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Application of the Bonn potential to proton-proton scattering

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The Bonn meson-exchange *NN* potential is made applicable to *pp* scattering. Its predictions are compared with a variety of *pp* scattering observables for energies below the pion-production threshold. The resulting χ^2 for the world *pp* data in the energy range 3–300 MeV is 1.72 per data point.

INTRODUCTION

In a recent paper the Bonn group presented their most refined model for the fundamental nucleon-nucleon (*NN*) interaction.[1] This model—the full Bonn potential—is derived completely within meson theory. The particular field-theoretical approach, namely relativistic, time-ordered perturbation theory, leads to an interaction model that is nonlocal as well as energy dependent.

Such a potential can be conveniently represented only in momentum space. As a consequence, however, one has to deal with the difficulty of incorporating also the Coulomb interaction into momentum-space calculations. In the original work this problem was avoided and the *NN* results (phase shifts) given there are, strictly speaking, valid for neutron-proton (*np*) scattering only. Indeed, the description of *np* phenomenology utilizing these phase shifts is rather satisfactory leading to a χ^2 of 1.87 per data point for 1884 *np* data in the energy interval 0–300 MeV.[2] In order to allow at least a qualitative comparison with proton-proton (*pp*) data, a few observables have been shown in Ref. 1, where the Coulomb interaction was treated approximately, namely by multiplying the purely nuclear amplitude with the Coulomb phase factor. This status is certainly somewhat unsatisfactory, in particular in view of the wealth and high accuracy of available *pp* data which would provide an additional and rather sensitive test for the quality of the potential model developed by the Bonn group.

In this paper we want to remedy this situation. Our aim is to modify the Bonn potential in such a way that it is applicable also to *pp* scattering. However, it is not our intention to perform and present a completely new fit. Rather we take the interaction as published in Ref. 1, add the Coulomb interaction and make small adjustments to guarantee a reasonable confrontation with *pp* phenomenology.

The paper is structured as follows: First, we give a brief introduction to the main features of the Bonn potential. Then we explain how we treat the Coulomb interaction in momentum space. Subsequently, we present our results for *pp* scattering where emphasis is laid on a direct comparison of the predictions with experimental data. Finally, the quality of the fit is discussed by means of the χ^2 for the world *pp* data set.[3–20]

THE BONN *NN* INTERACTION MODEL

A comprehensive presentation of the Bonn meson-exchange model for the *NN* interaction has been given in Ref. 1; here we restrict ourselves to a brief description. As mentioned earlier, this model is derived in the framework of relativistic, time-ordered perturbation theory. It contains the well-established single meson-exchange processes shown in Fig. 1(a). (Note that the σ' corresponds to the empirically determined correlated $\pi\pi$ *S*-wave interaction.) Furthermore, it includes two-boson exchange contributions involving the N as well as the Δ isobar in intermediate states, as represented by the box and crossed-box diagrams in Figs. 1(b) and 1(c).

Each meson-baryon-baryon vertex is supplied with a form factor in order to account for the extended hadron structure. These form factors are of conventional monopole type; the cutoff masses Λ_α (which can in principle be related to the hadronic size), have been fixed by a fit to *np* data below pion-production threshold. A qualitative description of the deuteron properties, *np* scattering phase shifts, and observables has been achieved.[1,2]

THE COULOMB PROBLEM IN MOMENTUM SPACE

We handle the Coulomb interaction by means of a method proposed by Vincent and Phatak.[21] This method relies on essentially the same boundary-matching condition as applied in coordinate-space calculations.

Asymptotically the scattering solution for a hadronic potential V_N plus the long-range Coulomb interaction, $V_C = Z_1 Z_2 e^2/r$, is given by

$$\Psi_l(k,r) \approx F_l(kr,\eta) + \tan\delta_l^{SC} G_l(kr,\eta) \ , \quad (1)$$

where F_l and G_l are regular and irregular Coulomb functions and $\eta = Z_1 Z_2 e^2 \mu/\hbar k$. The (Coulomb-distorted) nuclear phase shifts δ_l^{SC} (δ_{N+C}^C in the notation of Ref. 22) are the quantities which we need for the calculation of scattering observables.[23]

In usual configuration space calculations it is assumed that we can choose a radius R where

$$V_N \Psi_l(k,r) = 0 \quad \text{for} \quad r \geq R \ , \quad (2)$$

so that Eq. (1) is the exact solution. Then the

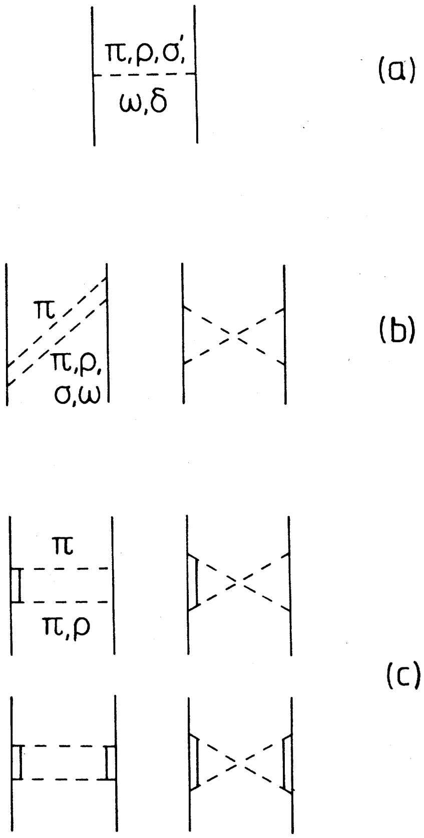

FIG. 1. Diagrams included in the full Bonn NN interaction model. (a) One-boson-exchange contributions. σ' stands for the correlated isoscalar $\pi\pi$ S-wave contribution. (b) Irreducible (stretched- and crossed-box) parts of the two-boson-exchange contributions involving nucleons in intermediate states. (c) Two-boson-exchange contributions involving the Δ isobar in intermediate states.

Schrödinger equation is integrated outwards from the origin and at the boundary R the logarithmic derivative of the obtained wave function is matched to Eq. (1) in order to determine the phase shifts.[24] Thus, only the potential for $r \le R$ enters into the actual calculation while the long-range part is absorbed into the boundary condition. Consequently the wave function at the boundary can also be obtained from a momentum space solution where the interaction is of short range, namely given by V_N and the Coulomb potential truncated at R. The resulting phase shifts, let us call them $\tilde{\delta}_l$, correspond, of course, to an asymptotic solution

$$\tilde{\Psi}_l(k,r) \approx F_l(kr,\eta=0) + \tan\tilde{\delta}_l G_l(kr,\eta=0) \ , \tag{3}$$

but they are just needed to calculate the wave function at the boundary, $\tilde{\Psi}_l(k,R)$, that by matching it to Eq. (1) allows again the determination of δ_l^{SC}.

The formalism can be easily extended to the case of coupled partial waves by appropriately replacing the quantities in Eqs. (1)–(3) by 2×2 matrices containing the wave functions for the $l=J+1$ and $l=J-1$ channels, respectively.[25]

One technical point should be mentioned in connection with the application of Vincent-Phatak's method as described above to the Bonn potential. This potential is defined within the unitarizing equation of time-ordered perturbation theory, which contains relativistic energies in the Green's function [see, e.g., Eq. (C3) in Ref. 1]; the static Coulomb potential V_C, on the other hand, belongs to the conventional nonrelativistic Lippmann-Schwinger propagator. Consistency can be obtained, however, by multiplying V_C with a factor[25] analogous to the transformation which relates the Blankenbecler-Sugar equation to the Lippmann-Schwinger equation.[1]

For numerical reasons R should be chosen as small as possible but still consistent with Eq. (2). In our calculations we found $R=10$ fm to be a reasonable value. In order to check the reliability of the method of Vincent and Phatak we carried out test calculations with the Paris NN potential.[26] We could reproduce the pp results of the configuration-space solution with an accuracy of $\Delta\delta_l^{SC} \approx 0.01$ degrees.

PROTON-PROTON SCATTERING

For pp scattering we cannot take over directly the Bonn (np) potential as presented in Ref. 1 due to the charge-independence breaking of the NN force. This breaking of the charge independence manifests itself most strikingly in the scattering length of the $^1S_0$ partial wave which is -23.75 fm for the np case but around -18 fm for nn (or purely nuclear pp) scattering.

A natural way would be the replacement of the averaged nucleon and pion masses used in Ref. 1 by the ones of the proton and the neutral pion. Actually, in a recent study with the Bonn potential the effects of the pion-mass difference on the $^1S_0$ scattering length were evaluated[27] and it was found that it can explain most (namely 80%) of the observed discrepancy between the nn and np scattering lengths.

However, for a quantitative description of pp scattering, in particular in the low-energy region, it is inevitable

TABLE I. Effective range parameters of the $^1S_0$ state.

	Without Coulomb		With Coulomb	
	a_s(fm)	r_s(fm)	a_s(fm)	r_s(fm)
Our model	-17.19	2.81	-7.82	2.75
Experiment	-17.9 ± 0.8[a]	2.82[a]	-7.8196 ± 0.0029[b]	2.790 ± 0.014[b]

[a]Reference 35.
[b]Reference 22.

TABLE II. Effective range parameters of the ^3P waves.

	$^3P_0$		$^3P_1$		$^3P_2$	
	a_{10} (fm)	r_{10} (fm^{-1})	a_{11} (fm)	r_{11} (fm^{-1})	a_{12} (fm)	r_{12} (fm^{-1})
Our model	−3.24	3.41	1.95	−7.57	−0.27	4.83
Experiment	−3.03±0.11[a]	4.22±0.11[a]	−2.013±0.053[a]	−7.92±0.17[a]	−0.306±0.015[a]	4.2±1.6[a]

[a]Reference 22.

to account for the experimental $^1S_0$ low-energy parameters (*pp* scattering length) exactly. Thus, we have chosen a different approach. We kept for convenience all the baryon and meson masses and cutoffs like in the full Bonn potential defined in Table 4 of Ref. 1 and varied only the coupling constants of the $NN\sigma'$ and $NN\delta$ vertices. These parameters were changed in such a way that in the *pp* channel we reproduce the $^1S_0$ scattering length while in the $^3S_1-{}^3D_1$ partial wave we still get the same deuteron properties as for the potential in Ref. 1. Since we are not concerned with the *np* channel here, the latter requirement ensures only that despite such adjustments we stay as close as possible to the original interaction model. The so obtained (new) coupling constants are 5.6224 for $g_{\sigma'}^2/4\pi$ and 2.7263 for $g_\delta^2/4\pi$ (cf. the former values 5.6893 and 2.8173, respectively).

The resulting effective-range parameters for the $^1S_0$ partial waves are listed in Table I. Note that since we included only the Coulomb potential but no other electromagnetic corrections in our calculations the appropriate quantities to compare with are those supplied with the superscript C in Ref. 22. Table II contains the effective range parameters obtained for the triplet P waves ($^3P_0, {}^3P_1, {}^3P_2$).

In the present considerations we took into account the Coulomb-distortion effect on the nuclear phase shifts only for partial waves with $J \le 2$. For higher partial waves this effect is already negligibly small. The predicted *pp* phase shifts are tabulated for several energies in Table III; they are in excellent agreement with recent phase shift analyses.[28]

For an accurate test of any theoretical model one should compare the predictions directly with experimental data rather than with phase shifts. Therefore, we show here also a large variety of *pp* observables for laboratory energies below the pion-production threshold (Figs. 2–7). Our predictions are depicted together with the ones from the Paris *NN* potential[26] and results of recent phase shift analyses of the Nijmegen group[22] (for $E_{\rm lab} \le 30$ MeV) and of R. A. Arndt (SM87).[29] The experimental information was mainly drawn from R. A. Arndt's code SAID.[29] It can be seen that an excellent overall description of the data is achieved by our model. Furthermore, it is noteworthy that in general the predictions for this model and the Paris potential are rather similar. Somewhat larger differences occur only in the Wolfenstein parameter D (Fig. 5) and, for higher energies, also in R [Figs. 4(c) and 4(d)] and A [Fig. 6(d)]. Note a peculiarity in the low-energy data of the analyzing power A_y [Figs. 2(a)–(d)]. The rather precise Wisconsin 82 (Ref. 17) data and the Los Alamos 75 (Ref. 6) data are nicely reproduced by the Nijmegen phase shifts while the potential model predictions are somewhat too low around the minimum, in particular at $E_{\rm lab}=10$ MeV [Fig. 2(b)] and 16 MeV [Fig. 2(d)]. On the other hand, the Erlangen 86 analyzing power data at 12 MeV (Ref. 30) [Fig. 2(c)] favor the theoretical models. Indeed, the Nijmegen group did not include these data in their analysis since they are not published in a regular journal. They found them also incompatible with the Wisconsin 82 data.[22]

The energy dependence of the spin-correlation parameters A_{yy} and A_{xx} at 90° c.m. angle [Figs. 7(a) and 7(b)] is very well described by both potential models—except for one A_{xx} data point at $E_{\rm lab}=47.5$ MeV given in Ref. 31.

TABLE III. *pp* phase shifts (in degrees) predicted by our model.

$E_{\rm lab}$ (MeV)	25	50	100	142	150	200	210	300	325
$^1S_0$	48.28	38.25	23.82	14.68	13.14	4.66	3.14	−8.50	−11.22
$^3P_0$	9.23	12.66	10.98	7.03	6.20	0.88	−0.19	−9.43	−11.86
$^3P_1$	−4.98	−8.34	−13.21	−16.79	−17.46	−21.57	−22.39	−29.73	−31.76
$^1D_2$	0.69	1.68	3.69	5.26	5.53	6.96	7.18	8.27	8.29
$^3P_2$	2.33	5.56	10.70	13.39	13.77	15.43	15.65	16.69	16.80
$^3F_2$	0.10	0.34	0.80	1.09	1.14	1.28	1.28	0.88	0.64
E_2	−0.82	−1.75	−2.75	−2.99	−3.00	−2.89	−2.85	−2.30	−2.13
$^3F_3$	−0.24	−0.72	−1.58	−2.13	−2.22	−2.68	−2.76	−3.35	−3.49
$^1G_4$	0.04	0.16	0.43	0.65	0.70	0.97	1.03	1.57	1.72
$^3F_4$	0.02	0.10	0.42	0.79	0.87	1.37	1.48	2.39	2.62
$^3H_4$	0.00	0.03	0.11	0.20	0.22	0.33	0.35	0.54	0.59
E_4	−0.05	−0.20	−0.55	−0.82	−0.87	−1.12	−1.17	−1.48	−1.54

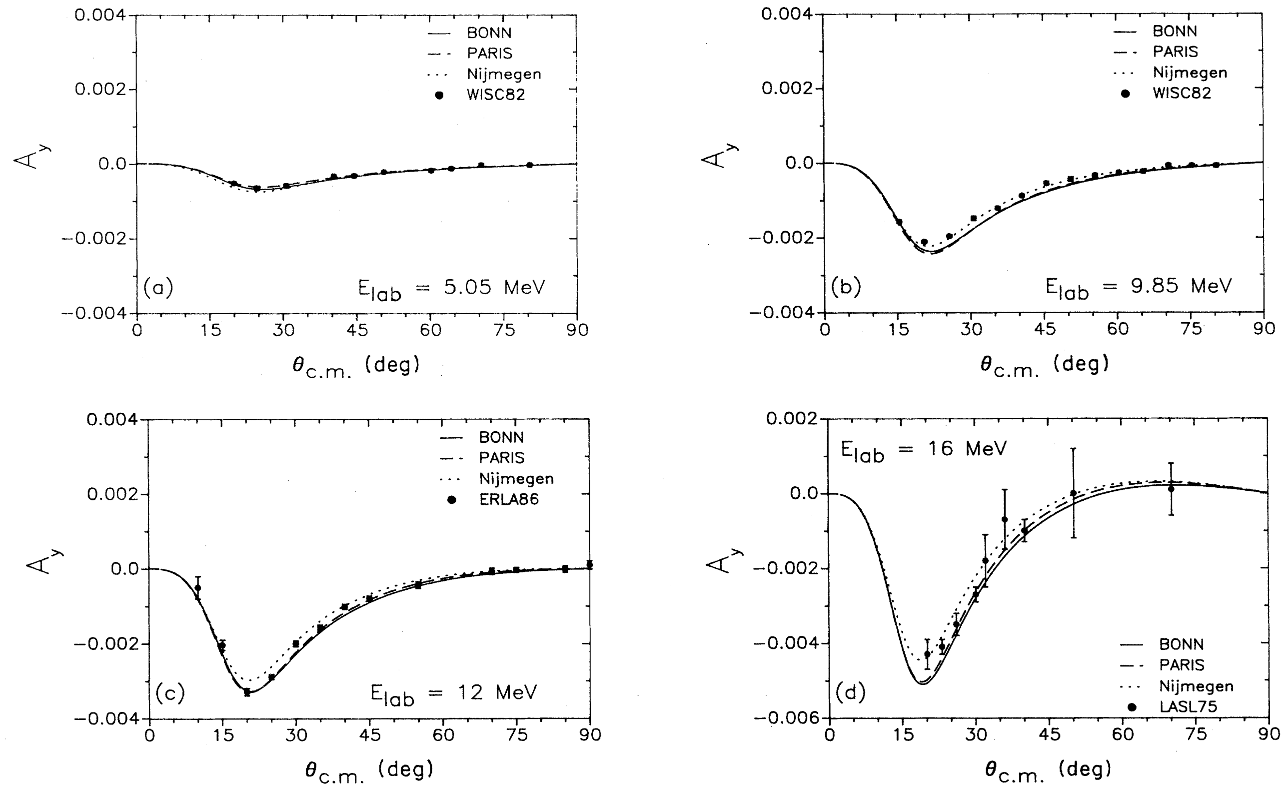

FIG. 2. *pp* analyzing powers at selected energies. Nijmegen refers to the results of the 0–30 MeV phase shift analysis of the Nijmegen group (Ref. 22).

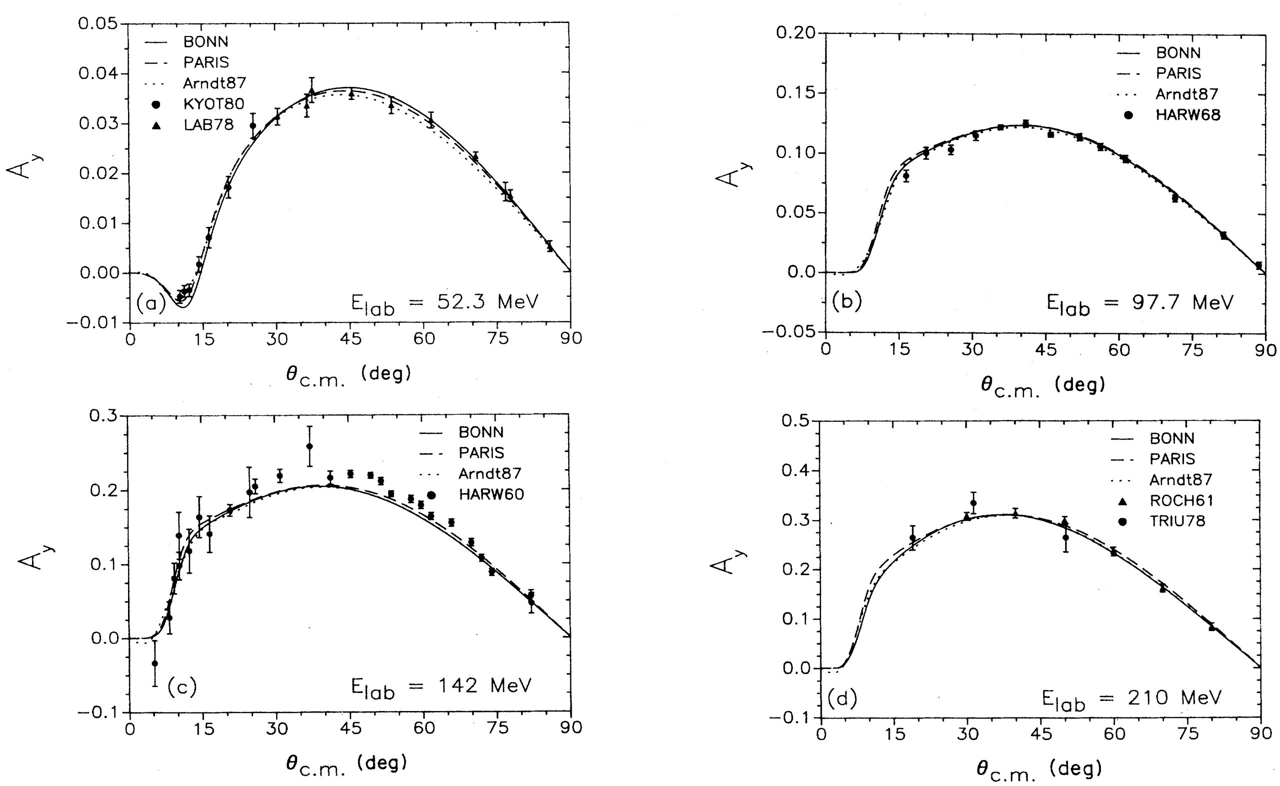

FIG. 3. *pp* analyzing powers at selected energies. Arndt87 refers to the results of his 0–1.3 GeV phase shift analysis (SM87) (Ref. 29).

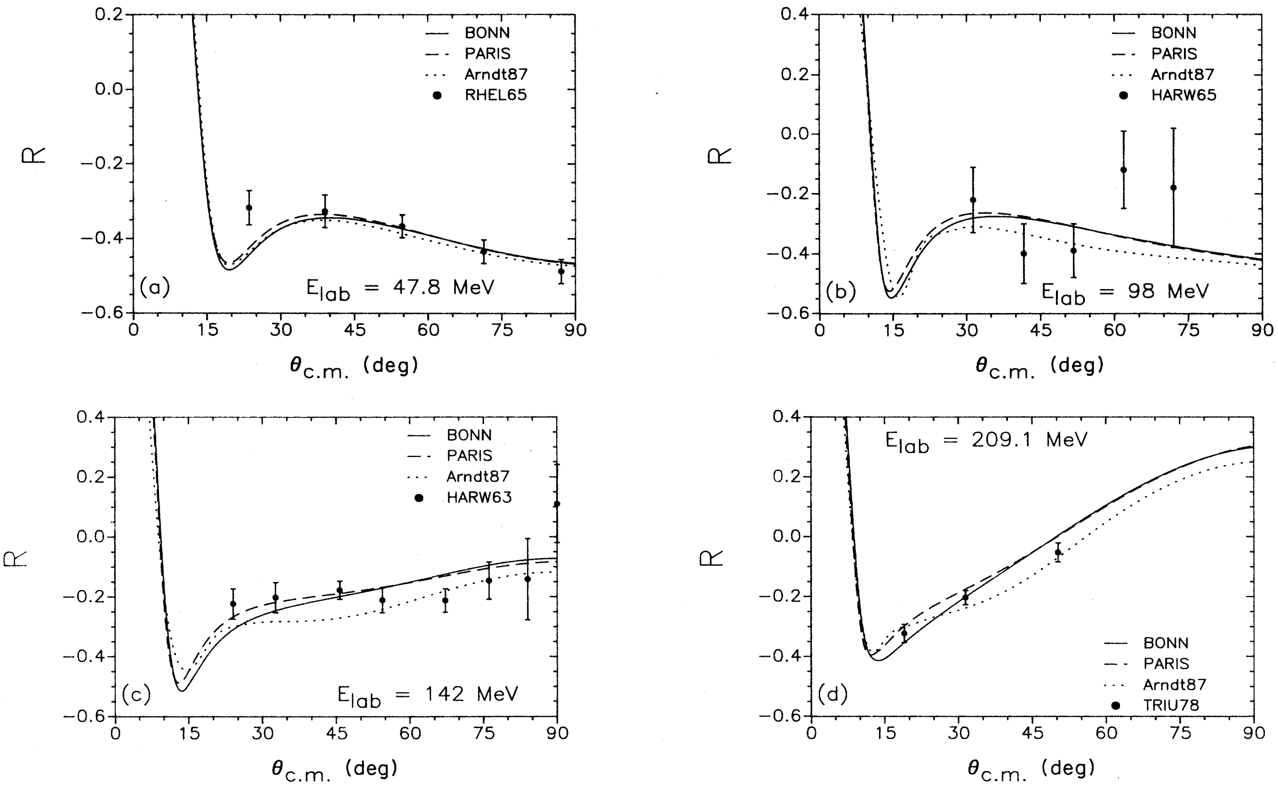

FIG. 4. *pp* Wolfenstein parameters *R* at selected energies. Same description as in Fig. 3.

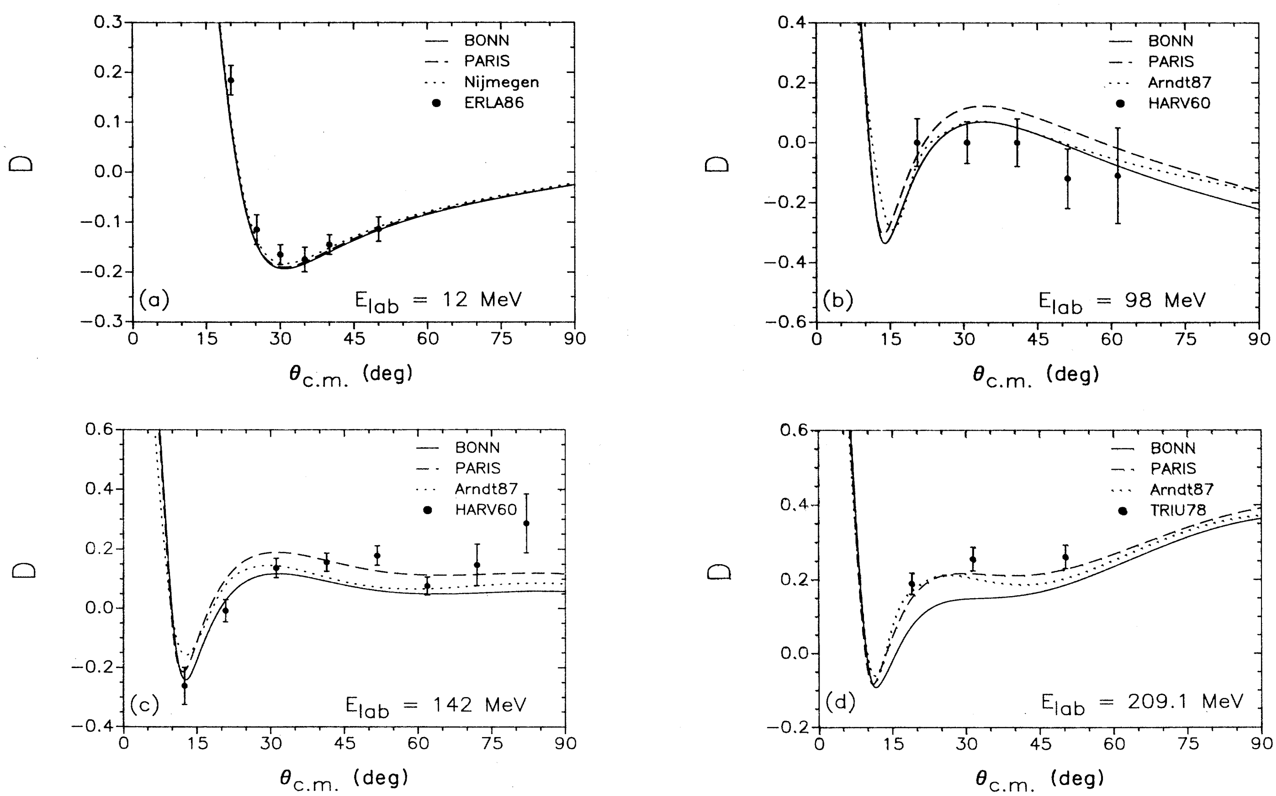

FIG. 5. *pp* Wolfenstein parameters *D* at selected energies. Same description as in Fig. 2 and Fig. 3, respectively.

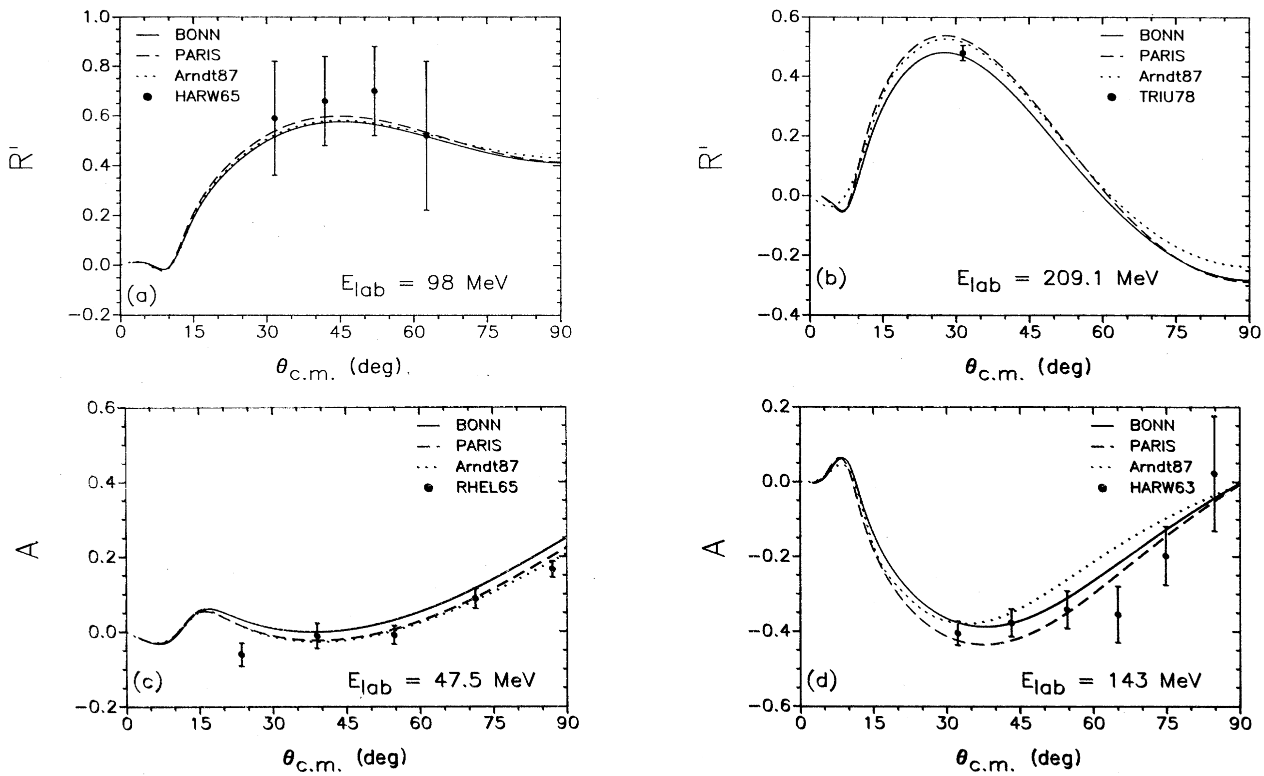

FIG. 6. *pp* Wolfenstein parameters R' and A at selected energies. Same description as in Fig. 3.

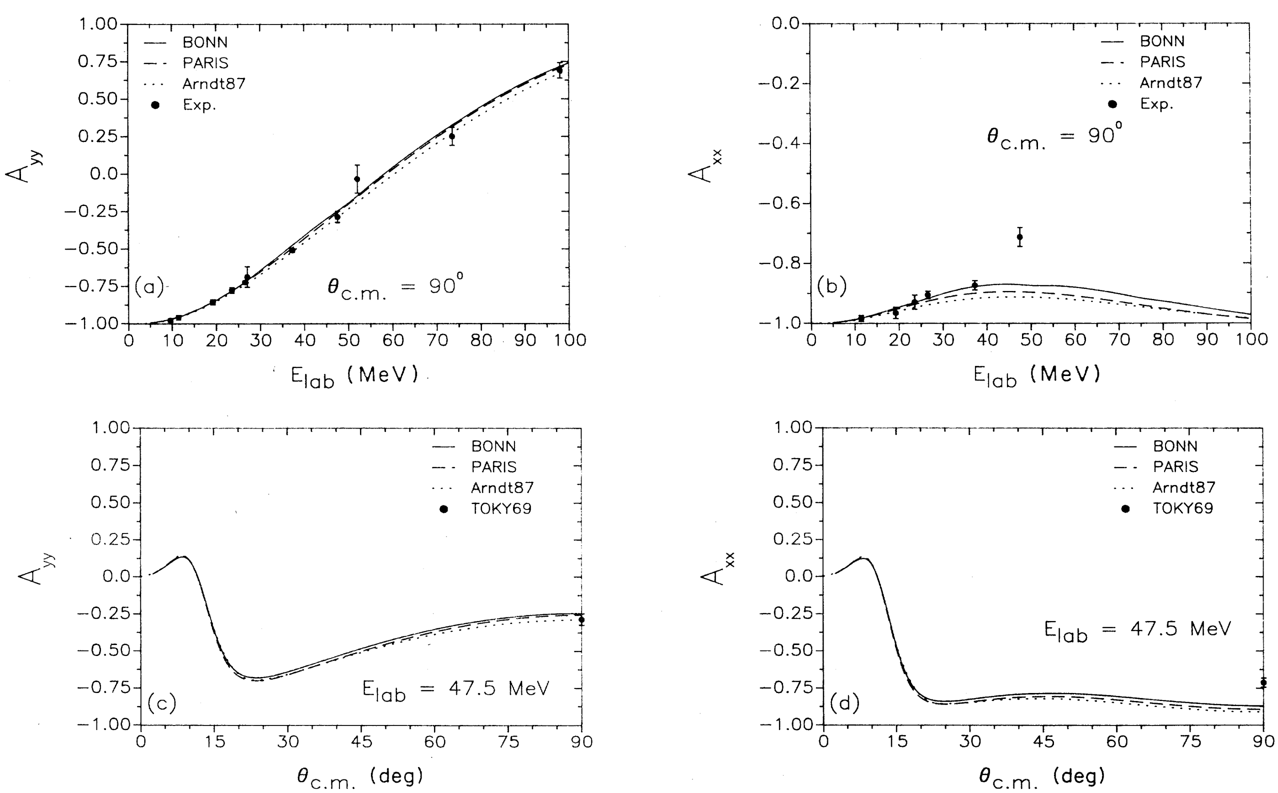

FIG. 7. *pp* spin correlation parameters A_{yy} and A_{xx}. Same description as in Fig. 3.

SUMMARY: THE QUALITY OF THE FIT

In order to provide an objective criterion for the quality of a potential model fit we present the χ^2 for the predicted *pp* observables. We calculated this quantity by means of the computer code SAID.[29] The obtained total χ^2 and the χ^2 per data point are listed in Table IV together with the values for Arndt's '87 phase shift analysis.[29] We considered two different energy intervals because there is a delicate problem connected with the very low-energy region. For $0.3 \leq E_{\text{lab}} \leq 1$ MeV a set of rather accurate differential cross-section data is available which, in principle, would constitute a very stringent test of theoretical *pp* models. On the other hand, for such energies not only the Coulomb potential but also other electromagnetic effects (vacuum polarization, etc.), usually not included in potential model calculations, become very important.

In the course of their low-energy phase shift analysis the Nijmegen group evaluated these additional electromagnetic corrections and tabulated them in their paper.[22] Their values can be regarded as a sound model-independent estimation of such corrections. Accordingly, we modified our ${}^1S_0$ phase appropriately (for $E_{\text{lab}} \leq 3$ MeV)—following relation Eq. (74) of Ref. 22—to take into account these electromagnetic effects.

The changes in the ${}^1S_0$ phase shift due to these corrections are typically of the order of 1%. Nevertheless they have a tremendous influence on the resulting χ^2. For the interval 0–3 MeV the χ^2 per data point for the amended ${}^1S_0$ is 2.5; in the uncorrected case, however, it would amount to 33.86. In view of this it is, of course, very misleading to confront potential models which have been fitted to the effective range parameters of the ${}^1S_0$ *np* channel with those low-energy *pp* data.[32] Any χ^2 evaluated in such a way does not really represent a criterion for the quality of the hadronic force model, but reflects primarily the extent to which electromagnetic corrections have been implemented.

For the presently available *pp* data up to $E_{\text{lab}}=300$ MeV (Refs. 3–20) we reach a χ^2 of 1.85 per data point. This value compares favorably with other modern *NN* potential models. For example, the values usually quoted for the Paris[33] and the Nijmegen (1978) (Ref. 34) potentials obtained in a comparable energy range are 5–10% higher. Thus, we can say that the Bonn potential provides for a quantitative description not only of *np* scattering but also, as has been demonstrated in this paper, when applied to *pp* scattering.

TABLE IV. χ^2 for our potential model and the phase shift analysis of Arndt (SM87), with respect to *pp* data.

		Total χ^2		χ^2/N_{data}	
E_{lab} (MeV)	N_{data}	Bonn	Arndt	Bonn	Arndt
3–300	1024	1759	1427	1.72	1.39
0–300	1240	2298[a]	2043	1.85[a]	1.65

[a]Corrections for other electromagnetic effects beyond Coulomb have been included (cf. text).

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