of neutrons and protons, respectively.
[b]Due to $C_f=0.73$.

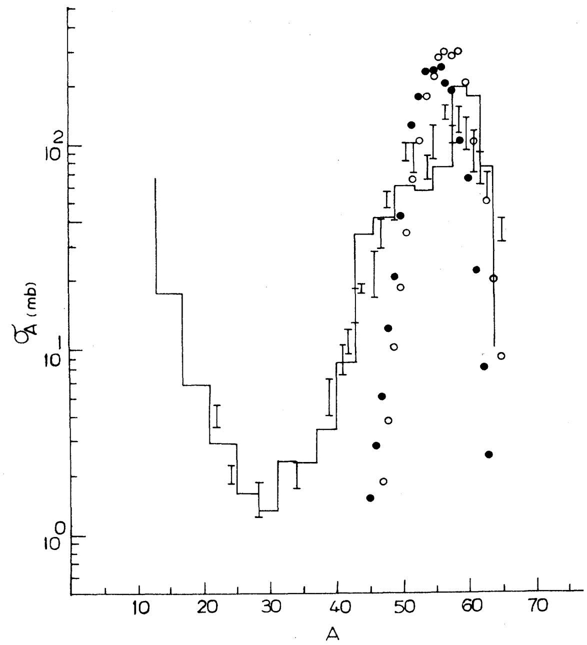

FIG. 1. Mass-yield distribution of fragments arising in ^{12}C ($E/A=35$ MeV)$+^{63}Cu$ reactions. I: Experimental (Ref. 4) data. ——: Results of this paper (with parameters $C_f=0.73$ and $R_{T0}=2.05$ fm). ●: Results of evaporation model (Ref. 4). ○: Results of preequilibrium model (Ref. 4).

$$E_B=\sum_{1}^{N_c} E_{Bi} \ , \tag{7}$$

here E_{Bi} denotes the binding energy of fragment i, and

$$E_{in}^{*}=\sum_{1}^{N_c} \epsilon_i \ , \tag{8}$$

$$E_{kf}=\sum_{1}^{N_c} p_i^2/2m_i \ , \tag{9}$$

$$E_{kn}=\sum_{1}^{N_n} p_j^2/2m_0 \ . \tag{10}$$

The configurations allowed by mass, charge, momentum, and energy conservations are assumed to conform to a distribution of canonical[7] or microcanonical[8] ensemble. By means of the Monte Carlo method and the corresponding Metropolis pass a large number of allowed configurations (10^6, say) were generated and the physical observables could then be calculated as a statistical average.

III. RESULTS

In Fig. 1 the histogram gives a calculated mass-yield distribution of fragments in reactions of ^{12}C ($E/A=35$ MeV)$+^{63}$cu averaging over six impact parameters (c.f. Table I) and normalizing the integration between $A_f=28$ and 68 to the experimental total reaction cross section $\sigma_R=1.27b$. The model parameters C_f and R_{T0} were taken to be equal to 0.73 and 2.05 fm, respectively. Error bars in Fig. 1 show the corresponding experimental data of Ref. 4. The solid and open circles were results of cascade evaporation and hybrid preequilibrium model calculations,[4] respectively. Contrary to both the evaporation and preequilibrium models our modified statistical model reproduces the data quite well.

In order to check the effect of model parameters we give the corresponding mass-yield distribution calculated with different values of C_f and fixed $R_{T0}=2.05$ fm in Fig. 2. The dotted, solid, and dashed histograms in this figure are due to the values of C_f equal to 0.8, 0.73, and 0.666, respectively.

Figure 3 shows the effect of parameter R_{T0} on the

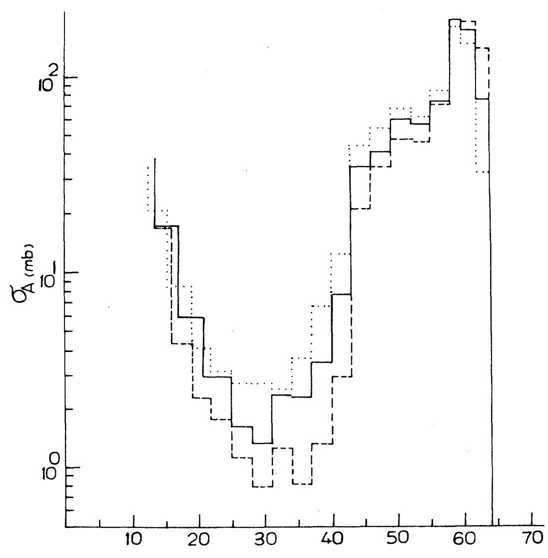

FIG. 2. Theoretical mass-yield distribution of fragments with fixed parameter $R_{T0}=2.05$ fm and different values of parameter C_f. Dotted histogram: $C_f=0.8$. Solid histogram: $C_f=0.73$. Dashed histogram: $C_f=0.666$.

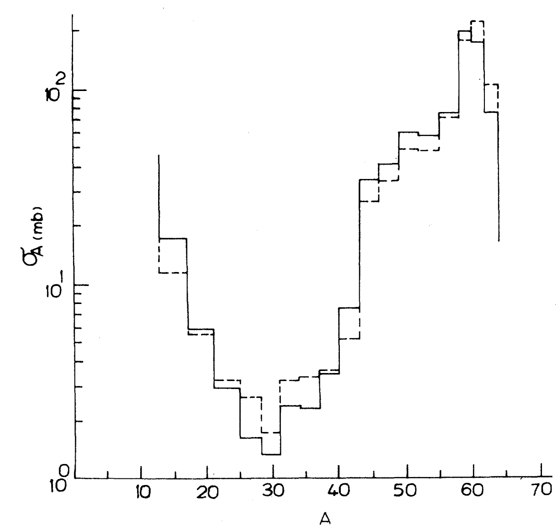

FIG. 3. Theoretical mass-yield distribution of fragments with fixed parameter $c_f=0.73$ and different values of parameter R_{T0}. Solid histogram: $R_{T0}=2.05$ fm. Dashed histogram: $R_{T0}=1.90$ fm.

mass-yield distribution of fragments under fixed $C_f = 0.73$ The solid and dashed histograms refer to mass-yield distribution with R_{T0} equal to 2.05 and 1.90 fm, respectively.

IV. DISCUSSION AND CONCLUSION

By inspecting the data of mass-yield distribution of fragments carefully one might confirm with the fact that the mass yields around the peak at $A_f \simeq 58$ are caused by the disassembly of hot nuclei in the pseudoevaporation mode which mainly leads to a pair of fragments with mass number $A_{f1} \lesssim 4$ and $A_{f2} \lesssim A_{RT}$. The severe increase of mass yield at $A_f \simeq 40$ should attribute to the effect of the pseudofission mode which mainly results in a pair of symmetrical fragments with mass number $A_f \lesssim A_{RT}/2$. The multifragmentation, which mainly leads to the case with more than three fragments with mass number $A_f \lesssim A_{RT}/4$, should be the source of the increase again of mass-yield distribution at lower mass numbers ($A_f \leq 20$, say).

The pseudoevaporation mode, pseudofission mode, multifragmentation mode, and vaporization mode, by which the hot nuclei disassembly into an assembly of protons, neutrons, and light charged fragments ($A \leq 4$, say), are all the possible decay modes of hot nuclei created in intermediate and high-energy heavy-ion reactions. The mass-yield distribution of fragments measured in the disassembly of hot nuclei, for example, is the result of the competition among those decay modes under given conditions of hot nuclei (E^* and volume, say) at freeze out. As mentioned above, our statistical model includes all of those modes, thus the data of mass-yield distribution of fragments could be reproduced by this model with suitable values of adjustable parameters C_f and R_{T0}.

However, the cascade evaporation model and the hybrid preequilibrium model are only taken pseudoevaporation mode into account therefore they did not predict the data of mass-yield distribution of fragments especially the considerable large yield at medium mass $A_f \simeq 40$ and the upturn again of mass-yield distribution at $A_f \simeq 30$.

One might see from Figs. 2 and 3 that the features of experimental mass-yield distribution of fragments, i.e., it peaks at $A_f \simeq 58$, decreases to a minimum at $A_f \simeq 30$ and increases again at lower mass number, all exist in theoretical histograms depending on the values of C_f and R_{T0}. Although the details of theoretical mass-yield distribution of fragments especially the ratio of the peak at $A_f \simeq 58$ and the valley at $A_f \simeq 30$ are sensitive to the values of parameters C_f and R_{T0}.

In summary, the proposed modified statistical model and corresponding Monte Carlo simulation technique are available in predicting the mass-yield distribution of fragments arising in the disassembly of hot nuclei created in intermediate-energy heavy-ion reactions. Together with the successes of its original version in high-energy heavy-ion reactions, it seems reasonable to conclude that the disassembly of hot nuclei might be depicted by a unified statistical model in spite of the energy of the collision, in which the hot nuclei were created, is high or intermediate. The most important problem is to give the suitable description of hot nuclei at freeze out including mass and charge number, volume and excitation energy, etc.

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