Relativistic transport theory of fluctuating fields for hadrons

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We analyze the physics of relativistic nuclear collisions and demonstrate that an adequate treatment of pions must include the quantum time-energy uncertainty principle and nonsequential three-body collisions. We apply relativistic quantum field theory to obtain exact equations determining the time evolution of hadronic fields and their fluctuations in terms of the effective interactions describing scattering in matter. These equations relate the main physical observables to the sought-after properties of nuclear matter, and involve only these quantities and the corresponding properties of free-space one- and two-body processes. We show how to regularize the singularities by employing the internal structure of the mesons, while maintaining causality and unitarity. We show that, in a very good approximation, the dynamic quantities reduce to functions of eight variables—those of the Boltzmann equation, supplemented by the energy of the hadron. Our method appears capable of deducing the properties of hot dense nuclear matter from data already measured in experiments on the collisions of heavy nuclei.

I. AIM OF THE PAPER

The properties of nuclear matter at high density and excitation remain unknown after more than a decade of relativistic heavy-ion experiments in which such matter is created and its decay products are observed. This paradoxical circumstance seems to be due mainly to theoretical rather than experimental inadequacies. Experiments have been able[1] to observe and record hundreds of particles in the final state of a collision of heavy nuclei at center-of-mass excitation energies up to several hundred MeV/nucleon. They have also succeeded in analyzing these observations quantitatively in terms that surely carry information about the properties of the hot dense matter: the degree of stopping, amount of transverse flow, number of pions and kaons per baryon, entropy of baryons measured by cluster probabilities, shapes and flow patterns measured by two-particle interferometry, and recently dilepton spectra. Simple theoretical models have demonstrated[2] that these observables are indeed sensitive to interesting properties of baryonic matter: compression energy, heat capacity, viscosity and thermal conductivity (or equivalently scattering cross sections in the matter), potential energies of mesons and baryons in the matter, and rates of production and absorption of mesons and excited baryons. Nevertheless, convincing quantitative interpretations of the observations are still lacking; the interesting theoretical quantities all remain uncertain by at least a factor of 2.

The source of the problem seems to be that each of the observed quantities is sensitive to several of the unknown properties of the hot dense matter. Thus no subset of the measurements can be interpreted validly without simultaneously interpreting the remaining measurements. (A possible exception is the dilepton data, which seem to be sensitive mainly to meson spectra;[3] unfortunately these measurements are as yet very incomplete.) As a result, a theoretical model is faced with very stringent requirements. First, it must be able to address all the important data just described; otherwise it will not be sufficiently constrained by the measurements to permit any unique interpretation. Second, it must contain a plausible account of all the theoretical properties mentioned; otherwise all its other inferences will be tainted by the residual uncertainty which in every case has been demonstrated to be very large. Finally, it must be quantitatively reliable at a good level of accuracy; otherwise the errors in each of the half-dozen important theoretical or experimental variables will compound to render them all so uncertain as to be uninteresting.

None of the currently available models of relativistic heavy-ion collisions meets these demands. The best models to date are based on the Boltzmann transport equation, with Fermi statistics included by blocking factors in the collision term. These models, often known by such labels as Boltzmann-Uehling-Uhlenbeck, Vlasov-Uehling-Uhlenbeck, or Nordheim-Vlasov, have been developed to address all the important data, and are able to give reasonable account of most of the theoretically necessary physics.[2] Their most conspicuous failure is in their treatment of pions, whose creation, absorption, and propagation in the matter is described as though the pions were classical particles interacting with the matter only by scattering off baryons and by being created or absorbed via the Δ excitation of the nucleon. Actually, most of the pions observed in nuclear collisions have momenta (relative to the baryonic matter) about equal to their rest mass; thus their wave packets must overlap many nucleons simultaneously instead of colliding with only one at a time. (The baryons, by contrast, have substantially larger momenta and thus can be somewhat better localized.) Furthermore, pions have a large, attractive potential energy in the matter due to their p-wave interaction with the nucleons; this interaction is so large that in the impulse (Kisslinger) approximation it roughly cancels their kinetic energy at normal nuclear density.[4] Therefore, any model which neglects the potential energy of the pions in nuclear matter cannot be considered to be even semiquantitative. The importance of an adequate account of pions in nuclear collisions around 1 GeV/nucleon is apparent: half the *NN* cross section consists of inelastic collisions in which the pion takes nearly all the available center-of-mass energy.

While clever tricks might make it possible to include the pions' potential energy and even their wave nature within the fundamentally classical transport theory based on extensions of the Boltzmann equation, there is an even more daunting obstacle to a quantitative treatment of their creation and absorption in the matter. When a pion in the hot dense matter collides with a nucleon, their relative kinetic energy typically places the πN system about halfway down the low-energy tail of the Δ resonance. Thus the amplitude for creation and absorption of pions will be sensitive to the width and shape of the Δ. This width, due in free space to the decay of the Δ into πN, must be modified by the presence of the matter in several ways:[5] the potential energies of the π, Δ, and N in the matter, the Pauli blocking of the nucleon in the final state into which the Δ decays, and the broadening of the Δ by collisions with other particles in the matter, which shares the pion's energy among several nucleons.[6] This latter effect is not only essentially three body in nature, casting serious doubt on the Boltzmann *stosszahlansatz*; it is also a fundamentally quantum effect resting on the time-energy uncertainty principle. The extension of the classical Boltzmann equation to include a quantitatively reliable description of the propagation, creation and absorption of pions in hot dense nuclear matter seems unpromising. We have elected instead to pursue a more fundamental approach.

In this paper we derive equations of motion for the relativistic Green's functions describing the quantum propagation of baryons and mesons in excited nuclear systems. We start from a low-energy effective Lagrangian containing nucleons, deltas, pions, and other heavier mesons motivated by the boson-exchange model of nuclear forces as well as the nuclear phenomenology of quantum hadrodynamics.[7] We show how to obtain a hierarchy of Dyson equations for one-, two-, and three-point Green's functions, and then truncate this hierarchy by parametrizing a four-point residual effective interaction in a manner introduced by Migdal.[8] This nonperturbative truncation leads to a coupled set of equations for functions of two space-time variables (i.e., eight real numbers), within the capability of the latest computers. Collisions of the mesons and baryons are included by way of self-energy loops; we show how to regulate the ultraviolet divergences of the loops by a unitary, causal cutoff procedure—necessary because our inclusion of derivative-coupled spin-$\frac{3}{2}$ particles precludes renormalization but perfectly acceptable in a low-energy effective theory. An overview of our method is presented in Sec. II, together with our choice of effective Lagrangian. Section III derives the transport equations for a general Lagrangian with cubic interactions. Section IV shows how to regularize the short-distance singularities we encounter in our equations of motion. We conclude in Sec. V with a summary of our results, a critique of the approximations involved, and a sketch of a plan to implement our method.

II. INGREDIENTS OF THE THEORY

To give a quantitatively reliable description of nuclear collisions at center-of-mass energies of a few hundred MeV/nucleon, a theoretical model must incorporate many aspects of our extensive experience of nuclear structure and forces. It must give a good account of *NN* and πN scattering in its energy regime because the particles in the final state of the collision collide with each other pairwise after the density of the matter has been attenuated but before they are detected. It must accurately describe the motion of nucleons and pions in normal nuclei, including the scattering of nucleons and pions from nuclei, not only because the model must be tested in known cases, but also because of the presence of spectator fragments of nuclear matter in the final states of many relativistic heavy-ion collisions. It must have free parameters to adjust the properties of hot and cold high-density nuclear matter because the comparison of a range of model predictions to measured data is essential to the process of inductive reasoning from which conclusions on the significance of experiments must be drawn. The theory also should include the main known aspects of nuclear collective motion, since we are hoping to uncover collective effects in the hot dense matter. Of course, it has to be built within the framework of relativistic quantum mechanics.

A. Degrees of freedom: the effective fields

The foregoing general considerations determine in large degree the ingredients of our model. The only quantitative, relativistic descriptions of nucleon-nucleon scattering[9] are based on the exchange of at least three bosons, corresponding to three of the spin-isospin channels of the *NN* system. It has been possible to successfully identify these bosons with the known π, ρ, and ω mesons. We add a fourth boson, a scalar σ, as a surrogate for the strong intermediate-range scalar attraction thought to arise[9] from the exchange of two pions correlated by their interactions, in order to avoid having to incorporate these complicated and controversial correlations into the structure of the model's equations. The short range of the ρ and ω exchange induces strong correlations between the nucleons at distances less than about 0.7 fm, where the effects of the structure of the hadrons may also be coming into play. We do not believe that the details of this short-distance behavior are crucial to intermediate-energy nuclear collisions; thus we are content to introduce additional parameters, at least one for each spin-isospin channel plus an overall cutoff, to summarize the effects of short-range correlations and hadronic substructure. These parameters are introduced in the manner of Migdal,[8] as described following; together with the meson masses and coupling constants, they give us at least three parameters per spin-isospin channel to describe *NN* elastic scattering over the relevant energy range of up to several hundred MeV in the center-of-mass (c.m.) system.

To provide a simple but realistic description of πN scattering[5] we include explicitly the $\Delta(J=\frac{3}{2},\ T=\frac{3}{2})$ as a fundamental field of our Lagrangian. While there is a long history of attempts to build this state as a resonance out of the interactions of pions and nucleons, we do not wish to encumber our theory with this additional burden; besides, the discovery of the quark substructure of the hadrons makes it clear that such an additional, independent degree of freedom does indeed exist. It may justly be argued that the physical Δ contains large admixtures of πN components; this feature is also reflected in our model, as we shall see. Because we include the Δ as a fundamental spin-$\frac{3}{2}$ field, our theory is not renormalizable. Recognizing this, we tentatively choose the phenomenologically simpler pseudovector form for the πN coupling instead of the renormalizable pseudoscalar coupling, which would have to rely on extensive cancellations of large terms to correctly describe the small s-wave πN scattering length (but our method does not depend on this choice). In order to provide a good description of the nonresonant spin-isospin amplitudes which play an important role in low-energy πN scattering, we include three-meson couplings $\rho\pi\pi$ and $\sigma\pi\pi$; the latter has the additional, desirable effect of broadening the σ, which is not observed as a well-defined resonance.

B. The model Lagrangian

We are now ready to write the effective Lagrangian of our model, after a few conventions of notation. We describe the Δ by the Rarita-Schwinger formalism,[10] and assume that the superfluous degrees of freedom are projected out as necessary. We denote Lorentz indices by Greek subscripts and superscripts, and isovectors by underlines. Thus the meson fields are written $\underline{\pi}$, σ, $\underline{\rho}_\mu$, and ω_μ. Repeated indices are summed, and isospin scalar and vector products are denoted by $\cdot$ and $\times$ respectively. Spinor fields are understood to include the isospin as well as Dirac components in their columns and rows; we use N for the nucleon and Δ_μ for the delta.

Our Lagrangian density is written as

$$\mathcal{L}(x)=\mathcal{L}^0(x)+\mathcal{L}^{\rm int}(x)\ , \tag{2.1}$$

where $\mathcal{L}^0(x)$ and $\mathcal{L}^{\rm int}(x)$ are the free-field and interaction Lagrangian densities. The free-field Lagrangian density is

$$\begin{aligned}\mathcal{L}^0(x)=&\overline{N}(x)(i\gamma^\mu\partial_\mu-M_N)N(x)+\overline{\Delta}_\nu(x)(i\gamma^\mu\partial_\mu-M_\Delta)\Delta^\nu(x)+\tfrac{1}{2}[\partial_\mu\underline{\pi}(x)\cdot\partial^\mu\underline{\pi}(x)-m_\pi^2\underline{\pi}(x)^2]\\&+\tfrac{1}{2}[\partial_\mu\sigma(x)\partial^\mu\sigma(x)-m_\sigma^2\sigma(x)^2]-\tfrac{1}{4}\underline{F}^{(\rho)}_{\mu\nu}(x)\cdot\underline{F}^{(\rho)\mu\nu}(x)+\tfrac{1}{2}m_\rho^2\underline{\rho}_\mu(x)\cdot\underline{\rho}^\mu(x)\\&-\tfrac{1}{4}F^{(\omega)}_{\mu\nu}(x)F^{(\omega)\mu\nu}(x)+\tfrac{1}{2}m_\omega^2\omega_\mu(x)\omega^\mu(x)\ ,\end{aligned} \tag{2.2}$$

where the field tensors for the rho and omega are given in terms of their potential fields by

$$\underline{F}^{(\rho)}_{\mu\nu}(x)\equiv\partial_\mu\underline{\rho}_\nu(x)-\partial_\nu\underline{\rho}_\mu(x) \tag{2.3}$$

and

$$F^{(\omega)}_{\mu\nu}(x)\equiv\partial_\mu\omega_\nu(x)-\partial_\nu\omega_\mu(x)\ . \tag{2.4}$$

The interaction Lagrangian $\mathcal{L}^{\rm int}(x)$ consists of meson-baryon and meson-meson terms,

$$\mathcal{L}^{\rm int}(x)=\mathcal{L}^{\rm int}_{\rm Bm}(x)+\mathcal{L}^{\rm int}_{\rm mm}(x)\ , \tag{2.5}$$

which are given by

$$\begin{aligned}\mathcal{L}^{\rm int}_{\rm Bm}(x)=&-ig_{\pi NN}\overline{N}(x)\gamma^\mu\gamma_5\underline{\tau}N(x)\cdot\partial_\mu\underline{\pi}(x)+g_{\sigma NN}\overline{N}(x)N(x)\sigma(x)-\tfrac{1}{2}g^V_{\rho NN}\overline{N}(x)\gamma^\mu\underline{\tau}N(x)\cdot\underline{\rho}_\mu(x)\\&-\tfrac{1}{2}g^T_{\rho NN}\overline{N}(x)\sigma^{\mu\nu}\underline{\tau}N(x)\cdot\underline{F}^{(\rho)}_{\mu\nu}(x)-g_{\omega NN}\overline{N}(x)\gamma^\mu N(x)\omega_\mu(x)\\&+g_{\pi N\Delta}[\overline{\Delta}^\mu(x)\underline{\mathcal{T}}N(x)\cdot\partial_\mu\underline{\pi}(x)+\overline{N}(x)\underline{\mathcal{T}}\Delta^\mu(x)\cdot\partial_\mu\underline{\pi}(x)]\\&+g_{\rho N\Delta}[\overline{\Delta}^\nu(x)\gamma_5\gamma^\mu\underline{\mathcal{T}}N(x)\cdot\underline{F}^{(\rho)}_{\mu\nu}+\overline{N}(x)\gamma_5\gamma^\mu\underline{\mathcal{T}}\Delta^\nu(x)\cdot\underline{F}^{(\rho)}_{\mu\nu}]-ig_{\pi\Delta\Delta}\overline{\Delta}^\mu(x)\gamma^\nu\gamma_5\underline{T}\Delta_\mu(x)\cdot\partial_\nu\underline{\pi}(x)\\&+g_{\sigma\Delta\Delta}\overline{\Delta}^\mu(x)\Delta_\mu(x)\sigma(x)-g_{\rho\Delta\Delta}\overline{\Delta}^\mu(x)\gamma^\nu\underline{T}\Delta_\mu(x)\cdot\underline{\rho}_\nu(x)-g_{\omega\Delta\Delta}\overline{\Delta}^\mu(x)\gamma^\nu\Delta_\mu(x)\omega_\nu(x)\end{aligned} \tag{2.6}$$

and

$$\begin{aligned}\mathcal{L}^{\rm int}_{\rm mm}(x)=&g_{\sigma\pi\pi}\sigma(x)\underline{\pi}(x)\cdot\underline{\pi}(x)\\&+g_{\rho\pi\pi}\underline{\rho}_\mu(x)\cdot[\partial^\mu\underline{\pi}(x)\times\underline{\pi}(x)]\ .\end{aligned} \tag{2.7}$$

In these expressions the operators $\underline{\tau}$ and $\underline{T}$ are the isospin operators in the nucleon and the delta sectors respectively, and $\underline{\mathcal{T}}$ is the nucleon-delta isospin transition operator;[11] $\sigma^{\mu\nu}$ is the spin tensor operator. With this choice our interaction Lagrangian density contains only cubic terms.

C. Green's functions as dynamical quantities

While the hadron fields provide the fundamental dynamic degrees of freedom of the model, we need to map them onto numbers in order to reduce the dynamics to computable form and thus permit comparisons to data.

This mapping is performed by forming expectation values, not only of the fields, but also of products of fields: the Green's functions of the theory. These Green's functions play a role in our theory analogous to the role of the probability density in the Boltzmann equation. Indeed, the probability density matrix may be obtained from the two-point Green's function $G^{(2)}_{\alpha\beta}(x_1,x_2)$ of two adjoint fields ϕ_α,ϕ_β, by setting its time arguments equal, as is evident from the definition,

$$G^{(2)}_{\alpha\beta}(x_1,x_2)\equiv\langle T\phi_\alpha(x_2)\phi_\beta(x_1)\rangle-\langle\phi_\alpha(x_2)\rangle\langle\phi_\beta(x_1)\rangle \ . \tag{2.8}$$

The two-point Green's function is a more relativistic dynamic quantity than the density matrix, which has two spatial arguments but only one time argument and thus inherently treats time and space asymmetrically. This point may be further appreciated by considering the Wigner transform of the propagator,

$$\hat{G}^{(2)}_{\alpha\beta}(x,p)\equiv\int d^4x' G^{(2)}_{\alpha\beta}\left[x+\frac{x'}{2},x-\frac{x'}{2}\right]e^{ipx'} \ . \tag{2.9}$$

The Wigner transform of the density matrix—the quantum analog of the phase-space probability distribution of classical statistical mechanics—is the integral of $\hat{G}^{(2)}_{\alpha\beta}(x,p)$ over the energy variable p_0. Here again we see that the Green's function treats the complementarity of time and energy on an equal footing with the complementarity of coordinates and momenta. By choosing the Green's functions as dynamic variables instead of the density matrix, we satisfy the requirements of both quantum mechanics and relativity.[12] Meanwhile, the fact that the density matrix may be easily extracted from the propagator permits us to adopt for comparison with observations the same techniques that have been developed in the context of the Boltzmann-derived models. We conclude that a theory describing the motion of two-point Green's functions could satisfy the requirements of being relativistic, quantum mechanical, and able to be compared with observations.

Dyson and Schwinger showed[13] that the equations of motion for n-point Green's functions involve a knowledge of the $(n+1)$-point Green's functions. As a well-known example, in mean-field theory the values of the mean meson fields, which are one-point Green's functions, are determined by the baryon densities, which are parts of the two-point Green's functions (see Fig. 1). Similarly, in the theory of pionic collective modes in nuclei, the Dyson equation for the two-point Green's function (Fig. 2) involves a three-field πNN vertex, closely related to the

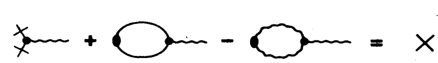

FIG. 1. Mean-field equation for boson field × in terms of bare three-point vertex (dot), free boson propagator (wavy line), dressed fermion propagator (straight line with solid oval), and dressed boson propagator (wavy line with solid oval). For an algebraic realization of these equations see text, Eq. (3.47) or (5.7).

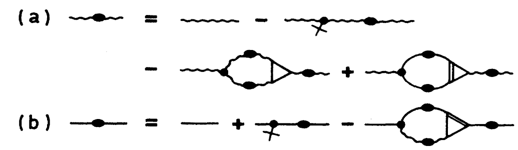

FIG. 2. Dyson equation for (a) boson and (b) fermion propagators in terms of dressed three-point vertex functions for three-boson vertex (open triangle) and fermion-boson vertex (triangle with bar). Notation as in Fig. 1. For an algebraic realization of these equations see (a) Eqs. (3.52) and (3.53) or (5.9) and (5.10) and (b) Eqs. (3.59) and (3.61) or (5.11) and (3.12).

three-point Green's function which is the expectation value of the product of the pion field with the baryon fields and their adjoints; the three-point Green's function and the three-field vertex function both describe the process of pion emission and absorption by baryons. This three-field vertex function, in turn, is determined (see Fig. 3) by a four-point vertex, the T matrix, describing the scattering of hadrons from each other.

D. The phenomenological residual effective interaction

The Dyson equations relating the n-point Green's functions to those with $n+1$ fields form an infinite hierarchy of coupled integral equations which clearly cannot be solved without approximations. For our problems, we believe that an appropriate approximation is to parameterize the four-point vertex describing two-body scattering. In fact, such a phenomenological parametrization procedure is highly desirable for our purposes, since a knowledge of two-body scattering in hot dense nuclear matter is an important part of the information we hope to learn from the data on nuclear collisions; as pointed out before, we have to be able to study how the observations depend on the unknown quantities if we are to draw conclusions from the data. Inspired by the usefulness of the Skyrme effective interaction in low-energy nuclear physics, then, we seek to introduce a simple parametrization of the short-range parts of two-body scattering that are not already determined by the one-, two-, and three-field Dyson equations (which for example build in the one-boson exchange forces, as seen in Fig. 3). As in the Skyrme program, we will permit our parameters to depend on the densities of baryons in the neighborhood of

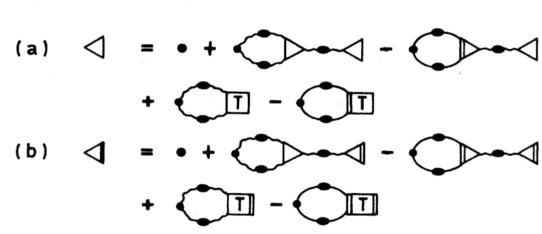

FIG. 3. Dyson equation for (a) three-boson and (b) fermion-boson vertices in terms of T matrices for two-boson scattering (open square), boson-fermion scattering (square with bar), and fermion-antifermion scattering (square with two bars). Notation as in Figs. 1 and 2. For an algebraic realization of these equations see (a) Eq. (3.64) and (b) Eq. (3.67).

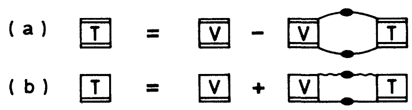

FIG. 4. Introduction of a two-particle irreducible interaction V in two cases: (a) Lippmann-Schwinger equation for two-fermion scattering; (b) Bethe-Salpeter equation for fermion-boson scattering.

the scattering; unlike that low-energy program, however, we will have to insist that the zero-density limit of our effective interactions should give a reasonable representation of free-space two-body scattering, when used in the few-point Dyson equations which (see Fig. 3) are constructed to include explicitly the longer-range boson-exchange forces in NN scattering as well as the direct-channel Δ resonance in πN. The aims of our program are thus complementary to that of ter Haar and Malfliet[14] who seek to compute from few-body data the in-medium scattering which we parametrize.

Dyson's hierarchy of equations is well suited to dealing with collective effects in the strong interactions, since each equation sums an infinite set of terms in the perturbative expansion of the Green's functions. There are several complementary ways to introduce an effective residual interaction into the hierarchy of Dyson equations. One of the most familiar is based on the Lippmann-Schwinger equation for two-fermion scattering, Fig. 4(a). This scheme sums ladders of repeated two-body interactions; its residual interaction is related to the full T matrix in the same way as the potential is related to the full scattering matrix in the Schrödinger equation. The Bethe-Salpeter equation performs a similar function for boson-fermion scattering, Fig. 4(b). These equations are the most important for a microscopic understanding of scattering processes in free space and in the medium. By contrast, Migdal introduces the effective residual interaction in the particle-antiparticle channel, Fig. 5. Migdal's formulation is especially suited to understanding the self-energies of mesons in matter, because here the

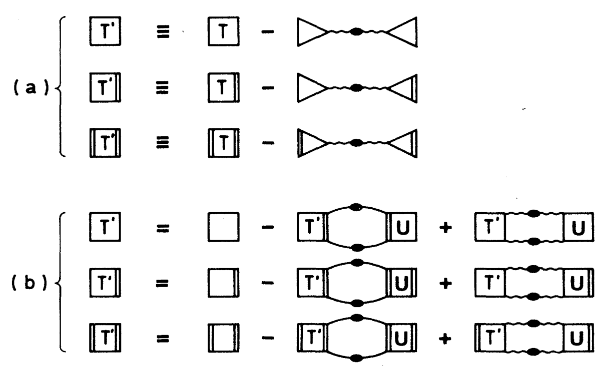

FIG. 5. Migdal reduction of effective interactions to eliminate (a) one-boson intermediate states, leaving one-body irreducible vertices; (b) two-boson and fermion-antifermion intermediate states, leaving one- and two-body irreducible four-point vertices. Notation as in Figs. 1, 2, and 3. For an algebraic realization of these equations see (a) Eqs. (3.68)–(3.70) or (5.15)–(5.17) and (b) Eqs. (3.73)–(3.75) or (5.18)–(5.20).

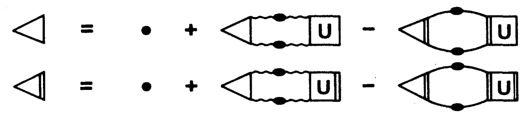

FIG. 6. Dyson equations for three-point vertices after introducing Migdal's effective interactions. Notation as in Figs. 1, 2, 3, and 5. For an algebraic realization of these equations see Eqs. (3.76) or (5.13) and (3.77) or (5.14).

fermion-hole states provide low-energy intermediate states for the interactions to mix with the meson fields.[5,8,11] These various schemes for truncating the Dyson hierarchy are related to each other by the crossing relations among scattering amplitudes of particles and antiparticles; they would all be equivalent if their residual interactions were evaluated exactly. However, when we introduce a simplified parametrization of the residual interaction, we are likely to break the crossing symmetry inherent in the otherwise-exact equations. We then have to choose the formalism based on which part of the physics we wish to emphasize. Since we are most concerned with the effects of the nuclear medium on the properties of mesons, we choose Migdal's way. First reducing out the single-meson state from the particle-antiparticle channel [Fig. 5(a)], we then eliminate also the two-body intermediate states, both two-meson and particle-antiparticle [Fig. 5(b)]. The remaining residual effective interaction, which is one- and two-body irreducible in the given channel, is then approximated by a local, zero-range force to be chosen phenomenologically.

By truncating the hierarchy of Dyson equations through the introduction of an effective residual two-body interaction, we gain an unexpected advantage of great practical importance: the numerical problem is greatly simplified. At first sight, it seems that we have to deal with functions of four space-time variables, or 16 real numbers, a task far beyond the capacity of existing computers. However, when we introduce the residual effective interaction as a zero-range local force in the Midgal formulation of the equation relating three- and four-body scattering, then the equations for the one-, two-, and three-point Green's functions are reduced[15] to a coupled set of equations for functions of only two space-time variables, or eight real numbers, as may be seen by inspecting Fig. 6. Such equations appear computationally approachable: for comparison, recall that the Boltzmann equation involves a function of seven real variables.

E. Program of applications

The equations of motion sketched previously and derived following may be applied to many different situations. In fact, it is essential to our program that the same theory should describe NN and πN scattering, nuclear ground states and their collective and independent-particle excitations, nuclear matter in its ground state and at high excitation, and the scattering of nucleons and pions from nuclei, as well as nuclei in collision. In each case, the Lagrangian, residual interactions, and equations of motion are the same; the different situations correspond to different wave packets used to form the expectation values represented by the Green's functions.

Of course, it is not necessary (and indeed would be impossible) to construct the wave packets themselves; instead, the various solutions are distinguished by the way their initial values and/or boundary conditions are specified. For example, the Green's functions describing two-particle scattering in free space are obtained by specifying the usual scattering boundary conditions on the Green's functions. Nuclear matter is produced by introducing an appropriate chemical potential to enforce the nucleon density. Nuclear ground states correspond to localized wave packets where the mean fields differ from their vacuum values in a localized region of space; to find these wave packets will probably require an iterative procedure similar to that commonly employed in Hartree-Fock computations. The construction of a nuclear ground state will necessarily involve the propagators of nucleons and mesons in that state; thus the scattering of nucleons and mesons from nuclei will be automatically included as part of the construction of the ground state.

All the previously described situations correspond to stationary states in the sense that the Green's functions depend only on time differences, which should help simplify the computations. To describe nuclear collisions, we would begin by constructing such stationary-state wave packets to describe the initial target and projectile; we would then find an initial condition for the collision problem by boosting these wave packets along classical trajectories aimed to make them collide, as is commonly done for the time-dependent Hartree-Fock model of low-energy nuclear collisions.

The equations we derive following go far beyond the mean-field approximation,[7] which also involves two-point functions for the fermions: by including the two-point functions describing the fluctuations of the meson fields, the equations are able not only to include collective modes at the level of random-phase approximation (RPA) theory, consistent with the best descriptions of pions in nuclei, but also to incorporate the effects of collisions on the motion of both baryons and mesons, as in the most advanced theories of the optical model for nucleon-nucleus and pion-nucleus scattering; since these collisions are responsible for the lifetime of single-particle excitations; they provide a natural description of the collision broadening of the Δ in the hot matter which is so important for pion creation and absorption in nucleus-nucleus collisions. Furthermore, since our theory encompasses the four-point function, the T matrix, it can naturally account for the correlations among pairs of hadrons in the final state which are usually called interferometry. Our theory is thus a theory not only of mean fields but also of their fluctuations; it is a transport theory, since it describes the time evolution of probability densities. We refer to it as the transport theory of fluctuating fields.

III. DERIVATION OF THE TRANSPORT EQUATIONS FOR A CUBIC LAGRANGIAN

In this section we derive integral equations for Green's functions starting from the Lagrangian of a theory consisting of a set of boson fields, ϕ_α, interacting to cubic order with a set of fermion spinors ψ_a. The theory described in Sec. II is an example of such a theory. The Lagrangian density for this case has the form

$$\mathcal{L}(x)=\frac{1}{2}\sum_{\alpha,\beta}[\partial_\mu\phi_\alpha(x)B^{\mu\beta}_{\alpha\nu}\partial^\nu\phi_\beta(x)-\phi_\alpha(x)m^2_{\alpha\beta}\phi_\beta(x)]+\sum_a\bar{\psi}_a(x)(i\gamma^\mu\partial_\mu-M_a)\psi_a(x)$$
$$-\sum_{\alpha,\beta,\gamma}\int dy\,dz\,\phi_\gamma(z)\Gamma^{(3)}_{(0)\alpha\beta\gamma}(x,y,z)\phi_\beta(y)\phi_\alpha(x)-\sum_{\alpha,b,c}\int dy\,dz\,\bar{\psi}_c(z)\Gamma^{(3)}_{(0)abc}(x;y,z)\psi_b(y)\phi_\alpha(x)\;. \quad (3.1)$$

The bilinear metric tensor $B^{\mu\beta}_{\alpha\nu}$ may be identified by comparing Eq. (3.1) with Eq. (2.2). The vertices $\Gamma^{(3)}_{(0)}$ are of infinitesimal range, proportional to delta functions or the derivatives of delta functions. We give the explicit forms of these vertices in Appendix A.

A. Functional-integral definitions and identities

We shall derive the Dyson equations using functional-integral methods.[16] We define a generating functional $W(j,\eta,\bar{\eta})$ for connected Green's functions by introducing sources $j_\alpha(x)$ for the boson fields and anticommuting sources $\bar{\eta}_a(x)$ and $\eta_b(x)$ for the fermion spinor fields $\psi_a(x)$ and their adjoints $\bar{\psi}_b(x)$. In applications, the source currents j_α, $\bar{\eta}_a$, and η_b vanish; they are merely introduced as devices for systematizing the field theory. Once the source currents are set to zero, the expectation values of fermion fields also vanish, as do all expectation values involving an odd number of fermion fields. We define

$$W(j;\eta,\bar{\eta})\equiv\frac{Z(j;\eta,\bar{\eta})}{Z(j=\eta=\bar{\eta}=0)}\;, \quad (3.2)$$

$$Z(j;\eta,\bar{\eta})\equiv e^{G_c(j;\eta,\bar{\eta})} \quad (3.3)$$

$$=\int\mathcal{D}(\phi;\psi,\bar{\psi})e^{iA}\;, \quad (3.4)$$

$$A\equiv I(\phi;\psi,\bar{\psi})+\int d^4x\left\{\sum_\alpha j_\alpha(x)\phi_\alpha(x)+\sum_a[\bar{\eta}_a(x)\psi_a(x)+\bar{\psi}_a(x)\eta_a(x)]\right\}, \quad (3.5)$$

$$I(\phi;\psi,\bar{\psi})\equiv\int d^4x\,\mathcal{L}[\phi(x);\psi(x),\bar{\psi}(x)]\;, \quad (3.6)$$

in which the symbol $\mathcal{D}(\phi;\psi,\bar{\psi})$ denotes functional integration over the boson and fermion fields. $I(\phi;\psi,\bar{\psi})$ is the classical action of the theory. The fields are obtained by varying $G_c(j;\eta,\bar{\eta})$ with respect to the sources,

$$\phi_\alpha(x)=\frac{\delta G_c(j;\eta,\bar{\eta})}{i\delta j_\alpha(x)}\ , \tag{3.7}$$

$$\psi_a(x)=\frac{\delta G_c(j;\eta,\bar{\eta})}{i\delta \bar{\eta}_a(x)}\ , \tag{3.8}$$

$$\bar{\psi}_a(x)=\frac{\delta G_c(j;\eta;\eta,\bar{\eta})}{-i\delta \eta_a(x)}\ . \tag{3.9}$$

The two-point Green's functions for the boson fields are given by

$$G^{(2)}_{\alpha\beta}(x_1,x_2)\equiv\langle T\phi_\beta(x_2)\phi_\alpha(x_1)\rangle-\langle\phi_\beta(x_2)\rangle\langle\phi_\alpha(x_1)\rangle =\left.\frac{\delta^2 G_c}{i\delta j_\beta(x_2)i\delta j_\alpha(x_1)}\right|_{j\eta\bar{\eta}\to 0} =-\left.\frac{\delta^2 G_c}{\delta j_\beta(x_2)\delta j_\alpha(x_1)}\right|_{j\eta\bar{\eta}\to 0}\ . \tag{3.10}$$

For the fermion fields we have

$$G^{(2)}_{ab}(x_1,x_2)\equiv\langle T\psi_b(x_2)\bar{\psi}_a(x_1)\rangle =\left.\frac{\delta^2 G_c}{i\delta\bar{\eta}_b(x_2)-i\delta\eta_a(x_1)}\right|_{j\eta\bar{\eta}\to 0} =\left.\frac{\delta^2 G_c}{\delta\bar{\eta}_b(x_2)\delta\eta_a(x_1)}\right|_{j\eta\bar{\eta}\to 0}\ . \tag{3.11}$$

Similarly, the three-point Green's functions for the three-boson vertex and for the boson-fermion vertex are given by

$$\begin{aligned}G^{(3)}_{\alpha\beta\gamma}(x_1,x_2,x_3)\equiv&\langle T\phi_\gamma(x_3)\phi_\beta(x_2)\phi_\alpha(x_1)\rangle-\langle\phi_\gamma(x_3)\rangle\langle\phi_\beta(x_2)\rangle\langle\phi_\alpha(x_1)\rangle\\&-\langle\phi_\gamma(x_3)\rangle G^{(2)}_{\alpha\beta}(x_1,x_2)-\langle\phi_\beta(x_2)\rangle G^{(2)}_{\alpha\gamma}(x_1,x_3)-\langle\phi_\alpha(x_1)\rangle G^{(2)}_{\beta\gamma}(x_2,x_3)\\=&\left.\frac{\delta^3 G_c}{i\delta j_\gamma(x_3)i\delta j_\beta(x_2)i\delta j_\alpha(x_1)}\right|_{j\eta\bar{\eta}\to 0}=i\left.\frac{\delta^3 G_c}{\delta j_\gamma(x_3)\delta j_\beta(x_2)\delta j_\alpha(x_1)}\right|_{j\eta\bar{\eta}\to 0}\end{aligned} \tag{3.12}$$

for the three-boson vertex and

$$\begin{aligned}G^{(3)}_{\alpha bc}(x_1;x_2,x_3)\equiv&\langle T\psi_c(x_3)\bar{\psi}_b(x_2)\phi_\alpha(x_1)\rangle-\langle\phi_\alpha(x_1)\rangle G^{(2)}_{bc}(x_2,x_3)\\=&\left.\frac{\delta^3 G_c}{i\delta\bar{\eta}_c(x_3)-i\delta\eta_b(x_2)i\delta j_\alpha(x_1)}\right|_{j\eta\bar{\eta}\to 0}=-i\left.\frac{\delta^3 G_c}{\delta\bar{\eta}_c(x_3)\delta\eta_b(x_2)\delta j_\alpha(x_1)}\right|_{j\eta\bar{\eta}\to 0}\end{aligned} \tag{3.13}$$

for the boson-fermion vertex.

The four-point Green's functions for boson-boson scattering are

$$\begin{aligned}G^{(4)}_{\alpha\beta\gamma\delta}(x_1,x_2,x_3,x_4)\equiv&\langle T\phi_\delta(x_4)\phi_\gamma(x_3)\phi_\beta(x_2)\phi_\alpha(x_1)\rangle-G^{(2)}_{\gamma\delta}(x_3x_4)G^{(2)}_{\alpha\beta}(x_1,x_2)\\&-G^{(2)}_{\beta\delta}(x_2,x_4)G^{(2)}_{\alpha\gamma}(x_1,x_3)-G^{(2)}_{\alpha\delta}(x_1,x_4)G^{(2)}_{\beta\gamma}(x_2,x_3)\\&-G^{(3)}_{\beta\gamma\delta}(x_2,x_3,x_4)\langle\phi_\alpha(x_1)\rangle-G^{(3)}_{\alpha\gamma\delta}(x_1,x_3,x_4)\langle\phi_\beta(x_2)\rangle\\&-G^{(3)}_{\alpha\beta\delta}(x_1,x_2,x_4)\langle\phi_\gamma(x_3)\rangle-G^{(3)}_{\alpha\beta\gamma}(x_1,x_2,x_3)\langle\phi_\delta(x_4)\rangle\\&-G^{(2)}_{\gamma\delta}(x_3,x_4)\langle\phi_\beta(x_2)\rangle\langle\phi_\alpha(x_1)\rangle-\langle\phi_\delta(x_4)\rangle\langle\phi_\gamma(x_3)\rangle G^{(2)}_{\alpha\beta}(x_1,x_2)\\&-G^{(2)}_{\beta\gamma}(x_2,x_4)\langle\phi_\gamma(x_3)\rangle\langle\phi_\alpha(x_1)\rangle-G^{(2)}_{\alpha\delta}(x_1,x_4)\langle\phi_\gamma(x_3)\rangle\langle\phi_\beta(x_2)\rangle\\&-\langle\phi_\delta(x_4)\langle\phi_\beta(x_2)\rangle G^{(2)}_{\alpha\gamma}(x_1,x_3)-\langle\phi_\delta(x_4)\rangle\langle\phi_\alpha(x_1)\rangle G^{(2)}_{\beta\gamma}(x_2,x_3)\\&-\langle\phi_\delta(x_4)\rangle\langle\phi_\gamma(x_3)\rangle\langle\phi_\beta(x_2)\rangle\langle\phi_\alpha(x_1)\rangle\\=&\left.\frac{\delta^4 G_c}{i\delta j_\delta(x_4)i\delta j_\gamma(x_3)i\delta j_\beta(x_2)i\delta j_\alpha(x_1)}\right|_{j\eta\bar{\eta}\to 0}=\left.\frac{\delta^4 G_c}{\delta j_\delta(x_4)\delta j_\gamma(x_3)\delta j_\beta(x_2)\delta j_\alpha(x_1)}\right|_{j\eta\bar{\eta}\to 0}\ ,\end{aligned} \tag{3.14}$$

and for $\psi\bar{\psi}$ annihilation to mesons by

$$\begin{aligned}G^{(4)}_{\alpha\beta cd}(x_1,x_2;x_3,x_4)\equiv&\langle T\psi_d(x_4)\bar{\psi}_c(x_3)\phi_\beta(x_2)\phi_\alpha(x_1)\rangle-[G^{(2)}_{\alpha\beta}(x_1,x_2)+\langle\phi_\beta(x_2)\rangle\langle\phi_\alpha(x_1)\rangle]G^{(2)}_{cd}(x_3,x_4)\\&-\langle\phi_\alpha(x_1)\rangle G^{(3)}_{\beta cd}(x_2;x_3,x_4)-\langle\phi_\beta(x_2)\rangle G^{(3)}_{\alpha cd}(x_1;x_3,x_4)\\=&\left.\frac{\delta^4 G_c}{i\delta\bar{\eta}(x_4)-i\delta\eta(x_3)i\delta j_\beta 9x_2)i\delta j_\alpha(x_1)}\right|_{j\eta\bar{\eta}\to 0}=-\left.\frac{\delta^4 G_c}{\delta\bar{\eta}(x_4)\delta\eta(x_3)\delta j_\beta(x_2)\delta j_\alpha(x_1)}\right|_{j\eta\bar{\eta}\to 0}\ .\end{aligned} \tag{3.15}$$

The four-point Green's function for fermion-antifermion scattering is

$$G^{(4)}_{abcd}(x_1,x_2,x_3,x_4)\equiv\langle T\psi_d(x_4)\bar{\psi}_c(x_3)\psi_b(x_2)\bar{\psi}_a(x_1)\rangle-G^{(2)}_{cd}(x_3,x_4)G^{(2)}_{ab}(x_1,x_2)+G^{(2)}_{ad}(x_1,x_4)G^{(2)}_{cb}(x_3,x_2)$$
$$=\left.\frac{\delta^4 G_c}{i\delta\bar{\eta}(x_4)-i\delta\eta(x_3)i\delta\bar{\eta}(x_2)-i\delta\eta(x_1)}\right|_{j\eta\bar{\eta}\to 0}=\left.\frac{\delta^4 G_c}{\delta\bar{\eta}(x_4)\delta\eta(x_3)\delta\bar{\eta}(x_2)\delta\eta(x_1)}\right|_{j\eta\bar{\eta}\to 0}. \tag{3.16}$$

Higher-order Green's functions for five or more fields may be defined analogously; we shall not need to refer to them explicitly.

In addition to the Green's functions $G^{(n)}$ we shall also need to introduce the proper (i.e., one-body irreducible) vertices $\Gamma^{(n)}$. $\Gamma^{(2)}$ is the inverse propagator or wave operator [see Eqs. (3.24) and (3.26) following]; for $n>2$, the $\Gamma^{(n)}$ can be thought of as effective interactions. To introduce them, we define the generating functional for proper vertices, $\Gamma_c(\phi;\psi,\bar{\psi})$, which is obtained by taking the Legendre transform of the generating functional of connected Green's functions,

$$\Gamma_c(\phi;\psi,\bar{\psi})\equiv\frac{1}{i}G_c(j;\eta,\bar{\eta})-\int dx\left\{\sum_\alpha j_\alpha(x)\phi_\alpha(x)+\sum_a[\bar{\eta}_a(x)\psi_a(x)+\bar{\psi}_a(x)\eta_a(x)]\right\} \tag{3.17}$$

The sources are obtained by varying $\Gamma_c(\phi;\psi,\bar{\psi})$ with respect to the fields,

$$j_\alpha(x)=-\frac{\delta\Gamma_c(\phi;\psi,\bar{\psi})}{\delta\phi_\alpha(x)}, \tag{3.18}$$

$$\eta_b(x)=-\frac{\delta\Gamma_c(\phi;\psi,\bar{\psi})}{\delta\bar{\psi}_b(x)}, \tag{3.19}$$

$$\bar{\eta}_a(x)=\frac{\delta\Gamma_c(\phi;\psi,\bar{\psi})}{\delta\psi_a(x)}. \tag{3.20}$$

The two-point vertex is defined for the boson field by

$$\Gamma^{(2)}_{\alpha\beta}(x_1,x_2)\equiv\left.\frac{\delta^2\Gamma_c}{\delta\phi_\beta(x_2)\delta\phi_\alpha(x_1)}\right|_{j\eta\bar{\eta}\to 0}, \tag{3.21}$$

and for the fermion field by

$$\Gamma^{(2)}_{ab}(x_1,x_2)\equiv\left.\frac{\delta^2\Gamma_c}{\delta\bar{\psi}_b(x_2)\delta\psi_a(x_1)}\right|_{j\eta\bar{\eta}\to 0}. \tag{3.22}$$

The relation between $G^{(2)}_{\alpha\beta}(x_1,x_2)$ and $\Gamma^{(2)}_{\alpha\beta}(x_1,x_2)$ can be obtained by differentiating (3.7) with respect to the field ϕ_β:

$$\frac{\delta}{\delta\phi_\beta(x_2)}\left[\frac{\delta G_c}{i\delta j_\alpha(x_1)}\right]=\delta_{\alpha\beta}\delta^4(x_1-x_2)$$
$$=i\int dx_3\left[\sum_\gamma\frac{\delta^2 G_c}{\delta j_\gamma(x_3)\delta j_\alpha(x_1)}\frac{\delta^2\Gamma_c}{\delta\phi_\beta(x_2)\delta\phi_\gamma(x_3)}+\sum_c\frac{\delta^2 G_c}{\delta\eta_c(x_3)\delta j_\alpha(x_1)}\frac{\delta^2\Gamma_c}{\delta\phi_\beta(x_2)\delta\bar{\psi}_c(x_3)}\right.$$
$$\left.-\sum_c\frac{\delta^2 G_c}{\delta\bar{\eta}_c(x_3)\delta j_\alpha(x_1)}\frac{\delta^2\Gamma_c}{\delta\phi_\beta(x_2)\delta\psi_c(x_3)}\right]. \tag{3.23}$$

Taking the sources to zero, we find using (3.10) and (3.21)

$$i\delta_{\alpha\beta}\delta^4(x_1-x_2)=\sum_\gamma{}'\int dx_3 G^{(2)}_{\alpha\gamma}(x_1,x_3)\Gamma^{(2)}_{\gamma\beta}(x_3,x_2). \tag{3.24}$$

A similar relation can be obtained for the fermion field by differentiating (3.9) with respect to $\bar{\psi}_b$,

$$\frac{\delta}{\delta\bar{\psi}_b(x_2)}\left[\frac{\delta G_c}{-i\delta\eta_a(x_1)}\right]=\delta_{ab}\delta^4(x_1-x_2)$$
$$=i\int dx_3\left[-\sum_\gamma\frac{\delta^2 G_c}{\delta j_\gamma(x_3)\delta\eta_a(x_1)}\frac{\delta^2\Gamma_c}{\delta\bar{\psi}_\beta(x_2)\delta\phi_\gamma(x_3)}-\sum_c\frac{\delta^2 G_c}{\delta\eta_c(x_3)\delta\eta_a(x_1)}\frac{\delta^2\Gamma_c}{\delta\bar{\psi}_b(x_2)\delta\bar{\psi}_c(x_3)}\right.$$
$$\left.+\sum_c\frac{\delta^2 G_c}{\delta\bar{\eta}_c(x_3)\delta\eta_a(x_1)}\frac{\delta^2\Gamma_c}{\delta\bar{\psi}_b(x_2)\delta\psi_c(x_3)}\right]. \tag{3.25}$$

Taking the sources to zero, we find using (3.11) and (3.22)

$$-i\delta_{ab}\delta^4(x_1-x_2)=\sum_c \int dx_3 G^{(2)}_{ac}(x_1,x_3)\Gamma^{(2)}_{cb}(x_3,x_2) \ . \tag{3.26}$$

We see that the two-point vertex functions $\Gamma^{(2)}$ are the inverses of the Green's functions $G^{(2)}$.

We now define three-point dressed couplings for the three-boson vertex and for the boson-fermion vertex. We have

$$\Gamma^{(3)}_{\alpha\beta\gamma}(x_1,x_2,x_3)\equiv -\left.\frac{\delta^3\Gamma_c}{\delta\phi_\gamma(x_3)\delta\phi_\beta(x_2)\delta\phi_\alpha(x_1)}\right|_{j\eta\bar{\eta}\to 0} , \tag{3.27}$$

$$\Gamma^{(3)}_{\alpha ab}(x_1;x_2,x_3)\equiv \left.\frac{\delta^3\Gamma_c}{\delta\bar{\psi}_b(x_3)\delta\psi_a(x_2)\delta\phi_\alpha(x_1)}\right|_{j\eta\bar{\eta}\to 0} . \tag{3.28}$$

To derive the relationship between $G^{(3)}_{\alpha\beta\gamma}(x_1,x_2,x_3)$ and $\Gamma^{(3)}_{\alpha\beta\gamma}(x_1,x_2,x_3)$, we differentiate Eq. (3.23) with respect to ϕ_γ. Taking the sources to zero, we find

$$0=i\int dx_4 \sum_\delta \left[-\int dx_5 \sum_\varepsilon \frac{\delta^3 G_c}{\delta j_\varepsilon(x_5)\delta j_\delta(x_4)\delta j_\alpha(x_1)} \frac{\delta^2\Gamma_c}{\delta\phi_\gamma(x_3)\delta\phi_\varepsilon(x_5)} \frac{\delta^2\Gamma_c}{\delta\phi_\beta(x_2)\delta\phi_\delta(x_4)} + \frac{\delta^2 G_c}{\delta j_\delta(x_4)\delta j_\alpha(x_1)} \frac{\delta^3\Gamma_c}{\delta\phi_\gamma(x_3)\delta\phi_\beta(x_2)\delta\phi_\delta(x_4)} \right] . \tag{3.29}$$

We now insert the definitions of two- and three-point functions, preceding to get

$$i\int dx_4 dx_5 \sum_{\delta\varepsilon} G^{(3)}_{\alpha\delta\varepsilon}(x_1,x_4,x_5)\Gamma^{(2)}_{\varepsilon\gamma}(x_5,x_3)\Gamma^{(2)}_{\delta\beta}(x_4,x_2) = -\int dx_4 \sum_\delta G^{(2)}_{\alpha\delta}(x_1,x_4)\Gamma^{(3)}_{\delta\beta\gamma}(x_4,x_2,x_3) \ . \tag{3.30}$$

Multiplying by $\Gamma^{(2)}_{\zeta\alpha}(x_6,x_1)$, summing over α, integrating over x_1, and using (3.24), we get

$$\Gamma^{(3)}_{\zeta\beta\gamma}(x_6,x_2,x_3) = -\sum_{\alpha\delta\varepsilon}\int dx_4 dx_5 dx_1 \Gamma^{(2)}_{\zeta\alpha}(x_6,x_1) G^{(3)}_{\alpha\delta\varepsilon}(x_1,x_4,x_5)\Gamma^{(2)}_{\varepsilon\gamma}(x_5,x_3)\Gamma^{(2)}_{\delta\beta}(x_4,x_2) \ . \tag{3.31}$$

This expression can be inverted using (3.24). We find

$$G^{(3)}_{\alpha\delta\varepsilon}(x_1,x_4,x_5) = -i\sum_{\beta\gamma\zeta}\int dx_2 dx_3 dx_6 G^{(2)}_{\alpha\zeta}(x_1,x_6)\Gamma^{(3)}_{\zeta\beta\gamma}(x_6,x_2,x_3) G^{(2)}_{\beta\delta}(x_2,x_4) G^{(2)}_{\gamma\varepsilon}(x_3,x_5) \ . \tag{3.32}$$

A similar expression can be derived for the boson-fermion vertex starting from (3.25). Varying with respect to ϕ_α and then taking the sources to zero, we find

$$0=i\int dx_4 \sum_c \left[-\int d^4x_5 \sum_\beta \frac{\delta^3 G_c}{\delta j_\beta(x_5)\delta\bar{\eta}_c(x_4)\delta\eta_a(x_1)} \frac{\delta^2\Gamma_c}{\delta\phi_\alpha(x_3)\delta\phi_\beta(x_5)} \frac{\delta^2\Gamma_c}{\delta\bar{\psi}_b(x_2)\delta\psi_c(x_4)} + \frac{\delta^2 G_c}{\delta\bar{\eta}_c(x_4)\delta\eta_a(x_1)} \frac{\delta^3\Gamma_c}{\delta\phi_\alpha(x_3)\delta\bar{\psi}_b(x_2)\delta\psi_c(x_4)} \right] . \tag{3.33}$$

Inserting the definitions of two- and three-point functions, above, we get

$$i\int dx_4 dx_5 \sum_{\beta c} G^{(3)}_{\beta ac}(x_5;x_1,x_4)\Gamma^{(2)}_{\beta\alpha}(x_5,x_3)\Gamma^{(2)}_{cb}(x_4,x_2) = \sum_c \int dx_4 G^{2}_{ac}(x_1,x_4)\Gamma^{(3)}_{\alpha cb}(x_3;x_4,x_2) \ . \tag{3.34}$$

Multiplying by $\Gamma^{(2)}_{da}(x_6,x_1)$, summing over a, integrating over x_1, and using (3.26), we get

$$\Gamma^{(3)}_{\alpha db}(x_3;x_6,x_2) = -\sum_{a\beta c}\int dx_1 dx_4 dx_5 \Gamma^{(2)}_{\beta\alpha}(x_5,x_3)\Gamma^{(2)}_{cb}(x_4,x_2) G^{(3)}_{\beta ac}(x_5;x_1,x_4)\Gamma^{(2)}_{da}(x_6,x_1) \ . \tag{3.35}$$

This expression can be inverted using (3.24) and (3.26). We find

$$G^{(3)}_{\beta ac}(x_5;x_1,x_4) = -i\sum_{\alpha bd}\int dx_2 dx_3 dx_6 G^{(2)}_{ad}(x_1,x_6)\Gamma^{(3)}_{\alpha db}(x_3;x_6,x_2) G^{(2)}_{\alpha\beta}(x_3,x_5) G^{(2)}_{bc}(x_2,x_4) \ . \tag{3.36}$$

Finally, we define the effective interactions T which describe two-body scattering. The T matrix is defined by

$$T_{\alpha\beta\gamma\delta}(x_1,x_2,x_3,x_4)\equiv i\sum_{\varepsilon\zeta\eta\theta}\int dx_5 dx_6 dx_7 dx_8 \Gamma^{(2)}_{\alpha\varepsilon}(x_1,x_5)\Gamma^{(2)}_{\gamma\eta}(x_3,x_7) G^{(4)}_{\varepsilon\zeta\eta\theta}(x_5,x_6,x_7,x_8)\Gamma^{(2)}_{\zeta\beta}(x_6,x_2)\Gamma^{(2)}_{\theta\delta}(x_8,x_4) \tag{3.37}$$

for boson-boson scattering and by

$$T_{\alpha\beta cd}(x_1,x_2;x_3,x_4)\equiv i\sum_{\varepsilon\zeta gh}\int dx_5 dx_6 dx_7 dx_8 \Gamma^{(2)}_{\alpha\varepsilon}(x_1,x_5)\Gamma^{(2)}_{cg}(x_3,x_7) G^{(4)}_{\varepsilon\zeta gh}(x_5,x_6;x_7,x_8)\Gamma^{(2)}_{\zeta\beta}(x_6,x_2)\Gamma^{(2)}_{hd}(x_8,x_4) \tag{3.38}$$

for the fermion-antifermion annihilation vertex. The T matrix for $\psi\bar{\psi}$ scattering is

$$T_{abcd}(x_1,x_2,x_3,x_4)\equiv i\sum_{efgh}\int dx_5dx_6dx_7dx_8\Gamma^{(2)}_{ae}(x_1,x_5)\Gamma^{(2)}_{cg}(x_3,x_7)G^{(4)}_{efgh}(x_5,x_6,x_7,x_8)\Gamma^{(2)}_{fb}(x_6,x_2)\Gamma^{(2)}_{hd}(x_8,x_4)\ . \tag{3.39}$$

The effective interaction T has a simple relation to two-body scattering: its matrix elements between asymptotic plane waves, taken in first-order Born approximation, yield the exact on-shell scattering amplitude.

B. Equations of motion for the fields and two-point Green's functions

The equation of motion for the boson field is obtained by varying W with respect to ϕ_α

$$0=\int\mathcal{D}(\phi;\psi,\bar{\psi})\left[\frac{\delta I}{\delta\phi_\alpha(x)}+j_\alpha(x)\right]e^{iA}\ , \tag{3.40}$$

and expressing the fields as variations of the generating functional G_c with respect to the sources,

$$0=\left[\frac{\delta I}{\delta\phi_\alpha(x)}\left[\frac{\delta}{i\delta j};\frac{\delta}{i\delta\bar{\eta}},-\frac{\delta}{i\delta\eta}\right]+j_\alpha(x)\right]W(j;\eta,\bar{\eta})\ . \tag{3.41}$$

We can evaluate (3.41) for the Lagrangian (3.1) by inserting it in the definition of I, Eq. (3.6), to obtain

$$\begin{aligned}0=\Bigg[&j_\alpha(x)-\tfrac{1}{2}\sum_\beta(B^{\mu\beta}_{\alpha\nu}\partial^\nu\partial_\mu+B^{\mu\alpha}_{\beta\nu}\partial^\nu\partial_\mu\\&+m^2_{\alpha\beta}+m^2_{\beta\alpha})\frac{\delta}{i\delta j_\beta(x)}\\&-\sum_{\beta\gamma}\int dy\,dz\,\Gamma^{(3)}_{(S)\alpha\beta\gamma}(x,y,z)\frac{\delta}{i\delta j_\gamma(z)}\frac{\delta}{i\delta j_\beta(y)}\\&-\sum_{bc}\int dy\,dz\,\Gamma^{(3)}_{(0)abc}(x;y,z)\frac{\delta}{-i\delta\eta_c(z)}\frac{\delta}{i\delta\bar{\eta}_b(y)}\Bigg]e^{G_c}\end{aligned} \tag{3.42}$$

where the symmetrized three-boson bare vertex is defined by

$$\begin{aligned}\Gamma^{(3)}_{(S)\alpha\beta\gamma}(x,y,z)=\Gamma^{(3)}_{(0)\alpha\beta\gamma}(x,y,z)+\Gamma^{(3)}_{(0)\beta\gamma\alpha}(y,z,x)\\+\Gamma^{(3)}_{(0)\gamma\alpha\beta}(z,x,y)\ .\end{aligned} \tag{3.43}$$

Carrying out the differentiations, we have

$$\begin{aligned}0=j_\alpha(x)+&\sum_{\alpha_1}\int dx_1\Gamma^{(2)}_{(0)\alpha\alpha_1}(x,x_1)\frac{\delta G_c}{i\delta j_{\alpha_1}(x_1)}-\sum_{\beta\gamma}\int dy\,dz\,\Gamma^{(3)}_{(S)\alpha\beta\gamma}(x,y,z)\left[\frac{\delta^2G_c}{i\delta j_\gamma(z)i\delta j_\beta(y)}+\frac{\delta G_c}{i\delta j_\gamma(z)}\frac{\delta G_c}{i\delta j_\beta(y)}\right]\\&-\sum_{bc}\int dy\,dz\,\Gamma^{(3)}_{(0)abc}(x;y,z)\left[\frac{\delta^2G_c}{-i\delta\eta_c(z)i\delta\bar{\eta}_b(y)}+\frac{\delta G_c}{-i\delta\eta_c(z)}\frac{\delta G_c}{i\delta\bar{\eta}_b(y)}\right]\ ,\end{aligned} \tag{3.44}$$

where we have introduced the free-boson wave operator

$$\Gamma^{(2)}_{(0)\alpha\alpha_1}(x,x_1)\equiv-\tfrac{1}{2}(B^{\mu\alpha_1}_{\alpha\nu}\partial^\mu\partial_\nu+B^{\mu\alpha}_{\alpha_1\nu}\partial^\nu\partial_\mu+m^2_{\alpha\alpha_1}+m^2_{\alpha_1\alpha})\delta(x-x_1)\ . \tag{3.45}$$

Taking the source currents to zero, we find the equation of motion for the expectation value of the ϕ_α field (i.e., the one-point Green's function for the ϕ_α field). Using Eqs. (3.7)–(3.11) and recalling that the fermion fields and sources anticommute, we obtain

$$\begin{aligned}\sum_{\alpha_1}\int dx_1\Gamma^{(2)}_{(0)\alpha\alpha_1}(x,x_1)\langle\phi_{\alpha_1}(x_1)\rangle=&\sum_{\beta\gamma}\int dy\,dz\,\Gamma^{(3)}_{(S)\alpha\beta\gamma}(x,y,z)[G^{(2)}_{\beta\gamma}(y,z)+\langle\phi_\beta(y)\rangle\langle\phi_\gamma(z)\rangle]\\&-\sum_{bc}\int dy\,dz\,\Gamma^{(3)}_{(0)abc}(x;y,z)G^{(2)}_{cb}(z,y)\ .\end{aligned} \tag{3.46}$$

Notice that Eq. (3.46) requires the Green's functions at equal times and coordinates, because of the locality of the bare vertices. Since the Green's function is the expectation value of the time-ordered product of fields, Eqs. (3.10)–(3.11), there is an apparent ambiguity in the diagonal part of $G^{(2)}_{cb}$. We can resolve the ambiguity by noticing that the term containing $G^{(2)}_{cb}$ arises from the term in the Lagrangian proportional to the density of fermions: the source of the ϕ_α field is the fermion density n. Similarly, Eq. (3.46) states that the source of the mean boson field is the average fermion density. This observation determines the time ordering to be used: for particles, we need $\bar{\psi}\psi$, for antiparticles or holes we need $\psi\bar{\psi}$; i.e., we need the density matrix $n_{cb}(z,y)$ instead of $G^{(2)}_{cb}(z,y)$. Thus Eq. (3.46) has to be supplemented with this information:

$$G^{(2)}_{cb}(z,y)\rightarrow-n_{bc}(y,z),\quad\text{where } n_{bc}(y,z)\equiv-G^+_{bc}(y,z)+G^-_{cb}(z,y) \tag{3.46a}$$

and $G^\pm_{cb}(z,y)$ are the propagators for particles and antiparticles defined in Sec. IV A. By multiplying with the free-particle Green's function $G^{(2)}_{(0)}$, integrating, and using (3.24), Eq. (3.46) may be cast in the form of an integral equation

$$\langle\phi_\alpha(x)\rangle=i\sum_{\alpha_1}\int dx_1dy_1dz_1G^{(2)}_{(0)\alpha\alpha_1}(x,x_1)\left\{-\sum_{b_1c_1}\Gamma^{(3)}_{(0)\alpha_1\beta_1c_1}(x_1;y_1,z_1)n_{b_1c_1}(y_1,z_1)\right.$$
$$\left.+\sum_{\beta_1\gamma_1}\Gamma^{(3)}_{(S)\alpha_1\beta_1\gamma_1}(x_1,y_1,z_1)[G^{(2)}_{\beta_1\gamma_1}(y_1,z_1)+\langle\phi_{\beta_1}(y_1)\rangle\langle\phi_{\gamma_1}(z_1)\rangle]\right\}. \tag{3.47}$$

which is the familiar Hartree equation for the mean field illustrated in Fig. 1. The equations defining the free-particle Green's functions $G^{(2)}_{(0)}$ are given explicitly in Appendix A.

We now derive Dyson equations for the boson two-point Green's functions. Differentiating (3.44) with respect to ij_{α_2}, we find

$$i\delta_{\alpha_2\alpha}\delta(x_2-x)=\sum_{\alpha_1}\int dx_1\Gamma^{(2)}_{(0)\alpha\alpha_1}(x,x_1)\frac{\delta G_c}{i\delta j_{\alpha_2}(x_2)i\delta j_{\alpha_1}(x_1)}$$
$$-\sum_{\beta\gamma}\int dy\,dz\,\Gamma^{(3)}_{(S)\alpha\beta\gamma}(x,y,z)\left[\frac{\delta^3G_c}{i\delta j_{\alpha_2}(x_2)i\delta j_\gamma(z)i\delta j_\beta(y)}+\frac{\delta^2G_c}{i\delta j_{\alpha_2}(x_2)i\delta j_\gamma(z)}\frac{\delta G_c}{i\delta j_\beta(y)}\right.$$
$$\left.+\frac{\delta G_c}{i\delta j_\gamma(z)}\frac{\delta^2G_c}{i\delta j_{\alpha_2}(x_2)i\delta j_\beta(y)}\right]$$
$$-\sum_{bc}\int dy\,dz\,\Gamma^{(3)}_{(0)\alpha bc}(x;y,z)\left[\frac{\delta^3G_c}{i\delta j_{\alpha_2}(x_2)-i\delta\eta_c(z)i\delta\bar\eta_b(y)}+\frac{\delta^2G_c}{i\delta j_{\alpha_2}(x_2)-i\delta\eta_c(z)}\frac{\delta G_c}{i\delta\bar\eta_b(y)}\right.$$
$$\left.+\frac{\delta G_c}{-i\delta\eta_c(z)}\frac{\delta^2G_c}{i\delta j_{\alpha_2}(x_2)i\delta\bar\eta_b(y)}\right]. \tag{3.48}$$

Taking the limit $\eta,\bar\eta,j\to0$ and using the definitions (3.7)–(3.13) of the one-, two-, and three-point Green's functions, (3.48) becomes

$$i\delta_{\alpha_2\alpha}\delta(x_2-x)=\sum_{\alpha_1}\int dx_1\Gamma^{(2)}_{(0)\alpha\alpha_1}(x,x_1)G^{(2)}_{\alpha_1\alpha_2}(x_1,x_2)$$
$$-\sum_{\beta\gamma}\int dy\,dz\Gamma^{(3)}_{(S)\alpha\beta\gamma}(x,y,z)[G^{(3)}_{\beta\gamma\alpha_2}(y,z,x_2)+G^{(2)}_{\gamma\alpha_2}(z,x_2)\langle\phi_\beta(y)\rangle+G^{(2)}_{\beta\alpha_2}(y,x_2)\langle\phi_\gamma(z)\rangle]$$
$$+\sum_{bc}\int dy\,dz\,\Gamma^{(3)}_{(0)\alpha bc}(x;y,z)G^{(3)}_{\alpha_2cb}(x_2;z,y)\,. \tag{3.49}$$

Using relations (3.32) and (3.36) between three-point Green's functions and three-point vertices we find

$$i\delta_{\alpha\alpha_2}\delta(x_2-x)=\sum_{\alpha_1}\int dx_1\Gamma^{(2)}_{(0)\alpha\alpha_1}(x,x_1)G^{(2)}_{\alpha_1\alpha_2}(x_1,x_2)$$
$$-\sum_{\beta_1\gamma_1}\int dy_1dz_1\Gamma^{(3)}_{(S)\alpha\beta_1\gamma_1}(x,y_1,z_1)$$
$$\times\left[\langle\phi_{\beta_1}(y_1)\rangle G^{(2)}_{\gamma_1\alpha_2}(z_1,x_2)+\langle\phi_{\gamma_1}(z_1)\rangle G^{(2)}_{\beta_1\alpha_2}(y_1,x_2)\right.$$
$$\left.-i\sum_{\alpha_3\beta_3\gamma_3}\int dx_3dy_3dz_3G^{(2)}_{\beta_1\beta_3}(y_1,y_3)G^{(2)}_{\gamma_3\gamma_1}(z_3,z_1)\Gamma^{(3)}_{\beta_3\gamma_3\alpha_3}(y_3,z_3,x_3)G^{(2)}_{\alpha_3\alpha_2}(x_3,x_2)\right]$$
$$-\sum_{b_1c_1\alpha_3b_3c_3}\int dy_1dz_1dx_3dy_3dz_3\Gamma^{(3)}_{(0)\alpha b_1c_1}(x;y_1,z_1)$$
$$\times G^{(2)}_{c_1c_3}(z_1,z_3)G^{(2)}_{b_3b_1}(y_3,y_1)\Gamma^{(3)}_{\alpha_3c_3b_3}(x_3;z_3,y_3)G^{(2)}_{\alpha_3\alpha_2}(x_3,x_2)\,. \tag{3.50}$$

Equation (3.50) may be rewritten

$$i\delta_{\alpha_2\alpha_1}\delta(x_2-x_1)=\sum_{\alpha_3}\int dx_3[\Gamma^{(2)}_{(0)\alpha_1\alpha_3}(x_1,x_3)-\Pi_{\alpha_1\alpha_3}(x_1,x_3)]G^{(2)}_{\alpha_3\alpha_2}(x_3,x_2) \tag{3.51}$$

in terms of the polarization function

$$\Pi_{\alpha_1\alpha_3}(x_1,x_3)=\sum_{\beta_1}\int dy_1[\Gamma^{(3)}_{(S)\alpha_1\beta_1\alpha_3}(x_1,y_1,x_3)+\Gamma^{(3)}_{(S)\alpha_1\alpha_3\beta_1}(x_1,x_3,y_1)]\langle\phi_{\beta_1}(y_1)\rangle$$
$$-i\sum_{\beta_1\gamma_1\beta_3\gamma_3}\int dy_1dz_1dy_3dz_3\Gamma^{(3)}_{(S)\alpha_1\beta_1\gamma_1}(x_1,y_1,z_1)G^{(2)}_{\beta_3\beta_1}(y_3,y_1)G^{(2)}_{\gamma_1\gamma_3}(z_1,z_3)\Gamma^{(3)}_{\alpha_3\gamma_3\beta_3}(x_3,z_3,y_3)$$
$$+i\sum_{b_1c_1b_3c_3}\int dy_1dz_1dy_3dz_3\Gamma^{(3)}_{(0)\alpha_1b_1c_1}(x_1;y_1,z_1)G^{(2)}_{b_3b_1}(y_3,y_1)G^{(2)}_{c_1c_3}(z_1,z_3)\Gamma^{(3)}_{\alpha_3c_3b_3}(x_3;z_3;y_3)\ . \tag{3.52}$$

In transcribing Eq. (3.52) from Eq. (3.50) we have made use of the permutation symmetry of the boson Green's functions and vertices in order to emphasize their parallelism with the fermion terms. Equation (3.51) is often written in terms of the bare two-point Green's function $G^{(2)}_{(0)}$ in the form

$$G^{(2)}_{\alpha_2\alpha_1}(x_2,x_1)=G^{(2)}_{(0)\alpha_2\alpha_1}(x_2,x_1)-\sum_{\beta_1\gamma_1}i\int dx_3dx_4G^{(2)}_{(0)\alpha_2\alpha_4}(x_2,x_4)\Pi_{\alpha_4\alpha_3}(x_4,x_3)G^{(2)}_{\alpha_3\alpha_1}(x_3,x_1)\ , \tag{3.53}$$

which is obtained from Eq. (3.51) by multiplying through by $G^{(2)}_{(0)}$, summing, and integrating. Equation (3.53) is the Dyson equation for the boson two-point Green's function. Its graphical representation is shown in Fig. 2(a).

To derive the Dyson equation for the fermion two-point Green's function, we start from the equation of motion for the fermion field. Varying W with respect to $\bar{\psi}_a(x)$, we find analogous to Eq. (3.41)

$$0=\left[\frac{\delta I}{\delta\bar{\psi}_a(x)}\left[\frac{\delta}{i\delta j};\frac{\delta}{i\delta\bar{\eta}},-\frac{\delta}{i\delta\eta}\right]+\eta_a(x)\right]W(j;\eta,\bar{\eta})\ . \tag{3.54}$$

Inserting the definitions of I and W, Eqs. (3.2)–(3.6) together with the Lagrangian (3.1) we obtain analogous to Eq. (3.44)

$$0=\eta_a(x)+(i\gamma^\mu\partial_\mu-M_a)\frac{\delta G_c}{i\delta\bar{\eta}_a(x)}-\sum_{\beta c}\int dy\,dz\,\Gamma^{(3)}_{(0)\beta ca}(y;z,x)\left[\frac{\delta^2G_c}{i\delta\bar{\eta}_c(z)i\delta j_\beta(y)}+\frac{\delta G_c}{i\delta\bar{\eta}_c(z)}\frac{\delta G_c}{i\delta j_\beta(y)}\right]\ . \tag{3.55}$$

In the absence of sources the fermion fields vanish, so Eq. (3.55) is trivially satisfied when $j\eta\bar{\eta}\to 0$.

To find the equations of motion for the two-point Green's functions we vary (3.55) with respect to $-i\eta$, then set the sources to zero. We obtain for $j\eta\bar{\eta}\to 0$

$$0=i\delta_{aa_1}\delta(x-x_1)-(i\gamma^\mu\partial_\mu-M_a)G^{(2)}_{a_1a}(x_1,x)+\sum_{\beta c}\int dy\,dz\,\Gamma^{(3)}_{(0)\beta ca}(y;z,x)[G^{(3)}_{\beta a_1c}(y;x_1,z)+G^{(2)}_{a_1c}(x_1,z)\langle\phi_\beta(y)\rangle]\ . \tag{3.56}$$

Introducing the three-point dressed couplings $\Gamma^{(3)}$ via Eq. (3.36) we have

$$i\delta_{aa_1}\delta(x-x_1)=(i\gamma^\mu\partial_\mu-M_a)G^{(2)}_{a_1a}(x_1,x)$$
$$-\sum_{\beta_1c_1}\int dy_1dz_1\Gamma^{(3)}_{(0)\beta_1c_1a}(y_1;z_1,x)$$
$$\times\left[G^{(2)}_{a_1c_1}(x_1,z_1)\langle\phi_{\beta_1}(y_1)\rangle\right.$$
$$\left.-i\sum_{a_3\beta_3c_3}\int dx_3dy_3dz_3G^{(2)}_{c_3c_1}(z_3,z_1)G^{(2)}_{\beta_3\beta_1}(y_3,y_1)\Gamma^{(3)}_{\beta_3a_3c_3}(y_3;x_3,z_3)G^{(2)}_{a_1a_3}(x_1,x_3)\right]\ . \tag{3.57}$$

Equation (3.57) may be rewritten

$$-i\delta_{a_2a_1}\delta(x_2-x_1)=\sum_{a_3}\int dx_3G^{(2)}_{a_2a_3}(x_2,x_3)[\Gamma^{(2)}_{(0)a_3a_1}(x_3,x_1)+\Sigma_{a_3a_1}(x_3,x_1)] \tag{3.58}$$

in terms of the fermions' self-energy

$$\Sigma_{a_3a_1}(x_3,x_1)=\sum_{\beta_1}\int dy_1\Gamma^{(3)}_{(0)\beta_1a_3a_1}(y_1;x_3,x_1)\langle\phi_{\beta_1}(y_1)\rangle$$
$$-i\sum_{\beta_1c_1\beta_3c_3}\int dy_1dz_1dy_3dz_3\Gamma^{(3)}_{\beta_3a_3c_3}(y_3;x_3,z_3)G^{(2)}_{\beta_3\beta_1}(y_3,y_1)G^{(2)}_{c_3c_1}(z_3,z_1)\Gamma^{(3)}_{(0)\beta_1c_1a_1}(y_1;z_1,x_1)\ , \tag{3.59}$$

where the free-fermion wave operator is

$$\Gamma^{(2)}_{(0)a_3a_1}(x_3,x_1)\equiv\delta_{a_3a_1}(i\gamma^\mu\partial_{3\mu}+M_{a_3})\delta(x_3-x_1)\ . \tag{3.60}$$

We can invert Eq. (3.58) by multiplying on the left by $G^{(2)}_{(0)}\Gamma^{(2)}$ and on the right by $G^{(2)}$, then integrating and using (3.26) to obtain

$$G^{(2)}_{a_2a_1}(x_2,x_1)=G^{(2)}_{(0)a_2a_1}(x_2,x_1)-i\int dx_3dx_4G^{(2)}_{(0)a_2a_4}(x_2,x_4)\Sigma_{a_3a_4'}(x_3,x_4)G^{(2)}_{a_3a_1}(x_3,x_1)\ . \tag{3.61}$$

The graphical representation of Eq. (3.61) is shown in Fig. 2(b). We note that there is no ambiguity of time-ordering analogous to the one arising in the Hartree equation (3.47), since no Green's functions need to be evaluated at equal times; this is because contact terms in the equations for two-point and higher Green's functions would be factorizable, and thus are eliminated in terms of lower-order functions.

C. Dyson equations for the dressed vertices

We have now found equations for the two-point Green's functions which describe the mixing of the fundamental fermion and boson degrees of freedom with two-particle composite states appearing inside the self-energy insertion Σ or polarizability Π for fermions and bosons, respectively. The strength of this mixing is given in terms of the three-point vertices $\Gamma^{(3)}$. These vertices are themselves dynamical quantities satisfying Dyson equations.

The Dyson equation for the three-boson vertex is obtained by differentiating Eq. (3.48) with respect to the boson source current ij_{α_3}. Setting the sources equal to zero, we have

$$\begin{aligned}0=&\sum_{\alpha_1}\int dx_1\Gamma^{(2)}_{(0)\alpha\alpha_1}(x,x_1)G^{(3)}_{\alpha_1\alpha_2\alpha_3}(x_1,x_2,x_3)\\&-\sum_{\beta\gamma}\int dy\,dz\,\Gamma^{(3)}_{(S)\alpha\beta\gamma}(x,y,z)[G^{(4)}_{\beta\gamma\alpha_2\alpha_3}(y,z,x_2,x_3)+G^{(3)}_{\gamma\alpha_2\alpha_3}(z,x_2,x_3)\langle\phi_\beta(y)\rangle\\&+G^{(3)}_{\beta\alpha_2\alpha_3}(y,x_2,x_3)\langle\phi_\gamma(z)\rangle+G^{(2)}_{\gamma\alpha_2}(z,x_2)G^{(2)}_{\beta\alpha_3}(y,x_3)+G^{(2)}_{\beta\alpha_2}(y,x_2)G^{(2)}_{\gamma\alpha_3}(z,x_3)]\\&+\sum_{bc}\int dy\,dz\,\Gamma^{(3)}_{(0)\alpha bc}(x;y,z)G^{(4)}_{\alpha_2\alpha_3cb}(x_2,x_3;z,y)\ .\end{aligned}\tag{3.62}$$

We can eliminate $G^{(3)}$ and $G^{(4)}$ in favor of $\Gamma^{(3)}$ and T via Eqs. (3.32) and (3.37)–(3.38):

$$\begin{aligned}0=\sum_{\beta\gamma}\int dy\,dz\Bigg\{&i\sum_{\alpha_1\alpha_4}\int dx_1dx_4\Gamma^{(2)}_{(0)\alpha\alpha_1}(x,x_1)G^{(2)}_{\alpha_1\alpha_4}(x_1,x_4)\Gamma^{(3)}_{\alpha_4\beta\gamma}(x_4,y,z)\\&-i\sum_{\beta_1\gamma_1\gamma_2}\int dy_1dz_1dz_2\Gamma^{(3)}_{(S)\alpha\beta_1\gamma_1}(x,y_1,z_1)[\langle\phi_{\beta_1}(y_1)\rangle G^{(2)}_{\gamma_1\gamma_2}(z_1,z_2)\Gamma^{(3)}_{\gamma_2\beta\gamma}(z_2,y,z)\\&+\langle\phi_{\gamma_1}(z_1)\rangle G^{(2)}_{\beta_1\gamma_2}(y_1,z_2)\Gamma^{(3)}_{\gamma_2\beta\gamma}(z_2,y,z)]+\Gamma^{(3)}_{(S)\alpha\beta\gamma}(x,y,z)+\Gamma^{(3)}_{(S)\alpha\gamma\beta}(x,z,y)\\&-i\sum_{\beta_1\beta_2\gamma_1\gamma_2}\int dy_1dy_2dz_1dz_2\Gamma^{(3)}_{(S)\alpha\beta_1\gamma_1}(x,y_1,z_1)G^{(2)}_{\beta_2\beta_1}(y_2,y_1)G^{(2)}_{\gamma_1\gamma_2}(z_1,z_2)T_{\beta_2\gamma_2\beta\gamma}(y_2,z_2,y,z)\\&+i\sum_{b_1b_2c_1c_2}\int dy_1dy_2dz_1dz_2\Gamma^{(3)}_{(0)\alpha b_1c_1}(x;y_1,z_1)\\&\times G^{(2)}_{b_2b_1}(y_2,y_1)G^{(2)}_{c_1c_2}(z_1,z_2)T_{\beta\gamma c_2b_2}(y_2,z_2;y,z)\Bigg\}G^{(2)}_{\beta\alpha_2}(y,x_2)G^{(2)}_{\gamma\alpha_3}(z,x_3)\ .\end{aligned}\tag{3.63}$$

We can use Eq. (3.50) to evaluate the first term; then, multiplying from the right by $\Gamma^{(2)}$ twice and integrating, we find

$$\begin{aligned}\Gamma^{(3)}_{\alpha\beta\gamma}(x,y,z)=&\Gamma^{(3)}_{(S)\alpha\beta\gamma}(x,y,z)+\Gamma^{(3)}_{(S)\alpha\gamma\beta}(x,z,y)\\&+\sum_{\alpha_3\alpha_4}\int dx_3dx_4dy_1dy_3dz_1dz_3\\&\times\Bigg[\sum_{\beta_1\beta_3\gamma_1\gamma_3}\Gamma^{(3)}_{(S)\alpha\beta_1\gamma_1}(x,y_1,z_1)G^{(2)}_{\beta_1\beta_3}(y_1,y_3)G^{(2)}_{\gamma_3\gamma_1}(z_3,z_1)\Gamma^{(3)}_{\beta_3\gamma_3\alpha_3}(y_3,z_3,x_3)\\&-\sum_{b_1b_3c_1c_3}\Gamma^{(3)}_{(0)\alpha b_1c_1}(x;y_1,z_1)G^{(2)}_{c_1c_3}(z_1,z_3)G^{(2)}_{b_3b_1}(y_3,y_1)\Gamma^{(3)}_{\alpha_3c_3b_3}(x_3;z_3,y_3)\Bigg]G^{(2)}_{\alpha_3\alpha_4}(x_3,x_4)\Gamma^{(3)}_{\alpha_4\beta\gamma}(x_4,y,z)\\&-i\sum_{\beta_1\beta_2\gamma_1\gamma_2}\int dy_1dy_2dz_1dz_2\Gamma^{(3)}_{(S)\alpha\beta_1\gamma_1}(x,y_1,z_1)G^{(2)}_{\beta_2\beta_1}(y_2,y_1)G^{(2)}_{\gamma_1\gamma_2}(z_1,z_2)T_{\beta_2\gamma_2\beta\gamma}(y_2,z_2,y,z)\\&+i\sum_{b_1b_2c_1c_2}\int dy_1dy_2dz_1dz_2\Gamma^{(3)}_{(0)\alpha b_1c_1}(x;y_1,z_1)G^{(2)}_{b_2b_1}(y_2,y_1)G^{(2)}_{c_1c_2}(z_1,z_2)T_{\beta\gamma c_2b_2}(y,z;y_2,z_2)\ .\end{aligned}\tag{3.64}$$

Equation (3.64) is represented in Fig. 3(a).

To derive Dyson equations for the three-point vertex $\Gamma^{(3)}_{\alpha bc}$ we start from Eq. (3.44) and differentiate it first with respect to $-i\eta_b$ and then with respect to $i\bar{\eta}_c$. Taking the sources to zero, we have

$$\begin{aligned}0=&\sum_{\alpha_1}\int dx_1\Gamma^{(2)}_{(0)\alpha\alpha_1}(x,x_1)G^{(3)}_{\alpha_1 bc}(x_1;y,z)\\&-\sum_{\beta_1\gamma_1}\int dy_1dz_1\Gamma^{(3)}_{(S)\alpha\beta_1\gamma_1}(x,y_1,z_1)[G^{(4)}_{\beta_1\gamma_1 bc}(y_1,z_1;y,z)+G^{(3)}_{\gamma_1 bc}(z_1;y,z)\langle\phi_{\beta_1}(y_1)\rangle+G^{(3)}_{\beta_1 bc}(y_1;y,z)\langle\phi_{\gamma_1}(z_1)\rangle]\\&+\sum_{b_1c_1}\int dy_1dz_1\Gamma^{(3)}_{(0)\alpha b_1c_1}(x;y_1,z_1)[G^{(4)}_{b_1c_1bc}(y_1,z_1,y,z)-G^{(2)}_{bb_1}(y,y_1)G^{(2)}_{c_1c}(z_1,z)]\;.\end{aligned}\tag{3.65}$$

We can eliminate $G^{(3)}$ and $G^{(4)}$ in favor of $\Gamma^{(3)}$ and T via Eqs. (3.36), (3.38), and (3.39):

$$\begin{aligned}0=\sum_{bc}\int dy\,dz\Bigg\{&i\sum_{\alpha_1\alpha_4}\int dx_1dx_4\Gamma^{(2)}_{(0)\alpha\alpha_1}(x,x_1)G^{(2)}_{\alpha_1\alpha_4}(x_1,x_4)\Gamma^{(3)}_{\alpha_4bc}(x_4;y,z)\\&-i\sum_{\beta_1\gamma_1\gamma_2}\int dy_1dz_1dz_2\Gamma^{(3)}_{(S)\alpha\beta_1\gamma_1}(x,y_1,z_1)[\langle\phi_{\beta_1}(y_1)\rangle G^{(2)}_{\gamma_1\gamma_2}(z_1,z_2)\Gamma^{(3)}_{\gamma_2bc}(z_2;y,z)\\&+\langle\phi_{\gamma_1}(z_1)\rangle G^{(2)}_{\beta_1\gamma_2}(y_1,z_2)\Gamma^{(3)}_{\gamma_2bc}(z_2;y,z)]\\&+\Gamma^{(3)}_{(0)\alpha bc}(x;y,z)-i\sum_{\beta_1\beta_2\gamma_1\gamma_2}\int dy_1dy_2dz_1dz_2\Gamma^{(3)}_{(S)\alpha\beta_1\gamma_1}(x,y_1,z_1)G^{(2)}_{\beta_2\beta_1}(y_2,y_1)G^{(2)}_{\gamma_1\gamma_2}(z_1,z_2)T_{\beta_2\gamma_2bc}(y_2,z_2;y,z)\\&+i\sum_{b_1b_2c_1c_2}\int dy_1dy_2dz_1dz_2\Gamma^{(3)}_{(0)\alpha b_1c_1}(x;y_1,z_1)G^{(2)}_{b_2b_1}(y_2,y_1)G^{(2)}_{c_1c_2}(z_1,z_2)T_{c_2b_2bc}(y_2,z_2,y,z)\Bigg\}\\&\times G^{(2)}_{b_1b}(y_1,y)G^{(2)}_{cc_1}(z,z_1)\;.\end{aligned}\tag{3.66}$$

We can use Eq. (3.50) to evaluate the first term; then, multiplying from the right by $\Gamma^{(2)}$ twice and integrating, we find

$$\begin{aligned}\Gamma^{(3)}_{\alpha bc}(x;y,z)=&\Gamma^{(3)}_{(0)\alpha bc}(x;y,z)\\&+\sum_{\alpha_3\alpha_4}\int dx_3dx_4dy_1dy_3dz_1dz_3\\&\times\Bigg[\sum_{\beta_1\beta_3\gamma_1\gamma_3}\Gamma^{(3)}_{(S)\alpha\beta_1\gamma_1}(x,y_1,z_1)G^{(2)}_{\beta_1\beta_3}(y_1,y_3)G^{(2)}_{\gamma_3\gamma_1}(z_3,z_1)\Gamma^{(3)}_{\beta_3\gamma_3\alpha_3}(y_3,z_3,x_3)\\&-\sum_{b_1b_3c_1c_3}\Gamma^{(3)}_{(0)\alpha b_1c_1}(x;y_1,z_1)G^{(2)}_{c_1c_3}(z_1,z_3)G^{(2)}_{b_3b_1}(y_3,y_1)\Gamma^{(3)}_{\alpha_3c_3b_3}(x_3;z_3,y_3)\Bigg]\\&\times G^{(2)}_{\alpha_3\alpha_4}(x_3,x_4)\Gamma^{(3)}_{\alpha_4bc}(x_4;y,z)\\&-i\sum_{\beta_1\beta_2\gamma_1\gamma_2}\int dy_1dy_2dz_1dz_2\Gamma^{(3)}_{(S)\alpha\beta_1\gamma_1}(x,y_1,z_1)G^{(2)}_{\beta_2\beta_1}(y_2,y_1)G^{(2)}_{\gamma_1\gamma_2}(z_1,z_2)T_{\beta_2\gamma_2bc}(y_2,z_2;y,z)\\&+i\sum_{b_1b_2c_1c_2}\int dy_1dy_2dz_1dz_2\Gamma^{(3)}_{(0)\alpha b_1c_1}(x;y_1,z_1)G^{(2)}_{b_2b_1}(y_2,y_1)G^{(2)}_{c_1c_2}(z_1,z_2)T_{c_2b_2bc}(y_2,z_2,y,z)\;.\end{aligned}\tag{3.67}$$

The graphical representation of Eq. (3.67) is shown in Fig. 3(b).

D. The irreducible effective interaction

Like the Dyson equations for the two-point Green's functions, the Dyson equations for the three-point vertices $\Gamma^{(3)}$ take account of the dynamic role of two-body intermediate states in the interactions between two of the three fields involved at each vertex. When these three-point vertices are inserted in the Dyson equations for the two-point Green's functions, their two-body intermediate states give an additional admixture of two-body components into the quasiparticle wave functions beyond the admixtures displayed explicitly in the Dyson equations for the two-point Green's functions. For a successful description of the many-body effects on quasiparticle propagation, it is clearly essential to include all the sources of two-body admixtures: both the two-point Green's functions and the three-point vertices must be dressed in a consistent way. Furthermore, the effective interactions T appearing in the three-point Dyson equations produce a similar admixture of the same set of two-body states; we must also include these intermediate states in our description of T.

The equations we have obtained, namely, Eqs. (3.64) and (3.67), are not the only such equations satisfied by the vertex functions: they represent a choice of which pair of fields are allowed to interact with each other via the T matrix. Had we so chosen, we might just as well have obtained similar equations exhibiting the interactions of any given pair of fields. Of course, all these equations would be equally true, describing the same vertex in different ways. The fact that they must be equivalent illustrates how complex an object the T matrix is, since it must bear the burden of ensuring the equality of the different forms of the vertex equations.

Nevertheless, despite this complexity, we are compelled to seek an appropriate approximation for the T matrix. Two reasons motivate this need: (a) two-body scattering is a convenient place to introduce a phenomenological parametrization, since it is among the fundamental processes whose strength we wish to adjust to fit the measured data, and (b) a complete numerical treatment of a function of four space-time variables (16 scalar quantities) will probably remain beyond the capacity of even the largest computers for many years to come.

To motivate our choice of approximations for the two-body scattering matrices T—and by the extension the three-point vertices $\Gamma^{(3)}$—we appeal to an argument similar to the one we used to motivate our choice of the effective Lagrangian: we will treat explicitly those degrees of freedom whose modest energy permits them to be strongly excited in intermediate-energy nuclear collisions, while tolerating an abbreviated description of degrees of freedom with larger excitation energies. The lowest-energy degrees of freedom of a many-fermion system are the particle-hole excitations and the collective oscillations constructed by superposing them coherently. Note the contrast with the baryon-free vacuum, where nucleon-antinucleon states play a much less dynamic role at low energies. In vacuum, the pion is the lowest-energy hadronic excitation, followed by two pions, which are so strongly correlated as to require the introduction of a scalar field σ to describe their interactions; in baryonic matter, the excitation $\Delta\bar{N}$ of a nucleon to a delta must be included with the pionic degrees of freedom. All other strong-interaction degrees of freedom require more than 0.5 GeV to excite and therefore may reasonably be subjected to a simplified description in terms of few parameters: they are not expected to be strongly excited in collisions where the available energy per nucleon is less than 0.3 GeV in the center of mass (laboratory kinetic energy ≈ 1.4 GeV). We conclude that our first priority must be the explicit inclusion of $N\bar{N}$, $\Delta\bar{N}$, π, σ, and 2π degrees of freedom and their mixing with each other.

We can now see how to choose the best set of Dyson equations for the three-point vertices: they should be chosen to explicitly exhibit the lowest-energy two-body intermediate states. We see that our choices in Eqs. (3.64) and (3.67) do indeed fulfill this criterion, when we recall that $N\bar{N}$ includes particle-hole excitations in the nucleonic medium.

We now apply these considerations to the problem of approximating the T matrix for two-body scattering. Since the same T matrix describes, for example, $N\bar{N}$ and NN scattering, or πN scattering and $N\bar{N}$ annihilation to $\pi\pi$, different sets of intermediate states will appear depending on which process is being described. In principle, we would like to eliminate all the low-energy states in all channels; in practice this leads to equations beyond the reach of numerical computations. Faced with a choice of which channel to favor, we choose to extract the intermediate states in the channels in which T is used in the Dyson equations for the three-point vertices, namely, those channels which include $N\bar{N}$, the lowest-energy states in baryonic matter.

We begin by separating out in the T matrix all processes in which a single meson appears as an intermediate state. We therefore define the one-body irreducible part T' of the scattering matrix as illustrated in Fig. 5(a):

$$T'_{\beta_2\gamma_2\beta\gamma}(y_2,z_2,y,z)\equiv T_{\beta_2\gamma_2\beta\gamma}(y_2,z_2,y;z)+i\sum_{\alpha_1\alpha}\int dx_2dx\,\Gamma^{(3)}_{\beta_2\gamma_2\alpha_2}(y_2,z_2,x_2)G^{(2)}_{\alpha_2\alpha}(x_2,x)\Gamma^{(3)}_{\alpha\beta\gamma}(x,y,z)\ , \tag{3.68}$$

$$T'_{\beta_2\gamma_2bc}(y_2,z_2;y,z)\equiv T_{\beta_2\gamma_2bc}(y_2,z_2;y,z)+i\sum_{\alpha_1\alpha}\int dx_2dx\,\Gamma^{(3)}_{\beta_2\gamma_2\alpha_2}(y_2,z_2,x_2)G^{(2)}_{\alpha_2\alpha}(x_2,x)\Gamma^{(3)}_{\alpha bc}(x;y,z)\ , \tag{3.69}$$

and

$$T'_{c_2b_2bc}(y_2,z_2,y,z)\equiv T_{c_2b_2bc}(y_2,z_2,y,z)+i\sum_{\alpha_1\alpha}\int dx_2dx\,\Gamma^{(3)}_{\alpha_2c_2b_2}(x_2;z_2,y_2)G^{(2)}_{\alpha_2\alpha}(x_2,x)\Gamma^{(3)}_{\alpha bc}(x;y,z)\ . \tag{3.70}$$

The relation between T and T' is represented graphically in Fig. 5(a). In terms of the one-particle irreducible effective interaction T', Eqs. (3.64) and (3.67) become, respectively,

$$\begin{aligned}\Gamma^{(3)}_{\alpha\beta\gamma}(x,y,z)=&\Gamma^{(3)}_{(S)\alpha\beta\gamma}(x,y,z)+\Gamma^{(3)}_{(S)\alpha\gamma\beta}(x,z,y)\\&-i\sum_{\beta_1\beta_2\gamma_1\gamma_2}\int dy_1dy_2dz_1dz_2\,\Gamma^{(3)}_{(S)\alpha\beta_1\gamma_1}(x,y_1,z_1)G^{(2)}_{\beta_2\beta_1}(y_2,y_1)G^{(2)}_{\gamma_1\gamma_2}(z_1,z_2)T'_{\beta_2\gamma_2\beta\gamma}(y_2,z_2,y,z)\\&+i\sum_{b_1b_2c_1c_2}\int dy_1dy_2dz_1dz_2\,\Gamma^{(3)}_{(0)\alpha b_1c_1}(x;y_1,z_1)G^{(2)}_{b_2b_1}(y_2,y_1)G^{(2)}_{c_1c_2}(z_1,z_2)T'_{\beta\gamma c_2b_2}(y,z;y_2,z_2)\end{aligned} \tag{3.71}$$

and

$$\Gamma^{(3)}_{abc}(x;y,z)=\Gamma^{(3)}_{(0)abc}(x;y,z)$$
$$-i\sum_{\beta_1\beta_2\gamma_1\gamma_2}\int dy_1dy_2dz_1dz_2\Gamma^{(3)}_{(S)\alpha\beta_1\gamma_1}(x,y_1,z_1)G^{(2)}_{\beta_2\beta_1}(y_2,y_1)G^{(2)}_{\gamma_1\gamma_2}(z_1,z_2)T'_{\beta_2\gamma_2bc}(y_2,z_2;y,z)$$
$$+i\sum_{b_1b_2c_1c_2}\int dy_1dy_2dz_1dz_2\Gamma^{(3)}_{(0)ab_1c_1}(x;y_1,z_1)G^{(2)}_{b_2b_1}(y_2,y_1)G^{(2)}_{c_1c_2}(z_1,z_2)T'_{c_2b_2bc}(y_2,z_2;y,z)\ . \tag{3.72}$$

Because it no longer contains the single-particle intermediate states, T' is a better candidate for simplified parametrization than T was. But T' still contains the low-energy two-body states which we also want to describe explicitly. In order to eliminate these parts of T', we have to introduce yet another four-point function U, which will be called the residual interaction. This residual interaction is defined by the integral equations

$$T'_{\beta_2\gamma_2\beta\gamma}(y_2,z_2,y,z)=U_{\beta_2\gamma_2\beta\gamma}(y_2,z_2,y,z)$$
$$-i\int dy_3dy_4dz_3dz_4T'_{\beta_2\gamma_2\beta_4\gamma_4}(y_2,z_2,y_4,z_4)G^{(2)}_{\beta_4\beta_3}(y_4,y_3)G^{(2)}_{\gamma_3\gamma_4}(z_3,z_4)U_{\beta_3\gamma_3\beta\gamma}(y_3,z_3,y,z)$$
$$+i\int dy_3dy_4dz_3dz_4T'_{\beta_2\gamma_2b_4c_4}(y_2,z_2;y_4,z_4)G^{(2)}_{b_4b_3}(y_4,y_3)G^{(2)}_{c_3c_4}(z_3,z_4)U_{\beta\gamma c_3b_3}(y,z;y_3,z_3)\ , \tag{3.73}$$

$$T'_{\beta_2\gamma_2bc}(y_2,z_2;y,z)=U_{\beta_2\gamma_2bc}(y_2,z_2;y,z)$$
$$-i\int dy_3dy_4dz_3dz_4T'_{\beta_2\gamma_2\beta_4\gamma_4}(y_2,z_2,y_4,z_4)G^{(2)}_{\beta_4\beta_3}(y_4,y_3)G^{(2)}_{\gamma_3\gamma_4}(z_3,z_4)U_{\beta_3\gamma_3bc}(y_3,z_3;y,z)$$
$$+i\int dy_3dy_4dz_3dz_4T'_{\beta_2\gamma_2b_4c_4}(y_2,z_2;y_4,z_4)G^{(2)}_{b_4b_3}(y_4,y_3)G^{(2)}_{c_3c_4}(z_3,z_4)U_{c_3b_cbc}(y_3,z_3,y,z)\ , \tag{3.74}$$

$$T'_{c_2b_2bc}(y_2,z_2,y,z)=U_{c_2b_2bc}(y_2,z_2,y,z)$$
$$-i\int dy_3dy_4dz_3dz_4T'_{\beta_4\gamma_4b_2c_2}(y_2,z_2;y_4,z_4)G^{(2)}_{\beta_4\beta_3}(y_4,y_3)G^{(2)}_{\gamma_3\gamma_4}(z_3,z_4)U_{\beta_3\gamma_3bc}(y_3,z_3;y,z)$$
$$+i\int dy_3dy_4dz_3dz_4T'_{c_2b_2b_4c_4}(y_2,z_2,y_4,z_4)G^{(2)}_{b_4b_3}(y_4,y_3)G^{(2)}_{c_3c_4}(z_3,z_4)U_{c_3b_3bc}(y_3,z_3,y,z)\ . \tag{3.75}$$

The relation between T' and U is shown graphically in Fig. 5(b). These definitions ensure that the residual interaction U we have introduced is irreducible in the $N\bar{N}$ or, equivalently, the meson channel with respect to one- and two-body intermediate states.

With these definitions, the equations for the three-point vertices take on the form used by Migdal:[8] using these definitions in Eqs. (3.71) and (3.72), we obtain

$$\Gamma^{(3)}_{\alpha\beta\gamma}(x,y,z)-\Gamma^{(3)}_{(S)\alpha\gamma\beta}(x,z,y)=\Gamma^{(3)}_{(S)\alpha\beta\gamma}(x,y,z)$$
$$-i\sum_{\beta_1\beta_2\gamma_1\gamma_2}\int dy_1dy_2dz_1dz_2[\Gamma^{(3)}_{\alpha\beta_1\gamma_1}(x,y_1,z_1)-\Gamma^{(3)}_{(S)\alpha\gamma_1\beta_1}(x,z_1,y_1)]$$
$$\times G^{(2)}_{\beta_2\beta_1}(y_2,y_1)G^{(2)}_{\gamma_1\gamma_2}(z_1,z_2)U_{\beta_2\gamma_2\beta\gamma}(y_2,z_2,y,z)$$
$$+i\sum_{b_1b_2c_1c_2}\int dy_1dy_2dz_1dz_2\Gamma^{(3)}_{ab_1c_1}(x;y_1,z_1)$$
$$\times G^{(2)}_{b_2b_1}(y_2,y_1)G^{(2)}_{c_1c_2}(z_1,z_2)U_{\beta\gamma c_2b_2}(y,z;y_2,z_2) \tag{3.76}$$

and

$$\Gamma^{(3)}_{abc}(x,y,z)=\Gamma^{(3)}_{(0)abc}(x;y,z)$$
$$-i\sum_{\beta_1\beta_2\gamma_1\gamma_2}\int dy_1dy_2dz_1dz_2[\Gamma^{(3)}_{\alpha\beta_1\gamma_1}(x,y_1,z_1)-\Gamma^{(3)}_{(S)\alpha\gamma_1\beta_1}(x,z_1,y_1)]$$
$$\times G^{(2)}_{\beta_2\beta_1}(y_2,y_1)G^{(2)}_{\gamma_1\gamma_2}(z_1,z_2)U_{\beta_2\gamma_2bc}(y_2,z_2;y,z)$$
$$+i\sum_{b_1b_2c_1c_2}\int dy_1dy_2dz_1dz_2\Gamma^{(3)}_{ab_1c_1}(x;y_1,z_1)G^{(2)}_{b_2b_1}(y_2,y_1)G^{(2)}_{c_1c_2}(z_1,z_2)U_{b_2c_2bc}(y_2,z_2,y,z)\ . \tag{3.77}$$

The graphical representations of Eqs. (3.76) and (3.77) are shown in Fig. 6.

We have now introduced the main equations of our theory, which determine the mean field $\langle\phi_\alpha(x)\rangle$ [Eq. (3.47)], the two-point Green's functions $G^{(2)}$ for the nucleons and mesons [Eqs. (3.53) and (3.61)], and the vertices $\Gamma^{(3)}$ describing the coupling of mesons to nucleons and to other mesons [Eqs. (3.76) and (3.77)]. This set of equations determines the evolution of the dynamical quantities in terms of the free-two-point Green's functions $G^{(2)}_{(0)}$ and bare vertices $\Gamma^{(3)}_{(0)}$ of the Lagrangian, together with the residual effective interactions $U(x_1,x_2;x_3,x_4)$. When these quantities are known, the two-body scattering matrices $T(x_1,x_2;x_3,x_4)$ may be found from Eqs. (3.68)–(3.75).

E. The Migdal phenomenological effective interaction

In principle the residual effective interactions are determined by a further set of Dyson equations which could be obtained by additional variation of the generating functionals; these Dyson equations would in turn introduce further unknowns, the five-point functions. Instead, for our purposes, as we have explained in Sec. II, it is more useful to regard the effective residual interactions as quantities to be determined inductively by fitting experimental information. Thus we close the hierarchy of Dyson equations by making the main approximation of our method. Following Migdal[8] and Pines and Aldrich[17] we shall approximate the residual interactions as local functions of their coordinates, proportional to delta functions of the coordinate differences of the particle and antiparticle or to the derivatives of delta functions, where the coefficient functions have to depend on the medium in which the effective interaction is occurring. It seems reasonable to suppose that the properties of the medium will be reasonably well represented by the mean boson fields, which according to the Hartree equation (3.47) are proportional to the fermion densities; thus we set

$$\begin{aligned} U_{\beta_2\gamma_2\beta\gamma}(y_2,z_2,y,z) &= U_{\beta_2\gamma_2\beta\gamma}(y_2-y;\{\langle\phi_\alpha(y)\rangle,\langle\phi_\alpha(y_2)\rangle\})\delta(y_2-z_2)\delta(y-z)\ ,\\ U_{\beta_2\gamma_2 bc}(y_2,z_2;y,z) &= U_{\beta_2\gamma_2 bc}(y_2-y;\{\langle\phi_\alpha(y)\rangle,\langle\phi_\alpha(y_2)\rangle\})\delta(y_2-z_2)\delta(y-z)\ ,\\ U_{c_2b_2bc}(y_2,z_2;y,z) &= U_{c_2b_2bc}(y_2-y;\{\langle\phi_\alpha(y)\rangle,\langle\phi_\alpha(y_2)\rangle\})\delta(y_2-z_2)\delta(y-z)\ , \end{aligned} \tag{3.78}$$

where the delta functions may in some cases be chosen as the derivatives of delta functions to allow for gradient couplings. Alternatively, U could be taken to depend on the mean fields in between its endpoints

$$U(y_2-y;\{\langle\phi_\alpha(y+y_2/2)\rangle\})$$

instead of

$$U(y_2-y;\{\langle\phi_\alpha(y)\rangle,\langle\phi_\alpha(y_2)\rangle\})\ .$$

This form of U is a relativistic generalization of the pseudopotential of Pines and Aldrich;[17] when he originally introduced the phenomenological residual interaction, Migdal also made the further simplification of approximating U as zero range, i.e., proportional to $\delta(y_2-y)$. Clearly we would prefer the simplest form that can adequately represent the effective interaction. There are some physical constraints on the choice of U, which will be discussed in Sec. V.

If we insert the zero-range local residual interactions (3.78) in Eqs. (3.76) and (3.77), we find that the three-point vertices $\Gamma^{(3)}_{abc}$ are local in the fermion coordinates while the vertex $\Gamma^{(3)}_{\alpha\beta\gamma}$ is local in the coordinates of bosons β and γ.

$$\Gamma^{(3)}_{abc}(x_1;x_2,x_3)=\Gamma^{(3)}_{abc}(x_1;x_2,x_3)\delta^4(x_2-x_3)\ , \tag{3.79}$$

$$\Gamma^{(3)}_{\alpha\beta\gamma}(x_1,x_2,x_3)=\Gamma^{(3)}_{\alpha\beta\gamma}(x_1,x_2,x_3)\delta^4(x_2-x_3)\ , \tag{3.80}$$

or similar relations involving the derivatives of delta functions. The three-point vertices are now functions of only two space-time coordinates, just as are the two-point Green's functions. It is this feature which gives hope of a numerical treatment of these coupled integral equations. The structure of these equations is displayed in Fig. 7. Similarly, Eqs. (3.68)–(3.70) and (3.73)–(3.75) imply that also the T matrix itself is only a function of two space-time coordinates, and thus equally amenable to numerical treatment. We will return to the discussion of the physical consequences and plausibility of the Migdal locality approximation in Sec. V. First, however, we have to deal with an inevitable consequence of our use of zero-range interactions in both the Lagrangian and the Migdal residual interactions: the loops which dress the two-point Green's functions and vertices in our theory diverge. We therefore devote the next chapter to the regularization of loops.

IV. REGULARIZATION OF LOOPS

The approximation of the residual interaction by a local effective force reduces the equations of motion to a set of coupled equations for functions of two space-time variables $F_\alpha(x,y)$ in which all the terms can be constructed by successive combinations of two such functions into a single new function of two space-time variables. For the moment we will pretend that the effective interactions and bare vertices are truly local, ignoring the derivative couplings; the extension to derivative couplings is

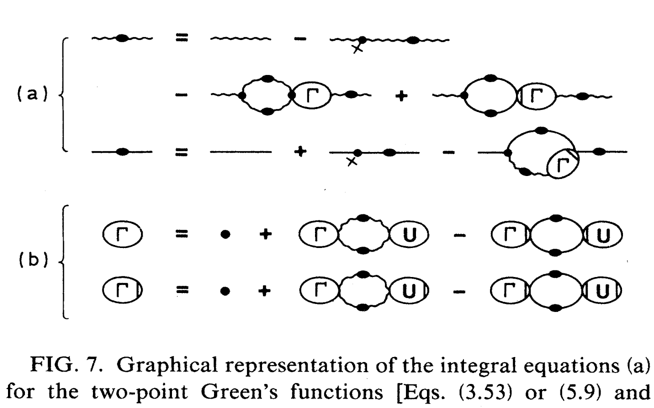

FIG. 7. Graphical representation of the integral equations (a) for the two-point Green's functions [Eqs. (3.53) or (5.9) and (3.61) or (5.11)] and (b) for the three-point vertices [Eqs. (3.76) or (5.13) and (3.77) or (5.14)], in every case assuming the local form (3.78) for the residual interaction U.

straightforward and will be discussed at the end of Sec. IV D. For local couplings, there are two operations of combination: convolution,

$$C(x,y,F_\alpha(x,z),F_\beta(z,y))=\int dz\, F_\alpha(x,z)F_\beta(z,y)\ , \tag{4.1}$$

and loop formation,

$$L(x,y,F_\alpha,F_\beta)=F_\alpha(x,y)F_\beta(y,x)\ . \tag{4.2}$$

While the convolution of two singular functions presents no special difficulties, it is to be expected that the formation of loops leads to ultraviolet divergences associated with the singular short-distance behavior of the effective interactions. The preferred treatment of these divergences would be renormalization, a procedure unavailable in our case because of the presence of spin-$\frac{3}{2}$ particles and their derivative couplings. We therefore must seek another way of regularizing the ultraviolet divergences of the loops. Since the divergences arise from the assumption that the hadrons are point particles, we find a satisfying resolution by introducing their structure in an appropriate way.

A. Causality, unitarity, and dispersion relations for loops

Since the purpose of our equations is to investigate the flow of probability with time, it is essential that any regularization method must preserve, in the resulting equations, the features of unitarity and causality which are, of course, automatically present in the equations derived above. These requirements are quite stringent, as we can see by considering the simplified case of stationary or steady-state motion in which the Green's functions depend only on the difference of times,

$$G^{(2)}_{\alpha\alpha'}(x,x')=G^{(2)}_{\alpha\alpha'}(\mathbf{x},\mathbf{x}',x_0-x_0')\ . \tag{4.3}$$

In the following development we will suppress the spatial variables, assuming only that a real representation has been chosen for them. Thus we introduce the shorthand notation

$$F_\alpha(t)=F_\alpha(\mathbf{x},x_0,\mathbf{x}',x_0-t)\ , \tag{4.4a}$$

$$L_{\alpha\beta}(t)=L(\mathbf{x},x_0,\mathbf{x}',x_0-t,F_\alpha,F_\beta)$$
$$=F_\alpha(t)F_\beta(-t)\ . \tag{4.4b}$$

We consider first the case where F_α and F_β are both two-point Green's functions G_α and G_β, where in the interests of brevity and legibility we suppress the superscript (2) and compress the subscripts α and α' of Eq. (4.3) into a single generic subscript α; we will eventually establish that all other quantities that appear as F_α in loops have similar properties so that the discussion applies to all loops.

Since the Green's functions are the expectation values of time-ordered products, they are the "causal" functions which propagate particles and antiparticles, respectively, forward and backward in time:

$$G_\alpha(t)=\theta(t)G^+_\alpha(t)+(-)^\alpha\theta(-t)G^-_\alpha(t)\ , \tag{4.5}$$

where G^+_α and G^-_α are the propagators for particles and antiparticles respectively, and $(-)^\alpha$ is -1 when α is a fermion and $+1$ when α is a boson. The associated propagators $\tilde{G}^+_\alpha(\omega)$, defined as i times the time Fourier transforms of the Green's functions

$$\tilde{G}^\pm_\alpha(\omega)\equiv i\int dt\, e^{i\omega t}G^\pm_\alpha(t)\ , \tag{4.6}$$

are, like all physical propagators, analytic in ω except along the real axis. Because $\tilde{G}^+_\alpha(\omega)$ represents the propagation of particles, its singularities lie along the real axis above the chemical potential μ_α; conversely, $\tilde{G}^-_\alpha(\omega)$ has singularities only for $\omega<\mu_\alpha$ representing antiparticles or, for fermions, "holes" in the Fermi sea. Using the theorem that a function $F^\pm(t)$ is proportional to $\theta(\pm t)$ if and only if $\tilde{F}$ [defined analogous to Eq. (4.6) as i times its Fourier transform] obeys a dispersion relation

$$\mathrm{Re}\tilde{F}^\pm(\omega)=\pm\int\frac{d\omega'}{\pi}\frac{\mathcal{P}}{\omega'-\omega}\mathrm{Im}\tilde{F}^\pm(\omega')\ , \tag{4.7}$$

where $\mathcal{P}$ denotes a principal-value integral, we realize that each term in Eq. (4.5) obeys such a relation with the sign determined by whether we are dealing with particles or antiparticles, i.e., whether the frequency ω' is greater than or less than the chemical potential μ_α. We can therefore combine the two terms to find[18] a dispersion relation for $\tilde{G}_\alpha(\omega)$,

$$\mathrm{Re}\tilde{G}_\alpha(\omega)=\int\frac{d\omega'}{\pi}\frac{\mathcal{P}}{\omega'-\omega}\mathrm{Im}\tilde{G}_\alpha(\omega')\cdot\mathrm{sgn}(\omega'-\mu_\alpha) \tag{4.8}$$

or, equivalently,

$$\tilde{G}_\alpha(\omega)=\int\frac{d\omega'}{\pi}\left[\frac{\mathcal{P}}{\omega'-\omega}\mathrm{sgn}(\omega'-\mu_\alpha)+i\pi\delta(\omega'-\omega)\right]\mathrm{Im}\tilde{G}_\alpha(\omega')\ . \tag{4.8$'$}$$

We see that $\mathrm{Re}\tilde{G}_\alpha(\omega)$ is determined by $\mathrm{Im}\tilde{G}_\alpha(\omega)$, which we will concentrate on because it is the most closely related to the probability current.

The decomposition (4.5) immediately implies a similar decomposition for the loop, Eq. (4.4b):

$$L_{\alpha\beta}(t)=(-)^\beta\theta(t)G^+_\alpha(t)G^-_\beta(-t)+(-)^\alpha\theta(-t)G^-_\alpha(t)G^+_\beta(-t)\ . \tag{4.9}$$

We see that $L_{\alpha\beta}$ only has terms where one of the Green's functions represents a particle and the other an antiparticle; there are no particle-particle or antiparticle-antiparticle terms. Using Eq. (4.8$'$) in the definition (4.4b) we obtain, after a brief computation (see Appendix B), the dispersion relation for $\tilde{L}$, defined like Eq. (4.6) as i times the Fourier transform of L:

$$\tilde{L}_{\alpha\beta}(\omega)=\int\frac{d\omega'}{\pi}\left[\frac{\mathcal{P}}{\omega'-\omega}\mathrm{sgn}(\omega'-\mu_\alpha+\mu_\beta)+i\pi\delta(\omega'-\omega)\right]\mathrm{Im}\tilde{L}_{\alpha\beta}(\omega')\ . \tag{4.10}$$

This result is completely analogous to Eq. (4.8a), if we identify the chemical potential of the loop as the difference of the chemical potentials of the internal lines, as would be expected from equilibrium thermodynamics.

We conclude that the loops have an analytic structure completely analogous to that of the Green's functions, allowing their real parts to be readily computed from their imaginary parts.

The same computation (Appendix B) which leads to Eq. (4.10) also gives us a simple expression for $\mathrm{Im}\tilde{L}_{\alpha\beta}(\omega)$:

$$\mathrm{Im}\tilde{L}_{\alpha\beta}(\omega)=\int \frac{d\omega_\alpha d\omega_\beta}{\pi}\delta(\omega_\alpha-\omega_\beta-\omega)\mathrm{Im}\tilde{G}_\alpha(\omega_\alpha) \times \mathrm{Im}\tilde{G}_\beta(\omega_\beta)\{\theta(\omega_\alpha-\mu_\alpha)\theta(\mu_\beta-\omega_\beta) +\theta(\mu_\alpha-\omega_\alpha)\theta(\omega_\beta-\mu_\beta)\} . \tag{4.11}$$

As expected from Eq. (4.9), we see that $\tilde{L}_{\alpha\beta}(\omega)$ has only particle-antiparticle terms but no particle-particle or antiparticle-antiparticle terms. Of course, Eq. (4.10) and (4.11) together imply the structure (4.9). Inspecting Eq. (4.11) verifies that its two terms have singularities only for $\omega>\mu_\alpha-\mu_\beta$ and $\omega<\mu_\alpha-\mu_\beta$, respectively, in harmony with the discussion in the preceding paragraph. Because of the identification of the imaginary parts of Green's functions with the probability densities, Eq. (4.11) provides an explicit realization of unitarity, the conservation of probability density: the probability density associated with a loop is the sum of terms representing various possibilities (i.e., combinations of particles and antiparticles) for intermediate states which can occur in the propagation of the particle in whose Green's function the loop appears; each of these terms is the product of the probability densities of the independently-propagating components of the corresponding intermediate state.

B. Implications of causality and unitarity for cutoff procedures

We have established that the loop formed from two Green's functions has the same analytic properties, particle-antiparticle decompositions, and dispersion relations as a Green's function, provided that its chemical potential is chosen in the natural way. It is easier to see that the same is true for the convolution of two Green's functions: since the Fourier transform of a convolution is the product of the Fourier transforms of its constituents, all evaluated at the same frequency, we only need to note that the fact that particle α can turn into particle β, which is implied by the diagram leading to the convolution, requires that their chemical potentials be equal; this common chemical potential may then be identified as the chemical potential of the convolution. Typically, one of the factors in the convolution will be a loop; the requirement then is that the net chemical potential of the particles (minus antiparticles) of the loop should equal the chemical potential of the other factor in the convolution, as expected.

We are now in a position to discuss the choice of cutoff procedures, which need to respect causality and unitarity and therefore must preserve the relations (4.10) and (4.11). We will eventually want to determine the cutoffs by considering processes occurring in vacuum, so the cutoffs have to be introduced covariantly. For this reason it is useful to complete the Fourier transformation to momentum space, including the spatial variables as well as the time variables. Introducing the notation

$$\bar{G}(p,p')\equiv\int dx\, dx' e^{ipx-ip'x'}G(x,x') \equiv 2\pi\delta(p_0'-p_0)\bar{G}(\mathbf{p},\mathbf{p}',p_0) , \tag{4.12}$$

we have from Eq. (4.6).

$$\bar{G}(\mathbf{p},\mathbf{p}',p_0)=\frac{1}{i}\int d\mathbf{x}\, d\mathbf{x}' e^{-i\mathbf{p}\cdot\mathbf{x}+i\mathbf{p}'\cdot\mathbf{x}'}\tilde{G}(\mathbf{x},\mathbf{x}',p_0) . \tag{4.13}$$

Noting that $G_\alpha(x,x')$ and $G_\beta(x',x)$, and therefore $L_{\alpha\beta}(x,x')$, are real, we can eliminate $\mathrm{Im}\tilde{G}_\alpha$ in favor of $\tilde{G}_\alpha$:

$$\mathrm{Im}\tilde{G}_\alpha=\mathrm{Im}\, i\int dt\, e^{i\omega t}G_\alpha(t) =\frac{1}{2i}[\tilde{G}_\alpha(\omega)+\tilde{G}_\alpha(-\omega)] \tag{4.14}$$

with a similar relation for $\tilde{L}_{\alpha\beta}$. Equations (4.13) and (4.14) enable us to rewrite Eqs. (4.10) and (4.11) in momentum representation:

$$\bar{L}_{\alpha\beta}(\mathbf{q},\mathbf{q}',\omega)=\int\frac{d\omega'}{i\pi}\left[\frac{\mathcal{P}}{\omega'-\omega}\mathrm{sgn}(\omega'-\mu_\alpha+\mu_\beta)+i\pi\delta(\omega'-\omega)\right]\tfrac{1}{2}[\bar{L}_{\alpha\beta}(\mathbf{q},\mathbf{q}',\omega')+\bar{L}_{\alpha\beta}(\mathbf{q},\mathbf{q}',-\omega')] , \tag{4.15}$$

and

$$\tfrac{1}{2}[\bar{L}_{\alpha\beta}(\mathbf{q},\mathbf{q}',\omega')+\bar{L}_{\alpha\beta}(\mathbf{q},\mathbf{q}',-\omega')]=\int\frac{d\mathbf{p}\,d\mathbf{p}'d\mathbf{k}\,d\mathbf{k}'}{(2\pi)^6}\delta(\mathbf{p}-\mathbf{k}-\mathbf{q})\delta(\mathbf{p}'-\mathbf{k}'-\mathbf{q}') \times\frac{1}{4\pi}\int d\omega_\alpha d\omega_\beta\delta(\omega_\alpha-\omega_\beta-\omega')\{\theta(\omega_\alpha-\mu_\alpha)\theta(\mu_\beta-\omega_\beta)+\theta(\mu_\alpha-\omega_\alpha)\theta(\omega_\beta-\mu_\beta)\} \times[\bar{G}_\alpha(\mathbf{p},\mathbf{p}',\omega_\alpha)+\bar{G}_\alpha(\mathbf{p},\mathbf{p}',-\omega_\alpha)] \times[\bar{G}_\beta(\mathbf{k}',\mathbf{k},\omega_\beta)+\bar{G}_\beta(\mathbf{k}',\mathbf{k},-\omega_\beta)] . \tag{4.16}$$

The most obvious move is to try to associate cutoff factors with the propagators inside the loop, following Feynman and Pauli-Villars. If these cutoffs are introduced covariantly, then Eqs. (4.15) and (4.16) imply that they are ineffectual, as is most readily seen in a momentum representation. Covariance requires the cutoff factor $\bar{f}(p^2)$ to depend only on the invariant mass p^2 of the momentum of the propagator. Inspecting first for simplicity the approximation of free propagators, we realize that their imaginary parts are nonzero only for $p^2=m^2$ so that the integrals (4.15) and (4.16) are simply multiplied by a constant factor $\bar{f}(m^2)$, conventionally chosen to be unity. Since we expect at least some of our self-consistent propagators to exhibit quasiparticle poles with finite residues, we must anticipate that a similar difficulty will persist in the case of loops computed from self-consistent solutions.

Another commonplace method of regularizing the propagator involves subtracting from it the propagators of one or more fictional, very massive particles $(p^2-M^2)^{-1}$. This procedure, which reduces to a special case of the previously mentioned method

$$\bar{f}(p^2)=(M^2-m^2)/(p^2-M^2)$$

when applied to a free propagator, has the additional disadvantage that the coefficient of the singularity at M^2 corresponds to a negative probability density. This latter problem is unimportant for stationary-state applications where high energies are never realized; however, in our application in the time domain all frequencies contribute to the short-time propagation, and thus unphysical behavior cannot be avoided in the presence of such a subtraction.

Because the cutoffs cannot readily be associated with the propagators, we have to try to incorporate them in the vertices. We begin by inspecting the three-point irreducible vertex. Defining its Fourier transform

$$\bar{\Gamma}(q,p,k)\equiv\int d^4x\, d^4y\, d^4z\, \Gamma(x,y,z)\exp i(xq+yp+zk)\ , \tag{4.17}$$

we realize from this argument that it will be useless to introduce any cutoff depending on only one momentum. Of the size combinations of two momenta, the sums of pairs of momenta are equal to the remaining individual momentum for the case of translational invariance; the differences are reasonable candidates. Inspecting the Migdal approximation

$$\Gamma(x,y,z)=\Gamma(x,(y+z)/2)\delta(y-z)\ ,$$

Eq. (3.78), we see that $\bar{\Gamma}(q,p,k)=\bar{\Gamma}(q,p+k)$ is independent of the combination $p-k$, while it already falls off with $p-q$ or $q-k$. Apparently we need to have our cutoff depend on $(p-k)^2$.

The simplest way, then, to cut off the loop integrals is to multiply the vertex function by a factor $\bar{f}[(p-k)/2]$ which depends only on $(p-k)^2$:

$$\bar{\Gamma}(q,p,k)=\bar{\Gamma}(q,p+k)\bar{f}[(p-k)/2]\ , \tag{4.18a}$$

which would lead to a corresponding factor in coordinate representation

$$\Gamma(x,y,z)\approx\Gamma(x,(y+z)/2)f(y-z)\ . \tag{4.18b}$$

We see that the Migdal approximation corresponds to setting $f(y-z)=\delta(y-z)$. Relaxing the locality assumption is necessary to make the integrals converge. We can interpret $f(y-z)$ as a structure function for decay of the boson at $(y+z)/2$ into hadrons at y and z.

Unfortunately, however, the simplest ansatz (4.18) leads to an acausal self-energy loop. For example, if x is the external coordinate, a typical self-energy term becomes [for fermion-loop insertions in a boson propagator, Eq. (3.52)]

$$\begin{aligned}\Pi_{\alpha\alpha'}(x,x')&=i\sum_{bc}\int dy\,dz\,\Gamma_{abc}(x;y,z)G_c(y,x')G_b(x',z)\Gamma^0_{\alpha cb}\\&=i\sum_{bc}\int dY\,\Gamma_{abc}(x;Y)\int dZ\,f(Z)G_c\left[Y+\frac{Z}{2},x'\right]G_b\left[x',Y-\frac{Z}{2}\right]\Gamma^0_{\alpha cb}\\&=i\sum_{bc}C(x,x',\Gamma_{abc}(x;Y),L_f(Y,x',G_c,G_b))\Gamma^0_{\alpha bc}\ .\end{aligned}\tag{4.19}$$

The cutoff loop L_f defined by the last two lines of Eq. (4.19) may be evaluated using Eq. (4.5):

$$\begin{aligned}L_f(Y,x',G_c,G_b)=\int dZ\,f(Z)\Bigg[&G_c^+\left[Y+\frac{Z}{2},x'\right]G_b^-\left[x',Y-\frac{Z}{2}\right]\theta\left[x_0'-Y_0<\frac{Z_0}{2}\right]\theta\left[x_0'-Y_0<-\frac{Z_0}{2}\right]\\&-G_c^+\left[Y+\frac{Z}{2},x'\right]G_b^+\left[x',Y-\frac{Z}{2}\right]\theta\left[x_0'-Y_0<\frac{Z_0}{2}\right]\theta\left[x_0'-Y_0>-\frac{Z_0}{2}\right]\\&-G_c^-\left[Y+\frac{Z}{2},x'\right]G_b^-\left[x',Y-\frac{Z}{2}\right]\theta\left[x_0'-Y_0>\frac{Z_0}{2}\right]\theta\left[x_0'-Y_0<-\frac{Z_0}{2}\right]\\&+G_c^-\left[Y+\frac{Z}{2},x'\right]G_b^+\left[x',Y-\frac{Z}{2}\right]\theta\left[x_0'-Y_0>\frac{Z_0}{2}\right]\theta\left[x_0'-Y_0>-\frac{Z_0}{2}\right]\Bigg]\ .\end{aligned}\tag{4.20}$$

While the first and last terms have the appropriate causality/unitarity structure, the middle terms permit particles and antiparticles, respectively, to propagate both directions in time unless $f(Z)$ contains a δ function in Z_0. Apparently a more subtle procedure is called for.

C. Proposed cutoff procedure: Momentum representation for stationary states

Fortunately, Eqs. (4.10) and (4.11) allow us to construct a cutoff procedure which leads to causal functions. We construct the self-energy operator $\Pi(x,x')$ from the loop L_f as given in the last line of Eq. (4.19), but to ensure that L_f is causal we evaluate the real part of $\bar{L}_f$ from the dispersion relation, Eq. (4.10), and introduce the cutoff explicitly only in the computation of the imaginary parts. Using the momentum representation (4.14) and (4.15), we thus define the cutoff loop L_f by introducing the cutoff factors into Eq. (4.16):

$$\bar{L}_f(\mathbf{q},\mathbf{q}',\omega,F_\alpha,F_\beta)=\int\frac{d\omega'}{i\pi}\left[\frac{\mathcal{P}}{\omega'-\omega}\mathrm{sgn}(\omega'-\mu_\alpha+\mu_\beta)+i\pi\delta(\omega'-\omega)\right]\tfrac{1}{2}[\bar{L}_f(\mathbf{q},\mathbf{q}',\omega',F_\alpha,F_\beta)+\bar{L}_f(\mathbf{q},\mathbf{q}',-\omega',F_\alpha,F_\beta)]\ , \tag{4.21}$$

where

$$\begin{aligned}\tfrac{1}{2}[\bar{L}_f(\mathbf{q},\mathbf{q}',\omega,F_\alpha,F_\beta)&+\bar{L}_f(\mathbf{q},\mathbf{q}'-\omega,F_\alpha,F_\beta)]\\ &=\int\frac{d\mathbf{p}\,d\mathbf{p}'d\mathbf{k}\,d\mathbf{k}'}{(2\pi)^6}\delta(\mathbf{p}-\mathbf{k}-\mathbf{q})\delta(\mathbf{p}'-\mathbf{k}'-\mathbf{q}')\frac{1}{4\pi}\int d\omega_\alpha d\omega_\beta\delta(\omega_\alpha-\omega_\beta-\omega)\\ &\quad\times\{\theta(\omega_\alpha-\mu_\alpha)\theta(\mu_\beta-\omega_\beta)+\theta(\mu_\alpha-\omega_\alpha)\theta(\omega_\beta-\mu_\beta)\}[\bar{G}_\alpha(\mathbf{p},\mathbf{p}',\omega_\alpha)+\bar{G}_\alpha(\mathbf{p},\mathbf{p}',-\omega_\alpha)]\\ &\quad\times[\bar{G}_\beta(\mathbf{k}',\mathbf{k},\omega_\beta)+\bar{G}_\beta(\mathbf{k}',\mathbf{k},-\omega_\beta)]\bar{f}\left\{\left[\frac{\mathbf{p}+\mathbf{k}}{2}\right]^2-\left[\frac{\omega_\alpha+\omega_\beta}{2}\right]^2\right\}\bar{f}\left\{\left[\frac{\mathbf{p}'+\mathbf{k}'}{2}\right]^2-\left[\frac{\omega_\alpha+\omega_\beta}{2}\right]^2\right\}\ .\end{aligned} \tag{4.22}$$

Here, we have introduced the cutoff symmetrically at both ends of the loop, in order to preserve the symmetries of the self-energies and vertices. The symmetrization of the cutoff is physically acceptable because the locality of the bare vertex, like that of the Migdal interaction, is also only a low-energy, long-wavelength approximation to an interaction which ought to take account of the hadrons' structure.

Our prescription, then, for regularizing loop integrals consists of introducing a factorized smearing (in coordinate space) or form factor (in momentum space) in the computation of the imaginary part of the loop; where the smearing or form factor is related to the distance which the Migdal locality approximation had set to zero. We have illustrated our prescription for the case when the particles whose coordinates were chosen for smearing are both interior to the loop; the loops appearing in the equations for the dressed three-point vertices Γ and the T matrix are all of this type. The other case, when one of the smeared coordinates is external, may be obtained analogously. For example, in the case of a boson-loop insertion in a fermion propagator, we have a term [Eq. (3.59)]

$$\begin{aligned}\Sigma_{aa'}(x,x')&=-i\sum_{\beta c}\int dy\,dz\,\Gamma_{\beta ac}(y,x,z)G_\beta(y,x')G_c(x',z)\Gamma^0_{\beta a'c}\\ &=-i\sum_{\beta c}L'_f(x,x',G_\beta,C(x,x',\Gamma_{\beta ac}(x,y),G_c(y,x'))\Gamma^0_{\beta a'c}\end{aligned} \tag{4.23}$$

where

$$\begin{aligned}\tfrac{1}{2}[\bar{L}'_f(\mathbf{q},\mathbf{q}',\omega,F_\alpha,F_\beta)&+\bar{L}'_f(\mathbf{q},\mathbf{q}',-\omega,F_\alpha,F_\beta)]\\ &=\int\frac{d\mathbf{p}\,d\mathbf{p}'d\mathbf{k}\,d\mathbf{k}'}{(2\pi)^6}\delta(\mathbf{p}-\mathbf{k}-\mathbf{q})\delta(\mathbf{p}'-\mathbf{k}'-\mathbf{q}')\frac{1}{4\pi}\int d\omega_\alpha d\omega_\beta\,\delta(\omega_\alpha-\omega_\beta-\omega)\\ &\quad\times\{\theta(\omega_\alpha-\mu_\alpha)\theta(\mu_\beta-\omega_\beta)+\theta(\mu_\alpha-\omega_\alpha)\theta(\omega_\beta-\mu_\beta)\}[\bar{G}_\alpha(\mathbf{p},\mathbf{p}',\omega_\alpha)+\bar{G}_\alpha(\mathbf{p},\mathbf{p}',-\omega_\alpha)]\\ &\quad\times[\bar{G}_\beta(\mathbf{k}',\mathbf{k},\omega_\beta)+\bar{G}_\beta(\mathbf{k}',\mathbf{k},-\omega_\beta)]\bar{f}\left\{\left[\frac{\mathbf{p}+\mathbf{q}}{2}\right]^2-\left[\frac{\omega_\alpha+\omega}{2}\right]^2\right\}\bar{f}\left\{\left[\frac{\mathbf{p}'+\mathbf{q}'}{2}\right]^2-\left[\frac{\omega_\alpha+\omega}{2}\right]^2\right\}\ ,\end{aligned} \tag{4.24}$$

and $\bar{L}'_f$, like $\bar{L}_f$, obeys the relations (4.10), (4.15), and (4.21). The boson loops appearing in the equations for the dressed boson propagators are either of the form (4.22)—in the case of 2π insertions in the σ and ρ propagators—or of the form (4.24) when the loops appear in the pion propagator.

D. Cutoff as smearing in coordinate representation; nonstationary states

We can further illuminate our cutoff procedure by considering its consequences for the construction of loops in coordinate representation. This will also allow us to generalize our procedure to the case of nonstationary states, where we cannot use the simple dispersion relations employed in the case of stationary states.

We will show in detail the case when the smeared-cutoff variables are internal to the loop. We begin by performing inverse Fourier transforms on the spatial variables of the cutoff loop obtained earlier for the case of stationary states. This leaves us with relations among the time-Fourier-transformed functions $\tilde{G}$ and $\tilde{L}$. Equation (4.22) becomes

$$\mathrm{Im}\tilde{L}_f(\mathbf{r},\mathbf{r}',\omega,G_\alpha,G_\beta)=\int \frac{d\omega_\alpha d\omega_\beta}{\pi}\delta(\omega_\alpha-\omega_\beta-\omega)\{\theta(\omega_\alpha-\mu_\alpha)\theta(\mu_\beta-\omega_\beta)+\theta(\mu_\alpha-\omega_\alpha)\theta(\omega_\beta-\mu_\beta)\}$$
$$\times\int d\mathbf{s}\,d\mathbf{s}'\tilde{f}\left[\mathbf{s},\frac{\omega_\alpha+\omega_\beta}{2}\right]\tilde{f}\left[\mathbf{s}',\frac{\omega_\alpha+\omega_\beta}{2}\right]\mathrm{Im}\tilde{G}_\alpha\left[\mathbf{r}+\frac{\mathbf{s}}{2},\mathbf{r}'+\frac{\mathbf{s}'}{2},\omega_\alpha\right]$$
$$\times\mathrm{Im}\tilde{G}_\beta\left[\mathbf{r}'-\frac{\mathbf{s}'}{2},\mathbf{r}-\frac{\mathbf{s}}{2},\omega_\beta\right]. \tag{4.25}$$

Here we have used the fact that G_α and G_β are real and have assumed that $\bar{f}$ is a real function depending only on the magnitude of its argument; $\tilde{f}$ is defined as i times the time-Fourier transform of f, as in Eq. (4.6). As intended, Eq. (4.21) reverts to the dispersion relation (4.10) upon inverse Fourier transformation of the spatial variables. Using the dispersion relation, we have

$$L_f(x,x',G_\alpha,G_\beta)=\frac{1}{i}\int\frac{dq_0}{2\pi}e^{-iq_0(x_0-x_0')}\tilde{L}_f(\mathbf{x},\mathbf{x}',q_0,G_\alpha,G_\beta)$$
$$=\frac{1}{i}\int\frac{dq_0}{2\pi}e^{-iq_0(x_0-x_0')}\int\frac{d\omega}{\pi}\left[\frac{\mathcal{P}}{\omega-q_0}\mathrm{sgn}(\omega-\mu_\alpha+\mu_\beta)+i\pi\delta(\omega-q_0)\right]\mathrm{Im}\tilde{L}_f(\mathbf{x},\mathbf{x}',q_0,G_\alpha,G_\beta). \tag{4.26}$$

We now introduce the cutoff expression $\mathrm{Im}\tilde{L}_f$ from Eq. (4.25):

$$L_f(x,x',G_\alpha,G_\beta)=\frac{1}{i}\int\frac{dq_0}{2\pi}e^{-iq_0(x_0-x_0')}\int\frac{d\omega}{\pi}\left[\frac{\mathcal{P}}{\omega-q_0}\mathrm{sgn}(\omega-\mu_\alpha+\mu_\beta)+i\pi\delta(\omega-q_0)\right]$$
$$\times\int\frac{d\omega_\alpha d\omega_\beta}{\pi}\delta(\omega_\alpha-\omega_\beta-\omega)\{\theta(\omega_\alpha-\mu_\alpha)\theta(\mu_\beta-\omega_\beta)+\theta(\mu_\alpha-\omega_\alpha)\theta(\omega_\beta-\mu_\beta)\}$$
$$\times\int d\mathbf{s}\,d\mathbf{s}'\tilde{f}\left[\mathbf{s},\frac{\omega_\alpha+\omega_\beta}{2}\right]\tilde{f}\left[\mathbf{s}',\frac{\omega_\alpha+\omega_\beta}{2}\right]\mathrm{Im}\tilde{G}_\alpha\left[\mathbf{r}+\frac{\mathbf{s}}{2},\mathbf{r}'+\frac{\mathbf{s}'}{2},\omega_\alpha\right]$$
$$\times\mathrm{Im}\tilde{G}_\beta\left[\mathbf{r}'-\frac{\mathbf{s}'}{2},\mathbf{r}-\frac{\mathbf{s}}{2},\omega_\beta\right]. \tag{4.27}$$

Using the theorem

$$\int\frac{dq_0}{\pi}\frac{\mathcal{P}}{q_0}e^{iq_0\tau}=i\,\mathrm{sgn}\tau, \tag{4.28}$$

the integrals over first q_0 and then ω are readily performed:

$$L_f(x,x',G_\alpha,G_\beta)=\int\frac{d\omega}{2\pi}e^{-i\omega(x_0-x_0')}[\mathrm{sgn}(x_0-x_0')\mathrm{sgn}(\omega-\mu_\alpha+\mu_\beta)+1]$$
$$\times\int\frac{d\omega_\alpha d\omega_\beta}{\pi}\delta(\omega_\alpha-\omega_\beta-\omega)\{\theta(\omega_\alpha-\mu_\alpha)\theta(\mu_\beta-\omega_\beta)+\theta(\mu_\alpha-\omega_\alpha)\theta(\omega_\beta-\mu_\beta)\}$$
$$\times\int d\mathbf{s}\,d\mathbf{s}'\tilde{f}\left[\mathbf{s},\frac{\omega_\alpha+\omega_\beta}{2}\right]\tilde{f}\left[\mathbf{s}',\frac{\omega_\alpha+\omega_\beta}{2}\right]\mathrm{Im}\tilde{G}_\alpha\left[\mathbf{r}+\frac{\mathbf{s}}{2},\mathbf{r}'+\frac{\mathbf{s}'}{2},\omega_\alpha\right]\mathrm{Im}\tilde{G}_\beta\left[\mathbf{r}'-\frac{\mathbf{s}'}{2},\mathbf{r}-\frac{\mathbf{s}}{2},\omega_\beta\right]$$
$$=\int\frac{d\omega_\alpha d\omega_\beta}{2\pi^2}e^{-i(\omega_\alpha-\omega_\beta)(x_0-x_0')}[\mathrm{sgn}(x_0-x_0')\mathrm{sgn}(\omega_\alpha-\omega_\beta-\mu_\alpha+\mu_\beta)+1]$$
$$\times\{\theta(\omega_\alpha-\mu_\alpha)\theta(\mu_\beta-\omega_\beta)+\theta(\mu_\alpha-\omega_\alpha)\theta(\omega_\beta-\mu_\beta)\}$$
$$\times\int d\mathbf{s}\,d\mathbf{s}'\tilde{f}\left[\mathbf{s},\frac{\omega_\alpha+\omega_\beta}{2}\right]\tilde{f}\left[\mathbf{s}',\frac{\omega_\alpha+\omega_\beta}{2}\right]\mathrm{Im}\tilde{G}_\alpha\left[\mathbf{r}+\frac{\mathbf{s}}{2},\mathbf{r}'+\frac{\mathbf{s}'}{2},\omega_\alpha\right]\mathrm{Im}\tilde{G}_\beta\left[\mathbf{r}'-\frac{\mathbf{s}'}{2},\mathbf{r}-\frac{\mathbf{s}}{2},\omega_\beta\right]$$

$$=\int \frac{d\omega_\alpha d\omega_\beta}{\pi^2} e^{-i(\omega_\alpha-\omega_\beta)(x_0-x_0')}\{\theta(x_0-x_0')\theta(\omega_\alpha-\mu_\alpha)\theta(\mu_\beta-\omega_\beta)+\theta(x_0'-x_0)\theta(\mu_\alpha-\omega_\alpha)\theta(\omega_\beta-\mu_\beta)\}$$
$$\times\int ds\,ds'\tilde{f}\left[\mathbf{s},\frac{\omega_\alpha+\omega_\beta}{2}\right]\tilde{f}\left[\mathbf{s}',\frac{\omega_\alpha+\omega_\beta}{2}\right]\mathrm{Im}\tilde{G}_\alpha\left[\mathbf{r}+\frac{\mathbf{s}}{2},\mathbf{r}'+\frac{\mathbf{s}'}{2},\omega_\alpha\right]\mathrm{Im}\tilde{G}_\beta\left[\mathbf{r}'-\frac{\mathbf{s}'}{2},\mathbf{r}-\frac{\mathbf{s}}{2},\omega_\beta\right]. \tag{4.29}$$

The factor in curly braces in Eq. (4.29) affirms that our procedure enforces the required structures of causality and unitarity.

To complete our computation we have to perform inverse Fourier transforms in order to express the result in terms of the coordinate representations of G_α, G_β, and f. We first introduce the coordinate representation of the smearing function f:

$$L_f(x,x',G_\alpha,G_\beta)=\int ds\,ds'f(s)f(s')$$
$$\times\int\frac{d\omega_\alpha}{\pi}\mathrm{Im}\tilde{G}_\alpha\left[\mathbf{r}+\frac{\mathbf{s}}{2},\mathbf{r}'+\frac{\mathbf{s}'}{2},\omega_\alpha\right]e^{-i\omega_\alpha(x_0-x_0'+s_0/2-s_0'/2)}$$
$$\times\int\frac{d\omega_\beta}{\pi}\mathrm{Im}\tilde{G}_\beta\left[\mathbf{r}'-\frac{\mathbf{s}'}{2},\mathbf{r}-\frac{\mathbf{s}}{2},\omega_\beta\right]e^{+i\omega_\beta(x_0-x_0'-s_0/2+s_0'/2)}$$
$$\times\{\theta(x_0-x_0')\theta(\omega_\alpha-\mu_\alpha)\theta(\mu_\beta-\omega_\beta)+\theta(x_0'-x_0)\theta(\mu_\alpha-\omega_\alpha)\theta(\omega_\beta-\mu_\beta)\}. \tag{4.30}$$

To perform the inverse Fourier transform of $\tilde{G}_\alpha$, we start from the spectral representation of G_α,

$$G_\alpha(y,y')=\frac{1}{i}\int\frac{d\omega_\alpha}{2\pi}e^{-i\omega_\alpha(y_0-y_0')}\tilde{G}_\alpha(\mathbf{y},\mathbf{y}',\omega_\alpha)$$
$$=2\int\frac{d\omega_\alpha}{2\pi}e^{-i\omega_\alpha(y_0-y_0')}[\theta(y_0-y_0')\theta(\omega_\alpha-\mu_\alpha)+\theta(y_0'-y_0)\theta(\mu_\alpha-\omega_\alpha)]\mathrm{Im}\tilde{G}_\alpha(\mathbf{y},\mathbf{y}',\omega_\alpha). \tag{4.31}$$

The last equality in Eq. (4.31) follows from Eq. (4.8′) and (4.28), together with the discussion following Eq. (4.6). The expression (4.31) enables us to evaluate the integrals appearing in (4.30): by considering separately the cases $y_0>y_0'$ and $y_0<y_0'$ and recalling that $G_\alpha(y',y)=G_\alpha(y',y)^*$ we readily verify that

$$\int\frac{d\omega_\alpha}{\pi}e^{-i\omega_\alpha(y_0-y_0')}\theta(\omega_\alpha-\mu_\alpha)\mathrm{Im}\tilde{G}_\alpha(\mathbf{y},\mathbf{y}',\omega_\alpha)=G_\alpha(\mathbf{y},y_0,\mathbf{y}',y_0')\theta(y_0-y_0')+G_\alpha(\mathbf{y},y_0',\mathbf{y}',y_0)\theta(y_0'-y_0) \tag{4.32a}$$

and

$$\int\frac{d\omega_\alpha}{\pi}e^{-i\omega_\alpha(y_0-y_0')}\theta(\mu_\alpha-\omega_\alpha)\mathrm{Im}\tilde{G}_\alpha(\mathbf{y},\mathbf{y}',\omega_\alpha)=G_\alpha(\mathbf{y},y_0,\mathbf{y}',y_0')\theta(y_0'-y_0)+G_\alpha(\mathbf{y},y_0',\mathbf{y}',y_0)\theta(y_0-y_0'). \tag{4.32b}$$

These results allow us to evaluate the integrals in (4.30). Identifying y and y' by inspecting the arguments in (4.30) we find

$$L_f(x,x',G_\alpha,G_\beta)=\int ds\,ds'dy\,dy'dz\,dz'f(s)f(s')\delta\left[x+\frac{s}{2}-y\right]\delta\left[x'+\frac{s'}{2}-y'\right]\delta\left[x-\frac{s}{2}-z\right]\delta\left[x'-\frac{s'}{2}-z'\right]$$
$$\times\{\theta(x_0-x_0')[G_\alpha(\mathbf{y},y_0,\mathbf{y}',y_0')\theta(y_0-y_0')+G_\alpha(\mathbf{y},y_0',\mathbf{y}',y_0)\theta(y_0'-y_0)]$$
$$\times[G_\beta(\mathbf{z}',z_0',\mathbf{z},z_0)\theta(z_0-z_0')+G_\beta(\mathbf{z}',z_0,\mathbf{z},z_0')\theta(z_0'-z_0)]$$
$$+\theta(x_0'-x_0)[G_\alpha(\mathbf{y},y_0,\mathbf{y}',y_0')\theta(y_0'-y_0)+G_\alpha(\mathbf{y},y_0',\mathbf{y}',y_0)\theta(y_0-y_0')]$$
$$\times[G_\beta(\mathbf{z}',z_0',\mathbf{z},z_0)\theta(z_0'-z_0)+G_\beta(\mathbf{z}',z_0,\mathbf{z},z_0')\theta(z_0-z_0')]\}. \tag{4.33}$$

An analogous computation gives for the case when one of the smeared variables is exterior to the loop, the other being the argument of G_α

$$L'_f(x,x',G_\alpha,G_\beta)=\int ds\, ds'dy\, dy'dz\, dz'f(s)f(s')\delta\left[z+\frac{s}{2}-y\right]\delta\left[z'+\frac{s'}{2}-y'\right]\delta\left[z-\frac{s}{2}-x\right]\delta\left[z'-\frac{s'}{2}-x'\right]$$
$$\times\{\theta(x_0-x'_0)[G_\alpha(\mathbf{y},y_0,\mathbf{y}',y'_0)\theta(y_0-y'_0)+G_\alpha(\mathbf{y},y'_0,\mathbf{y}',y_0)\theta(y'_0-y_0)]$$
$$\times[G_\beta(\mathbf{z}',z'_0,\mathbf{z},z_0)\theta(z_0-z'_0)+G_\beta(\mathbf{z}',z_0,\mathbf{z},z'_0)\theta(z'_0-z_0)]$$
$$+\theta(x'_0-x_0)[G_\alpha(\mathbf{y},y_0,\mathbf{y}',y'_0)\theta(y'_0-y_0)+G_\alpha(\mathbf{y},y'_0,\mathbf{y}',y_0)\theta(y_0-y'_0)]$$
$$\times[G_\beta(\mathbf{z}',z'_0,\mathbf{z},z_0)\theta(z'_0-z_0)+G_\beta(\mathbf{z}',z_0,\mathbf{z},z'_0)\theta(z_0-z'_0)]\}\ . \tag{4.34}$$

Note that Eqs. (4.33) and (4.34) reduce to Eq. (4.4b) in the limit when $f(s)$ approaches $\delta(s)$.

Motivated by our study of the stationary-state case, we propose to apply the cutoff prescriptions (4.33) and (4.34) in all cases, even when the wave packets have the more complicated time dependence needed for describing nuclear collisions. We observe that our cutoff prescription consists of two elements. (1) The ends of the loop are smoothed by convolution with the smearing function f. This is the feature that eliminates the ultraviolet infinities. f results from the structure of the boson whose hadronic decay is described by the vertex. (2) During this convolution, the time arguments of the loop's factors are interchanged as necessary to preserve the causality/unitarity structure of the diagram. The covariance of this procedure is assured by its construction in the momentum representation, where the covariance is manifest. Covariance is obscured in the coordinate representation by the frame-dependent parametrization of the light cone.

We have until now ignored the fact that some of the pointlike interactions contain derivative couplings. Those can be reduced to the cases considered above by starting with a partial integration to move the derivatives out of the vertices and onto the Green's functions: then the functions F_α and F_β may contain derivatives of Green's functions. After the smearing procedure, the derivatives may again be moved back to the vertices by partial integration. The result will be that the smearing function $f(s)$ is replaced by its gradient, in precise analogy to the gradient of the delta function which appeared in the original point interaction; no additional computation is necessary.

E. Choice of cutoff function

The covariant cutoff function $f(s^2)$ or $\bar{f}(q^2)$ may be chosen to fulfill simultaneously a number of roles. First, and most important, it has to ensure convergence of the dispersion integrals and therefore of the loops. Obviously this requires that $\bar{f}(q^2)$ falls off rapidly enough for large q^2. Since we have couplings proportional to the gradients of boson fields, the most divergent loops are those with one fermion and one boson propagator, which go for large q like $\int d^4q\, q^{-1}\bar{f}(q^2)^2$. Thus it is necessary to have $\bar{f}(q^2)$ fall off faster than $q^{-3/2}$ for large q^2, for example like $1/q^2$. However, the large-q^2 behavior of $\bar{f}$ is not sufficient to make the integrals converge; it also has to be integrable on the real axis, in order to give finite results in the dispersion relation. In fact, we surely ought to require that both $f(s^2)$ and $\bar{f}(q^2)$ are smooth and monotonically falling for real values of their arguments. This mild criterion excludes the most familiar form, which is the dipole

$$\bar{f}_d(q^2)\equiv(M^2+q^2_{\rm ref})/(M^2+q^2)\ ,$$

where M is the cutoff momentum scale and $q^2_{\rm ref}$ is the reference point at which the form factor is normalized to unity (of course this normalization could be absorbed into the coupling constants which always multiply the cutoff function). Obviously the dipole form factor is singular at $q^2=-M^2$ and therefore unusuable for our purposes. A modified dipole form,

$$\bar{f}_{\rm md}(q^2)\equiv\left[\frac{M^4+(q^2_{\rm ref})^2}{M^4+(q^2)^2}\right]^{1/2} \tag{4.35}$$

fulfills the criteria and has the added advantage that its square, which appears in free-space and nuclear-matter loops, is convenient for analytical manipulations such as inverse Fourier transformation or complex integration. Of course, in many cases a more rapid falloff for large q^2 may be desirable; then a power larger than $\frac{1}{2}$ could be chosen in (4.35).

Besides the mathematical criteria introduced above, we can also try to use physical insight to help with the choice of the cutoff. In fact, we can be glad for the cutoff or smearing in our theory of hadron fields, since these fields are not elementary and therefore should not be used to describe short-distance or high-momentum phenomena. As a result neither the Lagrangian vertices Γ^0 nor the residual effective interactions U ought to be pointlike, but ought to become smooth on a distance scale of the order of hadronic sizes. The electromagnetic form factors of hadrons are dominated at long distances by the pion clouds surrounding them, which are built into our theory by the self-energies that dress our bare fields. Thus the scale on which we should expect structure is not the 0.8 fm of the rms charge radii, but rather the much shorter distance scale where quark-gluon substructure becomes evident, about 0.1–0.2 fm corresponding to a cutoff mass M of 1–2 GeV. This cutoff mass has to be chosen at the same time and by the same methods as the parameters of the Lagrangian and the residual interaction, i.e., when fitting to two-body scattering and meson production data. We hope that most of these data will turn out to be relatively insensitive to the cutoff, as they would be for a large enough cutoff mass in a renormalizable theory. However, we have to anticipate that at least the nonrenormalizable parts of the theory, notably the $\pi N \Delta$ coupling, will surely be sensitive to the cutoff mass and perhaps even to the shape of the cutoff function. Thus we may expect that the structure of the hadrons plays a role in the results obtained, especially in the case of p-wave couplings.

V. DISCUSSION OF THE MODEL

We have now completed the formal field-theoretic derivations of our model. We conclude by discussing some physical implications of the equations and approximations we have introduced. To facilitate this discussion, we first introduce some simplifications in our notation.

A. Recapitulation of main equations in simplified notation

In the formal development above we have emphasized the parallels between n-point Green's functions and n-point irreducible vertices. We have thereby introduced a number of intermediate quantities along the way which do not appear in the set of equations with which we finally shall be working. These equations contain only the mean fields, the two-point Green's functions, the three-point vertices and the four-point functions T and U. Thus we may safely dispense with the superscripts that specify the orders of the Green's functions and vertices. Also, since all quantities are now expectation values instead of field operators, we can suppress the bras and kets. We therefore introduce the following simplified notations:

$$\phi_\alpha(x) \text{ for } \langle \phi_\alpha(x) \rangle , \quad (5.1)$$

$$G_{\alpha\beta}(x_1,x_2) \text{ for } G^{(2)}_{\alpha\beta}(x_1,x_2) , \quad (5.2)$$

$$G^0_{\alpha\beta}(x_1,x_2) \text{ for } G^2_{(0)\alpha\beta}(x_1,x_2) , \quad (5.3)$$

$$\Gamma_{\alpha\beta\gamma}(x_1,x_2,x_3) \text{ for } \Gamma^{(3)}_{\alpha\beta\gamma}(x_1,x_2,x_3) , \quad (5.4)$$

$$\Gamma^0_{\alpha\beta\gamma}(x_1,x_2,x_3) \text{ for } \Gamma^{(3)}_{(0)\alpha\beta\gamma}(x_1,x_2,x_3) , \quad (5.5)$$

$$\Gamma^S_{\alpha\beta\gamma}(x_1,x_2,x_3) \text{ for } \Gamma^{(3)}_{(S)\alpha\beta\gamma}(x_1,x_2,x_3) , \quad (5.6)$$

with similar substitutions in cases involving fermions.

Applying this simplified notation, we collect the transport equations from Sec. III. The Hartree equation (3.47) reads

$$\phi_\alpha(x) = i\sum_{\alpha_i}\int dx_1 dy_1 dz_1 G^0_{\alpha\alpha_1}(x,x_1)\Bigg\{ -\sum_{b_1c_1}\Gamma^0_{\alpha_1 b_1 c_1}(x_1;y_1,z_1) n_{b_1c_1}(y_1,z_1) + \sum_{\beta_1\gamma_1}\Gamma^S_{\alpha_1\beta_1\gamma_1}(x_1,y_1,z_1)[G_{\beta_1\gamma_1}(y_1,z_1)+\phi_{\beta_1}(y_1)\phi_{\gamma_1}(z_1)]\Bigg\} \quad (5.7)$$

where, from Eq. (3.43),

$$\Gamma^S_{\alpha\beta\gamma}(x,y,z) = \Gamma^0_{\alpha\beta\gamma}(x,y,z)+\Gamma^0_{\beta\gamma\alpha}(y,z,x)+\Gamma^0_{\gamma\alpha\beta}(z,x,y) . \quad (5.8)$$

The Dyson equation (3.53) for the boson two-point Green's function is

$$G_{\alpha_2\alpha_1}(x_2,x_1) = G^0_{\alpha_2\alpha_1}(x_2,x_1) - i\int dx_3 dx_4 G^0_{\alpha_2\alpha_4}(x_2,x_4)\Pi_{\alpha_4\alpha_3}(x_4,x_3) G_{\alpha_3\alpha_1}(x_3,x_1) , \quad (5.9)$$

where the polarization function is given by Eq. (3.52) which reads

$$\begin{aligned}\Pi_{\alpha_1\alpha_3}(x_1,x_3) = {} & \sum_{\beta_1}\int dy_1 [\Gamma^S_{\alpha_1\beta_1\alpha_3}(x_1,y_1,x_3)+\Gamma^S_{\alpha_1\alpha_3\beta_1}(x_1,x_3,y_1)]\phi_{\beta_1}(y_1) \\ & - i \sum_{\beta_1\gamma_1\beta_3\gamma_3}\int dy_1 dz_1 dy_3 dz_3 \Gamma^S_{\alpha_1\beta_1\gamma_1}(x_1,y_1,z_1) G_{\beta_3\beta_1}(y_3,y_1) G_{\gamma_1\gamma_3}(z_1,z_3)\Gamma_{\alpha_3\gamma_3\beta_3}(x_3,z_3,y_3) \\ & + i \sum_{b_1c_1b_3c_3}\int dy_1 dz_1 dy_3 dz_3 \Gamma^0_{\alpha_1 b_1 c_1}(x_1,y_1,z_1) G_{b_3b_1}(y_3,y_1) G_{c_1c_3}(z_1,z_3)\Gamma_{\alpha_3c_3b_3}(x_3;z_3,y_3) .\end{aligned} \quad (5.10)$$

The Dyson equation (3.61) for the fermion two-point Green's function becomes

$$G_{a_2a_1}(x_2,x_1) = G^0_{a_2a_1}(x_2,x_1) - i\int dx_3 dx_4 G^0_{a_2a_4}(x_2,x_4)\Sigma_{a_3a_4}(x_3,x_4) G_{a_3a_1}(x_3,x_1) , \quad (5.11)$$

where the fermion self-energy from Eq. (3.59) is

$$\begin{aligned}\Sigma_{a_3a_1}(x_3,x_1) = {} & \sum_{\beta_1}\int dy_1 \Gamma^0_{\beta_1a_3a_1}(y_1;x_3,x_1)\phi_{\beta_1}(y_1) \\ & - i\sum_{\beta_1c_1\beta_3c_3}\int dy_1 dz_1 dy_3 dz_3 \Gamma_{\beta_3a_3c_3}(y_3;x_3,z_3) G_{\beta_3\beta_1}(y_3,y_1) G_{c_3c_1}(z_3,z_1)\Gamma^0_{\beta_1c_1a_1}(y_1;z_1,x_1) .\end{aligned} \quad (5.12)$$

The Dyson equation (3.76) for the dressed three-boson vertex becomes

$$\Gamma_{\alpha\beta\gamma}(x,y,z)-\Gamma^S_{\alpha\gamma\beta}(x,z,y)=\Gamma^S_{\alpha\beta\gamma}(x,y,z)-i\sum_{\beta_1\beta_2\gamma_1\gamma_2}\int dy_1dy_2dz_1dz_2[\Gamma_{\alpha\beta_1\gamma_1}(x,y_1,z_1)-\Gamma^S_{\alpha\gamma_1\beta_1}(x,z_1,y_1)]$$
$$\times G_{\beta_2\beta_1}(y_2,y_1)G_{\gamma_1\gamma_2}(z_1,z_2)U_{\beta_2\gamma_2\beta\gamma}(y_2,z_2,y,z)$$
$$+i\sum_{b_1b_2c_1c_2}\int dy_1dy_2dz_1dz_2\Gamma_{\alpha b_1c_1}(x;y_1,z_1)G_{b_2b_1}(y_2,y_1)$$
$$\times G_{c_1c_2}(z_1,z_2)U_{\beta\gamma c_2b_2}(y,z;y_2,z_2)\ , \tag{5.13}$$

while the Dyson equation (3.77) for the boson-fermion vertex reads

$$\Gamma_{\alpha bc}(x;y,z)=\Gamma^0_{\alpha bc}(x;y,z)-i\sum_{\beta_1\beta_2\gamma_1\gamma_2}\int dy_1dy_2dz_1dz_2[\Gamma_{\alpha\beta_1\gamma_1}(x,y_1,z_1)-\Gamma^S_{\alpha\gamma_1\beta_1}(x,z_1,y_1)]$$
$$\times G^{(2)}_{\beta_2\beta_1}(y_2,y_1)G_{\gamma_1\gamma_2}(z_1,z_2)U_{\beta_2\gamma_2bc}(y_2,z_2;y,z)$$
$$+i\sum_{b_1b_2c_1c_2}\int dy_1dy_2dz_1dz_2\Gamma_{\alpha b_1c_1}(x;y_1,z_1)G_{b_2b_1}(y_2,y_1)G_{c_1c_2}(z_1,z_2)U_{c_2b_2bc}(y_2,z_2;y,z)\ . \tag{5.14}$$

The one-body irreducible effective interaction of Eqs. (3.68)–(3.70) becomes

$$T'_{\beta_2\gamma_2}(y_2,z_2,y,z)\equiv T_{\beta_2\gamma_2\beta\gamma}(y_2,z_2,y,z)+i\sum_{\alpha_1\alpha}\int dx_2dx\ \Gamma_{\beta_2\gamma_2\alpha_2}(y_2,z_2,x_2)G_{\alpha_2\alpha}(x_2,x)\Gamma_{\alpha\beta\gamma}(x,y,z)\ , \tag{5.15}$$

$$T'_{\beta_2\gamma_2bc}(y_2,z_2;y,z)\equiv T_{\beta_2\gamma_2bc}(y_2,z_2;y,z)+i\sum_{\alpha_1\alpha}\int dx_2dx\ \Gamma_{\beta_2\gamma_2\alpha_2}(y_2,z_2,x_2)G_{\alpha_2\alpha}(x_2,x)\Gamma_{\alpha bc}(x;y,z)\ , \tag{5.16}$$

and

$$T'_{c_2b_2bc}(y_2,z_2,y,z)\equiv T_{c_2b_cbc}(y_2,z_2,y,z)+i\sum_{\alpha_1\alpha}\int dx_2dx\,\Gamma_{\alpha_2c_2b_2}(x_2;z_2,y_2)G_{\alpha_2\alpha}(x_2,x)\Gamma_{\alpha bc}(x;y,z)\ . \tag{5.17}$$

The two-body irreducible residual effective interaction of Eqs. (3.73)–(3.75) is given by

$$T'_{\beta_2\gamma_2\beta\gamma}(y_2,z_2,y,z)=U_{\beta_2\gamma_2\beta\gamma}(y_2,z_2,y,z)$$
$$-i\int dy_3dy_4dz_2dz_4T'_{\beta_2\gamma_2\beta_4\gamma_4}(y_2,z_2,y_4,z_4)G_{\beta_4\beta_3}(y_4,y_3)G_{\gamma_3\gamma_4}(z_3,z_4)U_{\beta_3\gamma_3\beta\gamma}(y_3,z_3,y,z)$$
$$+i\int dy_3dy_4dz_3dz_4T'_{\beta_2\gamma_2b_4c_4}(y_2,z_2;y_4,z_4)G_{b_4b_3}(y_4,y_3)G_{c_3c_4}(z_3,z_4)U_{\beta\gamma c_4b_3}(y,z,y_3,z_3)\ , \tag{5.18}$$

$$T'_{\beta_2\gamma_2bc}(y_2,z_2;y,z)=U_{\beta_2\gamma_2bc}(y_2,z_2;y,z)$$
$$-i\int dy_3dy_4dz_3dz_4T'_{\beta_2\gamma_2\beta_4\gamma_4}(y_2,z_2,y_4,z_4)G_{\beta_4\beta_3}(y_4,y_3)G_{\gamma_3\gamma_4}(z_3,z_4)U_{\beta_3\gamma_3bc}(y_3,z_3;y,z)$$
$$+i\int dy_3dy_4dz_3dz_4T'_{\beta_2\gamma_2b_4c_4}(y_2,z_2;y_4,z_4)G_{b_4b_3}(y_4,y_3)G_{c_3c_4}(z_3,z_4)U_{c_3b_3bc}(y_3,z_3,y,z)\ , \tag{5.19}$$

$$T'_{c_2b_2bc}(y_2,z_2,y,z)=U_{c_2b_2bc}(y_2,z_2,y,z)$$
$$-i\int dy_3dy_4dz_3dz_4T'_{\beta_2\gamma_2b_4c_4}(y_2,z_2;y_4,z_4)G_{\beta_4\beta_3}(y_4,y_3)G_{\gamma_3\gamma_4}(z_3,z_4)U_{\beta_3\gamma_3bc}(y_3,z_3;y,z)$$
$$+i\int dy_3dy_4dz_3dz_4T'_{c_2b_2b_4c_4}(y_2,z_2,y_4,z_4)G_{b_4b_3}(y_4,y_3)G_{c_3c_4}(z_3,z_4)U_{c_3b_3bc}(y_3,z_3,y,z)\ . \tag{5.20}$$

B. Effects of symmetries on means fields, two-point Green's functions, and vertices

The dressed two-point Green's functions and vertices of the model are given by the general equations above with the substitutions of Appendix A for the bare two-point Green's functions and vertices. These equations respect the conservation laws implied by the symmetries of the Lagrangian; however, not all the solutions of the equations possess all the symmetries of the Lagrangian. In general, we need to be able to prepare wave packets corresponding to initial conditions which break the symmetries of the Lagrangian: for example, a scattering wave function singles out the beam direction. Furthermore, we may expect to find stationary solutions of the theory which break the symmetries spontaneously, or in which the explicit breaking of one symmetry leads to solutions which break another, as when a nonzero chemical potential for baryons leads to mean-field solitons that break translational and boost symmetries.[7] Since our theory includes the RPA or iterated-loop correlations, it goes a long way toward restoring the broken symmetries by projecting out spurious modes.[4] Although our theory goes beyond the mean field, the mean field has such a prominent part in our equations that we expect similar broken-symmetry solutions to play a comparable role.

In the absence of broken symmetries, we expect only the scalar field to have a mean value $\phi_\sigma(x)=\sigma_0$, which has to be independent of the coordinates. In the presence of symmetry-breaking mean fields, the two-point Green's functions and vertices will also describe processes that violate the symmetries of the Lagrangian. For example, a position-dependent mean field will lead to two-point Green's functions and vertices that do not conserve momentum. Similarly, solutions describing nuclei with unequal numbers of neutrons and protons can have two-point Green's functions and vertices in which isospin is not conserved, so that rhos can turn into omegas; if the nucleus is not spin saturated, nucleons can also turn into deltas. Similarly, the description of spin-polarized nuclei may lead to two-point Green's functions and vertices where parity and angular momentum are exchanged with the mean fields. In these cases, all of the terms in the Dyson equations will contribute to each two-point Green's function and vertex, and no simplifications can be counted on beyond those inherent in the zero-range effective interactions.

In simpler cases, however, we may hope to find solutions which break only a few of the fundamental symmetries, such as translational and boost invariance. For example, ignoring the effects of Coulomb forces and the tiny charge and isospin dependence of the nuclear forces, the ground state of an even-even nucleus with $N=Z$ has isospin zero both globally and locally. If we restrict our considerations to nuclei with equal and even numbers of neutrons and protons, then isospin and parity should be good quantum numbers conserved by the mean fields and therefore also by the dressed two-point Green's functions and vertices. Thus the mean fields of the isovector mesons π and ρ vanish. We expect this condition to persist dynamically in the collision of two such nuclei. The isoscalar mesons σ and ω, however, do possess nonvanishing mean fields.[7] For a time-independent stationary state (such as the nuclear ground state), the mean vector field ϕ_{ω^μ} will have only a timelike component $\mu=0$, but in time-dependent situations with spatial inhomogeneities other components of the mean vector field may appear. By inserting the bare vertices of Eqs. (A3) and (A4) in the Hartree equation (5.7), we see that ϕ_σ and ϕ_{ω^μ} originate from the scalar and vector densities of the nucleons and deltas.

Since each meson of the model has a unique combination of isospin and parity, its propagation also involves admixtures of a unique set of intermediate states when these quantities are conserved. Thus there are four different parity-isospin classes of one- and two-body bosonlike excitations which propagate independently of each other except for their mutual effects on the dressed quasiparticle excitations, which appear pairwise in the intermediate states of the boson two-point Green's functions and vertices and also contribute to the mean boson fields. The propagation of the scalar field σ involves intermediate states of $N\bar{N}$, $\Delta\bar{\Delta}$, and $\pi\pi$ coupled to isospin zero and natural parity. The propagation of the isoscalar vector meson ω involves only $N\bar{N}$ and $\Delta\bar{\Delta}$ pairs. The propagation of the pseudoscalar isovector field π involves intermediate states not only of $N\bar{N}$ and $\Delta\bar{\Delta}$, but also $N\bar{\Delta}$ and $\Delta\bar{N}$; in addition, there are intermediate states of $\pi\sigma$ and $\pi\rho$ resulting from the meson-meson coupling. The isovector vector meson propagates through intermediate states of $N\bar{N}$, $\Delta\bar{\Delta}$, $N\bar{\Delta}$, $\Delta\bar{N}$, and $\pi\pi$.

C. Critique of the Migdal approximation

The central approximation of our transport theory is Migdal's phenomenological parametrization of the residual effective interaction U by a zero-range local function of the fields. We have shown how this approximation truncates the heirarchy of Dyson equations, reducing them to a finite set of coupled equations for functions of two space-time variables. A main virtue of our formulation is that U is well defined in terms of the T matrix for hadron-hadron scattering. Thus we can compare the approximate form we need to our knowledge and expectations about the exact result.

The first, most striking objection to Migdal's approximation is that it breaks crossing symmetry. The equations relating T and U reduce out one- and two-particle intermediate states only in the channel with excitations carrying no baryon number; such states are still present in the crossed channels. In principle we could try to perform the reduction in a crossing-symmetric way, adapting the parquet-diagram techniques of Lande and Smith.[19] However, the resulting equations would surely involve functions of at least three space-time variables even if the fully irreducible residual interaction (analogous to U) were approximated by a zero-range force. Thus they would be numerically tractable only in situations of high symmetry, such as free space or uniform nuclear matter. While such equations would be interesting in their own right, they do not promise to help interpret data from nuclear collisions.

Since we must reconcile ourselves to the loss of crossing symmetry, we need to evaluate the seriousness of the resulting deficiency. Here we recall the argument used in Sec. III D to choose which channel to favor: we have tried to take explicit account of the lowest-energy excitations.

To judge the effect of the resulting approximations for hadron scattering, we begin with the single-meson ex-

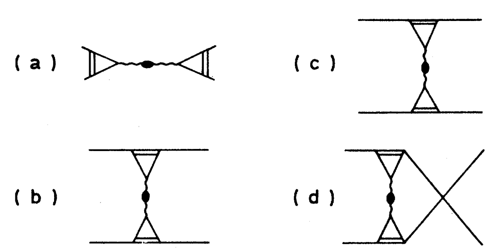

FIG. 8. Treatment of single-meson exchange graphs in the transport theory: (a) included in $N\bar{N}$ scattering, (b) not included in $N\bar{N}$ scattering, (c) included in NN scattering, (d) not included in NN scattering.

change graphs. Figure 8 shows the consequences for $N\bar{N}$ and NN scattering. The graph of Fig. 8(a) is generated explicitly by Eq. (5.17), while Fig. 8(b) has to be included in U. This is a reasonable approximation because the meson in Fig. 8(b) is far off its energy shell, making the resulting diagram a smooth function of the four-momenta of the fermions in the kinematic region of interest, which may reasonably be taken as a constant—the pure Migdal approximation—or as a slowly falling function—the result of the smearing procedure employed to regularize the loops. The corresponding crossed-channel diagrams for NN scattering are shown in Figs. 8(c) and 8(d) respectively, which together constitute the one-boson exchange force. The diagram of Fig. 8(c) is included explicitly in our approximation, while the locality approximation on Fig. 8(b) results only in a zero-range contribution to the one-boson exchange in Fig. 8(d). However, this unacceptable defect is easily remedied by taking care to use antisymmetrized wave functions when evaluating the scattering produced by the T matrix computed from the transport equations; when this is done, Fig. 8(d) will be correctly generated from Fig. 8(c). We must accept that we have to be careful to carry the antisymmetry in the wave packets, rather than allowing ourselves the conventional reliance on the exchange properties of the T matrix that normally make explicit antisymmetrization unnecessary. With this caveat, however, we see that the single-meson exchange is treated in an acceptable way.

The two-meson exchange graphs of $N\bar{N}$ and NN scattering are not treated as explicitly as the single-meson graphs, since we are missing the crossed-meson exchange diagrams that cannot be restored by antisymmetrization of the initial and final states. However, the correlated two-pion exchange is approximated by the scalar field σ; other two-meson graphs lead to forces of short enough range that they may reasonably be approximated by our smeared δ functions. The most important two-particle intermediate states appear instead in the meson self-energies, where we have taken pains to include them explicitly. Of course, these arguments show that our chosen truncation scheme is tailored to our intended applications. On the other hand, we cannot hope to give a satisfactory account of, e.g., $N\bar{N}$ annihilation without major extensions of the transport theory, since such processes reach far beyond the kinematic regime where our arguments apply.

Even in its realm of validity, the truncation of iterated meson exchanges means that our equations do not automatically enforce unitarity in the NN channel,[20] as they would if they summed ladders. Instead we have to ensure the unitarity of NN scattering as a constraint on its parametrization. This leads to a rather complicated condition on the phenomenological effective interaction.

A second objection to the Migdal type of phenomenological force is that the effective interaction U should depend in principle not only on the mean fields—or, equivalently, local density—but also on higher moments of the fields or momentum distributions, for example the temperature in an equilibrated system. Extending the model to include such dependences would be quite straightforward and ought to be included among the phenomenological investigations.

A third objection to the truncation scheme that we employ is that it does not automatically incorporate the thermodynamic or variational relationships among global and single-particle properties.[21] For example the chemical potential used to separate particle from antiparticle states in the Green's functions may not be equal to the derivative of the energy calculated from the one and two-body density matrices determined by the Green's functions and T matrices. It is unfortunate that this desirable characteristic is not guaranteed by the structure of the equations, as it would be in an exact theory; however, it can be enforced as a constraint on the density dependence of the phenomenological residual effective interaction U.

We conclude that the Migdal approximation should be not merely adequate but actually quite good for the applications we propose. Our arguments are in good agreement with decades of satisfactory experience with density-dependent local effective forces in nuclear-structure applications. There, the most serious deviation from the local approximation appears in magnetic collective excitations, which are sensitive to crossed-channel meson exchange.[22] Another major effect which we may not be able to reproduce quantitatively is pairing, which relies on high-order multiple scattering in the NN channel to build up the correlations. Thus we expect to be able to reproduce most but not all features of nuclear structure.

Our analysis of the truncation scheme suggests a reasonable simplification in the equations to be solved: we may as well limit the sums over intermediate states to only those lowest-energy states which we have argued to be specially important. Thus, for example, no important loss would be suffered by omitting most $N\bar{N}$ channels in the meson propagators, retaining only those corresponding to holes or retarded meson-nucleon scattering; of course the same states would have to be omitted from the three- and four-point vertex equations. Similarly, the relation between T' and U could be truncated to discard intermediate states of two heavy mesons, provided these were also consistently removed from the Dyson equations for the two-point Green's functions and dressed vertices. While these steps do not provide a simplification of principle, they could be of substantial practical advantage in the numerical treatment of the coupled equations.

D. Conclusions and outlook

We have presented a transport theory for hadrons which appears to be capable of giving a quantitative interpretation of nuclear collisions in the regime of laboratory energies around 1 GeV/nucleon. Since our theory includes the one-body density matrices of the hadrons, it can be used directly in the phenomenological context already developed around the Boltzmann equation; in addition our theory includes predictions for the correlations of hadrons, usually called interferometry. Before applying the equations to the case of relativistic heavy ions, it will first be necessary to address the simpler cases of hadron-hadron collisions at low and intermediate energies. There, we will fit the parameters of the Lagrangian and the free-space residual interaction to the observed scattering data. Our model incorporates barely enough parameters to permit success at this program. Corresponding to each of the four spin-isospin channels of NN scattering, we have four exchanged mesons, each described by at least three parameters: a bare meson mass, a bare meson-nucleon coupling constant, and one or more Migdal parameters giving a range to the meson-nucleon vertex. These parameters plus the meson-Δ couplings and two meson-meson couplings have to describe not only NN scattering but also πN scattering and pion production (including the physical pion mass); in addition we have at our disposal a cutoff mass scale used in smearing the vertices to regularize the loops. Success at this stage will be a good indication of the quality of our theory.

After fitting the Lagrangian and free-space Migdal parameters to these elementary processes, we plan to proceed to nuclear matter and finite nuclei. Here our model will be an extension both of the density-dependent mean-field method and of the self-consistent delta-hole model, including the binding forces on an equal footing with the explicit pion propagation. These studies will allow us to fix the low-density behavior of the Migdal parameters, and perhaps also to resolve any ambiguities that may remain from the two-body phenomenology. The Green's functions that solve the transport equations for nuclear ground states will automatically describe the scattering of nucleons and mesons from those nuclei as well as their normal modes of collective vibration. Thus the properties of the theory will be thoroughly tested before proceeding to the most demanding computations of nuclear collisions. We hope that experience gained in the preliminary studies will lead to further simplifications and clever techniques for treating our transport equations.

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APPENDIX A: BARE PROPAGATORS AND VERTICES OF THE MODEL

The bare vertices $\Gamma^{(0)}_{\alpha\beta\gamma}\equiv\Gamma^{(3)}_{(0)\alpha\beta\gamma}$ and $\Gamma^{(0)}_{abc}\equiv\Gamma^{(3)}_{(0)abc}$ are found by comparing Eqs. (3.1) and (2.6) and (2.7):

$$\Gamma^{(0)}_{\sigma\pi_i\pi_j}(x,y,z)\equiv -g_{\sigma\pi\pi}\delta_{ij}\delta(x-y)\delta(y-z)\ , \tag{A1a}$$

$$\Gamma^{(0)}_{\rho^\mu_i\pi_j\pi_k}(x,y,z)\equiv -g_{\rho\pi\pi}\epsilon_{ijk}\delta(x-y)\frac{\partial}{\partial y^\mu}\delta(y-z)\ ,$$

$$\Gamma^{(0)}_{\sigma NN}(x\,;y,z)\equiv -g_{\sigma NN}\delta(x-y)\delta(y-z)\ , \tag{A2a}$$

$$\Gamma^{(0)}_{\pi_i NN}(x\,;y,z)\equiv -ig_{\pi NN}\gamma^\mu\gamma_5\tau_i\frac{\partial}{\partial x^\mu}\delta(x-y)\delta(y-z)\ , \tag{A2b}$$

$$\Gamma^{(0)}_{\omega^\mu NN}(x\,;y,z)\equiv g_{\omega NN}\gamma^\mu\delta(x-y)\delta(y-z)\ , \tag{A2c}$$

$$\Gamma^{(0)}_{\rho^\mu_i NN}(x\,;y,z)\equiv\left[\tfrac{1}{2}g^V_{\rho NN}\gamma^\mu+g^T_{\rho NN}\sigma^{\mu\nu}\frac{\partial}{\partial x^\nu}\right]\tau_i \times\delta(x-y)\delta(y-z)\ , \tag{A2d}$$

$$\Gamma^{(0)}_{\pi_i N\Delta^\mu}(x\,;y,z)\equiv g_{\pi N\Delta}\mathcal{T}_i\frac{\partial}{\partial x^\mu}\delta(x-y)\delta(y-z)\ , \tag{A2e}$$

$$\Gamma^{(0)}_{\rho^\mu_i N\Delta^\nu}(x\,;y,z)\equiv -g_{\rho N\Delta}\gamma_5\left[\gamma^\mu\frac{\partial}{\partial x^\nu}-\gamma^\nu\frac{\partial}{\partial x^\mu}\right]\mathcal{T}_i \times\delta(x-y)\delta(y-z)\ , \tag{A2f}$$

$$\Gamma^{(0)}_{\sigma\Delta^\mu\Delta^\nu}(x\,;y,z)\equiv -g_{\sigma\Delta\Delta}g_{\mu\nu}\delta(x-y)\delta(y-z)\ , \tag{A2g}$$

$$\Gamma^{(0)}_{\pi_i\Delta^\mu\Delta^\nu}(x\,;y,z)\equiv -ig_{\pi\Delta\Delta}g_{\mu\nu}\gamma^\lambda\gamma_5 T_i\frac{\partial}{\partial x^\lambda}\delta(x-y)\delta(y-z)\ , \tag{A2h}$$

$$\Gamma^{(0)}_{\omega^\lambda\Delta^\mu\Delta^\nu}(x\,;y,z)\equiv g_{\omega\Delta\Delta}g_{\mu\nu}\gamma^\lambda\delta(x-y)\delta(y-z)\ , \tag{A2i}$$

$$\Gamma^{(0)}_{\rho^\lambda_i\Delta^\mu\Delta^\nu}(x\,;y,z)\equiv g_{\rho\Delta\Delta}g_{\mu\nu}\gamma^\lambda T_i\delta(x-y)\delta(y-z)\ , \tag{A2j}$$

remembering that the Lagrangian also includes terms Hermitian conjugate to (A2e) and (A2f).

The free two-body propagators $G^{(0)}_\alpha\equiv G^{(2)}_{(0)\alpha\alpha}$ of bosons and $G^{(0)}_a\equiv G^{(2)}_{(0)aa}$ of fermions may be found by solving Eqs. (3.51) and (3.58) in the case where $\Pi=\Sigma=0$. Identifying $\Gamma^{(2)}_{(0)}$ from Eqs. (3.45) and (3.60), using Eqs. (3.42) and (2.2)–(2.4), we find the defining equations for $G^{(2)}_{(0)}$:

$$i[\partial_\lambda\partial^\lambda+m^2_\sigma]G^{(0)}_\sigma(x,y)=\delta(y-x)\ , \tag{A3a}$$

$$i[\partial_\lambda\partial^\lambda+m^2_\pi]G^{(0)}_\pi(x,y)=\delta(x-y)\ , \tag{A3b}$$

$$i[\partial^\lambda\partial_\lambda+m^2_\rho]G^{(0)\mu\nu}_\rho(x,y)=g^{\mu\nu}\delta(x-y)\ , \tag{A3c}$$

$$i[\partial^\lambda\partial_\lambda+m^2_\omega]G^{(0)\mu\nu}_\omega(x,y)=g^{\mu\nu}\delta(x-y)\ , \tag{A3d}$$

$$-i[i\gamma^\mu\partial_\mu-M_N]G^{(0)}_N(x,y)=\delta(x-y)\ , \tag{A4a}$$

$$-i[i\gamma^\lambda\partial_\lambda-M_\Delta]G^{(0)\mu\nu}_\Delta(x,y)=g^{\mu\nu}\delta(x-y)\ , \tag{A4b}$$

where all the free Green's functions are diagonal in isospin and particle type. The free Green's functions solving Eqs. (A3) and (A4) are well known and need not be discussed further here except to note that the phase convention is determined by the inhomogeneous terms in the wave equations (A3) and (A4), and to recall that the Green's function for the Δ is proportional to the projector into the $S=\frac{3}{2}$ subspace of the $\frac{1}{2}\otimes 1$ Rarita-Schwinger representation, while the free propagators of the massive vector mesons are divergenceless leaving three independent degrees of freedom in contrast to the two transverse components of the massless vector photon.

APPENDIX B: DISPERSION RELATION FOR LOOP INTEGRALS

To obtain the dispersion relation for the loop integral, Eq. (4.10), we begin with the time Fourier transform of Eq. (4.4b). Suppressing spatial coordinates, we have

$$\tilde{L}_{\alpha\beta}(\omega)=-\frac{i}{2\pi}\int d\omega_\alpha d\omega_\beta \delta(\omega_\alpha-\omega_\beta-\omega)\tilde{G}_\alpha(\omega_\alpha)\tilde{G}_\beta(\omega_\beta)$$
$$=-\frac{i}{2\pi}\int d\omega' \tilde{G}_\alpha(\omega+\omega')\tilde{G}_\beta(\omega') \quad \text{(B1)}$$

from which we obtain

$$\mathrm{Im}\tilde{L}_{\alpha\beta}(\omega)=-\int \frac{d\omega'}{2\pi}[\mathrm{Re}\tilde{G}_\alpha(\omega+\omega')\mathrm{Re}\tilde{G}_\beta(\omega') - \mathrm{Im}\tilde{G}_\alpha(\omega+\omega')\mathrm{Im}\tilde{G}_\beta(\omega')] \ . \quad \text{(B2)}$$

$\tilde{G}_\alpha$ and $\tilde{G}_\beta$ each satisfy dispersion relations of the form

$$\mathrm{Re}\tilde{G}(\omega)=\int \frac{d\omega'}{\pi}\frac{\mathcal{P}}{\omega'-\omega}\mathrm{Im}\tilde{G}(\omega')\mathrm{sgn}(\omega'-\mu) \ , \quad \text{(B3)}$$

where μ is the chemical potential.

In order to evaluate (B2), we need the theorem of Poincare and Bertrand:[18,23]

$$\int_{-\infty}^{\infty} dx \frac{\mathcal{P}}{x-u}\int_{-\infty}^{\infty} dy \frac{\mathcal{P}}{y-x} f(x,y)=\int_{-\infty}^{\infty} dy\int_{-\infty}^{\infty} dx \frac{\mathcal{P}}{x-u}\frac{\mathcal{P}}{y-x}f(x,y)-\pi^2 f(u,u) \ , \quad \text{(B4)}$$

as well as the identity

$$\int_{-\infty}^{\infty} dx\int_{-\infty}^{\infty} dy \frac{\mathcal{P}}{y-x-\alpha}=\int_{-\infty}^{\infty} dy\int_{-\infty}^{\infty} dx \frac{\mathcal{P}}{y-x-\alpha} \ . \quad \text{(B5)}$$

Using this theorem we compute

$$\int d\omega' \mathrm{Re}\tilde{G}_\alpha(\omega+\omega')\mathrm{Re}\tilde{G}_\beta(\omega')=\frac{1}{\pi^2}\int d\omega'\int d\omega_\alpha \frac{\mathcal{P}}{\omega_\alpha-\omega'-\omega}\mathrm{Im}\tilde{G}_\alpha(\omega_\alpha)\mathrm{sgn}(\omega_\alpha-\mu_\alpha)$$
$$\times\int d\omega_\beta \frac{\mathcal{P}}{\omega_\beta-\omega'}\mathrm{Im}\tilde{G}_\beta(\omega_\beta)\mathrm{sgn}(\omega_\beta-\mu_\beta)$$
$$=-\frac{1}{\pi^2}\int d\omega_\alpha \mathrm{Im}\tilde{G}_\alpha(\omega_\alpha)\mathrm{sgn}(\omega_\alpha-\mu_\alpha)$$
$$\times\int d\omega' \frac{\mathcal{P}}{\omega'-\omega_\alpha+\omega}\int d\omega_\beta \frac{\mathcal{P}}{\omega_\beta-\omega'}\mathrm{Im}\tilde{G}_\beta(\omega_\beta)\mathrm{sgn}(\omega_\beta-\mu_\beta)$$
$$=\frac{1}{\pi^2}\int d\omega_\alpha \mathrm{Im}\tilde{G}_\alpha(\omega_\alpha)\mathrm{sgn}(\omega_\alpha-\mu_\alpha)\int d\omega_\beta \mathrm{Im}\tilde{G}_\beta(\omega_\beta)\mathrm{sgn}(\omega_\beta-\mu_\beta)$$
$$\times\int d\omega' \frac{\mathcal{P}}{\omega'-\omega_\alpha+\omega}\frac{\mathcal{P}}{\omega_\beta-\omega'}$$
$$+\frac{\pi^2}{\pi^2}\int d\omega_\alpha \mathrm{Im}\tilde{G}_\alpha(\omega_\alpha)\mathrm{sgn}(\omega_\alpha-\mu_\alpha)\mathrm{Im}\tilde{G}_\beta(\omega_\alpha-\omega)\mathrm{sgn}(\omega_\alpha-\omega-\mu_\beta)$$
$$=\int d\omega_\alpha \mathrm{Im}\tilde{G}_\alpha(\omega_\alpha)\mathrm{sgn}(\omega_\alpha-\mu_\alpha)\mathrm{Im}\tilde{G}_\beta(\omega_\alpha-\omega)\mathrm{sgn}(\omega_\alpha-\omega-\mu_\beta) \ , \quad \text{(B6)}$$

so that

$$\mathrm{Im}\tilde{L}_{\alpha\beta}(\omega)=\int \frac{d\omega'}{2\pi}\mathrm{Im}\tilde{G}_\alpha(\omega+\omega')\mathrm{Im}\tilde{G}_\beta(\omega')[1-\mathrm{sgn}(\omega+\omega'-\mu_\alpha)\mathrm{sgn}(\omega'-\mu_\beta)]$$
$$=\int \frac{d\omega_\alpha d\omega_\beta}{2\pi}\delta(\omega_\alpha-\omega_\beta-\omega)\mathrm{Im}\tilde{G}_\alpha(\omega_\alpha)\mathrm{Im}\tilde{G}_\beta(\omega_\beta)[1-\mathrm{sgn}(\omega_\alpha-\mu_\alpha)\mathrm{sgn}(\omega_\beta-\mu_\beta)] \ . \quad \text{(B7)}$$

Noting that the integrand vanishes unless ω_α and ω_β are on opposite sides of their respective chemical potentials, we confirm that only particle-antiparticle (hole) contributions survive. Equation (4.11) is a simple rewriting of Eq. (B7).

Turning now to the real part of the loop, we have from Eq. (B1)

$$\mathrm{Re}\tilde{L}_{\alpha\beta}(\omega)=\int \frac{d\omega_\alpha d\omega_\beta}{2\pi}\delta(\omega_\alpha-\omega_\beta-\omega)[\mathrm{Im}\tilde{G}_\alpha(\omega_\alpha)\mathrm{Re}\tilde{G}_\beta(\omega_\beta)+\mathrm{Re}\tilde{G}_\alpha(\omega_\alpha)\mathrm{Im}\tilde{G}_\beta(\omega_\beta)]$$
$$=\int \frac{d\omega_\alpha d\omega_\beta}{2\pi}\delta(\omega_\alpha-\omega_\beta-\omega)\left[\mathrm{Im}\tilde{G}_\alpha(\omega_\alpha)\int \frac{d\omega'_\beta}{\pi}\frac{\mathcal{P}}{\omega'_\beta-\omega_\beta}\mathrm{Im}\tilde{G}_\beta(\omega'_\beta)\mathrm{sgn}(\omega'_\beta-\mu_\beta)\right.$$
$$\left.+\int \frac{d\omega'_\alpha}{\pi}\frac{\mathcal{P}}{\omega'_\alpha-\omega_\alpha}\mathrm{Im}\tilde{G}_\alpha(\omega'_\alpha)\mathrm{sgn}(\omega'_\alpha-\mu_\alpha)\mathrm{Im}\tilde{G}_\beta(\omega_\beta)\right] \ .$$

Using the δ function to integrate over ω_β and ω_α in the first and second terms, respectively, we find after substituting variables

$$\mathrm{Re}\tilde{L}_{\alpha\beta}(\omega)=-\int \frac{d\omega_\alpha d\omega_\beta}{2\pi^2}\frac{\mathcal{P}}{\omega_\alpha-\omega_\beta-\omega}\mathrm{Im}\tilde{G}_\alpha(\omega_\alpha)\mathrm{Im}\tilde{G}_\beta(\omega_\beta)[\mathrm{sgn}(\omega_\beta-\mu_\beta)-\mathrm{sgn}(\omega_\alpha-\mu_\alpha)]$$
$$=-\int \frac{d\omega_\alpha d\omega_\beta}{\pi^2}\frac{\mathcal{P}}{\omega_\alpha-\omega_\beta-\omega}\mathrm{Im}\tilde{G}_\alpha(\omega_\alpha)\mathrm{Im}\tilde{G}_\beta(\omega_\beta)[\theta(\mu_\alpha-\omega_\alpha)\theta(\omega_\beta-\mu_\beta)-\theta(\omega_\alpha-\mu_\alpha)\theta(\mu_\beta-\omega_\beta)]\ . \quad \text{(B8)}$$

As before, we confirm the absence of the particle-particle terms.

Finally, we confirm that the loop obeys a dispersion relation of the form (B3). Using Eq. (4.11) we compute

$$\int \frac{d\omega'}{\pi}\frac{\mathcal{P}}{\omega'-\omega}\mathrm{Im}\tilde{L}_{\alpha\beta}(\omega')\mathrm{sgn}(\omega'-\mu_\alpha+\mu_\beta)=2\int\frac{d\omega'}{\pi}\int\frac{d\omega_\alpha d\omega_\beta}{2\pi}\frac{\mathcal{P}}{\omega'-\omega}\delta(\omega_\alpha-\omega_\beta-\omega')$$
$$\times\mathrm{sgn}(\omega'-\mu_\alpha+\mu_\beta)\mathrm{Im}\tilde{G}_\alpha(\omega_\alpha)\mathrm{Im}\tilde{G}_\beta(\omega_\beta)$$
$$\times[\theta(\omega_\alpha-\mu_\alpha)\theta(\mu_\beta-\omega_\beta)+\theta(\mu_\alpha-\omega_\alpha)\theta(\omega_\beta-\mu_\beta)]\ ,$$
$$=\int\frac{d\omega_\alpha d\omega_\beta}{\pi^2}\frac{\mathcal{P}}{\omega_\alpha-\omega_\beta-\omega}\mathrm{sgn}(\omega_\alpha-\omega_\beta-\mu_\alpha+\mu_\beta)\mathrm{Im}\tilde{G}_\alpha(\omega_\alpha)\mathrm{Im}\tilde{G}_\beta(\omega_\beta)$$
$$\times[\theta(\omega_\alpha-\mu_\alpha)\theta(\mu_\beta-\omega_\beta)+\theta(\mu_\alpha-\omega_\alpha)\theta(\omega_\beta-\mu_\beta)]$$
$$=\int\frac{d\omega_\alpha d\omega_\beta}{\pi^2}\frac{\mathcal{P}}{\omega_\alpha-\omega_\beta-\omega}\mathrm{Im}\tilde{G}_\alpha(\omega_\alpha)\mathrm{Im}\tilde{G}_\beta(\omega_\beta)$$
$$\times[\theta(\omega_\alpha-\mu_\alpha)\theta(\mu_\beta-\omega_\beta)-\theta(\mu_\alpha-\omega_\alpha)\theta(\omega_\beta-\mu_\beta)]\ . \quad \text{(B9)}$$

Comparison with Eq. (B8) verifies that the loop fulfills the dispersion relation (B3) characteristic of the propagators, with the appropriate choice of chemical potential as discussed in the text.

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