

Nucleon emission via electromagnetic excitation in relativistic nucleus-nucleus collisions: Reanalysis of the Weizsäcker-Williams method

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(Received 23 February 1989)

Previous analyses of the comparison of Weizsäcker-Williams theory to experiment for nucleon emission via electromagnetic excitations in nucleus-nucleus collisions have not been definitive because of different assumptions concerning the value of the minimum impact parameter. This situation is corrected by providing criteria that allow one to make definitive statements concerning agreement or disagreement between Weizsäcker-Williams theory and experiment.

I. INTRODUCTION

Collisions between relativistic nuclei can occur via the strong or electromagnetic interaction. There is an enormous literature on processes induced by the strong force,¹ but relatively few studies have been carried out on the electromagnetic (EM) aspects of relativistic nucleus-nucleus collisions.²⁻²⁵ This situation is surprising given the richness of applications of EM effects. These effects are of importance for the following reasons: (i) EM interactions between relativistic nuclei are interesting in their own right; (ii) they will form a significant background to the formation of a quark-gluon plasma at ultrarelativistic energies; there are other applications in physics such as (iii) subthreshold pion production,³ (iv) astrophysical applications,⁴ (v) interference effects between strong and EM amplitudes,⁵ (vi) studies of virtual photon theory,^{4,6-10} and (vii) applications in space radiation effects.¹¹ Bertulani and Baur²⁵ have written an outstanding review article on EM effects in nucleus-nucleus collisions to which the reader is referred.

This paper is concerned with nucleon emission via electromagnetic dissociation in relativistic nucleus-nucleus collisions. The first experiment of this kind was performed by Heckman and Lindstrom¹⁵ looking at excitations in ¹²C and ¹⁶O projectiles at energies of 1.05 and 2.1 GeV/nucleon on a variety of targets (¹²C, ²⁷Al, ⁶⁴Cu, ¹⁰⁸Ag, and ²⁰⁸Pb). Measured EM cross sections for nucleon emission ranged from 0 to 50 mb. Olson *et al.*¹³ later measured excitation of ¹⁸O projectiles at 1.7 GeV/nucleon on ⁴⁸Ti, ²⁰⁸Pb, and ²³⁸U with cross sections up to 140 mb. Studies of ¹⁹⁷Au and ⁵⁹Co target excitation^{14,20-22} were later reported with cross sections all the way up to 1970 mb for ¹³⁹La projectiles at 1.26 GeV/nucleon. Lighter projectiles were also used^{14,20-23} with smaller cross sections. All studies mentioned so far have been for projectile energies less than or equal to 2.1 GeV/nucleon. The measurements were made at the Berkeley Bevalac. However, some very interesting measurements have also been made for ¹⁹⁷Au target excitation at the CERN Super Proton Synchrotron (SPS) using ¹⁶O projectiles at 60 and 200 GeV/nucleon with cross sections of 820 and 440 mb, respectively. All these data

are summarized in Tables I and III.

The authors of the preceding experiments have generally made a comparison of their data to theoretical predictions based on the Weizsäcker-Williams (WW) method of virtual quanta.^{6,12,15,25} The basic idea is that the virtual photon spectrum $N(E)$ of one nucleus is calculated from WW theory.⁶ This is folded into the photonuclear cross section $\sigma_\nu(E)$ for processes induced in the excited nucleus and then numerically integrated over energy to give the total EM nucleus-nucleus cross section

$$\sigma_{EM} = \int \sigma_\nu(E) N(E) dE. \quad (1)$$

Expressions for $N(E)$ are given by Jackson⁶ for the WW theory. These expressions include a minimum impact parameter b , below which EM interactions are not supposed to take place, each interaction proceeding via the much stronger nuclear force. One might naively expect b to be just the sum of the two nuclear radii. Note also that in WW theory $N(E)$ is the same for all EM multipoles²⁵ and so $\sigma_\nu(E)$ does not need to be divided into its constituent multipoles. It is partly for this reason that the present paper concerns itself exclusively with WW theory. Alternative theories for $N(E)$ (Refs. 7, 8 and 25) require that $\sigma_\nu(E)$ be divided into its constituent multipoles which is very involved and beyond the scope of the present paper, although work is proceeding in this direction. Here I wish to analyze WW theory only.

Agreement between WW theory and experiment has generally been claimed to be good^{13-15,20-23,25} and upon reading the literature on the subject one is left with the impression that WW theory is an accurate theoretical tool. However, if one examines the calculations and data more carefully one is led into some serious doubts. The following concerns arise.

(1) Table I provides a list of some experimental cross sections versus the theoretical calculations presented by various authors. As can be seen there is, in fact, very noticeable disagreement between experiment and WW theory, which might lead one to conclude that WW theory is not valid at all.

(2) Each research group^{13-15,20-23} uses a different procedure for determining the minimum impact parameter, and hence agreement between theory and experiment de-

TABLE I. Selected data and theoretical calculations showing marked disagreement between theory and experiment.

Projectile	Target	Energy (GeV/nucleon)	Final state	σ_{expt}	Reference	σ_{theory}
				(mb) from reference		(mb) from reference
^{18}O	Ti	1.7	^{17}O	8.7 ± 2.7	13	12.5
^{18}O	Ti	1.7	^{17}N	-0.5 ± 1.0	13	2.4
^{18}O	Pb	1.7	^{16}O	65.2 ± 2.3	13	55.2
^{18}O	U	1.7	^{17}O	140.8 ± 4.1	13	167
^{18}O	U	1.7	^{17}N	25.1 ± 1.6	13	29.2
^{18}O	U	1.7	^{16}O	74.3 ± 1.7	13	68.1
^{12}C	^{197}Au	2.1	^{196}Au	75 ± 14	20	45
^{20}Ne	^{197}Au	2.1	^{196}Au	153 ± 18	20	121
^{139}La	^{197}Au	1.26	^{196}Au	1970 ± 130	22	2340
^{56}Fe	^{89}Y	1.7	^{88}Y	217 ± 20	20	248
^{56}Fe	^{59}Co	1.7	^{58}Co	88 ± 14	20	122
^{139}La	^{59}Co	1.7	^{58}Co	280 ± 40	22	430
^{16}O	^{197}Au	60	^{196}Au	280 ± 30	21	220
^{16}O	^{197}Au	200	^{196}Au	440 ± 40	21	300
^{20}Ne	^{197}Au	2.1	^{195}Au	49 ± 15	23	14
^{40}Ar	^{197}Au	1.8	^{195}Au	76 ± 18	23	38
^{139}La	^{197}Au	1.26	^{195}Au	73 ± 13	23	238

pends to some extent on whose impact parameter one chooses. The differences in this parameter are, in fact, large enough to change agreement between theory and experiment.

(3) The calculation of the EM nucleus-nucleus cross section depends heavily on the photonuclear cross section $\sigma_{\nu}(E)$ used in Eq. (1). These authors have always used experimental data for the photonuclear cross section $\sigma_{\nu}(E)$. The trouble is that various experimental data for $\sigma_{\nu}(E)$ for a particular nucleus are often in disagreement²⁶ and this can lead to significant differences in the calculated nucleus-nucleus EM cross section depending on whose data one chooses for $\sigma_{\nu}(E)$.

For these reasons it was decided to reanalyze the WW method applying a single method to all existing data paying particular attention to the following points.

(a) Some recent articles have made a detailed study of the conflicting experimental photonuclear data^{27,28} and have made recommendations concerning what are the correct data. The $\sigma_{\nu}(E)$ used in the present work is that recommended by these studies^{27,28} for ^{12}C , ^{16}O , and ^{197}Au . For ^{18}O and ^{59}Co the data of Refs. 29 and 30 are expected to be very accurate. The two sets of conflicting data^{31,32} for ^{89}Y are both used herein for two separate nucleus-nucleus calculations.

(b) Some incorrect calculations are listed correctly herein. (See Refs. 33 and 34 for a discussion of these corrections.)

(c) To get around the problem of different possible choices of the minimum impact parameter b , it was decided to use it simply as an adjustable parameter fitted so that the theoretical WW cross section is equal to the experimental one. If b is a reasonable (unreasonable) value then one can definitely say that WW theory does (does not) agree with experiment. A reasonable value would be the sum of the two nuclear radii, whereas an unreasonable value might be close to 0 or very much larger than

the sum of the radii. Note that there will be cases where one cannot make a definite statement concerning agreement between theory and experiment. However, the advantage of the foregoing criterion is that it does provide for an unambiguous comparison between theory and experiment. One will not be left wondering whether the agreement or disagreement with experiment is due to a choice of parameters. The value of the fitted parameter b will provide either a definitive conclusion regarding agreement or disagreement and in the cases in which such a conclusion is not possible it will be clear why such a conclusion is, in fact, not possible.

In summary, the present work goes beyond other studies^{13-15,20-23,25} in three respects. First, it provides a comprehensive comparison to all existing nucleon emission data. Second, problems due to inaccurate photonuclear input data are avoided. Third, obtaining the value of b needed to fit the data enables one to make definitive statements concerning the agreement with WW theory, thus sidestepping the problem of comparing the calculations of various authors using various values of b to calculate the cross section.

II. SELECTION OF PHOTONUCLEAR REACTION CROSS SECTION DATA

Following Refs. 1 and 2, $\sigma(\gamma, jn)_a$ is defined as the cross section in which j and only j neutrons are emitted and

$$\sigma(\gamma, n) \equiv \sigma(\gamma, 1n)_a \quad (2)$$

by definition. Also,

$$\sigma(\gamma, 1n) = \sigma(\gamma, n) + \sigma(\gamma, pn) + \dots, \quad (3)$$

$$\sigma(\gamma, 2n) = \sigma(\gamma, 2n)_a + \sigma(\gamma, p2n) + \dots, \quad (4)$$

where the $+\dots$ indicate unimportant additional contri-

butions (for present purposes). The total photoneutron cross section is

$$\sigma(\gamma, n_i) = \sigma(\gamma, 1n) + \sigma(\gamma, 2n) + \dots \quad (5)$$

In nucleus-nucleus collisions what is typically measured is $^{197}\text{Au}(\text{RHI}, X)^{196}\text{Au}$ for example, implying that $\sigma(\gamma, n)$ only is needed (RHI means relativistic heavy ion). Unfortunately, what photonuclear experimentalists usually measure is $\sigma(\gamma, 1n)$ or $\sigma(\gamma, n_i)$. Thus we must discuss how to arrive at $\sigma(\gamma, n)$ alone. Furthermore, many photonuclear experiments provide contradictory data. For this reason I have relied heavily on Refs. 26–28 and have followed their recommendations in the selection of data which are summarized in Table II.

How to obtain individual (γ, n) and (γ, p) data for various nuclei is now discussed.

$^{197}\text{Au}(\gamma, n)$. Following the suggestion of Berman *et al.*²⁷ I have used the data of Ref. 35 but multiplied them by a factor of 0.93 (the data were actually extrapolated out to where the cross section is zero). The data actually used (Fig. 2, Ref. 35) represent $\sigma(\gamma, 1n)$, but because of the large Coulomb barrier, $\sigma(\gamma, 1n)$ will equal $\sigma(\gamma, n)$.

$^{89}\text{Y}(\gamma, n)$. The data of Refs. 31 and 32 for ^{89}Y are somewhat different requiring separate calculations for each set of data. The data are taken from Fig. 8(b) of Ref. 32 and Fig. 3 of Ref. 31, both of which represent $\sigma(\gamma, 1n)$. As for ^{197}Au , $\sigma(\gamma, 1n)$ is equal to $\sigma(\gamma, n)$ for ^{89}Y . Following the suggestion of Berman *et al.*,²⁷ the data of Ref. 31 have been multiplied by 0.82.

$^{59}\text{Co}(\gamma, n)$. The data of Fig. 3(b), Ref. 30 are used and again $\sigma(\gamma, 1n)$ is measured but is very nearly equal to $\sigma(\gamma, n)$ for ^{59}Co .

$^{16}\text{O}(\gamma, n)$. The normalized data presented by Fuller²⁸ represent $\sigma(\gamma, n_{\text{tot}})$. However this is very close to $\sigma(\gamma, 1n)$ because $\sigma(\gamma, 2n)$ is only about 2% of $\sigma(\gamma, 1n)$ beyond 30 MeV. [This can be seen from Fig. 19A(d) of Ref. 26]. The data of Fuller²⁸ extend out to 37 MeV and, in principle, there are data beyond this that should be estimated and included. However, various measurements of this higher-energy data are not in agreement with the normalized data of Fuller, so it was decided to simply only use Fuller's data up to 37 MeV. The neglect of the higher-energy photonuclear cross section is compensated by the fact that $\sigma(\gamma, 1n)$ includes not only $\sigma(\gamma, n)$ but also a small component of $\sigma(\gamma, pn)$ which we have not

subtracted. Further, $N(E)$ is quite small in this energy region and so the error in the resulting EM nucleus-nucleus cross section will not be more than a few percent.

$^{18}\text{O}(\gamma, n)$. Here the data of Ref. 29, Fig. 3(b), are used which is actually $\sigma(\gamma, 1n)$. This cross section is used for $\sigma(\gamma, n)$ in the present work with no correction. However, based on the above discussion for ^{16}O , the (γ, np) correction is expected to be very small. Furthermore, the nucleus-nucleus calculation for ^{18}O presented herein is only done at 1.7 GeV/nucleon where the photon spectrum dominates at low energy, so that a small uncertainty at higher energy, in the region of $\sigma(\gamma, np)$ will again not affect the results.

$^{12}\text{C}(\gamma, n)$. The data of Fig. 2.1, Ref. 28, which are used here represent $\sigma(\gamma, n_{\text{tot}})$. These normalized data represent the original data of Fultz *et al.*³⁶ multiplied by 1.17 which have been used to get the data from 30 to 37.5 MeV. Dietrich and Berman²⁶ point out that $\sigma(\gamma, 2n)$ is measured to be consistent with zero so that $\sigma(\gamma, n_{\text{tot}}) \approx \sigma(\gamma, 1n)$. Again I have no way of subtracting $\sigma(\gamma, np)$ but this will not affect the results presented herein for the same reason as already discussed for $^{18}\text{O}(\gamma, n)$ and $^{16}\text{O}(\gamma, n)$.

$^{16}\text{O}(\gamma, p)$. I have used $\sigma(\gamma, p_i)$ from Fig. 4.4, Ref. 28, up to 30 MeV. The data missing beyond 30 MeV affect the results of the present work by only a few percent. Following the preceding discussion comparing $\sigma(\gamma, 2n)$ to $\sigma(\gamma, n_i)$ for ^{16}O , the contribution of $\sigma(\gamma, 2p)$ to $\sigma(\gamma, p_i)$ is expected to be negligible, as is the contribution of $\sigma(\gamma, np)$.

$^{18}\text{O}(\gamma, p)$. This is given directly in Fig. 3(a) of Ref. 29.

$^{12}\text{C}(\gamma, p)$. Figure 2.1 of Ref. 28 gives $\sigma(\gamma, p_i)$ up to 30 MeV. The same considerations for $^{16}\text{O}(\gamma, p)$ were followed for $^{12}\text{C}(\gamma, p)$.

$^{18}\text{O}(\gamma, 2n)$. This is given directly in Fig. 3(c) of Ref. 29.

$^{197}\text{Au}(\gamma, 2n)$. As for $^{197}\text{Au}(\gamma, n)$ (as previously stated) the data of Ref. 35, multiplied by a factor of 0.93, were used.

$^{59}\text{Co}(\gamma, 2n)$. These data were taken from Ref. 30.

III. CRITERIA FOR EVALUATING THEORETICAL ANALYSES

The only adjustable parameter that appears in the present theory is the minimum impact parameter b , below which the reaction proceeds via the strong interaction. This has been the subject of much discussion and

TABLE II. Choice of photonuclear reaction cross sections.

Nucleus	Data	Remarks
^{197}Au	Refs. 27 and 35	Data of Ref. 35 are multiplied by 0.93 following suggestions of Ref. 27.
^{89}Y	Refs. 31 and 32	Data for these two references differ. Thus two sets of calculations for both data sets are performed. Data of Ref. 31 are multiplied by 0.82 following suggestions of Ref. 27.
^{59}Co	Ref. 30	The only existing data.
^{18}O	Ref. 29	The most accurate data that exist.
^{16}O	Ref. 28	The normalized data presented in Ref. 28 are used.
^{12}C	Refs. 28 and 36	The normalized data presented in Ref. 28 are used.

TABLE III. Overlap parameters d required to fit data. Zero overlap corresponds to a minimum impact parameter equal to the sum of the nuclear radii. Where d is listed as 0, it means that σ calculated for $b = R_{0.1}(P) + R_{0.1}(T)$ already agrees within error. Data are from Refs. 13, 15, 20, 21, 22, and 23.

Projectile	$R_{0.1}(P)$ (fm)	Target	$R_{0.1}(T)$ (fm)	Energy (GeV/nucleon)	Final state	σ_{expt} (mb)	σ calculated for $b = R_{0.1}(P) + R_{0.1}(T)$ (i.e., $d=0$)			d (fm)	Percentage of $R_{0.1}(P) + R_{0.1}(T)$
							$\sigma(b)$ (mb)	$R_{0.1}(P) + R_{0.1}(T)$ (fm)			
^{12}C	3.30	Pb	7.83	2.1	^{11}C	51 ± 18	47	11.13	0	0	
^{12}C	3.30	Pb	7.83	2.1	^{11}B	50 ± 25	68	11.13	0	0	
^{12}C	3.30	Pb	7.83	1.05	^{11}C	39 ± 24	28	11.13	0	0	
^{12}C	3.30	Pb	7.83	1.05	^{11}B	50 ± 25	42	11.13	0	0	
^{16}O	3.68	Pb	7.83	2.1	^{15}O	50 ± 24	59	11.51	0	0	
^{16}O	3.68	Pb	7.83	2.1	^{15}N	96 ± 26	111	11.51	0	0	
^{12}C	3.30	Ag	6.37	2.1	^{11}C	21 ± 10	18	9.67	0	0	
^{12}C	3.30	Ag	6.37	2.1	^{11}B	18 ± 13	26	9.67	0	0	
^{12}C	3.30	Ag	6.37	1.05	^{11}C	21 ± 10	12	9.67	0	0	
^{12}C	3.30	Ag	6.37	1.05	^{11}B	25 ± 19	17	9.67	0	0	
^{16}O	3.68	Ag	6.37	2.1	^{15}O	26 ± 13	23	10.05	0	0	
^{16}O	3.68	Ag	6.37	2.1	^{15}N	30 ± 16	42	10.05	0	0	
^{12}C	3.30	Cu	5.45	2.1	^{11}C	10 ± 7	8	8.75	0	0	
^{12}C	3.30	Cu	5.45	2.1	^{11}B	4 \pm 8	11	8.75	0	0	
^{12}C	3.30	Cu	5.45	1.05	^{11}C	9 \pm 8	5	8.75	0	0	
^{12}C	3.30	Cu	5.45	1.05	^{11}B	5 \pm 8	8	8.75	0	0	
^{16}O	3.68	Cu	5.45	2.1	^{15}O	9 \pm 8	10	9.13	0	0	
^{16}O	3.68	Cu	5.45	2.1	^{15}N	15 \pm 8	18	9.13	0	0	
^{12}C	3.30	Al	4.09	2.1	^{11}C	0 \pm 5	2	7.39	0	0	
^{12}C	3.30	Al	4.09	2.1	^{11}B	0 \pm 5	3	7.39	0	0	
^{12}C	3.30	Al	4.09	1.05	^{11}C	1 \pm 6	1	7.39	0	0	
^{12}C	3.30	Al	4.09	1.05	^{11}B	1 \pm 7	2	7.39	0	0	
^{16}O	3.68	Al	4.09	2.1	^{15}O	0 \pm 5	2	7.77	0	0	
^{16}O	3.68	Al	4.09	2.1	^{15}N	-1 \pm 9	4	7.77	0	0	
^{12}C	3.30	C	3.30	2.1	^{11}C	-2 \pm 5	0.4	6.60	0	0	
^{12}C	3.30	C	3.30	2.1	^{11}B	-1 \pm 4	0.6	6.60	0	0	
^{12}C	3.30	C	3.30	1.05	^{11}C	-2 \pm 5	0.3	6.60	0	0	
^{12}C	3.30	C	3.30	1.05	^{11}B	-2 \pm 5	0.5	6.60	0	0	
^{16}O	3.68	C	3.30	2.1	^{15}O	-1 \pm 4	0.5	6.98	0	0	
^{16}O	3.68	C	3.30	2.1	^{15}N	-1 \pm 4	1	6.98	0	0	
^{18}O	3.78	Ti	5.00	1.7	^{17}O	8.7 \pm 2.7	15	8.78	-5.7 $^{+3.1}_{-4.8}$	-65 $^{+35}_{-35}$	
^{18}O	3.78	Ti	5.00	1.7	^{16}O	6.3 \pm 2.5	6	8.78	0	0	
^{18}O	3.78	Ti	5.00	1.7	^{17}N	-0.5 \pm 1.0	3	8.78	-15.5 a	-177 a	
^{18}O	3.78	Pb	7.83	1.7	^{17}O	136 \pm 2.9	155	11.61	-1.5 $^{+0.2}_{-0.3}$	-13 $^{+2}_{-3}$	
^{18}O	3.78	Pb	7.83	1.7	^{16}O	65.2 \pm 2.3	63	11.61	0	0	
^{18}O	3.78	Pb	7.83	1.7	^{17}N	20.2 \pm 1.8	28	11.61	-2.8 \pm 0.8	-24 \pm 7	
^{18}O	3.78	U	8.09	1.7	^{17}O	140.8 \pm 4.1	191	11.87	-3.6 \pm 0.4	-30 \pm 3	

TABLE III. (Continued).

Projectile	$R_{0.1}(P)$ (fm)	Target	$R_{0.1}(T)$ (fm)	Energy (GeV/nucleon)	Final state	σ_{expt} (mb)	σ calculated for $b = R_{0.1}(P) + R_{0.1}(T)$ (i.e., $d=0$)			d	Percentage of $R_{0.1}(P) + R_{0.1}(T)$
							$\sigma(b)$ (mb)	$R_{0.1}(P) + R_{0.1}(T)$ (fm)	(fm)		
^{18}O	3.78	U	8.09	1.7	^{16}O	74.3 ± 1.7	77	11.87	$-0.3^{+0.3}_{-0.3}$		-3^{+3}_{-3}
^{18}O	3.78	U	8.09	1.7	^{17}N	25.1 ± 1.6	34	11.87	$-2.6^{+0.5}_{-0.6}$		-22^{+4}_{-5}
^{18}O	3.78	Cu	5.45	1.7	^{16}O	9.0 ± 3.5	10	9.23	0		0
^{18}O	3.78	Sn	6.58	1.7	^{16}O	27.5 ± 4.0	27	10.36	0		0
^{18}O	3.78	W	7.69	1.7	^{16}O	50.0 ± 4.3	52	11.47	0		0
^{12}C	3.30	^{197}Au	7.56	2.1	^{196}Au	75 ± 14	38	10.86	$7.5^{+1.2}_{-1.8}$		69^{+11}_{-17}
^{12}C	3.30	^{197}Au	7.56	2.1	^{195}Au	9 ± 17	6	10.86	0		0
^{20}Ne	4.00	^{197}Au	7.56	2.1	^{196}Au	153 ± 18	100	11.56	$5.4^{+1.1}_{-1.5}$		47^{+10}_{-13}
^{20}Ne	4.00	^{197}Au	7.56	2.1	^{195}Au	49 ± 15	15	11.56	$10.1^{+0.8}_{-2.0}$		87^{+7}_{-17}
^{40}Ar	4.72	^{197}Au	7.56	1.8	^{196}Au	348 ± 34	289	12.28	$2.5^{+1.1}_{-1.4}$		20^{+9}_{-11}
^{40}Ar	4.72	^{197}Au	7.56	1.8	^{195}Au	76 ± 18	40	12.28	$6.2^{+1.7}_{-2.4}$		51^{+14}_{-20}
^{56}Fe	5.24	^{197}Au	7.56	1.7	^{196}Au	601 ± 54	565	12.80	0		0
^{56}Fe	5.24	^{197}Au	7.56	1.7	^{195}Au	73 ± 13	77	12.80	0		0
^{139}La	6.89	^{197}Au	7.56	1.26	^{196}Au	1970 ± 130	2076	14.45	0		0
^{139}La	6.89	^{197}Au	7.56	1.26	^{195}Au	335 ± 49	257	14.45	$2.4^{+1.3}_{-1.4}$		17^{+9}_{-10}
^{16}O	3.68	^{197}Au	7.56	60	^{196}Au	280 ± 30	215	11.24	$8.0^{+1.4}_{-2.5}$		71^{+13}_{-22}
^{16}O	3.68	^{197}Au	7.56	200	^{196}Au	440 ± 40	278	11.24	$10.7^{+0.3}_{-0.6}$		95^{+3}_{-3}
^{12}C	3.30	$^{89}\text{Y}^b$	6.02	2.1	^{88}Y	9 ± 12	11	9.32	0		0
^{20}Ne	4.00	$^{89}\text{Y}^b$	6.02	2.1	^{88}Y	43 ± 12	29	10.02	$4.2^{+2.0}_{-3.4}$		42^{+20}_{-34}
^{40}Ar	4.72	$^{89}\text{Y}^b$	6.02	1.8	^{88}Y	132 ± 17	82	10.74	$4.8^{+1.0}_{-1.2}$		36^{+10}_{-12}
^{56}Fe	5.24	$^{89}\text{Y}^b$	6.02	1.7	^{88}Y	217 ± 20	159	11.26	$3.3^{+0.8}_{-1.0}$		29^{+7}_{-9}
^{12}C	3.30	^{59}Co	5.33	2.1	^{58}Co	6 ± 9	7	8.63	0		0
^{12}C	3.30	^{59}Co	5.33	2.1	^{57}Co	6 ± 4	1	8.63	$8.5^{+0.1}_{-2.7}$		98^{+1}_{-31}
^{20}Ne	4.00	^{59}Co	5.33	2.1	^{58}Co	32 ± 11	18	9.33	$5.4^{+1.8}_{-3.7}$		58^{+19}_{-40}
^{20}Ne	4.00	^{59}Co	5.33	2.1	^{57}Co	3 ± 5	2	9.33	0		0
^{56}Fe	5.24	^{59}Co	5.33	1.7	^{58}Co	88 ± 14	98	10.57	0		0
^{56}Fe	5.24	^{59}Co	5.33	1.7	^{57}Co	13 ± 6	12	10.57	0		0
^{139}La	6.89	^{59}Co	5.33	1.26	^{58}Co	280 ± 40	339	12.22	$-1.8^{+1.3}_{-1.6}$		15^{+13}_{-13}
^{139}La	6.89	^{59}Co	5.33	1.26	^{57}Co	32 ± 16	36	12.22	0		0

^aFitted to $\sigma = 0.5$ mb only. Not included in 30% criterion.^bFor ^{89}Y two calculations are presented using the photonic data of Lepretre (Ref. 31) (L), multiplied by 0.82, and Berman (Ref. 32) (B).

every author chooses his own form. For instance, Heckman and Lindstrom¹⁵ and Olson *et al.*¹³ choose a form

$$b = R_{0.1}(P) + R_{0.1}(T) - d, \quad (6)$$

where $R_{0.1}$ is the 10 percent charge-density radii of the projectile and target and d is a parameter measuring the amount of overlap. These authors choose values of d ranging from 0 up to 3 fm. Hill *et al.*^{14,20-23} choose

$$b = r_0 [A_p^{1/3} + A_T^{1/3} - X(A_p^{-1/3} + A_T^{-1/3})], \quad (7)$$

where $X=0.75$ and $r_0=1.34$ fm. Further, Bertulani and Baur²⁵ use the form

$$b = R(P) + R(T) + (\pi/2)a, \quad (8)$$

where a is given by $Z_1 Z_2 e^2 / m_0 v^2$ with m_0 the reduced mass and v the relative speed. These authors variously suggest that a particular form of b accounts for Rutherford bending of the orbit (derivation from a straight line) and the effect of a finite-charge distribution.

The problem with choosing a particular form of b and then comparing a resultant theoretical cross section to experiment is that the theory incorporates (perhaps unjustifiably) assumptions concerning b . Then, when one compares theory to experiment and makes claims about the WW method's validity, it is not only the WW method that one is testing but also mixed in is a test of one's assumption for b . I believe that this approach which has been taken in the literature leads to ambiguity concerning whether the WW method agrees with experiment.

Let us now formulate a model-independent criterion that will enable us to firmly establish whether or not WW theory agrees with experiment. The simplest possible assumption that one can make concerning b is that it is the sum of the projectile and target charge radii equivalent to choosing $d=0$ in Eq. (6), $X=0$ in Eq. (7), or $a=0$ in Eq. (8). All cross section calculations listed in Table III are calculated using this naive assumption for b . If theory agrees within experimental error, then this is taken as indicating that WW theory agrees with experiment. However, what is also calculated is the value of d from Eq. (6) needed to make theory agree with experiment. In this case d simply determines the difference of b from our naive assumption of the sum of the radii. Remember that the theoretical cross section σ is calculated for $d=0$. If agreement between theory and experiment is found it simply means that the value of b_{\min} (or d) needed to fit experiment is the sum of the radii (or $d=0$). In this case d has a reasonable value and it can be claimed that WW theory agrees with experiment. If the theoretical cross section (for $d=0$) does not fit experiment, but does fit it for another reasonable value of d (say, 0.5 fm), then again agreement with WW theory and experiment is claimed given the inherent uncertainties in b . On the other hand, if the calculated cross section (for $d=0$) does not agree with experiment and if a ridiculously large value of d (say, 10 fm) is needed for agreement then we conclude that WW theory does *not* agree with experiment. Finally if an intermediate value of d (say, 3 fm) is needed for agreement then the validity of WW theory is uncertain.

These criteria, however, are best expressed in terms of percentages. The percentages listed in Table III for d are the percentage of d relative to $R_{0.1}(P) + R_{0.1}(T)$. Thus the criteria are restated as follows: (a) Where a zero value of d is listed, WW theory does agree with experiment. (b) Where d is, say, 30% or greater of $R_{0.1}(P) + R_{0.1}(T)$, WW theory definitely does not agree with experiment. (c) Where d is between 0% and 30%, a definitive statement concerning agreement or disagreement is not possible. Note that this 30% criterion was chosen to be as pessimistic as possible. One could argue that it should, in fact, be lowered. However, its main use is to highlight any disagreement between theory and experiment. This 30% criterion is based on the assumption that Rutherford bending or finite charge effects cannot account for differences in d larger than 30%.

The advantage of the above criterion is that we can make a model-independent definitive statement concerning whether WW theory is adequate. Further, we can also say where it is clearly inadequate. This latter point means that a clear delineation is possible of where any new physics may emerge. The adoption of this criteria is suggested in all future analyses both for WW theory and other theories of $N(E)$.

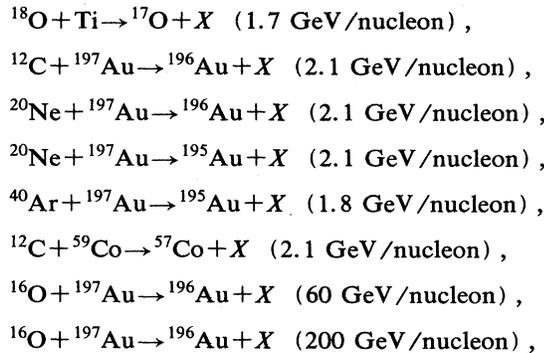
IV. RESULTS AND DISCUSSION

The basic calculational method has been described and criteria established for evaluating theoretical comparisons. Ten percent charge radii are determined from the compilations of Refs. 37 and 38 and are listed in Table III. For the sake of comparison the theoretical cross section has been calculated using the sum of the projectile and target radii as the minimum impact parameter. However, as just emphasized, *one should not focus on the calculated versus experimental cross section, but rather on the minimum impact parameter b needed to fit the data.* Listed in Table III is the difference between the fitted b and the sum of the nuclear radii. This difference is denoted by d (last two columns in Table III) as given in Eq. (6). Note that the values of the fitted b and d are not unique because of the nonzero size of the experimental errors. However, where d is listed as zero, it means rather that the calculated σ based on b as the sum of the radii already agrees with the experimental σ (in this case no fitting of b or d took place). d is also listed as a percentage of $R_{0.1}(P) + R_{0.1}(T)$.

Where a zero value of d is listed, I conclude that WW theory agrees with the data within the experimental uncertainty. It can be seen that this is the case with all the data on ¹²C and ¹⁶O projectile breakup.¹⁵ This is in agreement with the original conclusions of Heckman and Lindstrom,¹⁵ although more accurate data could conceivably change this situation. With the data for ¹⁸O projectile breakup and ¹⁹⁷Au, ⁸⁹Y, and ⁵⁹Co target breakup the situation is more complicated. First of all, note that even though the experimental and calculated cross section might only differ by a small amount, the value of the overlap parameter d required to fit the data may be enormous. This is another reason why one should concen-

trate on d and not σ in comparing WW theory to experiment.

Based on the above 30% criterion, I conclude from Table III that WW theory disagrees with experiment for the following reactions:



Note that if the error in d is such that the lower limit is below 30%, then that value of d is not included in the 30% criterion. For example, if d is $(45 \pm 20)\%$ then this is considered as lying *below* the 30% cutoff limit.

It can be seen that there is serious disagreement between WW theory and experiment for ^{197}Au target fragmentation both at low (2.1 GeV/nucleon) and high energies (60 and 200 GeV/nucleon), for both single and double neutron emission.

V. SUMMARY AND CONCLUSIONS

By considering the value of the overlap parameter d needed to fit the data, I have been able to clearly delineate the region where WW theory disagrees with ex-

periment. My basic conclusion is that target fragmentation of ^{197}Au is not understood either at high or low energy. This is very important. The results of Hill *et al.*²⁰⁻²³ did indicate that WW theory failed for ^{197}Au at 60 and 200 GeV/nucleon and thus one would naturally conclude that the failure is due to a high-energy effect. However, using the present criterion based on the 30% overlap parameter it is clear that the failure for ^{197}Au also occurs at low energy (2.1 GeV/nucleon). Thus I conclude that the problem may be with the nature of ^{197}Au EM fragmentation and not necessarily due to high-energy assumptions. (Note that the photonuclear data for ^{197}Au are extremely accurate.^{27,35}) High-energy data for lighter nuclei (say, ^{59}Co) would refute or verify this conclusion.

It has been pointed out that for heavy nuclei such as ^{197}Au and ^{238}U , the first order perturbation theory on which the WW theory is based is probably incorrect and that the WW formula is only the first approximation. This could explain why WW theory fails for ^{197}Au and suggests that it would be worthwhile to calculate higher-order effects.

Finally, the whole situation with respect to EM excitations could be very much clarified if we had a clear and precise way of calculating the overlap parameter d . Then one could use the actual cross section to compare theory and experiment.

ACKNOWLEDGMENTS

I wish to thank Gayle Norbury for extensive help with the photonuclear data and John Hill (Iowa State) for a very useful conversation. This work was supported in part by NASA Grant NAG-1-797.

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